Photon-Nucleus Total Cross Sections†

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Quantitative predictions for the energy and A dependence of the cross sections for nuclear photoabsorption and inelastic electron-nucleus scattering are given. In general, the nucleons do not contribute equally to the total photon-nucleus cross section when coherent contributions of photoproduced hadrons are taken into account. At low energies ($E_\gamma \sim 1$ BeV), the cross sections are proportional to nuclear number A, but at high energies, they become proportional to the number of surface nucleons—provided that the photon interactions are mediated by hadrons of sufficiently low mass. The condition on the masses is that the momentum transfers in forward photoproduction of these states should be small compared to the reciprocals of their mean free paths in nuclear matter. In the case of ρ dominance, the real-photon photoabsorption cross section has the same A dependence as hadron-nucleus total cross sections for photon energies above ≈ 10 BeV, whereas the cross section for virtual photon absorption at that energy, obtained from inelastic electron scattering is nearly proportional to A for spacelike momentum transfer $|Q^2| \gtrsim 5$ BeV^2. We then generalize to an arbitrary spectrum of intermediate particles, and discuss the sensitivity of feasible experiments to various models in which the spectrum contains important structure beyond the ρ. Measurements of the photon-nucleus cross sections will provide a fundamental test of "hadron dominance" in general, and of ρ-ω-ϕ dominance in particular, as well as help to determine the basic parameters of photon-nucleon and ρ-nucleon interactions. We also calculate the photon-deuteron cross section and discuss the multiple scattering approach to photon-nucleus interactions. This discussion provides insight into the many-body processes which underlie the eikonal, optical-model calculations; it is also relevant to the determination of $\sigma_{\gamma n}$ at high energies.

I. INTRODUCTION

DIRECT application of the ρ-dominance model[1] to the forward elastic amplitude for photons on nuclei, together with the optical theorem, yields the total-cross-section prediction

$$\sigma_{\gamma A} = (e/g)^2 \sigma_{\rho A}. \tag{1}$$

This result is paradoxical, because the mean free path of photons in nuclear matter [$=\sigma_{\gamma N}^{-1} \times$ (density of nucleons)$^{-1} \approx 700$ F] is large compared to nuclear sizes, so that one might have expected all of the nucleons to participate equally, and $\sigma_{\gamma A} \propto A$. However, one certainly expects $\sigma_{\rho A}$ not to be proportional to A because of shadow effects: The mean free path of ρ's in nuclear matter [$=\sigma_{\rho N}^{-1} \times$ (density of nucleons)$^{-1} \approx 3$ F] is comparable to nuclear sizes, so that nucleons deep inside the nucleus do not see the full incident ρ flux.[2] Nevertheless, Bell[3] and Stodolsky[4] have shown that Eq. (1) is not unthinkable and that, in particular, it follows at sufficiently high energies from assuming ρ dominance of the interactions on individual nucleons.

Our purpose in this paper is to develop a quantitative description of the energy and A dependence of the nuclear photoabsorption cross section.[5a,5b,6] At low energies (for our purposes, $E_\gamma \sim 1$ BeV), the photon cross section will be shown to be proportional to A. At very high energies it is predicted to be proportional to "$A^{2/3}$,"[2] provided only that the photon interactions are mediated by hadrons of sufficiently low mass. We wish to emphasize that this prediction of a hadron-like A-dependence for $\sigma_{\gamma A}$ at high energy does not rest on detailed assumptions of vector dominance. The energy of transition between A and "$A^{2/3}$" is related to the average mass of hadronic states which dominate the electromagnetic current: The critical condition for (1) to hold is that the momentum transfer in forward photoproduction of these states be small compared to the reciprocal of their mean free paths in nuclear matter. Measurements of the total photoabsorption cross section through the transition region (1–20 BeV) would, there-

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[1] For recent reviews, see J. J. Sakurai, *Lectures in Theoretical Physics* (Gordon and Breach, Science Publishers, Inc., New York, 1968), Vol. XI, and S. C. C. Ting in *Proceedings of the Fourteenth International Conference on High-Energy Physics in Vienna, 1968* (CERN, Geneva, 1968), p. 43. A Lagrangian formulation of vector dominance has been given by N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. **157**, 1376 (1967).

[2] If hadronic mean free paths were negligible compared to nuclear sizes, interactions would be confined to the surface, implying $\sigma \propto A^{2/3}$; nuclei are not that large, and experimentally $\sigma_{\pi A}$, σ_{pA}, and σ_{nA} grow like $\approx A^{0.8}$. See, e.g., M. L. Longo and B. Moyer, Phys. Rev. **125**, 701 (1962); J. Engler *et al.*, Phys. Letters **28B**, 64 (1968).

[3] J. S. Bell, Phys. Rev. Letters **18**, 57 (1964); CERN Report No. TH.877 (unpublished).

[4] L. Stodolsky, Phys. Rev. Letters **18**, 135 (1967).

[5] (a) We restrict our attention to the total photoabsorption cross section into hadronic final states, which is obtained via the optical theorem from the forward Compton scattering amplitude to order e^2. (b) After our work was completed we learned that M. Nauenberg [Phys. Rev. Letters **22**, 556 (1969)], B. Margolis (to be published), and K. Gottfried and D. R. Yennie [Phys. Rev. **182**, 1595 (1969)] have derived the ρ-dominance result for $Q^2=0$ given in Eq. (17). The latter authors also discuss incoherent photoproduction on nuclei.

[6] This ρ-dominance calculation parallels the calculations of particle photoproduction in nuclei given by S. D. Drell and J. S. Trefil, Phys. Rev. Letters **16**, 552 (1966); **16**, 832(E) (1966); M. Ross and L. Stodolsky, Phys. Rev. **149**, 1172 (1966); K. Gottfried and D. Yennie, Phys. Rev. **182**, 1595 (1969). Analyses in terms of a Glauber type of multiple scattering theory have been given by K. S. Kolbig and B. Margolis, Nucl. Phys. **B6**, 85 (1968) and J. S. Trefil, Phys. Rev. **180**, 1366 (1969). Final-state absorption effects in the photoproduction of particular hadron states makes the energy dependence of the effective number of nucleons for these processes less dramatic than in Compton scattering.

fore, constitute a fundamental test of the basic ideas of "hadron dominance" in general, and of ρ-ω-ϕ dominance in particular. They offer the possibility of confirming and understanding the breakdown of ρ dominance, which seems to have been observed in ρ photoproduction on complex nuclei.[7] The total cross section, i.e., the imaginary part of the forward Compton amplitude, is an especially useful physical quantity here, because it contains no background of energy-independent absorption such as occurs in the photoproduction of any given hadron through final state absorption, and because the forward Compton amplitude is totally coherent, involving only the ground state of the nucleus. We generalize our results to photons which are off-mass-shell in the spacelike region, i.e., to inelastic electron or muon scattering from nuclei. This generalization is important because it introduces a new parameter, the Q^2 of the photon, which can be varied in testing the theory, and because inelastic electron scattering experiments may be easier to perform than total photoabsorption experiments. Our calculations are first done within the framework of ρ dominance (Secs. II–IV). We then generalize to allow for an arbitrary spectrum of intermediate particles, and discuss the sensitivity of feasible experiments to various models in which that spectrum is related to the cross section for electron-positron annihilation into hadrons (Secs. VI and VII). We also calculate the photon-deuteron total cross section, and discuss the multiple-scattering approach to photon shadowing. This discussion provides insight into the many-body processes which underlie the optical-model calculations, and it clarifies our basic assumptions; it is also relevant to the determination of $\sigma_{\gamma n}$ at high energy.

