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Quark Model, Multiple Scattering, and Regge Theory*

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The quark model for meson-nucleon scattering is treated by means of Glauber theory, using Reggeized quark scattering amplitudes. The multiple-scattering effects are calculated, and numerical estimates of their magnitudes are obtained by fitting the resulting forms to the available data. We find that the correction terms obtained in this way are generally subtractive and amount to 15% of the total cross sections. They disappear at a rate intimately connected with the shrinking of the diffraction peak; when the latter phenomenon begins to occur, our model predicts that total cross sections will show an increase of several millibarns. The multiple-scattering terms are essentially Regge-cut effects, and therefore may be helpful in explaining the nonvanishing polarization observed in πp charge exchange. Using a simple model of this type, we obtain reasonably good fits to all types of pion-nucleon scattering data including both elastic and charge-exchange polarizations as well as total and differential cross sections.

I. INTRODUCTION

THE additive quark model, despite the naïveté of its physical content, has met with considerable success in predicting relations between cross sections observed for various scattering processes. Since the additivity assumption embodies the impulse approximation, it has been suggested in several recent papers[1–5] that corrections to this model may be calculable by means of the Glauber[6] formalism for scattering of composite particles.

The effects of these "multiple-scattering" corrections can be observed in three basic ways. First, since they are nonlinear and therefore nonadditive, the simple relations obtained from additivity will be replaced by more complicated nonlinear ones. Such equations have been obtained by Franco,[1] who derives nonlinear relations among total cross sections involving mesons, protons, and antiprotons, and by the author,[4] in a form supplying correction terms to the antisymmetric sum rule. Secondly, in the framework of multiple diffraction, the differential cross section will possess a characteristic "dip" structure resulting from the interference of the different orders of scattering. As Harrington and Pagnamenta[2] have emphasized, this mechanism may account for the angular behavior of proton-proton scattering. Finally, reactions forbidden by the additivity principle may be allowed through double-scattering terms; for example, in a previous paper[5] we have shown that the data on double charge or hypercharge exchange can be so interpreted. It is possible in that case to study the double-scattering amplitudes directly, rather than through interference effects with the leading terms.

In this paper we shall attempt a quantitative study of multiple-scattering effects in the quark model of meson-nucleon interactions. The interactions of the quarks are assumed to be described by Regge theory, in order to consider the energy variation of the resulting amplitudes. The multiple-scattering terms can be evaluated exactly by use of an exponential form of the Regge amplitude, and correspond in the well-known way to Regge cuts. In Sec. II we describe the details of our model, and in Sec. III we use it to estimate the magnitudes of these terms by fitting the measured meson-nucleon total cross sections. We find that the contributions of double scattering are subtractive and of magnitude about 15% of the observed cross sections.

The fact that the model provides a natural mechanism for generating a term with Regge-cut behavior suggests the consideration of the pion-nucleon charge-exchange polarization. We therefore extend the calculations to include spin in Sec. IV, and use the resulting forms to fit a large amount of data on pion-nucleon interactions, including polarizations, in Sec. V.

II. MODEL

Our calculations are based upon the formalism developed by Glauber[6] for the consideration of scattering

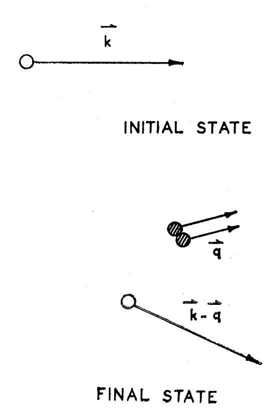

FIG. 1. Scattering by a two-component composite particle.

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[1] V. Franco, Phys. Rev. Letters **18**, 1159 (1967).
[2] D. Harrington and A. Pagnamenta, Phys. Rev. Letters **18**, 1147 (1967); Phys. Rev. **173**, 1599 (1968).
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[4] N. W. Dean, Nucl. Phys. **B4**, 534 (1968).
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[6] R. J. Glauber, Phys. Rev. **100**, 242 (1965); V. Franco and R. J. Glauber, *ibid.* **142**, 1195 (1966).

by a composite particle. Excellent reviews of the derivation and use of this theory already exist, so we shall not give a lengthy description of it here; the essential features can be shown by considering the simple process pictured in Fig. 1. An incident particle of momentum $\mathbf{k}$ is scattered from a bound state of two particles (for example, a deuteron). The form factor of the bound state is denoted by

$$S_D(\mathbf{q})=\int d^3r\,\psi_B{}^*(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}}\psi_B(\mathbf{r}),$$

where $\psi_B(\mathbf{r})$ is the bound-state wave function in the c.m. frame of the two particles, and the scattering of the incident particle by the free state of either of the constituent particles is described by a scattering amplitude $f_N(\mathbf{k},\mathbf{q})$, $\mathbf{q}$ being the momentum transfer. For the moment we neglect spin and isospin; then according to the Glauber theory, the scattering process shown is described by an amplitude

$$f_D(\mathbf{k},\mathbf{q})=f_N(\mathbf{k},\mathbf{q})[S_D(\tfrac{1}{2}\mathbf{q})+S_D(-\tfrac{1}{2}\mathbf{q})]+\frac{i}{2\pi k}\int d^2q'S_D(\mathbf{q}')f_N(\mathbf{k},\tfrac{1}{2}\mathbf{q}+\mathbf{q}')f_N(\mathbf{k},\tfrac{1}{2}\mathbf{q}-\mathbf{q}'). \quad (2.1)$$

This equation, which forms the basis for the calculation which we shall present, is familiar in lower-energy nuclear physics, where it has been used frequently with considerable success.

In order to evaluate simply the essential results of the Glauber model embodied in Eq. (2.1), we shall now make two simplifying assumptions. First, we assume that the scattering amplitudes $f_N(\mathbf{k},\mathbf{q})$ are effectively exponential in the square of the momentum transfer,

$$f_N(\mathbf{k},\mathbf{q})=f_Ne^{-\gamma q^2}.$$

This form is generally correct in most simple scattering processes at sufficiently high energies and small momentum transfer. Secondly, we shall assume that the form factor $S_D(\mathbf{q})$ is effectively unity in the region of interest. This assumption is strictly tenable only in the extreme forward direction. In fact, in the case of nucleon-deuteron scattering it is known that the variation of the deuteron form factor is responsible for a substantial part of the angular variations of the differential cross section, even in the diffraction region. Our consideration will not apply to the deuteron, however; we shall instead decompose either nucleons or mesons into their constituent quarks. The present understanding of the form factors of hadrons on the basis of the quark model is far from complete, and the experimental work on meson form factors is often meager. It is true, however, that the form factors of the hadrons seem to decrease less rapidly than that of the deuteron, corresponding to the expectation that the quarks forming the hadrons are tightly bound. Furthermore, it often seems in nuclear scattering that the results are rather insensitive to the form factor. This result is not surprising when we consider that if we make the exponential approximation in the double-scattering term, the main contribution to the scattering amplitude must come from momentum transfers for which both $|\tfrac{1}{2}\mathbf{q}+\mathbf{q}'|$ and $|\tfrac{1}{2}\mathbf{q}-\mathbf{q}'|$ are small, i.e., in which $|\mathbf{q}'|$ is small. This insensitivity is particularly significant in our applications to the quark model because the single-scattering amplitudes, which would involve free quarks, are not known. As an example we consider the possibility that the form factor is also exponential in q^2. A trivial calculation shows that the difference between the definitions

$$f_N(\mathbf{k},\mathbf{q})=fe^{-\gamma q^2},\quad S_D(\tfrac{1}{2}\mathbf{q})=1$$

and

$$f_N(\mathbf{k},\mathbf{q})=fe^{-(\gamma-\beta/4)q^2},\quad S_D(\tfrac{1}{2}\mathbf{q})=e^{-\beta q^2/4}$$

comes only from the double-scattering terms, and that these terms are in the ratio

$$[\gamma/(\gamma+\tfrac{1}{4}\beta)]e^{\beta q^2/8}.$$

Since we expect that $\beta\ll\gamma$ if the quarks are tightly bound, we see that for reasonably small momentum transfers the difference should be quite small. In all the applications we shall consider only small momentum transfers are involved, so it is reasonable to hope that the effects of this assumption will not be severe.

Making these two assumptions, then, we find from (2.1) that

$$f_D(\mathbf{k},\mathbf{q})=2f_Ne^{-\gamma q^2}+(if_N{}^2/2k\gamma)e^{-\gamma q^2/2}. \quad (2.2)$$

This relation serves as prototype for all the multiple-scattering processes we shall consider. It is easily extended to more general circumstances; for example, spin and isospin can be included quite naturally, as we shall presently show. If the scattering in question is that of a k-particle composite system by an l-particle one, a result analogous to (2.1) is similarly obtained, and contains $(kl)!/n!(kl-n)!$ terms describing n-tuple scattering.[2] The possibility of two different types of scattering process being involved in the multiple scattering (which is the case in the deuteron, of course) is also easily considered. The double-scattering integral can be performed even if the slopes of two diffraction peaks are different, since

$$\int d^2p\,e^{-\gamma_1(\frac{1}{2}\mathbf{q}+\mathbf{p})^2-\gamma_2(\frac{1}{2}\mathbf{q}-\mathbf{p})^2}=\frac{2\pi}{\gamma_1+\gamma_2}\exp\left(-\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}q^2\right). \quad (2.3)$$

This result can, in fact, be extended to n-tuple scattering, since it is quickly proved by induction that the relevant form is

$$\int d^2p_1\cdots d^2p_n\delta(\mathbf{q}-\sum_{i=1}^{n}\mathbf{p}_i)\exp(\sum_{i=1}^{n}\gamma_i p_i^2)$$

$$=[(2\pi)^{n-1}/(\prod_{i=1}^{n}\gamma_i)(\sum_{i=1}^{n}\gamma_i^{-1})]\exp[-(\sum_{i=1}^{n}\gamma_i^{-1})^{-1}q^2]. \quad (2.4)$$

In order to consider the energy variation and the phase of the multiple-scattering terms, we now assume that the amplitude $f_N(\mathbf{k},\mathbf{q})$ corresponds to a Regge pole. Since an integration over momentum transfer is involved in the multiple-scattering terms, however, it is necessary to employ a simplified version of the Regge-pole amplitude. The traditional form is

$$F(s,t)=[2\alpha(t)+1]\beta(t)\frac{1+\tau e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}s^{\alpha(t)}, \quad (2.5)$$

where $\beta(t)$ is the residue function, $\alpha(t)$ the pole's trajectory, τ the signature, and s and t have their usual meanings. The extraneous factors in (2.5) clearly prohibit the exact integration over t of this amplitude in any but pathological cases. We, therefore, adopt a much simpler form, which contains nonetheless all of the essential features of (2.5), by writing

$$F_R^{+}(s,t)=-c_+(s/is_0)^{\alpha+(t)}, \quad \text{for } \tau=+1 \quad (2.6a)$$

$$F_R^{-}(s,t)=ic_-(s/is_0)^{\alpha-(t)}, \quad \text{for } \tau=-1. \quad (2.6b)$$

It is simply a result of analyticity plus crossing symmetry that the dependence on s/i in Eqs. (2.6) correctly reproduces the phase of the Regge amplitude given, in (2.5), by the signature factor $1+\tau e^{-i\pi\alpha(t)}$.[7] The constants $c_\pm$ and s_0 are chosen so that they reproduce the t dependence of the residue function; that is, we assume that it is possible to write approximately, for $\tau=+1$,

$$2[2\alpha(t)+1]\beta(t)\frac{\cos\frac{1}{2}\pi\alpha(t)}{\sin\pi\alpha(t)}=-c_+(s_0)^{-\alpha+(t)}, \quad (2.7a)$$

and for $\tau=-1$, similarly,

$$2[2\alpha(t)+1]\beta(t)\frac{\sin\frac{1}{2}\pi\alpha(t)}{\sin\pi\alpha(t)}=-c_-(s_0)^{-\alpha-(t)}. \quad (2.7b)$$

The exponential t dependence in the residue function often invoked in order to fit the experimental diffraction peaks is thus pictured as representing an incorrect normalization of the energy. A trivial calculation shows that the value of s_0 necessary for compatibility with typical high-energy data is between 0.001 and 0.1 GeV^2, depending on the slope of the Regge trajectory.

