Compton Scattering Sum Rules and Their Saturation*

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Theoretical features of the many fixed-momentum-transfer dispersive sum rules which can be written for the 26 possible generalized nucleon Compton scattering amplitudes (retarded products of vector currents) are surveyed, and the sum rules are put to experimental test. Theoretical attention is focused on the occurrence of right-signature fixed poles in the angular momentum plane, such as the $j=1$ fixed poles whose couplings are related to electromagnetic form factors by current algebra. Unitarity is used to estimate the sum-rule integrands in terms of data for the photoproduction processes $\gamma N\rightarrow\pi N$ and $\gamma N\rightarrow\pi\Delta$. Limitations of the data require that the sum rules be cut off at photon lab energy $E_{\text{lab}}=1.12$ GeV. The main results are as follows: (a) Reasonable evidence is presented that two-time-component current-algebra sum rules involving the electric and magnetic isovector form factors $G_E{}^V(t)$ and $G_M{}^V(t)$ are correct for small spacelike $-t$. If they are also to be correct for $-t\gtrsim -0.6$ (GeV/c)^2, then the ρ Regge pole must choose nonsense at $\alpha=0$, and the associated wrong-signature fixed pole there must be multiplicative. A time-space current-algebra sum rule probably fails. (b) The separate isotopic components of the Drell-Hearn sum rule are investigated. Those with $I=0$ exchange in the t channel seem very successful, whereas the $I=1$ exchange sum rule clearly fails. The failure indicates an important contribution of a hitherto unsuspected $J^P(I^G)=1^+(1^-)$ fixed pole. (c) Detailed results on wrong-signature antialgebra sum rules, on Regge-pole sum rules (FESR's), and on sum rules testing conspiracy are presented.

I. INTRODUCTION

MANY fixed-momentum-transfer dispersive sum rules can be written for nucleon Compton amplitudes. These sum rules test various assumptions about high-energy behavior and about the equal-time algebra of vector current components. In this paper, we survey theoretical aspects of these sum rules and report on a systematic attempt to saturate them, at several t values, using presently available experimental data. Within the limits set by the extent and accuracy of this data, our goal is to milk from the sum rules all the theoretically interesting information they contain.

Since there is very little data on the Compton scattering process itself, we use the unitarity condition to express the integrands of the sum rules in terms of amplitudes for the photoproduction of hadronic states. We include the contributions of the πN and, in cruder form, the $\pi\pi N$ intermediate states. Specifically, we use the multipole analyses of $\gamma N\rightarrow\pi N$ by Berends, Donnachie, and Weaver[1] and by Walker,[2] and a modified Stichel-Scholz[3] model for the process $\gamma N\rightarrow\pi\Delta$. This gives us a description of the sum-rule integrands which seems reasonably accurate up to the laboratory energy $E_{\text{lab}}=1.12$ GeV (c.m. energy $\sqrt{s}=1.73$ GeV), and we cut off our sum rules at this value.

Because of spin and isospin complexity there are 26 independent amplitudes for the generalized Compton scattering process, and the use of photoproduction data decomposed into definite angular momentum and isospin components allows us to study sum rules for all of them. We study the sum rules derived from current algebra,[4] as well as superconvergence relations[5] and finite-energy sum rules[6] which give information on Regge-pole parameters and on the question of conspiracy. We are mainly interested in theoretical questions involving the presence of fixed j-plane poles.

Finite-energy sum rules have been much used recently to study meson-baryon scattering,[6,7] where there are two important advantages. First, good partial-wave analyses exist,[8] at least for πN scattering, up to the c.m. energy $\sqrt{s}=2.19$ GeV; and second there is considerable high-energy data with which to compare Regge-pole predictions. In our case, the low-energy data are unfortunately crude, and there are no high-energy experiments. However, because we study photon amplitudes with the possibility of double helicity flip, many of our sum rules are more convergent than their analogs in meson-baryon scattering. Further, we re-

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[1] F. A. Berends, A. Donnachie, and D. L. Weaver, Nucl. Phys. **B4**, 1 (1968). We will refer to this work as BDW.

[2] R. L. Walker, Phys. Rev. (to be published).

[3] P. Stichel and M. Scholz, Nuovo Cimento **34**, 1381 (1964); D. Lüke, M. Scheunert, and P. Stichel, *ibid.* **58**, 234 (1968).

[4] For a survey of these sum rules, see S. L. Adler and R. F. Dashen, *Current Algebras and Applications to Particle Physics* (W. A. Benjamin, Inc., New York, 1968).

[5] V. de Alfaro, S. Fubini, G. Rossetti, and G. Furlan, Phys. Letters **21**, 576 (1966).

[6] A. Logunov, L. D. Soloviev, and A. N. Tavkhelidze, Phys. Letters **24B**, 181 (1967); K. Igi and S. Matsuda, Phys. Rev. Letters **18**, 625 (1967); R. Dolen, D. Horn, and C. Schmid, *ibid.* **19**, 402 (1967); Phys. Rev. **166**, 1768 (1968).