II. BASIC DESCRIPTION

We begin by assuming that photons couple only through ρ mesons (see Fig. 1). The coupling constant is defined as em_ρ^2/g. Following Bell,[3] the incident photon wave function is $e^{-iq\cdot x}$, and the ρ wave equation in nuclear matter is

$$(\Box+m_\rho^2+V_{\rho\rho})\psi_\rho=(e/g)m_\rho^2e^{-iq\cdot x}. \tag{2}$$

$V_{\rho\rho}$ is an optical potential, and is related to the forward-scattering amplitude of ρ's on nucleons by

$$V_{\rho\rho}=-4\pi d f_{\rho N\to\rho N}{}^{(0°)}, \tag{3}$$

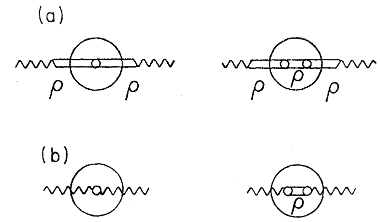

FIG. 1. Schematic representation of forward Compton scattering on nuclei; (a) corresponds to the vector-dominance description given in Sec. II in which photon-nucleon interactions are mediated by ρ mesons; (b) corresponds to the description used in Sec. III in which amplitudes are calculated by the "ρ-photon analogy." The two descriptions are related by a canonical transformation and give the same results (see Ref. 8). Only single- and double-scattering contributions to the order e^2 amplitude are shown; when the entire multiple-scattering series for the forward-produced ρ's is summed for large A in the Glauber approximation, the absorptive medium-eikonal approximation is obtained (see J. Trefil, Ref. 6).

where d is the nuclear density. In the case of pure absorption, f is imaginary and

$$V_{\rho\rho}=-ikd\sigma_{\rho N}\equiv-ik/\mu, \tag{4}$$

where $\sigma_{\rho N}$ is the total ρ-nucleon cross section for ρ mesons of momentum k, and μ is the mean free path ≈ 3 F. The index of refraction for ρ mesons in the nuclear medium is given by

$$n_\rho=1+2\pi d f_{\rho N\to\rho N}(0°)/k^2. \tag{5}$$

$V_{\rho\rho}$ is assumed not to mix spin states. Equation (2) describes a helicity component of the ρ wave function.

If $V_{\rho\rho}$ is constant within the nucleus, the solution has the form

$$\psi_\rho=\frac{e}{g}\left(\frac{m_\rho^2}{m_\rho^2+V_{\rho\rho}}e^{-iq\cdot x}+\chi\right), \tag{6}$$

where χ is a solution of the homogeneous equation corresponding to absorption of the ρ within the nucleus. Using the optical theorem and (4), the first term contributes to the cross section

$$\sigma_{\gamma A}{}^{(1)}=\left(\frac{e}{g}\right)^2 A\sigma_{\rho N}\operatorname{Re}\left(\frac{m_\rho^2}{m_\rho^2+V_{\rho\rho}}\right). \tag{7}$$

This term vanishes in the high-energy limit

$$|V_{\rho\rho}|\gg m_\rho^2,\quad \text{i.e., } E_\gamma=q\gg m_\rho^2\mu, \tag{8}$$

leaving only the contribution from χ, which is not proportional to A and produces the hadronlike behavior of the cross section. We give precise formulas below. On the other hand, for sufficiently low energies, χ becomes small and only the A term survives, corresponding appropriately to coherent scattering on a nearly transparent nucleus.

These results can be understood simply using an uncertainty principle argument. The description of Fig.

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[7] The first ρ-production (2–6 BeV) results were consistent with ρ dominance. See L. J. Lanzeretti *et al.*, Phys. Rev. **166**, 1365 (1968); J. G. Asbury *et al.*, Phys. Rev. Letters **19**, 865 (1967); **20**, 227 (1968); and H. Blechschmidt *et al.*, Nuovo Cimento **52A**, 1348 (1967). Two recent experiments at higher energies have indicated that the forward ρ photoproduction cross section is only half of the value predicted by ρ dominance using the value $\gamma_\rho^2/4\pi=\frac{1}{4}(g_\rho^2/4\pi)=0.52\pm0.07$ measured on the ρ mass shell; see G. McClellan *et al.*, Phys. Rev. Letters **22**, 374 (1969); **22**, 377 (1969); and F. Bulos *et al.*, Phys. Rev. Letters **22**, 490 (1969). We wish to thank Dr. D. W. G. Leith, Dr. W. Busza, and Dr. R. R. Larsen for discussion of these data. This possibility of violation of vector dominance would have striking consequences for the energy dependence of $\sigma_{\gamma A}$, as we discuss in Sec. VI.

1 in terms of old-fashioned perturbation theory is that energy is not conserved at the ρ-photon vertices by an amount $\Delta E=(m_\rho^2+q^2)^{1/2}-q\cong m_\rho^2/2q$. The longitudinal position of the γ-ρ conversion point must then be uncertain by a distance

$$\Delta x \gtrsim 1/\Delta E \sim 2q/m_\rho^2. \tag{9}$$

If Δx is large compared to the hadron mean free path in the nucleus, then the photon converts to hadrons well before reaching the nuclear surface, and shadow effects are to be expected. This results in the same high-energy condition (8) as obtained above. No details of ρ-dominance theory, such as the magnitude of the ρ-photon coupling constant, enter into this uncertainty principle argument. The necessary condition for $A^{2/3}$ behavior at high energy is, therefore, only that the photon interactions be mediated by hadronic states of finite mass.

III. ALTERNATIVE DESCRIPTION

Stodolsky[4] has given a somewhat different description, in which the photons are eigenstates of the vacuum; i.e., the direct ρ-photon coupling vanishes for on-mass-shell photons, and the photons are instead allowed to interact directly with the nucleons.

The two descriptions are related by a canonical transformation,[8] and, therefore, contain the same physics (see Fig. 1). The question of whether the photon changes into a ρ before or after reaching the nucleus in a ρ-dominance model is thus purely a matter of taste. In Stodolsky's description, the $A^{2/3}$ behavior of the photon-nucleus total cross section at high energy arises because a "downstream" nucleon feels, in addition to the full-strength incident photon beam, a beam of real ρ mesons of intensity $-f_{\gamma N\to\rho N}/f_{\rho N\to\rho N}$ which has been generated "upstream." (The factor $1/f_{\rho N\to\rho N}$ enters here because only ρ's generated within approximately one mean free path survive absorption.) The direct photon beam results in a contribution to the forward amplitude $\propto f_{\gamma N\to\gamma N}$, while the ρ "beam" gives a contribution $\propto -(f_{\gamma N\to\rho N}/f_{\rho N\to\rho N})f_{\gamma N\to\rho N}$, and the coefficients are such that these cancel each other leaving no term proportional to volume, provided that the relation

$$f_{\gamma N\to\gamma N}f_{\rho N\to\rho N}=f_{\gamma N\to\rho N}^2 \tag{10}$$

holds, as it does in the ρ-dominance model.[1,9]

At low energies, the cancellation is made imperfect by several effects which reduce the ρ term, so that (1) is only expected to hold at high energies. First of all, the ρ may decay before it can reconvert to a photon.[10] That will only be negligible when the distance a ρ can travel before decaying is long compared to its mean free path, i.e., when $k/m_\rho\Gamma\cong\mu$. Because of the time dilatation factor k/m_ρ, that inequality becomes true at energies of a few BeV. The most important effect results from the transfer of three-momentum $\mathbf{\Delta}$ to one of the nucleons, and $-\mathbf{\Delta}$ to another, where $|\mathbf{\Delta}|\gtrsim m_\rho^2/2q$. This effect depends, in general, on the nuclear wave function. In the eikonal model it takes the form of a coherence requirement: The photon wave is $\propto e^{iqz}$ and the ρ wave $\propto e^{ikz}$, where $k=q-\Delta$; in order for shadow effects to be important, these must stay approximately in step over at least a ρ mean free path, i.e., $m_\rho^2/2q\cong\Delta\ll 1/\mu$. This is the uncertainty principle condition (8) discussed above. A third effect is that (10) must be expected to fail at low energies, even if one believes in ρ dominance, because of minimum-momentum-transfer considerations—e.g., the ρ contribution to shadowing must vanish if the photon energy is less than ρ production threshold of 1.1 BeV, since, as we wish to emphasize, the shadowing is caused by real intermediate states.

IV. ρ-DOMINANT CALCULATION

We now use the Stodolsky description to calculate the photon-nucleus total cross section, assuming ρ dominance of photon-nucleon scattering. We use the eikonal approximation and treat the nucleus as a homogeneous sphere of radius $R=r_0A^{1/3}$, $r_0=\sim 1.3$ F. Begin by pretending that the ρ has zero width. The forward photon-nucleus amplitude is

$$T=(\phi,V\psi^{(+)}), \tag{11}$$

where

$$\phi=\begin{pmatrix} e^{iqz} \\ 0 \end{pmatrix} \tag{12}$$

is the incident photon wave with energy $E_\gamma=q$,

$$\psi^{(+)}=\begin{pmatrix} \psi_\gamma^{(+)} \\ \psi_\rho^{(+)} \end{pmatrix} \tag{13}$$

is the corresponding outgoing scattered wave, and V

[8] The distinction between the treatments of photon-hadron interactions in Secs. II and III can be characterized by the interaction Lagrangian used to represent vector dominance. In Sec. II, the analysis based on the usual vector-dominance Lagrangian in which photon interactions with the hadron current J^μ are always mediated by the ρ. In Sec. III, the analysis can be based on the Lagrangian model of Ref. 1; the ρ-photon coupling vanishes for real photons ($Q^2=0$) which, however, interact locally with the hadron current. As shown in Appendix B of Kroll, Lee, and Zumino (Ref. 1), the two Lagrangians are in fact related by a canonical transformation; the photon field of Sec. II is a linear combination of the photon and vector-meson fields of Sec. III. Indeed, in the case of photon-nucleon interactions, one takes precisely that linear combination of photon and vector mesons of Sec. III which has no direct interaction with the nucleon. We generalize this for arbitrary meson channels in Sec. VI A.