As a simple example in which the essential results following from the definitions (2.6) can easily be seen, we consider the exchange of a single Regge pole. To be definite we assume that the Regge pole involved is the Pomeranchuk, with positive signature and $\alpha(0)=1$. The required exponential dependence on q^2 is obtained by writing

$$\alpha(q^2)=1-\beta q^2. \quad (2.8)$$

The scattering amplitude can now be written by making in (2.2) the substitution

$$f_N e^{-\gamma q^2}=C(s/is_0)^{1-\beta q^2}, \quad (2.9)$$

that is,

$$f_N=-C(s/is_0), \quad \gamma=\beta\ln(s/is_0),$$

yielding

$$f_R(s,q^2)=-2C\left(\frac{s}{is_0}\right)^{1-\beta q^2}+\frac{iC^2}{2k\beta\ln(s/si_0)}\left(\frac{s}{is_0}\right)^{2-\frac{1}{2}\beta q^2}. \quad (2.10)$$

We notice at once that the double-scattering term has the logarithmic s dependence which typifies a Regge cut. It is, in fact, a well-known general result that the iteration of a Regge-pole term, whether as a multiple-scattering integral or in some other guise, leads to the appearance of terms having this behavior. This point was made as long ago as 1962 by Amati, Stanghellini, and Fubini[8] in considering the multiperipheral model of high-energy scattering. It was subsequently shown in a treatment of Feynman-diagram singularity structure by Mandelstam,[9] however, that the cut they found was canceled by other terms, and that only nonplanar diagrams could lead to Regge cuts. Such diagrams appear only if the structure of both particles involved in the scattering process is considered. This result is, therefore, difficult to reconcile with the Glauber formalism, which produces a cut regardless of the structure of the noncomposite particle. The contrast, even in language, between the physical assumptions of Glauber theory and the ponderous mathematical apparatus of diagrammatic techniques is a formidable one, and the resolution of the problems involved is far from apparent. The viewpoint we shall adopt is a phenomenological one; we regard the Glauber formalism as a tool supplying an eminently reasonable parametrization of the scattering amplitude. As such, it can be used independently of whether the actual form of the cut term is ultimately vindicated or not.

It is sufficient for our examination of the essential features of the amplitude (2.10) to study its asymptotic form, using the high-energy approximation $s\approx 2Mk$. Then (2.10) becomes

$$f_R(s,q^2)+-2C\left(\frac{s}{is_0}\right)^{1-\beta q^2}+\frac{C^2M}{\beta s_0\ln(s/is_0)}\left(\frac{s}{is_0}\right)^{1-\frac{1}{2}\beta q^2}. \quad (2.11)$$

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[8] D. Amati, A. Stanghellini, and S. Fubini, Nuovo Cimento **26**, 896 (1962).

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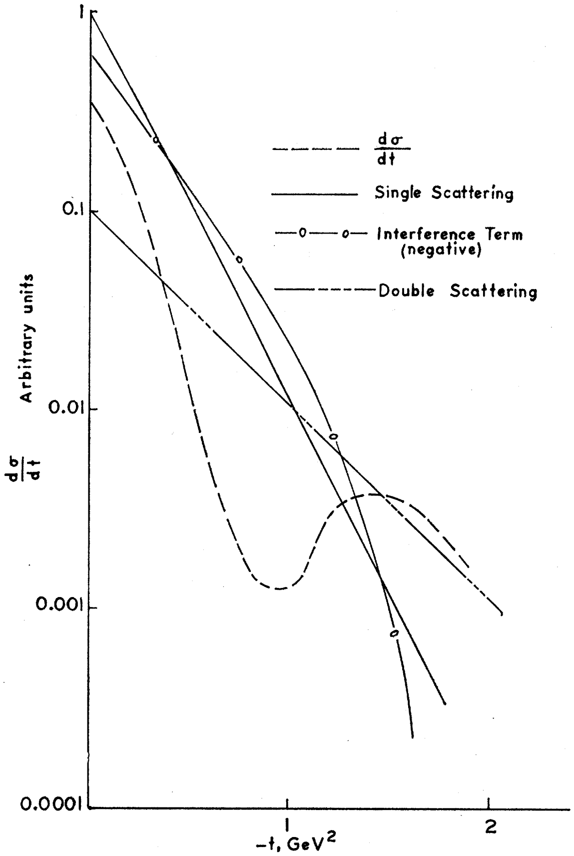

FIG. 2. Contributions of single and double scattering and of the interference between them to the differential cross section.

We examine first the total cross section, which is obtained by using the optical theorem. Assuming for the moment that $s \gg s_0$, so that $\ln(s/s_0)$ is essentially real, we have

$$\sigma_T(s) = (4\pi/k)\,\mathrm{Im} f_R(S,0) \approx 8\pi\left[2\left(\frac{MC}{s_0}\right) - \frac{1}{\beta\ln(s/s_0)}\left(\frac{MC}{s_0}\right)^2\right]. \quad (2.12)$$

The multiple-scattering contribution is immediately seen to be subtractive, implying that the total cross section increases to a constant value. The opposite situation is observed in experimental hadronic total cross sections. If several trajectories are involved, however, the increasing behavior may be hidden. More simply, a positive double-scattering contribution can result from a pole with negative signature because of the extra i in the definition (2.6b), or from the inclusion of double isospin-flip terms because of the Clebsch-Gordan coefficients. In any case, however, the multiple-scattering contribution vanishes with increasing energy at least logarithmically because of the shrinkage of the diffraction peak. We also note in (2.12) that the mass M and the constant C appear only in the combination MC/s_0, so that this result does not depend directly on the choice of M, the mass of the component particle. Except for the term $\beta\ln(s/s_0)$, which represents the slope of the single-scattering diffraction peak, the result is also insensitive to s_0.

We look next at the differential cross section. For the large-energy, small-momentum-transfer limit being considered, we obtain

$$\frac{d\sigma}{dt} \approx \frac{\pi}{k^2}|f_R|^2 \approx 16\pi\left(\frac{MC}{s_0}\right)^2\left[\left(\frac{s}{s_0}\right)^{-2\beta q^2} - \frac{\cos\frac{1}{4}\pi\beta q^2}{\ln(s/s_0)}\left(\frac{MC}{s_0}\right)\left(\frac{s}{s_0}\right)^{-\frac{3}{2}\beta q^2} + \frac{1}{4[\beta\ln(s/s_0)]^2}\left(\frac{MC}{s_0}\right)^2\left(\frac{s}{s_0}\right)^{-\beta q^2}\right]. \quad (2.13)$$

Once again M and C appear only as MC/s_0, and β and s_0 appear always as related to the single-scattering diffraction peak slope. Of particular interest in (2.13) is the energy dependence of the three terms. We have seen already in the total cross section that for $q^2=0$ the single scattering is dominant at $s\rightarrow\infty$. For nonzero values of q^2, however, the opposite result is obtained. The ratio of the double-scattering term to the single is proportional to $(s/s_0)^{\beta q^2}/[\ln(s/s_0)]^2$, and, therefore, for sufficiently high energies the former dominates. The behavior implied by (2.13) is pictured in Fig. 2; at very small q^2 we see mainly the single-scattering term. For larger q^2 the interference between single and double scattering becomes more important and may cause dips in the differential cross section. Finally, as sufficiently large values of q^2, the double-scattering term becomes dominant. A convenient measure of how rapidly the limiting behavior is approached is obtained by finding the value of q^2 for which the contributions of single and double scattering are equally important. Setting

$$\left(\frac{s}{s_0}\right)^{-2\beta q_0^2} = \frac{1}{4[\ln(s/s_0)]^2}\left(\frac{MC}{s_0}\right)^2\left(\frac{s}{s_0}\right)^{-\beta q_0^2}, \quad (2.14)$$

we find

$$q_0^2 = 2\left[\ln\left(\ln\frac{s}{s_0}\right) - \ln\left(\frac{MC}{2\beta s_0}\right)\right]\Big/\left(\beta\ln\frac{s}{s_0}\right), \quad (2.15)$$

which has only the slightest decreasing behavior with increasing s. The structure of the differential cross section therefore changes very slowly with increasing energy, but ultimately arrives at a limit in which the contribution of the single scattering term is visible only within an arbitrarily small range of momentum transfer.

III. APPLICATION TO MESON-NUCLEON TOTAL CROSS SECTIONS

In order to estimate more accurately the magnitudes of these effects, we have applied this Regge formalism to the parametrization of the meson-nucleon scattering amplitudes. An immediate problem in such an application is choosing which interaction to Reggeize. The most basic level of elementarity indicates that the quark-quark amplitudes should be chosen, but it is also conceivable that quark-nucleon or quark-meson amplitudes might be more appropriate.

We have attempted to fit the six meson-nucleon total cross sections using Regge amplitudes both for the quark-quark interaction and for the quark-nucleon; consideration of the quark-meson system is not pertinent here, since it does not yield any relations among the six amplitudes $F_{\pi^+p}(s,q^2)$, $F_{\pi^-p}(s,q^2)$, $F_{K^+p}(s,q^2)$, $F_{K^+n}(s,q^2)$, $F_{K^-p}(s,q^2)$, and $F_{K^-n}(s,q^2)$.

The details of our procedure are more easily written down in the case of Reggeized quark-nucleon interactions. Here the meson-nucleon scattering amplitudes are given by a sum of two terms

$$F_{MN}(s,q^2)=F_{MN}{}^1(s,q^2)+F_{MN}{}^2(s,q^2)\,, \qquad (3.1)$$

representing, respectively, single and double scattering of the nucleon N by the quark-antiquark pair. Each of these is given in terms of the six basic quark-nucleon interactions, which we denote by $f_{\mathcal{P}p}(s,q^2)$, $f_{\mathfrak{N}p}(s,q^2)$, $f_{\lambda p}(s,q^2)$, $f_{\bar{\mathcal{P}}p}(s,q^2)$, $f_{\bar{\mathfrak{N}}p}(s,q^2)$, and $f_{\bar{\lambda}p}(s,q^2)$ (we assume, of course, that isosymmetry is maintained on the quark-nucleon level).