[7] V. Barger and R. J. N. Phillips, Phys. Letters **26B**, 730 (1968); F. J. Gilman, H. Harari, and Y. Zarmi, Phys. Rev. Letters **21**, 323 (1968); M. G. Olsson and G. B. Yodh, *ibid.* **21**, 1022 (1968); G. V. Dass and C. Michael, *ibid.* **20**, 1066 (1968); C. Ferro Fontán, R. Odorico, and L. Masperi, Nuovo Cimento **58**, 534 (1968).

[8] A. Donnachie, R. G. Kirsopp, and C. Lovelace, Phys. Letters **26B**, 161 (1968).

mark that the analysis of Dolen, Horn, and Schmid[6] at cutoff $\sqrt{s}=1.73$, identical to ours, gave reasonable results for the couplings of the ρ trajectory, and we therefore have reason to hope for good results at this cutoff in the Compton case.[9]

The plan of the paper is the following. For the benefit of readers primarily interested in the results, a summary of the most important results is given in Sec. II together with references to that part of the text where specific sum rules are discussed. The kinematics of Compton scattering is presented in Sec. III. Theoretical questions pertaining to the sum rules are discussed in Sec. IV. In Sec. V, we explain our treatment of the experimental data, and in Sec. VI, we present and discuss the results of our attempt to saturate the sum rules. Section VII is reserved for some final methodological comments, while some necessary technical questions are treated in Appendices.

II. MAIN RESULTS

Our main results are summarized here, although we would caution that a wrong impression of the strength of our conclusions could well be gained without some study of the quantitative behavior of the sum rules. The quickest way to proceed would be via Sec. VI A, in which the graphical format of the results is given, to the part of Sec. VI where the specific questions are discussed and the appropriate graphs presented.

Regge-pole sum rules (VI B) From sum rules for amplitudes in which the P, P', and A_2 trajectories couple to photons with helicity flip 2, we find the following results. There is no particular evidence for important contributions to the sum rule from right signature fixed poles at $j=0$.[10] Factorization tests give values of the ratio of the nucleon flip and nucleon nonflip couplings of the trajectories which agree with the values deduced from meson-nucleon scattering, although there is an uncertainty of about a factor of 2 in this comparison. Our results are consistent with the nonsense-choosing mechanism for the A_2 at $\alpha_{A_2}(t)=0$.

Current-algebra sum rules. (VI C) Two well-known sum rules[11] can be obtained by studying the equal-time commutators of time components of the isovector current, taken between states with nucleon helicity nonflip and flip (measured in the t-channel c.m. system). The nonflip sum rule, whose right-hand side involves the electric form factor $G_E(t)$, coincides at $t=0$ with the sum rule of Cabibbo and Radicati.[12] The flip sum rule similarly tests the magnetic form factor $G_M(t)$, and seems to have been first written down by Muzinich.[13]

Our results indicate good agreement with current-algebra predictions near $t=0$. At large momentum transfer ($t\approx -0.6$), there is some evidence for a possible violation of current algebra, although we prefer an interpretation in which current algebra is valid. In this interpretation, the ρ trajectory chooses nonsense at $\alpha_\rho(t)=0$ and has a singular coupling to the currents there. Because of the singular coupling, the nonsense dips[14] associated with ρ exchange in hadronic processes are not present in the Compton amplitude.

Both these sum rules receive important contributions at low energies from nonresonating multipoles, a fact which suggests that theoretical models[15] in which saturation occurs purely with resonances may be unrealistic. We give some idea of the relative magnitude of resonant and nonresonant contributions to the sum rules in Sec. VI I.

A sum rule involving the commutator of the time and space components of the isovector current has been written down by Bég[16] and further studied by Adler and Dashen.[4] This sum rule has some peculiar features,[4] and it is perhaps not surprising that our numerical results show that it is probably violated.

Antialgebra sum rules. (VI D) We use this name (see Sec. IV B) for sum rules[17] sensitive to wrong-signature fixed poles. We find evidence for wrong-signature fixed poles (at $j=1$) which couple strongly to Pomeranchuk and A_2 exchange amplitudes. The theoretical significance of such fixed poles has been recently studied.[18]

Drell-Hearn sum rules. (VI E) Here we refer to sum rules for three different isospin symmetric amplitudes with t-channel photon helicity flip, antisymmetrized in the nucleon helicity indices. The sum rules are superconvergence relations (SCR's) which follow from the assumption that $j=1$ fixed poles are absent in these amplitudes. At $t=0$, the sum of our three SCR's coincides with the original sum rule written by Drell and Hearn[19] for the anomalous magnetic moment of the proton.