[9] The simple ρ-dominance prediction is $f_{\gamma N\to\rho N}(t=0)=(e/g_\rho)\times f_{\rho N\to\rho N}(t=0)$ and $f_{\gamma N\to\gamma N}(t=0)=(e/g_\rho)^2 f_{\rho N\to\rho N}(t=0)$. Consequences of breaking ρ dominance are discussed in Secs. VI and VII.

[10] K. Gottfried and D. Julius, Cornell Report (unpublished).

is the 2×2 matrix

$$\begin{pmatrix} V_{\gamma\gamma} & V_{\gamma\rho} \\ V_{\gamma\rho} & V_{\rho\rho} \end{pmatrix}, \tag{14}$$

where $V_{\gamma\rho}=-4\pi d f_{\gamma N\to\rho N}{}^{(0^\circ)}$, etc. To order e^2, $\psi_\gamma{}^{(+)}(z)=e^{iqz}$ and

$$(-\nabla^2+V_{\rho\rho}-k^2)\psi_\rho{}^{(+)}=-V_{\gamma\rho}\psi_\gamma{}^{(+)}, \tag{15}$$

where $k=(q^2-m_\rho{}^2)^{1/2}$. The ρ wave function, which satisfies the eikonal approximation to this equation, is[11]

$$\psi_\rho{}^{(+)}(z)=\frac{-V_{\gamma\rho}}{m_\rho{}^2+V_{\rho\rho}}(e^{iqz}-e^{ikn_\rho(z+a)}e^{-iqa}), \tag{16}$$

where $a=(R^2-b^2)^{1/2}$ for impact parameter b, so that the boundary condition $\psi_\rho(-a)=0$ is met; i.e., ψ_ρ vanishes at the left-hand edge of the nucleus. We emphasize that $V_{\gamma\rho}$ is proportional to the photoproduction amplitude for real ρ mesons with physical momentum $k=(\mathbf{q}^2-m_\rho{}^2)^{1/2}$. Employing the optical theorem, we obtain

$$\sigma_{\gamma A}=A\sigma_{\gamma N}\left(1-\frac{\operatorname{Im}[F(\zeta)V_{\gamma\rho}{}^2/(m_\rho{}^2+V_{\rho\rho})]}{\operatorname{Im}V_{\gamma\gamma}}\right), \tag{17}$$

where

$$\begin{aligned} F(\zeta)&=1-(3/\zeta^3)[(1+\zeta)e^{-\zeta}-1+\tfrac{1}{2}\zeta^2],\\ \zeta&=2iR[q-k+V_{\rho\rho}/2k]. \end{aligned} \tag{18}$$

At energies high enough that minimum-momentum-transfer effects are negligible, ρ dominance implies that[1,9]

$$V_{\gamma\rho}{}^2=V_{\rho\rho}V_{\gamma\gamma}. \tag{19}$$

If, in addition, the energy is high enough that $q-k$ times the ρ mean free path μ is small, then

$$V_{\rho\rho}/(m_\rho{}^2+V_{\rho\rho})\cong 1, \tag{20}$$

and the terms proportional to A in $\sigma_{\gamma A}$ vanish, leaving the surface contributions.

The expression (17) can be improved in several ways. We can take into account the instability of ρ approximately by adding $-im_\rho\Gamma$ to $V_{\rho\rho}$. This amounts to changing the mean free path by

$$1/\mu\to 1/\mu_{\text{absorption}}+1/\mu_{\text{decay}}. \tag{21}$$

We can include the nucleon recoil energy in computing the minimum momentum transfer $q-k$. Further, we can determine $V_{\gamma\rho}$ from the measured photoproduction amplitude for ρ mesons in order to incorporate the correct threshold dependence. These modifications have only minor effects on $\sigma_{\gamma A}$, because they are sizable only at low energies where the ρ contribution is small and $\sigma_{\gamma A}\approx A\sigma_{\gamma N}$. That fact also justifies our use of the eikonal approximation down to low energies. Real parts can be included in $V_{\rho\rho}$ and $V_{\gamma\rho}$; the values, of course, are not known, but assuming that (19) holds, we find the effects of changing $\operatorname{Re}f/\operatorname{Im}f$ from zero to ± 0.2 to be only about 5%.

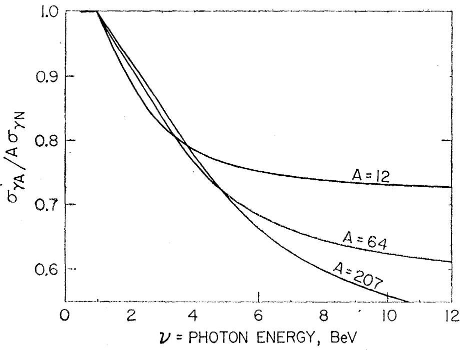

FIG. 2. Predictions of ρ dominance for the photoabsorption cross section as a function of incident lab energy. The effective number of nucleons $\sigma_{\gamma A}/A\sigma_{\gamma N}$ is given for carbon, copper, and lead, assuming $\sigma_{\rho N}=30$ mb and uniform sphere radii $R=1.3$ F $A^{1/3}$. The curves are lowered at 10 BeV by 0.04–0.05 (i.e., shadowing is increased) if R is decreased to 1.2 F $A^{1/3}$ or $\sigma_{\rho N}$ is increased to 35 mb. The sharp behavior at the ρ-photoproduction threshold is removed when the ρ width and threshold factors are taken into account. Nuclear effects become important at very low energies.

The energy dependence of $\sigma_{\gamma A}$ given by (17) is shown in Fig. 2. It is shown in the form of $\sigma_{\gamma A}/A\sigma_{\gamma N}$, which can be thought of as an effective number of nucleons; the strength of the γ-ρ coupling divides out in this expression, as does some of the energy dependence of $\sigma_{\gamma N}$. It is apparent that the effect we discuss is large and occurs in an energy region amenable to experiment. The transition energy increases with A, i.e., the shadowing comes in at lower energies in light nuclei. This is because the "thickness" of a light nucleus—especially at nonzero impact parameter—is less than a ρ mean free path, and the distance over which coherence between ρ and photon is required, Δx in Eq. (9), is determined by the smaller of (a) the ρ mean free path, and (b) the path-length through the nucleus.[12] The transition in energy from $\sigma\propto A$ to $\sigma\propto A^{\approx 0.8}$ is not monotonic in energy; the exponent at first rises above 1 because of the differing energy dependence on light and heavy nuclei just discussed. For this reason, the most effective way to study photon shadowing experimentally is by the energy dependence on a few nuclei rather than to measure A dependence

[11] A nonuniform nuclear density can readily be included. The general eikonal solution to (15) is

$$\psi_\rho{}^{(+)}(z,b)=\frac{2\pi i f_{\rho\gamma}}{k}\int_{-\infty}^{z}dz'd(b,z')e^{ik(z-z')}\times\exp\left[-\tfrac{1}{2}\sigma_{\rho N}\int_{z'}^{z}dz''d(b,z'')\right]e^{iqz'},$$

which reduces to (16) if d is constant. The cross section for forward Compton scattering on heavy nuclei should be insensitive to the nuclear model, as in the case for forward hadron-nucleus scattering (see Drell and Trefil, Ref. 6, and S. C. C. Ting, Ref. 1). Nuclear structure corrections have thus been ignored here.

[12] The transition energy where $\sigma_{\gamma A}/A\sigma_{\gamma N}$ is within ~10% of its asymptotic value is $\nu\sim m_\rho{}^2\sqrt{(R\mu)}$. See M. Nauenberg, Ref. 5(b).

at just a few energies. This recommended program also has the advantage of being relatively insensitive to deviations from simple $R\propto A^{1/3}$ nuclear models. On the other hand, the A dependence at high energy is sensitive to $\sigma_{\rho N}$, as in the case of ρ photoproduction, assuming ρ dominance holds.