For the nonstrange quarks the Reggeization of these amplitudes is equivalent to that of the nucleon-nucleon system, which involves basically four poles with different sets of quantum numbers. The four poles we denote as usual by P, ρ, ω, and R; their amplitudes are given in the form (2.6) by

$$F_P(s,q^2)=-P(s/is_0)^{\alpha_P(q^2)}\,, \qquad (3.2a)$$

$$F_\rho(s,q^2)=i\rho(s/is_0)^{\alpha_\rho(q^2)}\,, \qquad (3.2b)$$

$$F_\omega(s,q^2)=-i\omega(s/is_0)^{\alpha_\omega(q^2)}\,, \qquad (3.2c)$$

$$F_R(s,q^2)=R(s/is_0)^{\alpha_R(q^2)}\,. \qquad (3.2d)$$

The trajectories $\alpha_P(q^2)$, $\alpha_\rho(q^2)$, and $\alpha_R(q^2)$ are assumed to be linear in q^2,

$$\alpha_i(q^2)=\alpha_i+\beta_i q^2\,, \qquad (3.3)$$

for $i=P$, ρ, ω, R, and the residue constants P, ρ, ω, and R are real. In terms of these Regge amplitudes we have

$$f_{\mathcal{P}p}=F_P-F_\omega+F_\rho-F_R\,, \qquad (3.4a)$$

$$f_{\mathfrak{N}p}=F_P-F_\omega-F_\rho+F_R\,, \qquad (3.4b)$$

$$f_{\bar{\mathcal{P}}p}=F_P+F_\omega-F_\rho-F_R\,, \qquad (3.4c)$$

$$F_{\bar{\mathfrak{N}}p}=F_P+F_\omega+F_\rho+F_R\,. \qquad (3.4d)$$

For the scattering of the isosinglet strange quark the ρ and R poles, having $I=1$, cannot contribute; otherwise the situation is similar. To introduce independent amplitudes for the contributions of P and ω to $f_{\lambda p}$ and $f_{\bar{\lambda}p}$, however, would lead to an undetermined system of equations. The reason is that the six amplitudes we are considering have been shown to be related, in the presence of multiple scattering, by an equation similar to the weak Johnson-Treiman relation.[4] Consequently at any fixed energy only *five* independent quantities, i.e., five residue constants, can be determined from the six total cross sections. We have already introduced four residue constants in the amplitudes (3.2); to include both P and ω in $f_{\lambda p}$ and $f_{\bar{\lambda}p}$ would lead to a total of six independent parameters at each energy, and therefore no unique solution for the residues would be possible. A consistent assumption which has been invoked several times in earlier work on the quark model[10] is that the Pomeranchuk limit has been reached for the strange-quark interactions. It should be noted that the effects of this assumption on our model can be observed only as small changes in the energy variations of the double-scattering terms, and therefore are inconsequential. We therefore neglect the contribution of ω to $f_{\lambda p}$ and $f_{\bar{\lambda}p}$, so that the scattering of quark and antiquark is identical:

$$f_{\lambda p}(s,q^2)=f_{\bar{\lambda}p}(s,q^2)=F_P{}'(s,q^2)$$
$$\equiv -C_P(s/is_0)^{\alpha_P(q^2)}\,. \qquad (3.4e)$$

In terms of these Regge poles the contributions of single scattering to the six meson-nucleon amplitudes are given by

$$F_{\pi^+p}{}^1=2F_P+2F_\rho\,, \qquad (3.5a)$$

$$F_{\pi^-p}{}^1=2F_P-2F_\rho\,, \qquad (3.5b)$$

$$F_{K^+p}{}^1=F_P-F_\omega+F_\rho-F_R+F_P{}'\,, \qquad (3.5c)$$

$$F_{K^+n}{}^1=F_P-F_\omega-F_\rho+F_R+F_P{}'\,, \qquad (3.5d)$$

$$F_{K^-p}{}^1=F_P+F_\omega-F_\rho-F_R+F_P{}'\,, \qquad (3.5e)$$

$$F_{K^-n}{}^1=F_P+F_\omega+F_\rho+F_R+F_P{}'\,. \qquad (3.5f)$$

The double-scattering integrals can be performed as in Sec. II. We have noted there that the resulting terms depend upon both the slopes of the Regge trajectories and the normalization energy s_0, but only in that the slope of the diffraction peak must be correctly given. Since we shall consider only total cross sections here, it is probably sufficient to take all four slopes the same, $\beta_P=\beta_\rho=\beta_\omega=\beta_R=\beta$. Then the double-scattering amplitudes assume a much simpler form, especially at $q^2=0$; if we write

$$\xi=[4k\beta\ln(s/is_0)]^{-1}\,, \qquad (3.6)$$

then

$$F_{\pi^+p}{}^2(s,0)=\xi f_{\mathcal{P}p}(s,0)f_{\bar{\mathfrak{N}}p}(s,0)\,, \qquad (3.7a)$$

$$F_{\pi^-p}{}^2(s,0)=\xi f_{\mathfrak{N}p}(s,0)f_{\bar{\mathcal{P}}p}(s,0)\,, \qquad (3.7b)$$

$$F_{K^+p}{}^2(s,0)=\xi f_{\mathcal{P}p}(s,0)f_{\bar{\lambda}p}(s,0)\,, \qquad (3.7c)$$

$$F_{K^+n}{}^2(s,0)=\xi f_{\mathfrak{N}p}(s,0)f_{\bar{\lambda}p}(s,0)\,, \qquad (3.7d)$$

[10] For example, H. J. Lipkin, Phys. Rev. Letters **16**, 1015 (1966).

TABLE I. Summary of the results obtained in five different fits to the total cross-section data of Galbraith *et al.* (Ref. 11), using various forms of the Reggeized quark model.

Fit	A	B	C	D	E
Interaction Reggeized	Quark-nucleon			Quark-quark	
χ^2	65.5	59.6	190	90.1	65.2
$\sigma_\infty(\pi N)$ (mb)	19.6 ± 0.38	0	27.2 ±0.8	24.4 ±0.1	0
$\sigma_\infty(KN)$ (mb)	19.3 ± 0.38	0	20.1 ±1.3	18.7 ±0.1	0
α_P (GeV^{-2})	1	0.987± 0.006	1	1	0.935±0.001
$\alpha_{P'}$ (GeV^{-2})	...	...	...	0.507±0.009	...
α_ρ (GeV^{-2})	0.972± 0.001	0.969± 0.002	0.644±0.061	0.700±0.019	0.638± 0.027
α_ω (GeV^{-2})	0.336± 0.011	0.287± 0.027	0.532±0.035	0.483±0.009	0.129± 0.010
α_K (GeV^{-2})	0.225± 0.012	0.276± 0.029	0.529±0.138	0.423±0.008	0.803± 0.043
P (mb MeV)	0.831± 0.016	0.990± 0.081	0.385±0.001	0.346±0.001	0.704± 0.007
C_P (mb MeV)	0.811± 0.016	0.869± 0.043	0.182±0.002	0.183±0.001	0.331± 0.004
ρ (mb MeV)	−2.45 ± 0.06	−2.31 ± 0.14	−2.37 ±0.95	−1.80 ±0.20	−2.69 ± 0.47
ω (mb MeV)	−99.8 ± 9.2	−146 ±32	−0.753±0.194	−10.5 ±0.3	−199 ±16
R (mb MeV)	129 ±18	61.6 ±21.0	3.90 ±5.62	10.4 ±0.9	0.244± 0.101
P' (mb MeV)	...	...	...	5.22 ±0.48	...

$$F_{K^-p}{}^2(s,0)=\xi f_{\lambda p}(s,0) f_{\bar{\mathcal{P}}p}(s,0)\,, \tag{3.7e}$$

$$F_{K^-n}{}^2(s,0)=\xi f_{\lambda p}(s,0) f_{\bar{\mathcal{N}}p}(s,0)\,, \tag{3.7f}$$

where the quark-nucleon amplitudes are given by Eqs. (3.4).

If we wish to consider the Reggeization of the quark-quark amplitudes instead, the procedure will be similar; for parallel trajectories the results (3.7) will still hold, but the quark-nucleon amplitudes will contain terms corresponding to single, double, and triple scattering in the quark-quark system. Working out the resulting forms is tedious but straightforward, and we shall omit the equations obtained for the sake of brevity. In form they are very similar to the above, except that there are now terms corresponding to six orders of scattering instead of two.

Using these basic parametrizations and the MINROS function minimization program, we have obtained least-χ^2 fits to the total cross-section data of Galbraith *et al.*[11] Our choice of β and s_0 in all of these fits is $\beta=0.5$ GeV^{-2}, $s_0=0.002$ GeV^2, corresponding to a diffraction peak slope of about 9 GeV^{-2}. The results of the five different models are summarized in Table I.

For fits A and B, the quark-nucleon interaction was Reggeized using four poles, as described above. The Pomeranchuk trajectory was fixed at $\alpha_P(0)=1$ in fit A,

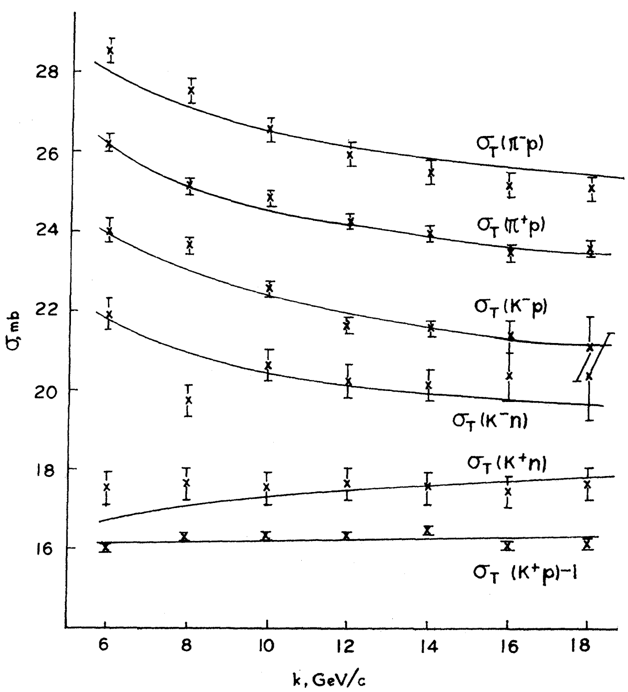

FIG. 3. Comparison with the experimental data of the fit obtained using the P, ω, ρ, and R poles to parametrize the quark-nucleon amplitude. The K^+p data have been displaced by 1 mb for clarity.

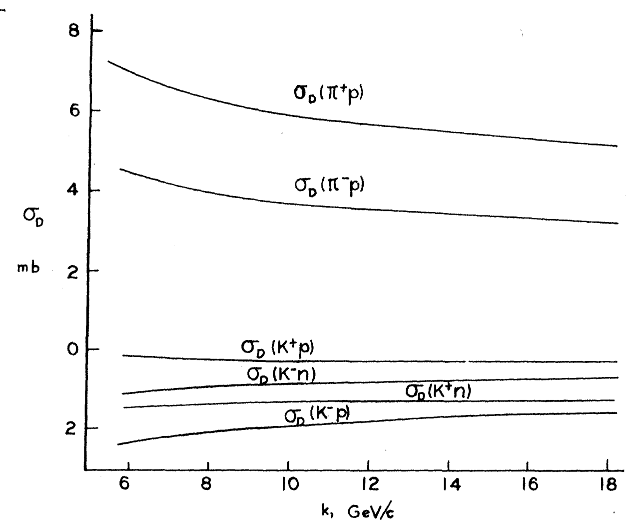

FIG. 4. Double-scattering contributions to the meson-nucleon total cross sections in the fit shown in Fig. 3.

[11] W. Galbraith *et al.*, Phys. Rev. **138**, B913 (1965).

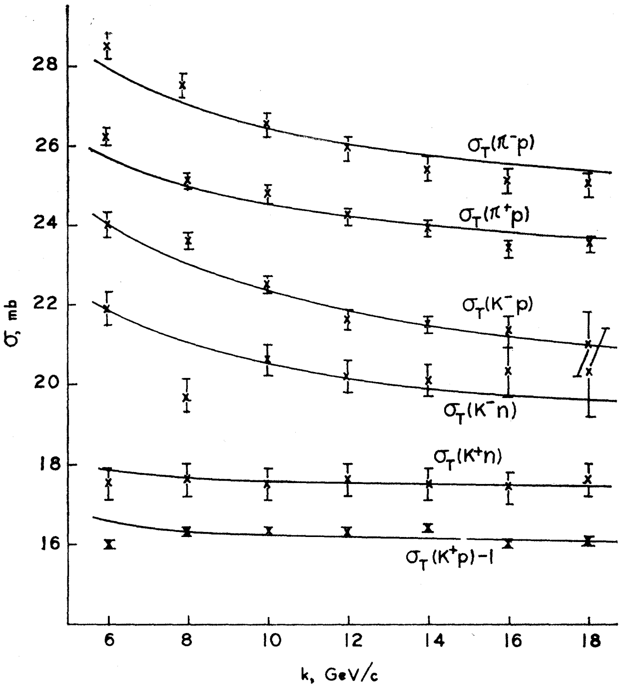

FIG. 5. Comparison with the experimental data of the fit obtained using P, P', ω, ρ, and R poles to parametrize the quark-quark amplitude. The K^+p data have been displaced by 1 mb for clarity.