Our results indicate that the two sum rules involving isoscalar exchange are very well satisfied, but that the sum rule involving isovector exchange is badly violated.

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[9] We are grateful to R. J. N. Phillips (private communication) for confirming that one does not get ridiculous results for the amplitude $B^{(+)}$ in πN scattering with a cutoff similar to that used here in our work on Compton scattering. The comparison with this amplitude is particularly relevant because it has similar convergence properties to our sum rules that test $j=0$ fixed poles.

[10] D. J. Gross and H. Pagels [Phys. Rev. **172**, 1381 (1968)] have suggested that an $I=1$, $j=0$ fixed pole couples to Compton amplitudes. See Sec. VI B for a further discussion of this point.

[11] We reserve the description "current algebra" for those sum rules carrying the same isospin and (G parity) in the t channel as the ρ meson. In fact, current algebra would appear to predict the fixed poles at $j=0$ and 1 in the other isospin states. These fixed poles are predicted to be zero for the quark algebra but they will be sensitive to Schwinger terms and technical assumptions (Ref. 4) that are perhaps less well studied than those in the ρ segment.

[12] N. Cabibbo and L. A. Radicati, Phys. Letters **19**, 697 (1966).

[13] I. J. Muzinich, Phys. Rev. **151**, 1206 (1966).

[14] G. Höhler, J. Baacke, H. Schlaile, and P. Sonderegger, Phys. Letters **20**, 79 (1966); F. Arbab and C. B. Chiu, Phys. Rev. **147**, 1045 (1966); G. Höhler and N. Zovko, Z. Physik **181**, 293 (1964).

[15] R. Dashen and M. Gell-Mann, Phys. Rev. Letters **17**, 340 (1966).

[16] M. A. B. Bég, Phys. Rev. Letters **17**, 333 (1966).

[17] J. H. Schwarz, Phys. Rev. **159**, 1269 (1967).

[18] H. D. I. Abarbanel, F. E. Low, I. J. Muzinich, S. Nussinov, and J. H. Schwarz, Phys. Rev. **160**, 1329 (1967).

[19] S. D. Drell and A. C. Hearn, Phys. Rev. Letters **16**, 908 (1966).

This last result was a surprise to us, and seems to indicate an important contribution from a $J^{PG}=1^{+-}$ fixed pole.

One negative result which may be of some interest is that neither of two sum rules sensitive to A_1 exchange showed any evidence for this Regge pole with an intercept near zero. (See also Sec. VI G).

Conspiracy sum rules. (VI F) By using a sum rule of Pagels[20] which relates the π^0 lifetime to an integral involving a Compton amplitude we infer that the effective π conspirator trajectory residue function $\beta_{c\pi}(t)$ in Compton scattering is a smooth function of momentum transfer near $t=0$. Unlike the photoproduction case we cannot write a sum rule sensitive to the t dependence of the pion residue function itself. However, comparison of the $t=0$ value obtained from the conspiracy condition with the value at the π pole (known from the π^0 lifetime) suggests a zero in β_π near $t=-m_\pi^2$. The behavior of both the pion and its conspirator is consistent with that found in strong interactions.

III. KINEMATICS

Using covariantly normalized states

$$\langle p_2|p_1\rangle=2E\delta^3(\mathbf{p}_2-\mathbf{p}_1)\,, \tag{1}$$

we define transition amplitudes for all two-body reactions

$$\langle p_2k_2|S|p_1k_1\rangle=\langle p_2k_2|p_1k_1\rangle+i(2\pi)^{-2}\delta^4 \times(p_2+k_2-p_1-k_1)T(p_2,k_2;\,p_1,k_1)\,. \tag{2}$$

Differential cross sections are given by, ignoring the spin summation,

$$d\sigma/d\Omega=|f(E,\theta)|^2\,,\quad f=(8\pi\sqrt{s})^{-1}(p_f/p_i)^{1/2}T\,, \tag{3}$$

where $s=(p_1+k_1)^2$ and p_i and p_f are the c.m. momenta of the initial and final states.

Compton scattering amplitudes are related to retarded products of currents by the formula

$$T(p_2,k_2;\,p_1,k_1)=(2\pi)^3e^2(\epsilon_2{}^\mu)^*\epsilon_1{}^\nu i \times\int d^4x\, e^{ik_2\cdot x}\langle p_2|\theta(x_0)[J_\mu{}^{\rm em}(x),J_\nu{}^{\rm em}(0)]|p_1\rangle\,, \tag{4}$$

where $e^2/4\pi=1/137$. We do not write explicitly the polynomial terms which may be required on the right-hand side of (4) to ensure covariance.