In addition to ρ dominance, the following approximations were made in obtaining Fig. 2: A purely absorptive potential was used corresponding to a uniform nuclear density of radius $1.3A^{1/3}$ F and $\sigma_{\rho N}=30$ mb. The width of the ρ was taken as 110 MeV. Nucleon recoil was included in calculating the minimum momentum transfer. The ρ photoproduction amplitudes were taken to be zero below threshold and constant at the ρ dominance value above threshold. Cross sections on neutrons were assumed the same as on protons. The ω and ϕ channels were neglected, but these states can contribute only very small amounts of shadowing, because the $\gamma\omega$ and $\gamma\phi$ couplings are relatively small; in addition, $\sigma_{\phi N}$ is probably substantially less than $\sigma_{\rho N}$, as one would expect by analogy with σ_{KN} versus $\sigma_{\pi N}$.[13]

Below 1.1 BeV, Fig. 2 shows a complete absence of shadowing. In fact, a small amount of shadowing is possible due to the low-mass tail of the ρ, or due to pions, whose contribution should be very small (see Secs. VI and VII). For energies substantially below 1 Bev, the photon wavelength becomes comparable to the internuclear separations, and the relation $\sigma_{\gamma A}{}^{(\mathrm{hadronic})}=A\sigma_{\gamma N}$, or $\sigma_{\gamma A}{}^{(\mathrm{hadronic})}=Z\sigma_{\gamma p}+(A-Z)\sigma_{\gamma n}$, again breaks down.

Our calculation can easily be extended to off-mass-shell photons. If Q^2 is the square of the photon invariant mass (negative for electron scattering), then in Eq. (17) we must include the correct minimum momentum transfer: $q-k\cong(m_\rho^2-Q^2)/2q$. The bracket then becomes

$$F(\zeta)V_{\gamma\rho}{}^2/(m_\rho^2-Q^2+V_{\rho\rho}),\qquad(22)$$

where the potentials $V_{\gamma\rho}$ and $V_{\gamma\gamma}$ are proportional to forward amplitudes for virtual photons. Because of the coherence requirement, $A^{2/3}$ behavior ensues only at relatively higher energy if $Q^2\neq 0$. The predictions are shown in Fig. 3. We assume spin-flip processes to be unimportant, and these curves therefore apply separately to the total absorption cross section for longitudinal photons and transverse photons $\sigma_L(Q^2,\nu)$ and $\sigma_T(Q^2,\nu)$, which are obtained from inelastic electron or muon scattering in the standard way.[14] The results are again divided by A times the corresponding nucleon cross section, and thus can be interpreted as the effective fraction of nucleons which absorb the photon. The mean free paths for longitudinal and transverse ρ's were assumed equal, corresponding to $\sigma_{\rho N}=30$ mb.

V. PHOTON-DEUTERON TOTAL CROSS SECTION

The simple eikonal model which we have used up to now is inappropriate for light nuclei and must be replaced by a multiple-scattering approach such as the Glauber approximation.[15] We shall discuss the simplest case: the photon-deuteron cross section. (A multiple-scattering analysis for $A\geq 2$, and its connection with the eikonal approximation in the limit $A\to\infty$, has been given by Trefil.[6]) This discussion should serve to clarify our basic physical assumptions and approximations and provide insight into the microscopic processes which underlie the optical-model calculations. Also, an understanding of this particular problem would enable one to extract the photon-neutron cross section from experiment.

The relevant multiple-scattering terms (see Fig. 4) are

$$f_{\gamma d\to\gamma d}(\mathbf{q},0\to\mathbf{q},0)=f_{\gamma n\to\gamma n}(\mathbf{q},0\to\mathbf{q},0)+f_{\gamma p\to\gamma p}(\mathbf{q},0,\mathbf{q},0)-\frac{1}{4\pi^2 q}\int d\mathbf{p}\,d\mathbf{s}\,\phi(\mathbf{s})\phi(\mathbf{s}-\mathbf{p})$$
$$\times\{|\mathbf{q}|-[(\mathbf{q}-\mathbf{p})^2+m_\rho^2]^{1/2}+\Delta E_{\mathrm{nuc}}+i\epsilon\}^{-1}[f_{\gamma n\to\rho n}(\mathbf{q},-\mathbf{s}\to\mathbf{q}-\mathbf{p},-\mathbf{s}+\mathbf{p})f_{\rho p\to\gamma p}(\mathbf{q}-\mathbf{p},\mathbf{s}\to\mathbf{q},\mathbf{s}-\mathbf{p})$$
$$+f_{\gamma p\to\rho p}(\mathbf{q},-\mathbf{s}\to\mathbf{q}-\mathbf{p},-\mathbf{s}+\mathbf{p})f_{\rho n\to\gamma n}(\mathbf{q}-\mathbf{p},\mathbf{s}\to\mathbf{q},\mathbf{s}-\mathbf{p})],\qquad(23)$$

where ϕ is the deuteron wave function in momentum space.

If we neglect the recoil and binding corrections (represented by ΔE_{nuc}) and assume that the energy variation of the scattering amplitudes is gradual enough to be neglected inside the integral (as is justified by experiment), the third term in $f_{\gamma d\to\gamma d}(0)$ becomes

$$-\frac{1}{2\pi^2 q}\int d\mathbf{p}\,S(\mathbf{p})f_{\gamma n\to\rho n}(\mathbf{p})f_{\gamma p\to\rho p}(\mathbf{p})\times\{|\mathbf{q}|-[(\mathbf{q}-\mathbf{p})^2-m_\rho^2]^{1/2}+i\epsilon\}^{-1},\qquad(24)$$

where the arguments of the amplitudes now refer to

[13] See also Ref. 4, J. J. Sakurai, Stanford Linear Accelerator Center Report No. SLAC-TN-103, 1967 (unpublished) and K. Kajantie and J. S. Trefil, Phys. Letters **24B**, 106 (1967) for calculations of $\sigma_{\gamma p}$ from ρ, ω, ϕ photoproduction data. A general discussion of contributions other than the ρ is given in Secs. VI and VII. Preliminary measurements of $\sigma_{\phi n}$ are reported in the rapporteur's summary by S. C. C. Ting (Ref. 1).

[14] See, e.g., L. H. Hand, in *Proceedings of the 1967 International Symposium on Electron and Photon Interactions at High Energies* (Stanford Linear Accelerator Center, Stanford, Calif., 1967).

[15] R. J. Glauber, *Lectures in Theoretical Physics* (Interscience Publishers, Inc., New York, 1959), Vol. 1, and Phys. Rev. **100**, 242 (1955). Corrections to the Glauber theory are discussed in J. Pumplin, Phys. Rev. **173**, 1651 (1968) and D. R. Harrington, Rutgers Report (unpublished).

the magnitude of the three-momentum transfer and we have used the relation $f_{\gamma N\to\rho N}=f_{\rho N\to\gamma N}$ from time reversal to combine the double-scattering terms. Here

$$S(p)=\int d\mathbf{s}\,\phi(\mathbf{s})\phi(\mathbf{s}-\mathbf{p})=\int d\mathbf{r}\,(\psi(\mathbf{r}))^2 e^{i\mathbf{p}\cdot\mathbf{r}} \qquad (25)$$

is the nonrelativistic deuteron form factor, and $\psi(r)$ is the deuteron wave function in coordinate space. We have ignored the spins of the nucleons and of the deuteron, as well as D-state and many-body components of the deuteron wave function, in accord with common practice.[16]

The Glauber approximation[15] now corresponds to dropping the principal-value part of the energy denominator (propagator), and keeping only the δ-function part. This approximation is good at high energy; it is also good at low energy to the extent that the real parts of the scattering amplitudes are negligible, because in that case the principal-value part does not contribute when the imaginary part of $f_{\gamma d\to\gamma d}$ is taken to use the optical theorem.[17] Changing to spherical coordinates and using the δ function,

$$f_{\gamma d\to\gamma d}(0)=f_{\gamma n\to\gamma n}(0)+f_{\gamma p\to\gamma p}(0)$$
$$+\frac{i}{q}\int_{q-(q^2-m_\rho^2)^{1/2}}^{q+(q^2-m_\rho^2)^{1/2}} dp\, pS(p)f_{\gamma n\to\rho n}(p)f_{\gamma p\to\rho p}(p). \qquad (26)$$

The upper limit of this integral is $\approx 2q$, which can be replaced by ∞, since the deuteron form factor must cut off at a few hundred MeV. The lower limit is $\Delta\equiv q-(q^2-m_\rho^2)^{1/2}\cong m_\rho^2/2q$, which is the minimum momentum transfer as calculated with neglect of the nucleon-recoil energy.