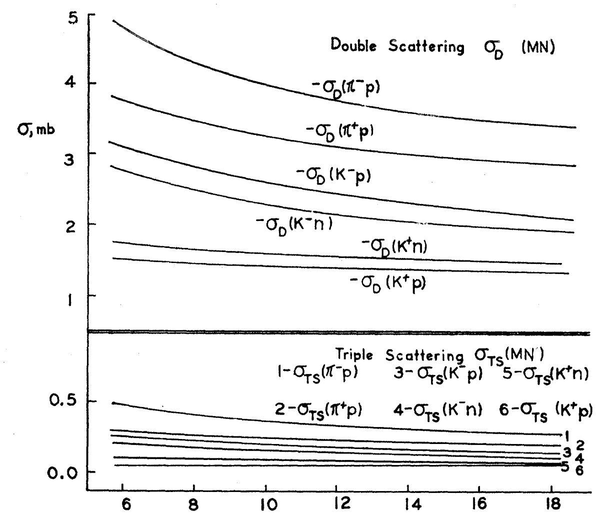

FIG. 6. Double- and triple-scattering contributions to the meson-nucleon total cross sections in the fit shown in Fig. 5.

yielding constant asymptotic total cross sections. The inclusion of a second vacuum trajectory P' in the usual way did not improve this fit noticeably. An alternative model in which the Pomeranchuk trajectory is a free parameter[12] produced a slight improvement, shown in fit B. Since the best fit chooses $\alpha_P<1$, the total cross sections vanish asymptotically in this model.

In both of these fits, the best value of the ρ trajectory intercept is surprisingly high. The reason for this result is that the double-scattering term corresponding to double ρ exchange has the form

$$\operatorname{Im}\{i[i\rho(s/is_0)^{\alpha_\rho}]^2\}=-\rho^2(s/s_0)^{2\alpha_\rho}\cos\pi\alpha_\rho\,,$$

which is *positive* for $\alpha_\rho>\frac{1}{2}$ and quite important for α_ρ near 1. In that case it acts effectively as a "second vacuum trajectory" contribution, thus explaining why addition of a P' was not helpful.

In order to show the general details of these fits we present in Fig. 3 the results obtained with fit A. The contributions of double scattering to the total cross sections are shown in Fig. 4.

In the other three fits, the Regge exchange was assumed to be in the quark-quark amplitude. A four-pole model with $\alpha_P(0)$ fixed at 1 to yield constant asymptotic total cross sections leads to fit C. This model is not capable of reproducing the decreasing behavior of the pion-nucleon data, and the resulting poor fit is manifested in the high χ^2 value obtained. The addition of a second vacuum trajectory P' results in fit D, which is substantially better than C; an even greater improvement is obtained in fit E, where the P intercept was allowed to vary. As in B, it chooses $\alpha_P<1$ for the best fit, leading thereby to vanishing asymptotes.

The general shape of the results obtained in these three fits can be seen in Fig. 5, where we show fit D. The contributions of double and triple scattering are shown in Fig. 6. The double-scattering terms are all subtractive and amount to about 15% of the single scattering; the triple scattering contributes a very small positive cross section. The three higher orders of scattering yield terms which are entirely negligible.

In all of these fits the phenomenological nature of the model should be kept in mind. Our intention has been to show that the quark model with multiple scattering is capable of describing meson-nucleon total cross sections, and to obtain an estimate of the magnitude of the multiple-scattering effects. The approximations that were necessary were not severe. Taking all of the Regge trajectories to be parallel, for example, may be inconsistent with the diffraction peak data; but the magnitudes of the multiple scattering terms should surely depend much more on the average behavior of these slopes than on their interplay.

Our principal conclusion in this section, therefore, is that a sizable part of presently observed total cross sections can be attributed to multiple-scattering effects in the quark model. This result is obtained by Reggeization of either the quark-nucleon or the quark-meson amplitude, and the same conclusion is reached regardless of whether the total cross section is asymptotically constant or vanishing.

If it is constant, however, our model predicts that, at higher energies, an increase toward an asymptotic

[12] N. Cabibbo, J. J. J. Kokkedee, L. Horwitz, and Y. Neeman, Nuovo Cimento 45A, 275 (1966).

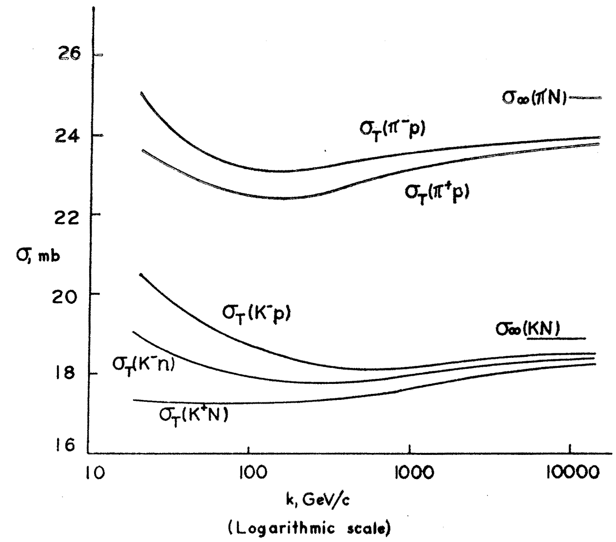

FIG. 7. Extension of the fit shown in Fig. 5 to superhigh energies.

value larger than those presently measured should become apparent as the double-scattering terms diminish in importance. This effect is present in all three fits (A, C, and D) for which $\alpha_P(0)=1$. The rate at which this asymptote is approached is determined by the shrinkage of the diffraction peak, i.e., by the constant s_0. The present nonshrinkage of the peaks leads to a very small s_0, and thereby to a slow disappearance of multiple-scattering effects. To illustrate this point we show in Fig. 7 the extension of fit D up to $k=10\,000$ GeV/c. More rapid shrinkage of the diffraction peak would, of course, lead to the appearance of increasing total cross sections at lower energies.

We expect that this property is a general one, and it should not depend crucially upon the numerical details of these fits. Similar conclusions have been reached by Frautschi and Margolis[13] on the basis of the Chou-Yang model and Regge cuts; their model corresponds to infinitely composite hadrons, however, and they estimate asymptotic cross sections much larger than those we find. If an increase in total cross sections should be observed experimentally, it would thus provide evidence for the composite nature of elementary particles, but not the degree of compositeness.

IV. INCLUSION OF SPIN EFFECTS

It is well known by now that although Regge theory is generally successful in describing a great many two-body processes, it is unable to explain simply the nonvanishing polarization observed in the reaction $\pi^- p \to \pi^0 n$. The nature of this charge-exchange process is such that a single set of quantum numbers, corresponding to the ρ pole, can be exchanged, but the polarization due to the exchange of a single Regge pole vanishes. In the usual Regge framework there have been attempts to explain the observed polarization of about 15% as resulting from the interference of the ρ with direct-channel resonances,[14] with a second pole ρ' having the same quantum numbers as the ρ,[15] or with a Regge-cut term.[16]

We have noted in the preceding sections that if the single-scattering amplitude is written in Regge form, then the multiple-scattering terms possess the s dependence characteristic of a Regge cut. Our model therefore provides a natural mechanism for the explanation of the charge-exchange polarization. It is clear, of course, that we cannot meaningfully analyze this effect without considering simultaneously the entire body of pion-nucleon elastic and charge-exchange phenomena in a formalism complete with all spin and isospin complications. To that task we shall devote Sec. V.

Before becoming deeply involved with the detailed calculations of the multiple-scattering effects which result when spin and isospin are included, however, we wish to show in a simpler model the essential features which arise. We therefore return to the basic deuteron model considered in Sec. II. The simplest technique for the inclusion of spin in this model is to keep the composite particle and its components spinless and assign spin $\frac{1}{2}$ to the incident particle. The Glauber model is valid even in the presence of spin provided any possible ordering ambiguities are resolved, which is accomplished by taking the anticommutator. The expected generalization of (2.1) is then

$$F_D(\mathbf{q})=F_N(\mathbf{q})[S_D(\tfrac{1}{2}\mathbf{q})+S_D(-\tfrac{1}{2}\mathbf{q})] +\frac{i}{4\pi k}\int d^2q'S(\mathbf{q}')\{F_N(\tfrac{1}{2}\mathbf{q}-\mathbf{q}'),F_N(\tfrac{1}{2}\mathbf{q}+\mathbf{q}')\}_+. \quad (4.1)$$

Both $F_D(\mathbf{q})$ and $F_N(\mathbf{q})$ are now taken to be matrices in the spin space of the system. A convenient form giving $F_N(\mathbf{q})$ in terms of scalar amplitudes is

$$F_N(\mathbf{q})=f(\mathbf{q})+2g(\mathbf{q})\mathbf{k}\times\mathbf{q}\cdot\boldsymbol{\sigma}, \quad (4.2)$$

where $\boldsymbol{\sigma}$ is the spin operator for the incident particle. In terms of this definition we find that

$$\begin{aligned}\{F_N(\tfrac{1}{2}\mathbf{q}+\mathbf{q}'),F_N(\tfrac{1}{2}\mathbf{q}-\mathbf{q}')\}_+ &=2[f(\tfrac{1}{2}\mathbf{q}+\mathbf{q}')f(\tfrac{1}{2}\mathbf{q}-\mathbf{q}') \\ &+g(\tfrac{1}{2}\mathbf{q}+\mathbf{q}')g(\tfrac{1}{2}\mathbf{q}-\mathbf{q}')k^2(\tfrac{1}{4}q^2-q'^2)] \\ &+\mathbf{k}\times\mathbf{q}\cdot\boldsymbol{\sigma}[f(\tfrac{1}{2}\mathbf{q}+\mathbf{q}')g(\tfrac{1}{2}\mathbf{q}-\mathbf{q}')+f(\tfrac{1}{2}\mathbf{q}-\mathbf{q}')g(\tfrac{1}{2}\mathbf{q}+\mathbf{q}')] \\ &+2\mathbf{k}\times\mathbf{q}'\cdot\boldsymbol{\sigma}[f(\tfrac{1}{2}\mathbf{q}+\mathbf{q}')g(\tfrac{1}{2}\mathbf{q}-\mathbf{q}') \\ &\qquad -f(\tfrac{1}{2}\mathbf{q}-\mathbf{q}')g(\tfrac{1}{2}\mathbf{q}+\mathbf{q}')]. \quad (4.3)\end{aligned}$$

We evaluate (4.1) now under the assumptions made in Sec. II, namely, that the amplitudes are exponentials in q^2 and that the form factor $S(\mathbf{q})$ is approximately

[13] S. C. Frautschi and B. Margolis, Nuovo Cimento **56A**, 1155 (1968).

[14] R. K. Logan and L. Sertorio, Phys. Rev. Letters **17**, 835 (1966).

[15] H. Hogaasen and A. Frisk, Phys. Letters **22**, 91 (1966); H. Hogaasen and W. Fischer, *ibid*, 22, 516 (1966).