The electromagnetic current operator $J_\mu{}^{\rm em}(x)$ can be decomposed into isotopic singlet and triplet parts

$$J_\mu{}^{\rm em}(x)=J_\mu{}^{I=0}(x)+J_\mu{}^{I=1,M=0}(x)\,. \tag{5}$$

In general, we are led to consider covariant amplitudes formed as in Eq. (4) from the individual pieces $J_\mu{}^0$ and $J_\mu{}^{1,M}$ with $M=\pm1$, 0 and construct these amplitudes according to the following isospin conventions.

First, we construct amplitudes $T_I(I_\gamma',I_\gamma)$ describing the transition between normalized states of total s-channel isospin I built up from nucleons and isoscalar ($I_\gamma=0$) or isovector ($I_\gamma=1$) photons. There are five independent amplitudes. Each $T_I(I_\gamma',I_\gamma)$ gives rise to a scattering

$$T_I(I_\gamma',I_\gamma)C(\tfrac{1}{2},I_\gamma',I;\,M_N',M_\gamma')C(\tfrac{1}{2},I_\gamma,I;\,M_N,M_\gamma) \tag{6}$$

in states specified by third component of isospin for the nucleon (M_N) and photon (M_γ). The C's are standard Clebsch-Gordan coefficients. Our sum rules are written for the following combinations of the $T_I(I_\gamma',I_\gamma)$ formed by symmetrizing or antisymmetrizing in the (t-channel) photon isospin labels:

$$\begin{aligned} T^1&=2T_{1/2}(0,0)\,,\\ T^2&=\tfrac{2}{3}[T_{1/2}(1,1)+2T_{3/2}(1,1)]\,,\\ T^3&=\tfrac{2}{3}\sqrt{3}[T_{1/2}(0,1)+T_{1/2}(1,0)]\,,\\ T^4&=\tfrac{2}{3}[T_{3/2}(1,1)-T_{1/2}(1,1)]\,,\\ T^5&=\tfrac{2}{3}\sqrt{3}[T_{1/2}(0,1)-T_{1/2}(1,0)]\,. \end{aligned} \tag{7}$$

Amplitudes 1 and 2 carry isospin 0 in the t channel while amplitudes 3, 4, and 5 carry isospin 1. The Compton scattering amplitudes of physical photons are related to ours by the equations

$$\begin{aligned} T(\gamma p\to\gamma p)+T(\gamma n\to\gamma n)&=T^1+T^2\,,\\ T(\gamma p\to\gamma p)-T(\gamma n\to\gamma n)&=T^3\,. \end{aligned} \tag{8}$$

To relate our amplitude T^4 to that of the current-algebra literature[4] we observe that T^4 is given by Eq. (4) with the commutator replacement

$$[J_\mu{}^{\rm em}(x),J_\nu{}^{\rm em}(0)]\to[J_\mu{}^{1,1}(x),J_\nu{}^{1,-1}(0)] -[J_\mu{}^{1,-1}(x),J_\nu{}^{1,1}(0)]\,. \tag{9}$$

Physical Compton scattering data cannot be used to resolve the individual contributions of T^1 and T^2 in Eq. (8), or to determine the amplitudes in isospin segments 4 and 5. The real part of T^2 can conceivably be measured only in neutrino processes. However, the imaginary parts of all amplitudes are related unambiguously by unitarity to experimentally measurable photoproduction processes. Isospin segment 5 has very peculiar kinematics, discussed below, and does not seem to have been mentioned in the literature.

We always express our sum rules in terms of regularized t-channel parity-conserving helicity amplitudes,[21] which are advantageous for us because they have simple analyticity and crossing properties and definite t-channel quantum numbers. Direct-channel helicity amplitudes $M_{\lambda_3\lambda_4;\lambda_1\lambda_2}$ can be defined from Eq. (4) by choosing nucleon states and photon polarization vectors according to standard conventions.[22] We take the nucleon as

[20] H. Pagels, Phys. Rev. **158**, 1566 (1967).

[21] L. L. Wang, Phys. Rev. **142**, 1187 (1966).

[22] M. Jacob and G. C. Wick, Ann. Phys. (N.Y.) **7**, 404 (1959).

"particle 1." We define t-channel helicity amplitudes through the crossing relations[23]

$$A_{\lambda_3\lambda_1;\lambda_4\lambda_2}{}^i=-i(-1)^{\lambda_3-\lambda_1}\sum_{\lambda_3'\lambda_3'} d_{\lambda_1'\lambda_1}{}^{1/2}(\pi-\chi)\times d_{\lambda_3'\lambda_3}{}^{1/2}(\chi)M_{\lambda_3'-\lambda_4;\lambda_1'\lambda_2}{}^i, \quad (10)$$

where

$$\cos\chi=\frac{(s+m^2)}{(s-m^2)}\left(\frac{-t}{4m^2-t}\right)^{1/2},\qquad \sin\chi=\frac{2m}{(s-m^2)}\left[\frac{(s-m^2)^2+st}{4m^2-t}\right]^{1/2}, \quad (11)$$

and the superscript i indicates a definite isospin amplitude formed according to Eq. (7).