In order to do a simple calculation, we use a Gaussian wave function for the deuteron,[18] which leads to $S(p)=e^{-\frac{1}{4}\alpha p^2}$ with $\alpha\cong 130\ \text{BeV}^{-2}$, and assume that the scattering amplitudes have a momentum-transfer dependence $\propto e^{-\frac{1}{2}\gamma p^2}$, $\gamma\cong 10\ \text{BeV}^{-2}$. Assuming pure imaginary amplitudes and neglecting the difference between $f_{\gamma n\to\rho n}$ and $f_{\gamma p\to\rho p}$, the optical theorem gives

$$\sigma_{\gamma d}=\sigma_{\gamma n}+\sigma_{\gamma p}-\left(\frac{d\sigma}{dt}\right)_{\gamma N\to\rho N\text{ at }0^\circ}\left(\frac{8}{4\gamma+\alpha}\right)\left(\frac{1-\eta^2}{1+\eta^2}\right)$$
$$\times\exp[-(\gamma+\tfrac{1}{4}\alpha)\Delta^2], \qquad (27)$$

where $\eta=\text{Re}f_{\gamma N\to\rho N}/\text{Im}f_{\gamma N\to\rho N}$. Note that 4γ is small compared to α, so that the result is rather insensitive to the value assumed for γ, i.e., to the assumed t dependence of the ρ production amplitude. (The sensitivity is even less for a large nucleus, so that essentially only the photoproduction cross section at zero degrees is important, as assumed in the eikonal model.) To the extent that the 4γ term can be completely ignored, the factor $2/(4\gamma+\alpha)$ represents $\langle r^{-2}\rangle$ for the deuteron wave function; however, for analyzing experimental data we would advocate performing the integral in (26), using a better wave function than the Gaussian, rather than going in for the still more crude $\langle r^{-2}\rangle$ approximation. The shadow correction $(\sigma_{\gamma n}+\sigma_{\gamma p}-\sigma_{\gamma d})/\sigma_{\gamma d}$ should be $\approx$4–5% at high energy, i.e., the correction in obtaining $\sigma_{\gamma n}$ from measurements of $\sigma_{\gamma d}$ and $\sigma_{\gamma p}$ should be $\approx$10%. The shadowing is reduced at intermediate energies due to "incoherence," i.e., due to the intolerance of the wave function to the minimum momentum transfer as given by the exponential factor in (27). The shadowing due to intermediate ρ's falls in principle to zero at the ρ production threshold, except for smearing due to the Fermi motion and the ρ width.

We wish to reemphasize that it is the photoproduction amplitude for real ρ mesons which appears in the shadow contribution. This is also true in the multiple-scattering analyses of photoproduction on heavy

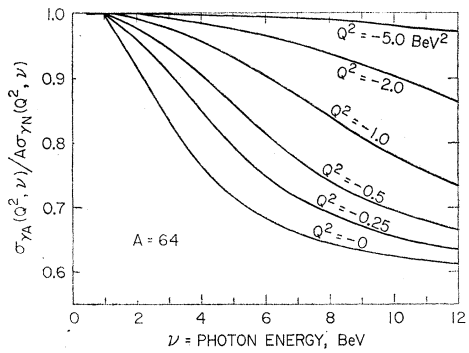

FIG. 3. Predictions of ρ dominance for inelastic electron scattering on nuclei are given as a function of the virtual photon lab energy and spacelike four-momentum squared, Q^2. The effective number of nucleons $\sigma_{\gamma A}(Q^2,\nu)/A\sigma_{\gamma N}(Q^2,\nu)$ is given for copper with $R=1.3\text{ F }A^{1/3}$, assuming $\sigma_{\rho N}=30$ mb for longitudinal or transverse ρ mesons. The results hold for the transverse or longitudinal photon cross sections $\sigma^i(Q^2,\nu)$ obtained from inelastic electron scattering.

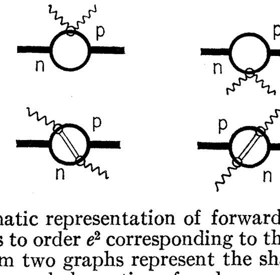

FIG. 4. Schematic representation of forward Compton scattering on deuterons to order e^2 corresponding to the four terms of Eq. (23). The bottom two graphs represent the shadow contributions from the emission and absorption of real ρ mesons (or, in general, any coherent hadron system photoproduced on the nucleons). The solid line represents the deuteron.

[16] Such corrections have been considered by D. Harrington, Phys. Rev. Letters **21**, 1496 (1968); Phys. Rev. **176**, 1982 (1968).

[17] Keeping the principal-value part would result in a correction to the shadow term in (27) which is down from the term given by $\sim\eta(m_\rho^2/2q)\langle r^2\rangle^{1/2}$, where η is proportional to the real part of the photoproduction amplitude.

[18] Simple, though unprecise, Gaussian wave functions for the deuteron have been given by M. Verde, Helv. Phys. Acta **22**, 339 (1949).

nuclei,[6] although that fact is somewhat obscure in the optical-model analysis. Since only on-shell photoproduction amplitudes enter, it should be possible to calculate the shadow correction in terms of photoproduction data, without ever making the assumption of vector dominance.

VI. BEYOND ρ DOMINANCE

A. Multichannel Effects in the Photoabsorption on Deuterium

So far we have discussed only the shadow effects which correspond to propagation of real intermediate ρ mesons. Other contributions are possible, however: e.g., ω, ϕ, and π many-particle systems. Our calculation on deuterium is easily generalized to include these possibilities. The approximate form (27), for example, becomes

$$\sigma_{\gamma d}=\sigma_{\gamma n}+\sigma_{\gamma p}-\int dm\frac{d\sigma}{dt\,dm}\bigg|_{\gamma N\to(\text{mass }m)N\text{ at }0^\circ}\times\frac{8}{4\gamma(m)+\alpha}\frac{1-[\eta(m)]^2}{1+[\eta(m)]^2}\times\exp[-(\gamma+\tfrac{1}{4}\alpha)(m^2/2q)^2]\times(\text{width factor}). \quad (28)$$

This generalization preserves the idea that the contribution of a state to shadowing is proportional to the cross section for photoproducing it on a nucleon. The exponential factor embodies the requirement that the shadowing be small if the minimum momentum transfer is too large for the nuclear wave function. The width factor, which we have not written explicitly, should reduce the cross section due to the spreading of intermediate several-particle states. For a *resonance* such as the ρ, the width factor is essentially 1, but for a nonresonant state at finite energy it may be substantially less than 1. In the limit $q\to\infty$, the width factor approaches 1 because of time dilatation. For example, a ρ of momentum q in free space will travel roughly $(q/m_\rho\Gamma)\approx 5$ F at 2 BeV and 25 F at 10 BeV before decaying. On the other hand, a completely nonresonant $\pi^+\pi^-$ system of invariant mass 500 MeV would spread by 1.0–1.2 F while traveling 3 F at 2 BeV and 0.04–0.25 F at 10 BeV. The two estimates given represent "decay" perpendicular or parallel to the direction of motion.

The effect of the real parts of the amplitudes could be important. If we write the phase of $f_{\gamma N\to(\text{state }n)N}$ as $ie^{i\phi}$, then the factor $(1-\eta^2)/(1+\eta^2)=\cos(2\phi)$. This can in principle be negative; i.e., if the photoproduction amplitude for a given state is predominantly *real*, then its contribution to the shadow effect actually adds to the cross section, tending to cancel the effect of the ρ. It could happen that the phases of the photoproduction amplitude for various final states are essentially random, in which case the contribution to the shadow effect would average to zero. This is expected to reduce the net effect of states whose photoproduction requires quantum number exchange. However, states which can be photoproduced by diffraction, i.e., with the exchange of no quantum number except for orbital angular momentum, are expected to be produced with essentially imaginary amplitudes. For example, the photoproduction of π-ρ systems with low invariant mass might be due to the "diffraction-dissociation" process[19] in which a γ-π-ρ vertex is followed by π or ρ elastic scattering. The phase of $f_{\gamma N\to\pi\rho N}$ is then the same as that for $f_{\pi N\to\pi N}$ and $f_{\rho N\to\rho N}$, which are known from experiment[20] to be predominantly imaginary.