[16] C. B. Chiu and J. Finkelstein, Nuovo Cimento **48A**, 820 (1967).

unity throughout the region in which there are important contributions to the amplitudes. Specifically, we write

$$f(\mathbf{q})=f_N e^{-\gamma q^2}, \tag{4.4a}$$

$$g(\mathbf{q})=g_N e^{-\gamma q^2}. \tag{4.4b}$$

We have taken the same slope γ for both spin-flip and spin-nonflip terms for the sake of simplicity; the results are not crucially dependent upon this equality. The integrations in the double-scattering term can be performed, yielding

$$F_D(\mathbf{q})=2f_N e^{-\gamma q^2}+\frac{i}{2k\gamma}\left[f_N^2+g_N^2k^2\left(\tfrac{1}{4}\mathbf{q}-\frac{1}{2\gamma}\right)\right]e^{-\frac{1}{2}\gamma q^2}$$
$$+\mathbf{k}\times\mathbf{q}\cdot\boldsymbol{\sigma}\left(2g_N e^{-\gamma q^2}+\frac{i}{2k\gamma}f_N g_N e^{-\frac{1}{2}\gamma q^2}\right). \tag{4.5}$$

We relate this result to the Regge model used in Sec. III in the obvious way. As before, we write for the nonflip amplitude

$$f_N e^{-\gamma q^2}=C(s/is_0)^{1-\beta q^2}, \tag{4.6}$$

i.e.,

$$f_N=-C(s/is_0), \tag{4.7a}$$

$$\gamma=\beta\ln(s/is_0). \tag{4.7b}$$

For the spin-flip amplitude we define a simplified version of the usual Regge formula

$$G_f(s,t)=\alpha R_f(t)\frac{1+\tau e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}s^{\alpha(t)-1} \tag{4.8}$$

by writing

$$kg_N e^{-\gamma q^2}=iD(s/is_0)^{1-\beta q^2}. \tag{4.9}$$

Then in the high-energy limit, using the approximation $s\approx 2Mk$ and $\ln(s/is_0)\approx\ln(s/s_0)$, we obtain for the Reggeized amplitude corresponding to $F_D(\mathbf{q})$

$$F_R(s,q^2)=-2C\left(\frac{s}{is_0}\right)^{1-\beta q^2}$$
$$+\frac{Ms_0}{\beta\ln(s/s_0)}\left[C^2-D^2\left(\tfrac{1}{4}\mathbf{q}^2-\frac{1}{2\beta\ln(s/s_0)}\right)\right]\left(\frac{s}{is_0}\right)^{1-\frac{1}{2}\beta q^2}$$
$$+\mathbf{k}\times\mathbf{q}\cdot\boldsymbol{\sigma}\left(\frac{2M}{s_0}\right)\left[2D\left(\frac{s}{is_0}\right)^{-\beta q^2}-\frac{Ms_0CD}{\beta\ln(s/s_0)}\left(\frac{s}{is_0}\right)^{-\frac{1}{2}\beta q^2}\right]. \tag{4.10}$$

It should be noted here that the spin-flip term in the single-scattering amplitude can contribute via double scattering to the forward nonflip amplitude, and therefore to the total cross section. The simple physical meaning of this fact is that two consecutive spin-flip processes with opposite momentum transfers will produce forward nonflip scattering. Should this effect be large, it would preclude the possibility of neglecting spin in considering forward scattering, as we did in Secs. II and III. We shall find, reassuringly, that the contributions to the total cross sections of such terms are very small.

The polarization resulting from (4.10) can be calculated using scalar amplitudes, which are conveniently defined by writing

$$F_R(s,q^2)=F(s,q^2)+\mathbf{k}\times\mathbf{q}\cdot\boldsymbol{\sigma}G(s,q^2). \tag{4.11}$$

The polarization parameter P is then determined by

$$P\frac{d\sigma}{d\Omega}=2\,\mathrm{Re}[F^*(s,q^2)G(s,q^2)]\lambda(s,q^2), \tag{4.12}$$

with the kinematical factor $\lambda(s,q^2)$ given by

$$\lambda(s,q^2)=kk'\sin\theta$$
$$=\{[(k+k')^2-q^2][q^2-(k-k')^2]\}^{1/2}, \tag{4.13}$$

k' and θ being the final momentum of the incident particle and the scattering angle. We note that the asymptotic behavior of $\lambda(s,q^2)$ for nonzero q^2 is given by

$$\lambda(s,q^2)\to 2kq \quad \text{as} \quad s\to\infty \tag{4.14}$$

and that

$$\lambda(s,0)=0. \tag{4.15}$$

From (4.10) we have

$$F(s,q^2)=-2C\left(\frac{s}{is_0}\right)^{1-\beta q^2}+\frac{Ms_0}{\beta\ln(s/s_0)}$$
$$\times\left[C^2-D^2\left(\tfrac{1}{4}\mathbf{q}^2-\frac{1}{2\beta\ln(s/s_0)}\right)\right]\left(\frac{s}{is_0}\right)^{1-\frac{1}{2}\beta q^2}, \tag{4.16}$$

$$G(s,q^2)=\left(\frac{2M}{s_0}\right)\left[2D\left(\frac{s}{is_0}\right)^{-\beta q^2}-\frac{Ms_0CD}{\beta\ln(s/s_0)}\left(\frac{s}{is_0}\right)^{-\frac{1}{2}\beta q^2}\right].$$

The single-scattering terms in $F(s,q^2)$ and $G(s,q^2)$ are 90° out of phase, so that they produce, as expected, no polarization. The same is true of the two double-scattering terms. Contributions to the polarization thus come only from interference between the single-scattering amplitude in $F(s,q^2)$ and the double-scattering in $G(s,q^2)$, and vice versa. Equation (4.12) then yields

$$P\frac{d\sigma}{d\Omega}\approx 8q\frac{DMs_0}{\beta\ln(s/s_0)}\left[2C^2-D^2\left(\tfrac{1}{4}q^2-\frac{1}{2\beta\ln(s/s_0)}\right)\right]$$
$$\times\left(\frac{s}{s_0}\right)^{2-\frac{3}{2}\beta q^2}\sin\tfrac{1}{4}\pi\beta q^2. \tag{4.17}$$

The differential cross section $d\sigma/d\Omega$ is given in this case by

$$d\sigma/d\Omega=|F(s,q^2)|^2+|\lambda(s,q^2)G(s,q^2)|^2 \tag{4.18}$$

and at high energy the dominant contribution for $q^2>0$ comes, as before, from the multiple-scattering terms. Asymptotically, then, we have

$$\frac{d\sigma}{d\Omega}\approx\left(\frac{Ms_0}{\beta\ln(s/s_0)}\right)^2\left\{\left[C^2-D^2\left(\tfrac{1}{4}q^2-\frac{1}{2\beta\ln(s/s_0)}\right)\right]^2+4q^2C^2D^2\right\}\left(\frac{s}{s_0}\right)^{2-\beta q^2}. \quad (4.19)$$

It follows from (4.17) and (4.19) that for $q^2>0$ the asymptotic behavior with s of the polarization parameter is given by

$$P\propto(s/s_0)^{-\frac{1}{2}\beta q^2}\ln(s/s_0). \quad (4.20)$$

The polarization thus goes to zero, but only extremely slowly. As an example, we take typical values of the parameters to be $\frac{1}{2}\beta q^2=0.2$, $s_0=0.002$; when s increases from 2 to 2000, the polarization decreases only by a factor $\frac{1}{2}$.

We see, therefore, that a nonvanishing polarization will result from multiple-scattering effects even if the single-scattering process involved permits the exchange of only one Regge pole, and that this polarization will decrease asymptotically toward zero at high energy so slowly as to appear almost constant. These observations are fully consistent with the experimental facts regarding the reaction $\pi^-p\to\pi^0n$. We, therefore, turn now to a detailed calculation of pion-nucleon interactions in terms of the Reggeized quark model with multiple scattering.

V. APPLICATION TO PION-NUCLEON SCATTERING

The general pion-nucleon scattering amplitude with full spin and isospin complexity included can be written conveniently in the form

$$F_{\pi N}(s,q^2)=F_{00}(s,q^2)+2F_{01}(s,q^2)\mathbf{T}_\pi\cdot\mathbf{T}_N+\mathbf{k}\times\mathbf{q}\cdot\boldsymbol{\sigma}_N[F_{10}(s,q^2)+2F_{11}(s,q^2)\mathbf{T}_\pi\cdot\mathbf{T}_N], \quad (5.1)$$

where $\mathbf{T}_\pi$, $\mathbf{T}_N$, and $\boldsymbol{\sigma}_N$ denote, respectively, the isospin operators of the pion and of the nucleon and the spin operator of the nucleon. In the scalar functions $F_{ij}(s,q^2)$ it is clear that i and j correspond to the spin and isospin exchanged. The actual amplitudes for the pertinent physical scattering processes are obtained in the obvious way, by taking the (matrix) from (5.1) between the appropriate spin-isospin states; for spin exchange i,

$$F_i^\pm(s,q^2)=F_{i0}(s,q^2)\pm F_{i1}(s,q^2) \quad (5.2a)$$

describes elastic $\pi^\pm P$ interactions, while

$$F_1^x(s,q^2)=\sqrt{2}F_{i1}(s,q^2) \quad (5.2b)$$

describes the charge-exchange reaction.

We shall review briefly here the connection between these amplitudes and the quantities measured experimentally. The total cross section for $(\pi^\pm p)$ scattering is given through the optical theorem,

$$\sigma^\pm(s)=(4\pi/k)\,\mathrm{Im}F_0^\pm(s,0). \quad (5.3)$$

The phases of the forward nonflip elastic amplitudes also are known and are usually specified by giving the ratio of the real to the imaginary part, which we shall denote by

$$\eta^\pm(s)=\frac{\mathrm{Re}F_0^\pm(s,0)}{\mathrm{Im}F_0^\pm(s,0)}. \quad (5.4)$$

At nonzero momentum transfer, the differential cross sections and the polarizations are measured for all three processes. The former are given in the laboratory frame by

$$\frac{d\sigma}{d\Omega_L}(s,q^2)=|F_0(s,q^2)|^2+|\lambda(s,q^2)F_1(s,q^2)|^2, \quad (5.5)$$

where $F_i(s,q^2)$ refers to any of the three amplitudes defined in (5.2) and $\lambda(s,q^2)$ is the kinematical factor defined in (4.13). This quantity is converted to an invariant distribution

$$\frac{d\sigma}{dt}(s,q^2)=C(s,q^2)\frac{d\sigma}{d\Omega_L}(s,q^2), \quad (5.6a)$$

with

$$C(s,q^2)=\frac{\pi}{kk'^3}\left\{[(k^2+\mu^2)(k'^2+\mu^2)]^{1/2}-\mu^2\left(1-\frac{q^2}{2M^2}\right)\right\}, \quad (5.6b)$$

in terms of the previously defined quantities, $\mu(M)$ being the pion (nucleon) mass. The polarization, finally, is calculated as in (4.12),

$$P(s,q^2)=2\,\mathrm{Re}[F_0^*(s,q^2)F_1(s,q^2)]\lambda(s,q^2)\bigg/\frac{d\sigma}{d\Omega_L}. \quad (5.7)$$

The first step in applying the multiple-scattering formalism in the pion-nucleon system is to decompose the nucleon into its quark structure, which we assume to be given by the $SU(6)$ spin-isospin wave functions

$$p_\pm=(18)^{-1/2}[\pm2(\mathcal{P}_\pm\mathcal{P}_\pm\mathcal{N}_\mp+\mathcal{P}_\pm\mathcal{N}_\mp\mathcal{P}_\pm+\mathcal{N}_\mp\mathcal{P}_\pm\mathcal{P}_\pm)\mp(\mathcal{P}_\pm\mathcal{P}_\mp\mathcal{N}_\pm+\mathcal{P}_\mp\mathcal{P}_\pm\mathcal{N}_\pm+\mathcal{P}_\pm\mathcal{N}_\pm\mathcal{P}_\mp+\mathcal{P}_\mp\mathcal{N}_\pm\mathcal{P}_\pm+\mathcal{N}_\pm\mathcal{P}_\pm\mathcal{P}_\mp+\mathcal{N}_\pm\mathcal{P}_\mp\mathcal{P}_\pm)], \quad (5.8a)$$

$$n_\pm=(18)^{-1/2}[\mp2(\mathcal{N}_\pm\mathcal{N}_\pm\mathcal{P}_\mp+\mathcal{N}_\pm\mathcal{P}_\mp\mathcal{N}_\pm+\mathcal{P}_\mp\mathcal{N}_\pm\mathcal{N}_\pm)\pm(\mathcal{N}_\pm\mathcal{N}_\mp\mathcal{P}_\pm+\mathcal{N}_\mp\mathcal{N}_\pm\mathcal{P}_\pm+\mathcal{N}_\pm\mathcal{P}_\pm\mathcal{N}_\mp+\mathcal{N}_\mp\mathcal{P}_\pm\mathcal{N}_\pm+\mathcal{P}_\pm\mathcal{N}_\pm\mathcal{N}_\mp+\mathcal{P}_\pm\mathcal{N}_\mp\mathcal{N}_\pm)]. \quad (5.8b)$$