For physical photons, the kinematic singularities of the amplitudes $A_{\lambda_3\lambda_1;\lambda_4\lambda_2}$ have recently been obtained.[23,24] The analysis of Ref. 23 depended on a simplification of the crossing relation (9) using the time-reversal constraint

$$M_{\lambda_3\lambda_4;\lambda_1\lambda_2}=(-1)^{\lambda_{12}-\lambda_{34}}M_{\lambda_1\lambda_2;\lambda_3\lambda_4}, \quad (12)$$

where $\lambda_{ij}=\lambda_i-\lambda_j$. An identical condition holds for our isospin amplitudes 1–4, and for these amplitudes the results of Ref. 23 apply completely with the single exception that the $s-u$ crossing properties of isospin 4 amplitudes are opposite to those of isospin 1–3 because of photon antisymmetry.

We give here the exact definition of the amplitudes for which our sum rules are written in isospin segments 1–4. In terms of reduced t-channel helicity amplitudes,

$$\hat{A}_{\lambda_3\lambda_1;\lambda_4\lambda_2}{}^i=(\cos\tfrac{1}{2}\theta_t)^{-|\lambda_{42}+\lambda_{31}|}(\sin\tfrac{1}{2}\theta_t)^{-|\lambda_{42}-\lambda_{31}|}A_{\lambda_3\lambda_1;\lambda_4\lambda_2}{}^i, \quad (13)$$

we take the following combinations which are kinematic singularity free in both s and t.

$$\begin{aligned}
B_1{}^i&=it^{-1}(4m^2-t)^{-1/2}\hat{A}_{\frac12\frac12;1-1}{}^i,\\
B_2{}^i&=-t^{-1}(t-4m^2)^{-1}(\hat{A}_{\frac12-\frac12;1-1}{}^i-\hat{A}_{-\frac12\frac12;1-1}{}^i),\\
B_3{}^i&=(-t)^{-3/2}(4m^2-t)^{-1/2}(\hat{A}_{\frac12-\frac12;1-1}{}^i+\hat{A}_{-\frac12\frac12;1-1}{}^i),\\
B_4{}^i&=\tfrac12 i(-t)^{-1/2}(\hat{A}_{\frac12\frac12;11}{}^i-\hat{A}_{-\frac12-\frac12;11}{}^i),\\
B_5{}^i&=it^{-1}(4m^2-t)^{1/2}(\hat{A}_{\frac12\frac12;11}{}^i+\hat{A}_{-\frac12-\frac12;11}{}^i),\\
B_6{}^i&=-2t^{-1}\hat{A}_{\frac12-\frac12;11}{}^i.
\end{aligned} \quad (14)$$

The $B_j{}^i(s,t)$ are independent except for the constraint condition at $t=0$,

$$\lim_{t\to 0}[B_4{}^i(s,t)+(8m)^{-1}(s-u)B_6{}^i(s,t)]=0, \quad (15)$$

and other constraints at $t=4m^2$ which are not relevant for our analysis. In Appendix A, we express the $B_j{}^i(s,t)$ in terms of s-channel helicity amplitudes and Hearn-Leader[25] invariant amplitudes.

In isospin segment 5, the situation is different. Because of antisymmetry, an extra minus sign must be inserted in the time-reversal condition (12), and this means that there are only two independent nonvanishing s-channel helicity amplitudes which we take to be $M_{\frac12 1;-\frac12 1}{}^5$ and $M_{\frac12-1;\frac12 1}{}^5$. There is an analogous restriction, due to charge conjugation invariance, to two nonvanishing t-channel amplitudes $A_{\frac12\frac12;1-1}{}^5$ and $A_{\frac12-\frac12;11}{}^5$. The crossing relations simplify to

$$\begin{aligned}
A_{\frac12\frac12;1-1}{}^5&=iM_{\frac12 1;-\frac12 1}{}^5,\\
A_{\frac12-\frac12;11}{}^5&=iM_{\frac12-1;\frac12 1}{}^5.
\end{aligned} \quad (16)$$

The kinematic singularities are easily obtained and we choose the following singularity-free amplitudes:

$$\begin{aligned}
B_7{}^5&=i(-t)^{-1/2}(m^4-us)^{-1}A_{\frac12\frac12;1-1}{}^5,\\
B_8{}^5&=-it^{-1}(m^4-us)^{-1/2}A_{\frac12-\frac12;11}{}^5.
\end{aligned} \quad (17)$$

The amplitudes $B_j{}^i$ satisfy dispersion relations in the variable $\nu=\frac12(s-u)$ which we write in the form

$$B_j{}^i(\nu,t)=C_j{}^i(t)\left(\frac{1}{\nu-\nu_B}-\frac{\eta_j{}^i}{\nu+\nu_B}\right)+\frac{1}{\pi}\int_{\nu_0}^{\infty}d\nu'\,\mathrm{Im}B_j{}^i(\nu',t)\left(\frac{1}{\nu'-\nu}+\frac{\eta_j{}^i}{\nu'+\nu}\right). \quad (18)$$

We have separated out the nucleon Born term, singular at $\nu_B=\frac12 t$, from the continuum threshold beginning at $\nu_0=2m\mu+\mu^2+\frac12 t$. Possible subtraction terms in (18) will be discussed later. The crossing phase $\eta_j{}^i=\pm1$.