The question of the phases of inelastic production amplitudes, and of the effects of the spreading of multiparticle intermediate states as discussed in the preceding paragraphs, are key problems in the theory of inelastic shadowing.[21] The shadowing predicted for the γ-A total cross section can be looked on as a special case of inelastic shadowing, which is made especially simple by the absence, to order α, of elastic shadowing.

The shadow effect due to states which are not produced diffractively, such as π^0 and $\pi^\pm$, are expected to be small. In the first place, the photoproduction amplitudes for these states are relatively small compared, for example, to the ρ, and are expected to continue to fall with energy. Second, nondiffractive processes, in particular π^0 and $\pi^\pm$ production, may involve spin flip in the forward direction and/or charge exchange, which will be suppressed by the requirement of leaving the nucleus in its ground state in the forward Compton amplitude.

B. Multichannel Shadow Effects for Heavy Nucleii

Returning now to the question of photon cross sections on large nuclei, and to A versus $A^{2/3}$ behavior, we find that the consideration of other intermediate states besides the ρ adds a great deal of complexity, which results from the many possibilities for inelastic scattering of the hadrons. For example, the incident photon might produce a ρ on nucleon 1; the ρ scatters on nucleon 2 to produce an $N\bar{N}$ state which scatters elastically on nucleon 3 and then produces a photon on nucleon 4; at high energy this could interfere with the amplitude for the photon simply to Compton scatter on one of the nucleons. In the face of such possibilities, a precise calculation of $\sigma_{\gamma A}$ seems impossible—if only

[19] M. L. Good and W. D. Walker, Phys. Rev. **120**, 1857 (1960).
[20] See, e.g., S. J. Lindenbaum, in *Proceedings of the Fourth Coral Gables Conference on Symmetry Principles at High Energy, 1967*, edited by A. Perlmutter and B. Kursunoglu (W. H. Freeman and Co., San Francisco, 1967). The phase of the ρ photoproduction amplitude was measured by J. Asbury *et al.*, Phys. Letters **25B**, 565 (1967).
[21] V. N. Gribov [English transl. by W. J. Zakrewski, University of Michigan, 1968 (unpublished)]; J. Pumplin and M. Ross, Phys. Rev. Letters **21**, 1778 (1968).

because it depends on a number of amplitudes which are not directly measurable. Nevertheless, it is possible to make some general statements on the A dependence, and, in particular, to demonstrate that $A^{2/3}$ behavior at high energy follows, as one would be led to expect from the uncertainty-principle argument of Sec. II, from any model in which the photon interactions are mediated by hadrons of bounded mass.[22]

Let us consider m hadronic channels instead of just the ρ. Define the matrices

$$\Lambda=\begin{bmatrix}\lambda_{\gamma\gamma} & \lambda_{\gamma 1} & \cdots & \lambda_{\gamma m}\\ \lambda_{\gamma 1} & \lambda_{11} & \cdots & \lambda_{1m}\\ \vdots & \vdots & & \\ \lambda_{\gamma m} & \lambda_{1m} & \cdots & \lambda_{mm}\end{bmatrix},$$
$$\Delta=\begin{bmatrix}0 & 0 & \cdots & 0\\ 0 & \Delta_1 & & \\ \vdots & & & \\ 0 & & & \Delta_m\end{bmatrix}, \qquad (29)$$

where the λ's are the forward-scattering amplitude normalized so that $1/\lambda_{ii}$ is the mean free path for channel i:

$$\lambda_{ab}=-(4\pi id/q)f_{aN\to bN}(0°)=(i/q)V_{ab}, \qquad (30)$$

and $\Delta_i\cong(m_i^2+|Q^2|)/2q$ is the minimum momentum transfer for photoproducing the state i. The generalization of Eqs. (13) and (16) for the wave function inside a homogeneous nucleus is

$$\psi^{(+)}(z)=\begin{bmatrix}\psi_\gamma(z)\\ \psi_1(z)\\ \vdots\\ \psi_m(z)\end{bmatrix}=e^{iqz}e^{-(z+a)(\frac{1}{2}\Lambda+i\Delta)}\begin{bmatrix}1\\0\\ \vdots\\0\end{bmatrix}. \qquad (31)$$

The forward-scattering amplitude is

$$f_{\gamma A\to\gamma A}(0°)=-(1/4\pi)(\phi,V\psi^+)$$
$$=\frac{iq}{4\pi}\int dr\,(1\;0\cdots0)\Lambda e^{-(z+a)(\frac{1}{2}\Lambda+i\Delta)}\begin{bmatrix}1\\0\\ \vdots\\0\end{bmatrix}, \qquad (32)$$

where the integral ranges over a sphere of radius $R=R_0A^{1/3}$ representing the nucleus and $z+a$ is the depth penetrated from the "left-hand" edge at $z=-a\equiv-(R^2-b^2)^{1/2}$ for impact parameter b.

In order to decide whether (32) contains a contribution proportional to A, i.e., proportional to volume, imagine diagonalizing the matrix $K\equiv\Lambda+2i\Delta$ which occurs in the exponential: In other words, expand the vector $(1\,0\cdots0)$ in eigenvectors of K. (This can certainly be done, since K is symmetric.) Working to leading powers in the electron charge e, K has an eigenvector

$$X=\begin{bmatrix}1\\x_1\\ \vdots\\x_m\end{bmatrix}, \qquad (33)$$

where

$$x_i=-\sum_j(T^{-1})_{ij}\lambda_{\gamma j}, \qquad (34)$$

and T is the $n\times n$ hadronic submatrix of K:

$$T_{ij}=\lambda_{ij}+2i\delta_{ij}\Delta_j. \qquad (35)$$

The eigenvalue associated with X is

$$\epsilon=\lambda_{\gamma\gamma}-\sum_{ij}\lambda_{\gamma i}(T^{-1})_{ij}\lambda_{\gamma j}=\det K/\det T. \qquad (36)$$

The eigenvalue ϵ is of order e^2, so X corresponds to a diagonal state with a mean free path which is long compared to the size of the nucleus. X can therefore contribute a volume term and will do so unless $\Lambda X=0$, in which case its contribution is killed by the Λ in (32) which is not exponentiated. In the high-energy limit $\Delta\to0$, so $K\to\Lambda$ and the condition for no volume term, i.e., $A^{2/3}$ behavior, becomes $\epsilon=0$, $\det\Lambda=0$. The other n eigenvectors of Λ must have eigenvalues of order 1, since the product of all the eigenvalues $=\det\Lambda\sim e^2$. They therefore correspond to states which are diagonal in the medium, but which are strongly absorbed, i.e., their mean free paths are a few fermis, so they contribute only "surface" terms. The generalization of the Bell picture (Sec. II) can now be obtained, if desired, by a canonical transformation in which the photon channel is made to correspond to the eigenvector X.[8]

A more explicit way to arrive at the condition for complete cancellation of the A term is simply to calculate the exponential in (32) to leading powers in e. This leads to

$$\psi_i(z)=\sum_j e^{iqz}[T^{-1}(e^{-\frac{1}{2}(z+a)T}-1)]_{ij}\lambda_{\gamma j} \qquad (37)$$

and to

$$f_{\gamma A\to\gamma A}(0°)=\frac{iq}{4\pi}\int dr\{\lambda_{\gamma\gamma}+\sum_{i,j}\lambda_{\gamma i}\times[T^{-1}(e^{-\frac{1}{2}(z+a)T}-1)]_{ij}\lambda_{\gamma j}\}. \qquad (38)$$

We assume on physical grounds that the eigenvalues of T are of order 1 F^{-1}, i.e., correspond to hadronic mean free paths; therefore T^{-1} necessarily exists, the exponential corresponds to "surface" terms, and the volume term is cancelled if and only if

$$\lambda_{\gamma\gamma}-\sum_{i,j}\lambda_{\gamma i}(T^{-1})_{ij}\lambda_{\gamma j}\equiv\det(\Lambda+2i\Delta)/\det T=0.$$

In the high-energy limit, $\Delta\to0$, leading again to the condition $\det\Lambda=0$.