In Eq. (5.8), p_+ (p_-) denotes a proton with spin up (down), and $n_\pm$, $\mathcal{P}_\pm$, and $\mathcal{N}_\pm$ analogously represent the spin states of the neutron and the nonstrange quarks. The total pion-nucleon scattering amplitude $F_{\pi N}(s,q^2)$

then consists of three terms,

$$F_{\pi N}(s,q^2)=\sum_{i=1}^{3} F_{\pi N}{}^i(s,q^2)\,, \tag{5.9}$$

corresponding to single, double, and triple scattering of the pion by the three quarks of the nucleon. If we denote the pion-quark scattering amplitude by $F_{\pi Q}(s,q)^2$, which is a matrix in the spin-isospin space of the pion-quark system, then in the strong-binding approximation the $F_{\pi N}{}^i(s,q^2)$ are given by

$$F_{\pi N}{}^1(s,q^2)=\sum_i F_{\pi Q_i}(s,q^2)\,, \tag{5.10a}$$

$$F_{\pi N}{}^2(s,q^2)=\frac{i}{4\pi k}\int d^2q' \sum_{i\neq j} F_{\pi Q_i}(s,(\tfrac{1}{2}\mathbf{q}-\mathbf{q}')^2) \times F_{\pi Q_j}(s,(\tfrac{1}{2}\mathbf{q}+\mathbf{q}')^2)\,, \tag{5.10b}$$

$$F_{\pi N}{}^3(s,q^2)=\frac{1}{6}\left(\frac{i}{2\pi k}\right)^2\int d^2q'\int d^2q'' \times \sum_{i\neq j\neq k} F_{\pi Q_i}(s,(\tfrac{1}{2}\mathbf{q}-\mathbf{q}')^2) \times F_{\pi Q_j}(s,(\tfrac{1}{2}\mathbf{q}-\mathbf{q}'')^2)F_{\pi Q_k}(s,(\mathbf{q}'+\mathbf{q}'')^2)\,. \tag{5.10c}$$

The summations over Q_i, etc., in (5.10) refer to the three quarks, and have been so taken that symmetry under the interchange of quark labels is guaranteed.

The pion-quark amplitude can be expressed by scalar functions precisely as was the pion-nucleon amplitude itself,

$$F_{\pi Q}(s,q^2)=f_{00}(s,q^2)+2f_{01}(s,q^2)\mathbf{T}_\pi\cdot\mathbf{T}_Q \quad \mathbf{k}\times\mathbf{q}\cdot\boldsymbol{\sigma}_Q[f_{10}(s,q^2)+2f_{11}(s,q^2)\mathbf{T}_\pi\cdot\mathbf{T}_Q]\,. \tag{5.11}$$

The contributions to the $F_{ij}(s,q^2)$ of single, double, and triple scattering, denoted hereafter by $F_{ij}{}^n(s,q^2)$, $n=1$, 2, 3, are then calculated in terms of the $f_{ij}(s,q^2)$ by inserting (5.11) into the expression (5.10). To evaluate the matrix elements of the spin and isospin operators, using the $SU(6)$ wave functions (5.8), is then a matter of a large amount of tedious but straightforward arithmetic. In fact, we shall neglect entirely the triple-scattering effects, which we expect to be small, and thus we summarize below the contributions of only those matrix elements necessary for considering single and double scattering. Using the (π^+p) states, we find that spin-nonflip terms arise only from the matrix elements

$$\langle\pi^+p_\pm|\sum_i 1_{Q_i}|\pi^+p_\pm\rangle=3\,, \tag{5.12a}$$

$$\langle\pi^+p_\pm|\sum_i \mathbf{T}_\pi\cdot\mathbf{T}_{Q_i}|\pi^+p\rangle=\tfrac{1}{2}\,, \tag{5.12b}$$

$$\langle\pi^+p_\pm|\sum_{i=j}[\mathbf{k}\times(\tfrac{1}{2}\mathbf{q}-\mathbf{q}')\cdot\boldsymbol{\sigma}_{Q_i}][\mathbf{k}\times(\tfrac{1}{2}\mathbf{q}+\mathbf{q}')\cdot\boldsymbol{\sigma}_{Q_j}]|\pi^+p_\pm\rangle =-\tfrac{1}{2}k^2(\tfrac{1}{4}\mathbf{q}^2-q'^2)\,, \tag{5.12c}$$

$$\langle\pi^+p_\pm|\sum_{i\neq j}(\mathbf{T}_\pi\cdot\mathbf{T}_{Q_i})(\mathbf{T}_\pi\cdot\mathbf{T}_{Q_j})|\pi^+p_\pm\rangle=-1\,, \tag{5.12d}$$

$$\langle\pi^+p_\pm|\sum_{i\neq j}[\mathbf{k}\times(\tfrac{1}{2}\mathbf{q}-\mathbf{q}')\cdot\boldsymbol{\sigma}_{Q_i}][\mathbf{k}\times(\tfrac{1}{2}\mathbf{q}+\mathbf{q}')\cdot\boldsymbol{\sigma}_{Q_j}] \times(\mathbf{T}_\pi\cdot\mathbf{T}_{Q_i}+\mathbf{T}_\pi\cdot\mathbf{T}_{Q_j})|\pi^+p_\pm\rangle=\tfrac{1}{6}k^2(\tfrac{1}{4}q^2-q'^2)\,, \tag{5.12e}$$

$$\langle\pi^+p_\pm|\sum_{i\neq j}[\mathbf{k}\times(\tfrac{1}{2}\mathbf{q}-\mathbf{q}')\cdot\boldsymbol{\sigma}_{Q_i}][\mathbf{k}\times(\tfrac{1}{2}\mathbf{q}+\mathbf{q}')\cdot\boldsymbol{\sigma}_{Q_j}] \times(\mathbf{T}_\pi\cdot\mathbf{T}_{Q_i})(\mathbf{T}_\pi\cdot\mathbf{T}_{Q_j})|\pi^+p_\pm\rangle=5k^2(\tfrac{1}{4}q^2-q'^2)/12\,, \tag{5.12f}$$

while spin-flip terms result from

$$\langle\pi^+p_\pm|\sum_i(\mathbf{k}\times\mathbf{q}\cdot\boldsymbol{\sigma}_{Q_i})|\pi^+p_\mp\rangle=-\tfrac{1}{2}kq\,, \tag{5.13a}$$

$$\langle\pi^+p_\pm|\sum_i(\mathbf{T}_\pi\cdot\mathbf{T}_{Q_i})(\mathbf{k}\times\mathbf{q}\cdot\boldsymbol{\sigma}_{Q_i})|\pi^+p_\mp\rangle =-(5/12)kq\,, \tag{5.13b}$$

$$\langle\pi^+p_\pm|\sum_{i\neq j}(\mathbf{T}_\pi\cdot\mathbf{T}_{Q_i})(\mathbf{k}\times\mathbf{q}\cdot\boldsymbol{\sigma}_{Q_j})|\pi^+p_\mp\rangle=\tfrac{1}{6}kq\,, \tag{5.13c}$$

$$\langle\pi^+p_\pm|\sum_{i\neq j}(\mathbf{T}_\pi\cdot\mathbf{T}_{Q_i})(\mathbf{T}_\pi\cdot\mathbf{T}_{Q_j})\mathbf{k}\times\mathbf{q}\cdot(\boldsymbol{\sigma}_{Q_i}+\boldsymbol{\sigma}_{Q_j})|\pi^+p_\pm\rangle =-\tfrac{1}{3}kq\,. \tag{5.13d}$$

[For simplicity in (5.13) we have defined $\mathbf{q}$ to be in the y direction. 1_{Q_i} is the unit operator in the Q_i space.]

It is easily verified that isosymmetry is maintained for the double-scattering terms. Consequently it suffices to calculate the (π^+p) amplitudes; those for (π^-p) are obtained by changing the sign of $\mathbf{T}_\pi$, and the amplitudes for the charge exchange are related to the elastic amplitudes by the familiar equation

$$\sqrt{2}M(\pi^-p\to\pi^0n)=M(\pi^+p\to\pi^+p)-M(\pi^-p\to\pi^-p)\,,$$

where M denotes any of the above matrix elements. This result is tantamount to f_{i0} (f_{i1}) being even (odd) under charge conjugation.

To carry out the integrations necessary in the double-scattering terms now requires a parametrization of the pion-quark amplitude. As we have pointed out in Sec. IV, the most basic premise would be the use of a simple representation for the quark-quark interactions; the pion-quark amplitude would then be obtained by applying the multiple-scattering formalism again, this time decomposing the pion into quark and antiquark. Effectively, however, this procedure leads only to an extremely complicated parametrization of the pion-quark interactions. The quark-quark scattering process has five helicity amplitudes, each of which involves at least four Regge poles. As a result, both the complexity of the algebra and the number of parameters are vastly larger than would result from simply Reggeizing the pion-quark amplitudes. In order that our parametrization be amenable to computerized fitting programs, we choose this less complicated technqiue.

For the pion-quark amplitude, analogously to the usual pion-nucleon Regge theory, only trajectories with positive G parity, namely, the vacuum and the

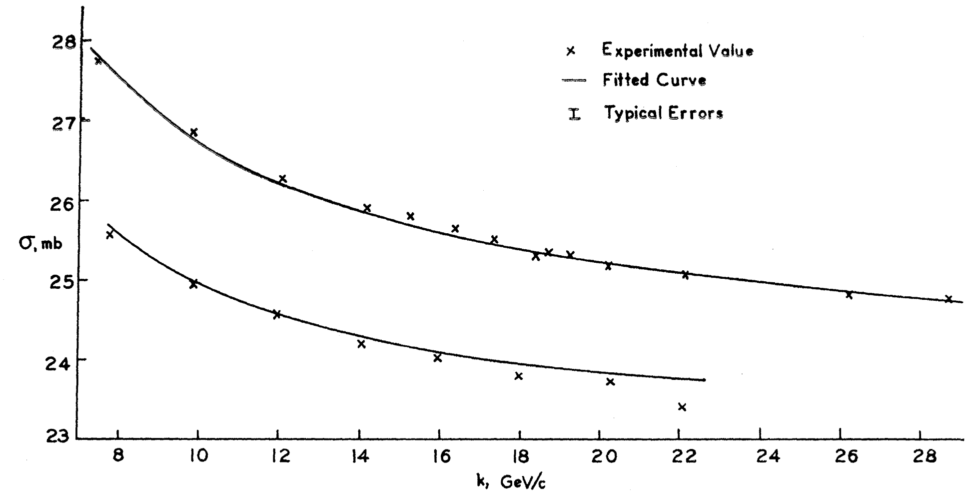

FIG. 8. Comparison with the experimental data of the fit obtained for the pion-nucleon total cross sections.

ρ trajectories, are allowed. We do not expect that the Pomeranchuk trajectory alone will be able to reproduce satisfactorily the decreasing total cross sections, so we shall invoke the traditional mechanism of a second vacuum trajectory P'. A substantial reduction of the computer time required can be achieved by taking the two vacuum trajectories to be parallel. We, therefore, parametrize the $I=0$ amplitudes as

$$f_{00}(s,q^2)=-R_0(s/is_0)^{1-\beta_P q^2}, \quad (5.14a)$$

$$f_{10}(s,q^2)=-R_1(s/is_0)^{-\beta_P q^2}, \quad (5.14b)$$

with residues R_0 and R_1 containing the energy dependence of both the P and P' poles, i.e.,

$$R_0=P_0+P_0'(s/is_0)^{\alpha_{P'}-1}, \quad (5.15a)$$

$$R_1=P_1+P_1'(s/is_0)^{\alpha_{P'}-1}. \quad (5.15b)$$

The $I=1$ amplitudes are those corresponding to the ρ pole only,

$$f_{01}(s,q^2)=2i\rho_0(s/is_0)^{\alpha_\rho-\beta_\rho q^2}, \quad (5.16a)$$

$$f_{11}(s,q^2)=2i\rho_1(s/is_0)^{\alpha_\rho-1-\beta_\rho q^2}. \quad (5.16b)$$

These equations define the real constants P_i, P_i', ρ_i, $\alpha_{P'}$, α_ρ, β_P, β_ρ, and s_0 on which our calculation will depend.