The nucleon pole residues have been calculated using Eq. (A2) and modifying the Hearn-Leader[25] residues to agree with our isospin conventions in segments 1–4. In isospin segment 5, the residues were calculated using unitarity to obtain the single-nucleon contribution to the imaginary part of the amplitudes.

The Born-pole residues and other important properties of our amplitudes are collected in Table I. The $s-u$ crossing phases $\eta_j{}^i$ and the t-channel quantum numbers carried by the amplitudes can be deduced by examining the t-channel partial-wave expansions and using parity and charge conjugation invariance properties and generalized statistics.

The leading asymptotic contribution to our amplitudes of a Regge pole of signature τ and position $j=\alpha(t)$ is

$$B_j{}^i(\nu,t)=-G_j{}^i(t)[(e^{-i\pi\alpha(t)}+\tau)/\sin\pi\alpha(t)]\nu^{\alpha(t)-\lambda}, \quad (19)$$

where $\lambda=\max(|\lambda_{31}|,|\lambda_{42}|)$. In spin segments 2 and 3 there are additional nonleading contributions from

[23] D. Z. Freedman, Phys. Rev. **168**, 1822 (1968). There is a minus sign error in the definition of $\sin\chi$ in this reference. Except for the choice of the Jacob-Wick phase (Ref. 22) for "particle two" helicities, our phase conventions and crossing path coincide with those of G. Cohen-Tannoudji, A. Morel, and H. Navelet, Ann. Phys. (N. Y.) **46**, 239 (1968).

[24] See also J. P. Ader, M. Capdeville, and H. Navelet, C.E.N.-Saclay Report (to be published).

[25] A. C. Hearn and E. Leader, Phys. Rev. **126**, 789 (1962).

TABLE I. The amplitudes $B_j^i(t)$. The meaning of the various quantities in the table is as follows. $B_j^i(t)$ are defined in Eqs. (14) and (17). η_j^i and $C_j^i(t)$ are defined by Eq. (18). In the $C_j^i(t)$ column κ_p, and κ_n are the anomalous magnetic moments of the proton and neutron, respectively. λ is defined after Eq. (19). $n_{\min}$ is the lowest value of n in Eq. (23) for the latter to be a right-signature sum rule. τ, P, and G are the signature, parity, and G parity of the allowed Regge-pole exchanges. Plausible candidates for the latter are listed in the next column; here we have taken the meson quantum numbers from the customary bible (Ref. 52). Further in this column c_X denotes the $\tau P=+$ partner of an $m=1$ conspiracy (Ref. 75) whose $\tau P=-1$ partner is the trajectory X_c. ($X=\eta$, π, B). X by itself denotes a nonconspiring η, π, or B trajectory.

(a)

Amplitude	η_j^i	λ	$n_{\min}$	τ	P	G	Regge pole
B_1^1	+	2	1	+	+	+	P, P' $c_\eta(\sim t)$
B_1^2	+	2	1	+	+	+	P, P' $c_\eta(\sim t)$
B_1^3	+	2	1	+	+	−	A_2 $c_\pi(\sim t)$
B_1^4	−	2	0	−	−	+	ρ $c_B(\sim t)$
B_2^1	−	2	0	−	+	+	$D, E(?)\ (\sim s^{\alpha-2})$
				+	+	+	$P, P', c_\eta(\sim ts^{\alpha-3})$
B_2^2	−	2	0	−	+	+	$D, E(?)(\sim s^{\alpha-2})$
				+	+	+	$P, P', c_\eta(\sim ts^{\alpha-3})$
B_2^3	−	2	0	−	+	−	$A_1(\sim s^{\alpha-2})$
				+	+	−	$A_2, c_\pi(\sim ts^{\alpha-3})$
B_2^4	+	2	1	+	−	+	?
				−	−	+	$\rho, c_B(\sim ts^{\alpha-3})$
B_3^1	+	2	1	+	+	+	$P, P', c_\eta(\sim s^{\alpha-2})$
				−	+	+	$D, E(?)(\sim s^{\alpha-3})$
B_3^2	+	2	1	+	+	+	$P, P', c_\eta(\sim s^{\alpha-2})$
				−	+	+	$D, E(?)(\sim s^{\alpha-3})$
B_3^3	+	2	1	+	+	−	$A_2, c_\pi(\sim s^{\alpha-2})$
				−	+	−	$A_1(\sim s^{\alpha-3})$
B_3^4	−	2	0	−	−	+	$\rho, c_B(\sim s^{\alpha-2})$
				+	−	+	?
B_4^1	+	0	1	+	−	+	$\eta(\sim t)\eta_c$
B_4^2	+	0	1	+	−	+	$\eta(\sim t)\eta_c$
B_4^3	+	0	1	+	−	−	$\pi(\sim t)\pi_c$
B_4^4	−	0	0	−	+	+	$B(\sim t)B_c$
B_5^1	+	0	1	+	+	+	P, P', c_η
B_5^2	+	0	1	+	+	+	P, P', c_η
B_5^3	+	0	1	+	+	−	A_2, c_π
B_5^4	−	0	0	−	−	+	ρ, c_B
B_6^1	−	1	0	+	+	+	$P, P'(\sim t), c_\eta$
B_6^2	−	1	0	+	+	+	$P, P'(\sim t), c_\eta$
B_6^3	−	1	0	+	+	−	$A_2(\sim t), c_\pi$
B_6^4	+	1	1	−	−	+	$\rho(\sim t), c_B$
B_7^5	+	2	1	+	−	−	π, π_c
B_8^5	+	1	1	−	+	−	A_1