Now let us generalize the vector dominance model by allowing the photon to couple directly to each of the

[22] This result was anticipated by Stodolsky, Ref. 4.

hadronic channels. Then

$$\lambda_{\gamma i}=\sum_j \frac{e}{g_j}\lambda_{ij},$$
$$\lambda_{\gamma\gamma}=\sum_{i,j} \frac{e}{g_i}\frac{e}{g_j}\lambda_{ij}, \tag{39}$$

where some of the coupling constants, which we write in the familiar form e/g, may be zero since some of the hadronic channels we consider may have quantum numbers different from the photon (such as, for example, the conjectured 3^- Regge recurrence of the ρ). Now (39) is equivalent to the condition $\det\Lambda=0$ to order e^2, which is needed for cancellation of the volume term at high energy. Note that this cancellation does not depend on any particular values for the coupling constants or for the hadronic scattering amplitudes. The only requirements are that the energy be so large that $(m_i^2+|Q^2|)/2q$ is negligible for all of the states involved, and that the hadronic amplitudes λ_{ij} not vary significantly when the square of the external four-momentum varies from m_i^2 or m_j^2 to Q^2.

In the "hadron-dominance" model discussed above, there are two distinct qualitative features to be observed in $\sigma_{\gamma A}/A\sigma_{\gamma N}$: the energy at which this quantity becomes substantially different from 1 (i.e., when the shadow effects become important, which is related to the average mass of the hadronic channels contributing importantly to the shadowing), and the limiting value at very high energy, which is related to the average cross sections of the hadronic states on nucleons. Quantitative results are discussed in Sec. VII.

C. Shadow Effects of a Hadron Spectrum

We have constructed a simple model to describe the shadow contributions of higher mass $J^P=1^-$ hadronic systems which could mediate the photon-nucleon interactions. The contribution of the ρ and the possibly important higher-mass systems is represented by a spectral function which is related to the electron-positron annihilation cross section $\sigma_{e^+e^-}$. This model is actually a special case of the many-channel model of Sec. VI B), in which the discrete states n are replaced by a continuum and all of the off-diagonal matrix elements between the hadronic channels are neglected.

First, we note that the propagator of the $(I=1)$ electromagnetic current of the hadrons is given by

$$\int e^{iQ\cdot x}\langle 0|T'[j_\mu(x),j_\nu(0)]|0\rangle d^4x =\frac{1}{16\pi^3\alpha^2}\int\frac{s^2\sigma_{e^+e^-}(s)[g_{\mu\nu}-Q_\mu Q_\nu/s]ds}{s-Q^2-i\epsilon}, \tag{40}$$

where $\sigma_{e^+e^-}(s)$ is the total $(I=1)$ hadron production cross section from electron-positron annihilation at c.m. energy $\sqrt{s}$.

We then compare this with virtual forward Compton scattering on a nucleus A: The Feynman amplitude is

$$m_{\mu\nu}{}^A(Q^2)=e^2\int e^{iQ\cdot x}\langle A|T'[j_\mu(x),j_\nu(0)]|A\rangle_{\text{conn}}d^4x. \tag{41}$$

Let us now use a "Furry picture" description putting the effects of the absorptive medium into the equation of motion for j_μ. Then, assuming a polarization-independent optical potential V,

$$m_{\mu\nu}{}^A(Q^2)=\frac{e^2}{16\pi^3\alpha^2}\int ds\, s^2\sigma_{e^+e^-}(s)\left(g_{\mu\nu}-\frac{Q_\mu Q_\nu}{s}\right)\times\left(\frac{1}{s-Q^2+V}-\frac{1}{s-Q^2}\right)ds$$
$$=\frac{-1}{4\pi^2\alpha}\int ds\, s^2\sigma_{e^+e^-}(s)\left(g_{\mu\nu}-\frac{Q_\mu Q_\nu}{s}\right)\times\frac{1}{s-Q^2}T(s,Q^2)\frac{1}{s-Q^2}, \tag{42}$$

where $T(s,Q^2)=V+V(Q^2-s-V+i\epsilon)^{-1}V$ is the virtual forward nuclear scattering amplitude for a vector meson of mass s, four-momentum Q. We have assumed that the potential V does not mix the various hadron components of the current, and we identify it with absorptive potential $V_{\rho\rho}$ of Sec. III. The result for the (virtual) total photoabsorption nuclear cross section is

$$\frac{\sigma_i{}^A(Q^2,\nu)}{A\sigma_i{}^N(Q^2,\nu)}=1-\text{Re}\left[\int ds\frac{\sigma_{e^+e^-}(s)s^2}{(s-Q^2)^2}\times\frac{V_{\rho\rho}}{s-Q^2+V_{\rho\rho}}F(\zeta)\bigg/\int ds\frac{\sigma_{e^+e^-}(s)s^2}{(s-Q^2)^2}\right], \tag{43}$$

where $\sigma_i{}^N(Q^2,\nu)$ is the total (transverse or longitudinal) photoabsorption cross section on nucleons as measured in inelastic electron or muon scattering ($i=$transverse or longitudinal).[14] $F(\zeta)$ is given by Eq. (17) with $m_\rho{}^2$ replaced by s.

Comparing with Eq. (39), if the annihilation cross section $\sigma_{e^+e^-}$ corresponds to a sum of narrow-width Breit-Wigner peaks for n vector mesons, then (43) corresponds to $\lambda_{ij}=\delta_{ij}\lambda_{ii}$ and

$$\sum_{i=1}^{n}\frac{\lambda_{\gamma i}{}^2}{\lambda_{ii}}=\lambda_{\gamma\gamma}, \tag{44}$$

where the λ_{ii} are all taken to be equal. Various possibilities for the hadron spectrum and the resulting implications for $\sigma_{\gamma A}(Q^2,\nu)$ are discussed in Sec. VII.

VII. NUMERICAL CALCULATION FOR VIOLATIONS OF VECTOR DOMINANCE

Let us now consider how the predictions based on ρ dominance which are shown in Figs. 2 and 3 are modified if vector dominance is broken.

As a first way to break vector dominance, assume that the effective ρ-photon coupling $em_\rho^2/2\gamma_\rho$ on the photon mass shell has strength given by $\gamma_\rho^2/4\pi=1.1$ to account for the small magnitude of nuclear ρ photoproduction observed by the Cornell and SLAC groups.[7] In the "pure" vector dominance models, $\gamma_\rho^2/4\pi$ has the same value at the photon mass as it does on the ρ mass shell where it is measured to be 0.52 ± 0.07 from colliding beam experiments $e^+e^-\to\pi^+\pi^-$ and the leptonic decay of ρ's.[1] If we continue to assume that $\sigma_{\rho N}\approx 30$ mb as obtained from the analysis of ρ photoproduction on nuclei, then the ρ-dominant part of $\sigma_{\gamma p}$ is $(e/2\gamma_p)^2\sigma_{\rho N}\approx 50\ \mu$b. The measured value of $\sigma_{\gamma p}$ is 110–130 μb,[23] so that a contribution in addition to the ρ must remain. Including the ω and ϕ, using γ_ω and γ_ϕ given by the Orsay experiments, and assuming $\sigma_{\phi N}$ is small like σ_{KN} adds $\sim 25\ \mu$b,[13] but this is probably overestimated, since if the effective γ-ρ coupling falls by a factor of 2 in going to $Q^2=0$, the γ-ω and γ-ϕ couplings would be expected to fall also. We must therefore assume some additional states mediate the photon interactions to account for the magnitude of $\sigma_{\gamma p}$. For simplicity, let us first assume those states are sufficiently massive ($\gtrsim 3$ BeV) that they do not contribute importantly to shadowing at energies below 20 BeV, where we can hope to have data in the near future. The effect of such states in this energy region is the same as for local coupling of the photon, which contributes to $\sigma_{\gamma N}$ and hence to the volume of A term but not to the shadowing. The effect is to essentially cut the shadowing in half compared to the pure ρ-dominance result. It would be very hard to understand an experimental result of much less shadowing than this, since we know the minimum strength and phase of ρ photoproduction at forward angles reasonably well.[1,7,20]

This reduction in shadowing is illustrated in detail in Fig. 5. The curves are calculated from Eq. (43) assuming that $\sigma_{e^+e^-}$ is dominated by the ρ with a Breit-Wigner distribution plus another similar resonance of higher mass M_V with equal magnitude. The results are shown for various values of M_V. For $\nu<20$ BeV, the results are nearly indistinguishable for any mass $M_V\geq 3$ BeV. We also note here that a similar calculation to include the ϕ meson shows that the effect of a pole at $M_V=m_\phi$ with $\frac{1}{10}$ the pole strength of the ρ is negligible. The use of a Breit-Wigner distribution for the two-pion states of the ρ produces a small amount of shadowing below $\nu=1.1$ BeV and thus removes the sharp behavior at threshold shown in Figs. 2 and 3.