The double-scattering integrations can be carried out using the parametrizations above, and employing these results along with the matrix elements summarized in (5.12) and (5.13), we find eventually that the various $F_{ij}^n(s,q^2)$ are given by

$$F_{00}^1(s,q^2)=-3R_0(s/s_0)^{1-\beta_P q^2}, \quad (5.17a)$$

$$F_{01}^1(s,q^2)=i\rho_0(s/is_0)^{\alpha_\rho-\beta_\rho q^2}, \quad (5.17b)$$

$$F_{10}^1(s,q^2)=-R_1(s/is_0)^{-\beta_P q^2}, \quad (5.17c)$$

$$F_{11}^1(s,q^2)=(5/3)i\rho_1(s/is_0)^{\alpha_\rho-1-\beta_\rho q^2}, \quad (5.17d)$$

$$F_{00}^2(s,q^2)=\frac{i}{2k}\left[\frac{F_0^{PP}(s,q^2)}{2\beta_P\ln(s/is_0)}\left(\frac{s}{is_0}\right)^{-\frac{1}{2}\beta_P q^2}+\frac{F_0^{\rho\rho}(s,q^2)}{2\beta_\rho\ln(s/is_0)}\left(\frac{s}{is_0}\right)^{-\frac{1}{2}\beta_\rho q^2}\right], \quad (5.18a)$$

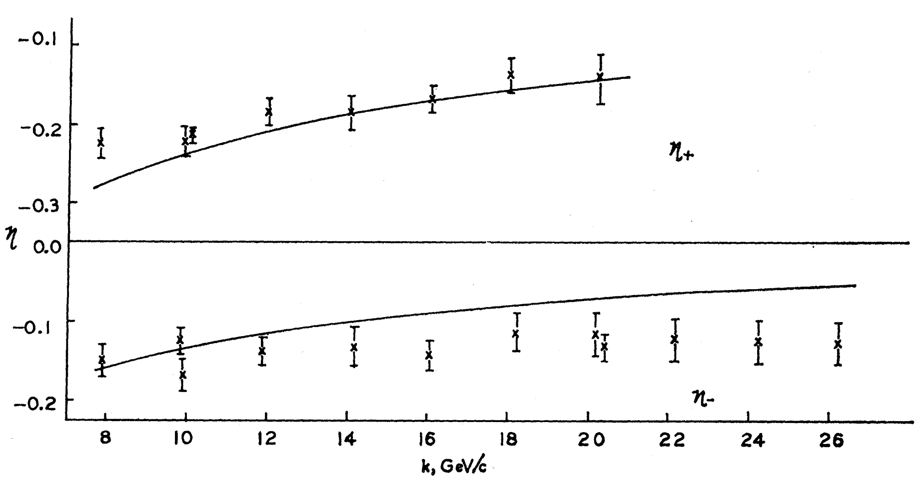

FIG. 9. Comparsion with the experimental data of the fit obtained for the ratio of real to imaginary part of the pion-nucleon amplitudes.

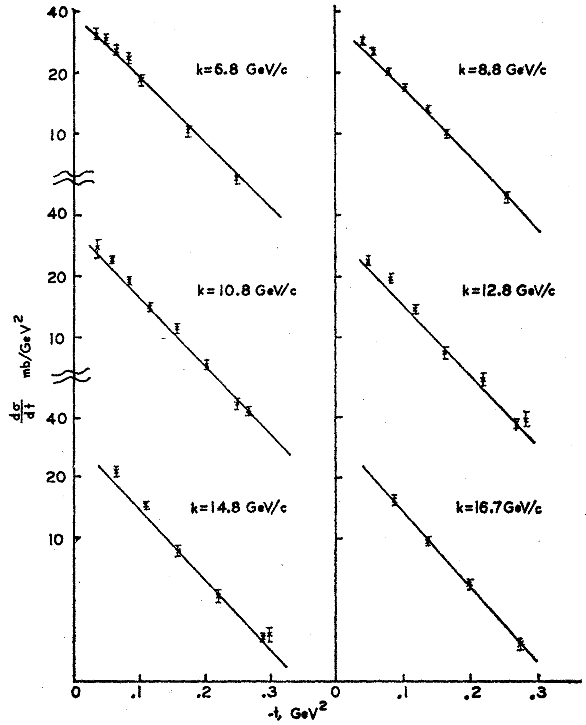

FIG. 10. Comparison with the experimental data of the fit obtained for the π^+p differential cross sections.

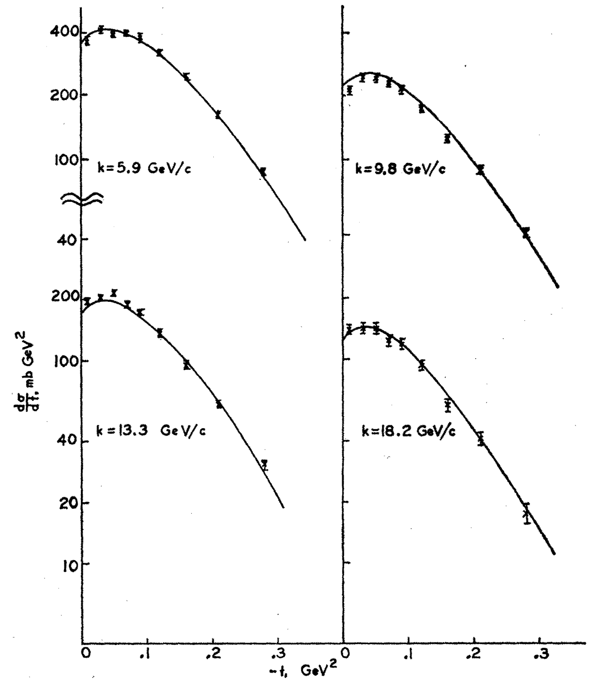

FIG. 12. Comparison with the experimental data of the fit obtained for πp charge-exchange differential cross sections.

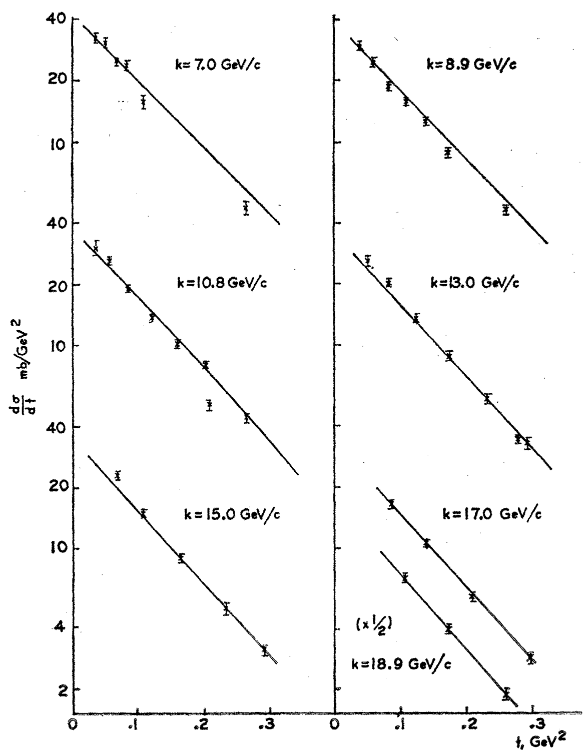

FIG. 11. Comparison with the experimental data of the fit obtained for the π^-p differential cross sections.

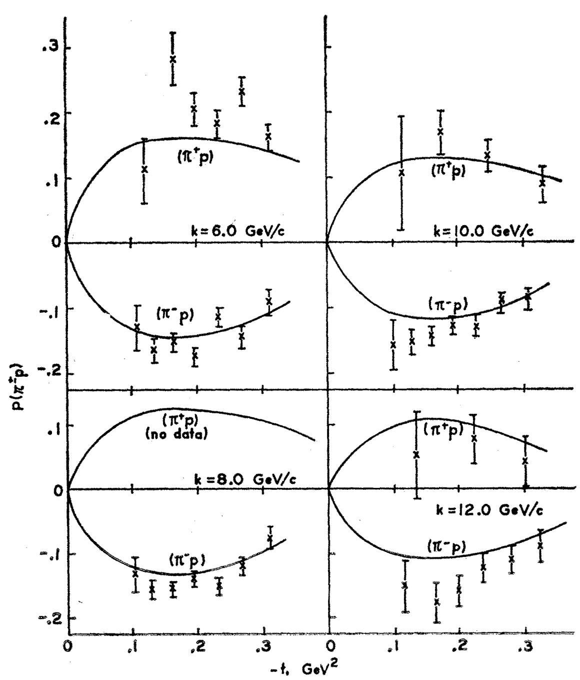

FIG. 13. Comparison with the experimental data of the fit obtained for the polarization in elastic πp scattering.

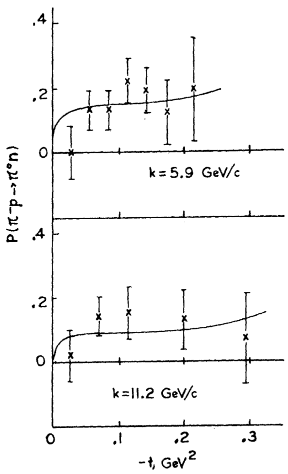

FIG. 14. Comparison with the experimental data of the fit obtained for the πp charge-exchange polarization.

$$F_0{}^{PP}(s,q^2)=3\left(R_0\frac{s}{is_0}\right)^2+\tfrac{1}{4}(kR_1)^2\left[\frac{1}{2\beta_P\ln(s/is_0)}-\tfrac{1}{4}q^2\right],$$

$$F_0{}^{\rho\rho}(s,q^2)=2\left[\rho_0\left(\frac{s}{is_0}\right)^{\alpha_\rho}\right]^2+\frac{2}{3}\left[k\rho_1\left(\frac{s}{is_0}\right)^{\alpha_\rho-1}\right]^2\times\left[\frac{1}{2\beta_\rho\ln(s/is_0)}-\tfrac{1}{4}q^2\right],$$

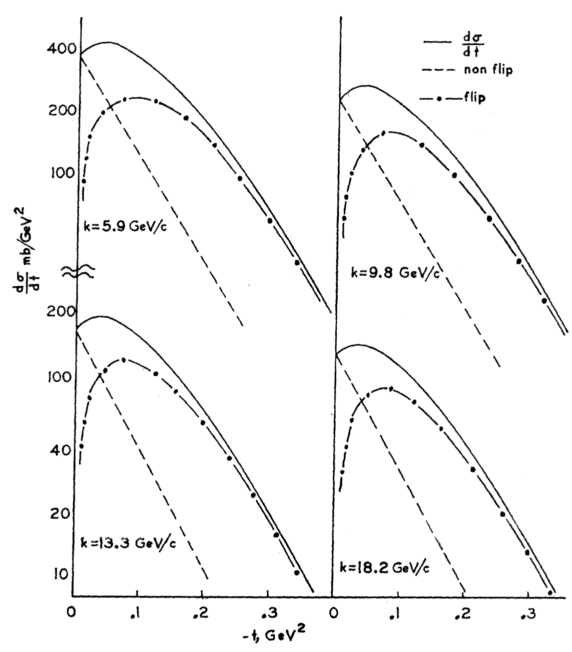

FIG. 15. Contributions of the spin-flip and nonflip terms to the charge-exchange differential cross sections.