(b)

Amplitude	Born residue $C_j^i(t)$
B_1^1	$-2me^2/t+(e^2/4m)[1-(1+\kappa_p+\kappa_n)^2]$
B_1^2	$-2me^2/t+(e^2/4m)[1-(1+\kappa_p-\kappa_n)^2]$
B_1^3	$-4me^2/t+(e^2/2m)[1-(1+\kappa_p)^2+\kappa_n^2]$
B_1^4	$2me^2/t-(e^2/4m)[1-(1+\kappa_p-\kappa_n)^2]$
B_2^1	$-(e^2/4m^2)(\kappa_p+\kappa_n)^2$
B_2^2	$-(e^2/4m^2)(\kappa_p-\kappa_n)^2$
B_2^3	$-(e^2/2m^2)(\kappa_p^2-\kappa_n^2)$
B_2^4	$(e^2/4m^2)(\kappa_p-\kappa_n)^2$
B_3^1	$-(2e^2/t)(1+\kappa_p+\kappa_n)-(e^2/4m^2)(\kappa_p+\kappa_n)^2$
B_3^2	$-(2e^2/t)(1+\kappa_p-\kappa_n)-(e^2/4m^2)(\kappa_p-\kappa_n)^2$
B_3^3	$-(4e^2/t)(1+\kappa_p)-(e^2/2m^2)(\kappa_p^2-\kappa_n^2)$
B_3^4	$(2e^2/t)(1+\kappa_p-\kappa_n)+(e^2/4m^2)(\kappa_p-\kappa_n)^2$
B_4^1	$\frac{1}{2}me^2(1+\kappa_p+\kappa_n)$
B_4^2	$\frac{1}{2}me^2(1+\kappa_p-\kappa_n)$
B_4^3	$me^2(1+\kappa_p)$
B_4^4	$-\frac{1}{2}me^2(1+\kappa_p-\kappa_n)$
B_5^1	$4m^3e^2/t-\frac{1}{2}me^2[1+(1+\kappa_p+\kappa_n)^2]$
B_5^2	$4m^3e^2/t-\frac{1}{2}me^2[1+(1+\kappa_p-\kappa_n)^2]$
B_5^3	$8m^3e^2/t-me^2[(1+\kappa_p)^2-\kappa_n^2+1]$
B_5^4	$-4m^3e^2/t+\frac{1}{2}me^2[1+(1+\kappa_p-\kappa_n)^2]$
B_6^1	$\frac{1}{2}e^2[(1+\kappa_p+\kappa_n)^2-1]$
B_6^2	$\frac{1}{2}e^2[(1+\kappa_p-\kappa_n)^2-1]$
B_6^3	$e^2[(1+\kappa_p)^2-1-\kappa_n^2]$
B_6^4	$-\frac{1}{2}e^2[(1+\kappa_p-\kappa_n)^2-1]$
B_7^5	$-e^2\kappa_n/(mt)$
B_8^5	$-e^2\kappa_n/t$

Regge poles $\alpha'(t)$ of signature τ' opposite to that of the leading poles, which take the form

$$-H_j^i(t)[(e^{-i\pi\alpha'(t)}+\tau')/\sin\pi\alpha'(t)]\nu^{\alpha'(t)-\lambda-1}. \quad (20)$$

In Appendices B and C, the detailed structure of the $G_j^i(t)$ and $H_j^i(t)$ is given in terms of the factorized couplings of Regge poles. Appendix D contains some further details of the Reggeization of spin segments 2 and 3.