[23] Aachen-Berlin-Bonn-Hamburg-Heidelberg-München Collaboration, Phys. Letters **27B**, 474 (1968); J. Ballam *et al.*, Phys. Rev. Letters **21**, 1544 (1968). We shall assume that the forward Compton amplitude is essentially diffractive.

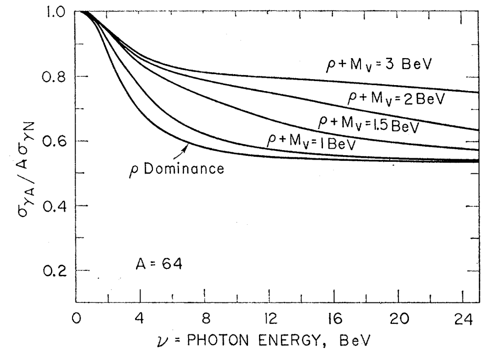

FIG. 5. Predictions for the photoabsorption cross section of copper assuming violation of vector dominance. The cross section for ρ photoproduction on nucleons is taken to be $\frac{1}{2}$ the ρ-dominance prediction (see Ref. 7). The effective number of nucleons $\sigma_{\gamma A}/A\sigma_{\gamma N}$ is given as a function of laboratory photon energy assuming shadow contributions from the ρ plus other hadron states, which are represented by a vector resonance at mass M_V photoproduced with the width and magnitude of the ρ. $\sigma_{\rho N}$ was taken as 30 mb, and R as $1.2\ \mathrm{F}\times A^{1/3}$. The result for $M_V\geqq 3$ BeV and $\nu<20$ BeV is to reduce the shadow contribution $1-\sigma_{\gamma A}/A\sigma_{\gamma N}$ to half of the value obtained from ρ dominance given in Fig. 2.

As another model for the spectral function, we can adopt the following form for the annihilation cross section:

$$\sigma_{e^+e^-}(s)=\mathrm{const}\left[\left(1-\frac{4m_\pi^2}{s}\right)^{3/2}\frac{m_\rho^2}{s}\frac{m_\rho^2\Gamma^2}{(s-m_\rho^2)^2+m_\rho^2\Gamma^2}+\epsilon\theta(s-bm_\rho^2)\left(\frac{m_\rho^2}{s}\right)^N\right]\theta(s-4m_\pi^2).\quad(45)$$

The extra tail, with adjustable parameters ϵ, b, and N, is added to reflect a large, slowly falling cross section beyond the ρ region.[24,25]

In order to restrict the possibilities we again assume that the ρ is responsible for roughly half of $\sigma_{\gamma p}$. Then, for example, with a charge tail falling like s^{-2} starting at $s=2m_\rho^2$ with $\epsilon=1$, the shadowing for real photons on copper ($A=64$) is 24% at $\nu=10$ BeV and 32% at $\nu=20$ BeV, compared to 31% and 41% shadowing implied by pure ρ dominance, respectively. Again, we note that from the magnitude of the observed ρ photoproduction the minimum shadowing will not be less than half of the pure ρ-dominance result.

VIII. SUMMARY

We have shown that measurements of the total hadronic cross section for photons up to 20 BeV on nuclei, and measurements of inelastic electron scat-

[24] For $\epsilon=0$, this form for $\sigma_{e^+e^-}$ is the experimental fit to the Orsay data ($m_\rho=0.76$, $\Gamma=0.11$ BeV, const$=1.7\pm0.2\ \mu$b), and corresponds to a simple Breit-Wigner pole in the pion form factor.

[25] The quark field algebra predicts $\sigma_{e^+e^-}\to s^{-1}$ for large s. See J. D. Bjorken, Phys. Rev. **148**, 1467 (1966). Also, a tail with $N>2$ would not contradict the gauge field algebra. See J. Dooher, Phys. Rev. Letters **19**, 600 (1967). Our form (43) is consistent with $\sigma_{T,L}(Q^2,\nu)\sim Q^{-2}$ in the diffractive energy region for inelastic e-p scattering if $\sigma_{e^+e^-}\to s^{-2}$.

tering in the same energy region, provide a sensitive test of the theoretical idea of vector-meson dominance, and of the more general idea of low-mass hadron dominance of the electromagnetic current.

In the case of the ρ dominance, the quantity $\sigma_{\gamma A}/A\sigma_{\gamma N}$, which amounts to an effective fraction of photoabsorbing nucleons, becomes energy-independent and equal to the corresponding fraction appropriate for hadron absorption when the momentum transfer for forward ρ photoproduction on a nucleon $\Delta\sim(m_\rho^2+|Q^2|)/2\nu$ is small compared to the reciprocal of the hadronic mean free path in the nucleus. If the nuclear photoabsorption data support this, then by making reasonable assumptions on nuclear models, real parts of amplitudes, and approximations of the ω and ϕ contributions, one will be able to extract $\sigma_{\rho N}$ for the ρ on the mass shell. This result should agree with the value for $\sigma_{\rho N}$ obtained from ρ photoproduction on nuclei. If ρ dominance is correct, $\sigma_{\gamma A}(Q^2,\nu)/A\sigma_{\gamma N}(Q^2,\nu)$ depends only on $t_{\min}$. Thus measurements of inelastic electron-nucleus scattering for values of Q^2 and ν which keep $t_{\min}$ constant, are sensitive to any deviation from the ρ dominance relation $f_{\gamma\rho}^2=f_{\gamma\gamma}f_{\rho\rho}$ for large spacelike Q^2.

On the other hand, the hadronlike behavior of the nuclear photoabsorption cross section at high energies is not a unique feature of ρ dominance, but also follows from a generalization of the vector-dominance model in which arbitrary hadron states are assumed to contribute to the electromagnetic current. The complete vanishing of the volume contribution to $\sigma_{\gamma A}$ only requires that the momentum transfer be negligible for all of the states involved and the determinant (29) of the forward-scattering amplitudes vanishes to order e^2; this condition is met for the "hadron-dominance" model in which photons interact with nucleons via a sum (possibly spectral) of $J=1^-$ hadron states, assuming the high-energy forward photoproduction amplitudes do not change appreciably when extrapolated to the Q^2 of the photon. In the energy region $\nu\lesssim 20$ BeV, dominance by hadron states of mass ≥ 3 BeV cannot be distinguished from dominance by states of very large mass, such as baryon-antibaryon pairs, quark-antiquark pairs, etc., or states of infinite mass which correspond to pointlike interactions. The energy dependence of $\sigma_{\gamma A}/A\sigma_{\gamma N}$, however, is sensitive to dominant states beyond the ρ in the 1–3 mass region.

In general, any state which can be produced (including higher spins, e.g., a ρ Regge recurrence at $J^P=3^-$) contributes to the shadowing in proportion to the square of its yet unmeasured nuclear forward photoproduction amplitude. In particular, if the real forward ρ photoproduction cross section is half[7] of what is predicted by simple vector dominance, i.e.,

$$2f_{\gamma\rho}^2=f_{\gamma\gamma}f_{\rho\rho}\,,$$

then the shadow contribution $1-\sigma_{\gamma A}/A\sigma_{\gamma N}$ is half of the ρ-ω-ϕ dominance prediction for $\nu<20$ BeV; eventually, it increases with photon energy as photoproduction for higher-mass states which contribute to $f_{\gamma\gamma}$ but not $f_{\gamma\rho}$ or $f_{\rho\rho}$, becomes coherent on the nucleus.

Finally, we emphasize that photoproduction of ρ mesons on nucleons implies shadowing for photon-nucleus interactions independent of any model, with the exception of the unlikely possibility that there are large cancellations due to photoproduction of low-mass states with real amplitudes.

By the time of journal publication of this paper, we expect relevant experimental data to be available. In addition to measurements of $\gamma+A\rightarrow$ hadrons and inelastic electron and muon-nucleus scattering, we would like to encourage measurements of total cross sections for pions on nuclei, in order to test the reliability of available methods for treating the nuclear physics and to look for the inelastic shadow effects which have been predicted for the scattering of hadrons, the theory of which is on the same footing as the many-channel calculation of Sec. III.

Note added in proof. Preliminary results of a Santa Barbara SLAC experiment to measure the photon-nucleus cross section at high energy indicate a shadow effect which lies roughly midway between zero (i.e., $\sigma_{\gamma A}\propto A$) and the vector-dominance prediction, for E_γ between 12 and 18 BeV. Thus it appears that the vector mesons "dominate" only about one half of the electromagnetic current, as is also indicated by the two high energy rho photoproduction experiments.[7] We wish to thank Dr. R. Morrison and Dr. F. Murphy for discussions of their data.

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