$$F_{01}{}^2(s,q^2)=\frac{i}{2k}\frac{F_0{}^{P\rho}(s,q^2)}{(\beta_P+\beta_\rho)\ln(s/is_0)}\times\left(\frac{s}{is_0}\right)^{-\beta_P\beta_\rho q^2/(\beta_P+\beta_\rho)},\quad(5.18\text{b})$$

$$F_0{}^{P\rho}(s,q^2)=-2i\rho_0R_0\left(\frac{s}{is_0}\right)^{\alpha_\rho+1}+\tfrac{1}{6}i\rho_1R_1k^2\left(\frac{s}{is_0}\right)^{\alpha_\rho-1}\times\left[\frac{1}{(\beta_P+\beta_\rho)\ln(s/is_0)}-\tfrac{1}{4}q^2\right],$$

$$F_{10}{}^2(s,q^2)=\frac{i}{2k}\left[\frac{F_1{}^{PP}(s)}{2\beta_P\ln(s/is_0)}\left(\frac{s}{is_0}\right)^{-\frac{1}{2}\beta_Pq^2}+\frac{F_1{}^{\rho\rho}(s)}{2\beta_\rho\ln(s/is_0)}\left(\frac{s}{is_0}\right)^{-\frac{1}{2}\beta_\rho q^2}\right],\quad(5.18\text{c})$$

$$F_1{}^{PP}(s)=R_0R_1(s/is_0),$$

$$F_1{}^{\rho\rho}(s)=-\tfrac{2}{3}\rho_0\rho_1(s/is_0)^{2\alpha_\rho-1},$$

$$F_{11}{}^2(s,q^2)=\frac{i}{2k}\frac{F_1{}^{P\rho}(s)}{(\beta_P+\beta_\rho)^2\ln(s/is_0)}\times\left(\frac{s}{is_0}\right)^{-\beta_P\beta_\rho q^2/(\beta_P+\beta_\rho)},\quad(5.18\text{d})$$

$$F_1{}^{P\rho}(s)=-[(5/3)i\beta_P\rho_1R_0-\tfrac{1}{3}i\beta_\rho\rho_0R_1](s/is_0)^{\alpha_\rho}.$$

The amplitudes for the relevant physical processes are obtained by inserting these forms into (5.2). Our Reggeized quark model can then be tested by performing a least-χ^2 fit to the experimental data using the amplitudes calculated in (5.17) and (5.18). For this purpose we choose from the extensive literature a selection of 230 data points describing all the physical quantities listed above at various energies and momentum transfers. The total cross sections and phase measurements are taken from the high-precision data recently obtained by Foley *et al.*,[17] which cover a range in laboratory momentum from $\sim$7 GeV/c to $\sim$22 GeV/c for (π^+p), and to $\sim$28 GeV/c for (π^-p), with errors of the order of only 0.3% in σ and of 15% in η. The differential cross sections are taken from the measurements by Foley *et al.*[18] for the elastic scattering and from those by Stirling *et al.*[19] for the charge-exchange process; the polarization data are due to Borghini *et al.*[20] for elastic scattering and to Bonamy *et al.*[21] for charge exchange. The range of laboratory momentum in these measurements is from $\sim$6 to 18 GeV/c for

[17] K. J. Foley *et al.*, Phys. Rev. Letters **19**, 193 (1967); **19**, 330 (1967).
[18] K. J. Foley *et al.*, Phys. Rev. Letters **11**, 425 (1963).
[19] A. V. Stirling *et al.*, Phys. Rev. Letters **14**, 763 (1965).
[20] M. Borghini *et al.*, Phys. Letters **24B**, 77 (1967).
[21] P. Bonamy *et al.*, Phys. Letters **23**, 501 (1966).

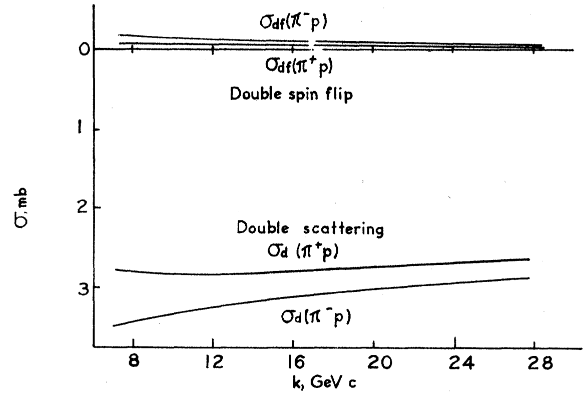

FIG. 16. Contributions to the total cross sections of double scattering and of double spin-flip terms.

$d\sigma/dt$, and to 12 GeV/c for the polarization. The range in momentum transfer has been arbitrarily limited; in order to reduce the importance of the strong-binding assumption, we used only points for which the invariant four-momentum transfer was such that $-t \leq 0.30$ GeV^2. All of the parameters defined in Eqs. (5.14)–(5.16) were taken to be free. The best fit obtained to the data produces a χ^2 value of 510.2 for 219 degrees of freedom, with the parameters given by

$$\alpha_{P'} = 0.1628 \pm 0.0002,$$
$$\alpha_\rho = 0.5242 \pm 0.0001,$$
$$\beta_P\ (=\beta_{P'}) = 0.4876 \pm 0.0005\ \text{GeV}^{-2},$$
$$\beta_\rho = 0.7839 \pm 0.0008\ \text{GeV}^{-2},$$
$$P_0 = 4.668 \pm 0.026\ \text{mb MeV},$$
$$P_1 = 0.07830 \pm 0.00092\ \text{mb}^{3/2},$$
$$P_0' = 0.6339 \pm 0.0013\ \text{mb GeV},$$
$$P_1' = 6.818 \pm 0.081\ \text{mb}^{3/2},$$
$$\rho_0 = 0.03009 \pm 0.00043\ \text{mb GeV},$$
$$\rho_1 = 3.937 \pm 0.042\ \text{mb}^{3/2},$$
$$s_0 = 0.01285 \pm 0.00007\ \text{GeV}^2.$$

Except for the rather low value of the P' intercept, the parameters of the Regge trajectories are in accord with the results of earlier Regge models. It is also interesting to note that the best-fit value of the normalization constant s_0 is roughly the pion mass squared.

The ratio of χ^2 to the number of degrees of freedom is 2.33, which is somewhat high to be considered a good fit. Qualitatively, however, the results are in reasonably good agreement with the experimental situation; differences of systematic errors between various experiments may have increased the value of χ^2. A detailed comparison of the model with the fitted data is given in Figs. 8–14.

The agreement with the data is quite satisfactory for the elastic and charge-exchange polarizations, and, in fact, for almost all of the other experimental quantities.

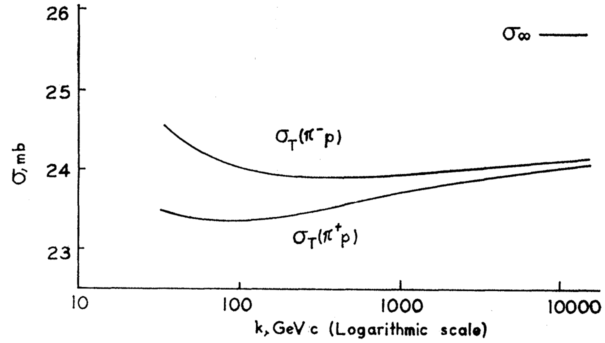

FIG. 17. Extension of the fit obtained for the total cross sections to the superhigh-energy region.

Only in the case of $\eta^-(s)$, where the measured values are generally larger than the model predicts, is there any consistent disparity between fit and data. The total and differential cross sections are very well fitted, including particularly the structure in the charge-exchange process at near-forward angles. To reproduce this dip by means of exponential amplitudes requires that spin-flip terms, negligible in the elastic reaction, must be quite important here (Fig. 15).

In general, the effects of double scattering are fairly significant in these results. Their contribution to the total cross sections are shown in Fig. 16, along with that resulting particularly from the double spin-flip terms.

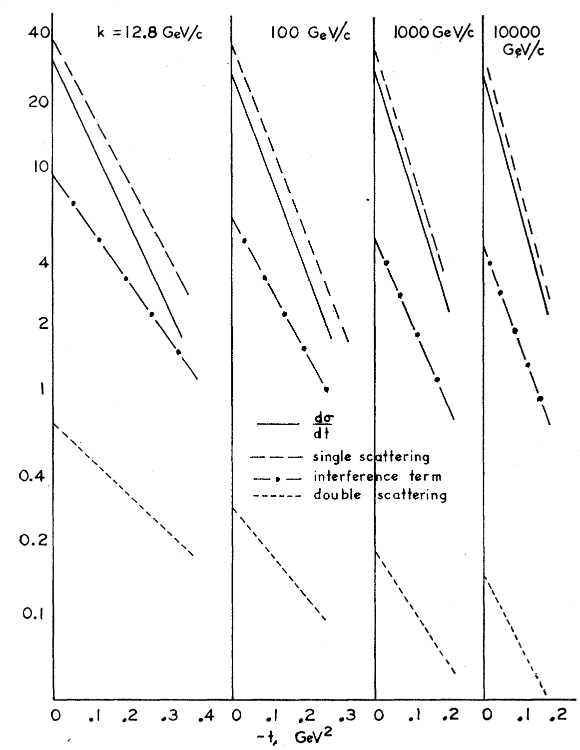

FIG. 18. Contributions of single scattering, double scattering, and the interference between them to the differential cross section at superhigh energies.

The smallness of the latter is reassuring evidence that the neglect of spin effects in earlier sections was not unreasonable. As noted in Sec. II, the disappearance of the double-scattering effects at higher energies will cause the total cross sections to increase slightly toward an asymptotically constant value; from the parameters of this model we deduce that this limit is

$$\sigma_\infty(\pi N)=25.66\pm0.28\text{ mb}.$$

The leading multiple-scattering contributions decrease only logarithmically, however, so that the increasing behavior becomes apparent only at superhigh energies. In order to see how rapidly $\sigma_\infty(\pi N)$ is approached, we give in Fig. 17 an extension of the model up to $k=10\,000$ GeV/c. It is again evident that the advent of the asymptotic region is still rather distant.

The importance of double scattering can also be seen in the differential cross sections. Interference between double and single scattering leads to a subtractive term with magnitude about $\frac{1}{3}$ of that resulting purely from the single-scattering term. The evolution with energy of the contributions of single and double scattering, and of their interference, to the (π^+p) curves can be seen in Fig. 18. The situation for the other reactions is very similar. In all three cases the range of t we are studying is still dominated by the single-scattering term.

It appears, then, that the quark model with multiple scattering is capable of reproducing, qualitatively and, to a reasonable extent, quantitatively, the pion-nucleon scattering amplitude.[22] In view of the approximations made, the results must be considered encouraging; in particular, we would hope that a proper treatment of the form factor and the use of a completely free P' trajectory would lead to a more comprehensive fit of the data, including even the dips observed at larger angles. A further extension of the model would be the calculation and comparison with experiment of the amplitudes for the production of nucleon resonances. Since the form factor in this case involves the overlap of the octet and decuplet spatial wave functions, the strong-binding assumption we have used may not be valid. We expect that future research efforts may clarify both of these possibilities.

ACKNOWLEDGMENTS

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[22] F. Henyey *et al.* [Phys. Rev. Letters **21**, 946 (1968)] obtain on the basis of a Reggeized absorption model results which are very similar to ours in their treatment of spin and isospin and in the appearance of the Regge-cut term. They attempt to fit the "dip" structure of pion-nucleon charge exchange as a double diffraction minimum, estimating the parameters of the elastic amplitude from experiment and including only the helicity flip amplitude in their calculations. As in Ref. 2, they concentrate on a region of larger momentum transfer, where the validity of the strong-binding approximation is less certain. Our results differ quantitatively from theirs for these reasons as well as the fact that they have not attempted to fit elastic scattering or polarization data; qualitatively, however, their approach is quite similar to ours.