IV. THEORETICAL MATTERS

A. Analyticity and Asymptotic Behavior

The sum rules which we study test both the analyticity properties and the high-energy behavior of Compton scattering amplitudes. Although the necessary analyticity—that underlying the dispersion relations (18)—can be proved rigorously from the axioms of quantum field theory, there is very little rigorous information on the asymptotic behavior. We review briefly here the types of asymptotic behavior which our present incomplete theoretical knowledge suggests.

For purely hadronic processes there are some rigorous asymptotic bounds on scattering amplitudes, such as the Froissart bound[26] which can be derived using analyticity and (s-channel) unitarity. For most applications this information is insufficient, and it is customary to assume that asymptotic behavior is determined by the singularities in the angular momentum variable of analytically continued t-channel partial-wave amplitudes. This hypothesis, called "analyticity of the

[26] M. Froissart, Phys. Rev. **123**, 1053 (1961).

second kind" by Chew,[27] effectively means an asymptotic structure of moving Regge poles and cuts.

In theories with analyticity of the second kind, t-channel unitarity plays an important role in determining asymptotic behavior. Its role is reviewed in the discussion of this subsection and references to the original literature are given. Further details, important in understanding our results, are presented in Sec. IV C.

Fixed poles in hadronic amplitudes are severely restricted by the t-channel unitarity condition,[28] they are allowed only at angular momentum values for which the unitarity cut is shielded by Regge cuts. Our present knowledge of this shielding mechanism[29] indicates that fixed poles occur because of the third double spectral function present in relativistic amplitudes and occur at wrong-signature nonsense values of angular momentum. These fixed poles do not contribute directly to asymptotic behavior, although they may modify the behavior of Regge-pole residues in an observable way. Schwarz sum rules[17] can be used to test for the presence of these fixed poles.

Compton scattering amplitudes are an example of the general class of "weak" amplitudes—those which never appear bilinearly in a unitarity relation. Because of the absence of bilinear unitarity in the s channel, the Froissart bound cannot be proved in the usual way, and there is at present no rigorous information on high-energy behavior. Further, the loss of bilinear unitarity in the t channel means that fixed poles in the angular momentum plane are no longer restricted.

Nevertheless, it is intuitively attractive to assume Regge asymptotic behavior for weak processes, and this was done in most early work on Compton scattering[30] and on more general weak amplitudes.[31] This Regge-pole picture led to puzzling features in the Pomeranchuk contribution to physical Compton scattering[30] and in the interpretation of current-algebra sum rules.[32] Fixed poles (and Kronecker delta terms[33]) provided the solution to these puzzles.

The known mechanisms for fixed poles in doubly weak amplitudes are discussed in Refs. 18 and 32, and we summarize them here. By doubly weak, we mean four-point amplitudes with two hadrons and two currents on external lines. In general, such amplitudes will have the j-plane behavior of their Born terms because this behavior is not smoothed by the weak unitarity condition. In particular, doubly weak amplitudes will have fixed poles at nonsense integers of both signatures. Usually the strong interactions—i.e., higher-order graphs—modify the residues of the fixed poles so that they differ from their Born values. Modification can be expected for both right- and wrong-signature fixed poles even if the third double spectral function (dsf) vanishes, although the third dsf mechanism will also contribute to wrong-signature fixed poles of weak amplitudes.

In general, therefore, the theory tells us the locations of fixed poles but is not powerful enough at present to predict their residues which depend on the details of strong interactions. Sum rules, as we will see, can be used to evaluate the fixed-pole residues directly from the experimental data.

There are two exceptions in which the general theory does give information about the fixed-pole residues. The first occurs in Compton scattering[18] where, because of photon masslessness, the Born terms of some amplitudes have a singular coefficient of t^{-1}. This may be observed in Table I for amplitudes B_1 B_3, B_5, B_7, and B_8. Since other contributions to the amplitude are regular at $t=0$, the residue of the fixed pole at the highest nonsense point is also singular at $t=0$ and is determined there by the Born term. This mechanism works in other kinematical configurations also.[34] Unfortunately, the corresponding sum rules reduce to simple identities at $t=0$, to which only the Born terms contribute, and are thus devoid of interest.

The second exception in which theory actually predicts the fixed-pole residue as a function of t concerns current algebra. It has been shown[32] that the well-known (and variously credited) Adler–Dashen–Fubini–Gell-Mann sum rules imply that the sum-rule amplitudes have fixed poles at $j=1$ and that the residues are given in terms of vector and axial-vector hadronic form factors. An observed failure of the sum rules would imply either (1) that the underlying algebra of currents must be modified, or (2) that the assumptions necessary to derive the sum rule from the algebra are incorrect,[4] or (3) both (1) and (2) are true. It may also be possible to relate the residues of fixed poles at $j=0$ and $j=-1$ to properties of the current algebra.[11,16,35]

We have stressed that the basic mechanism which permits fixed poles in weak amplitudes is the breakdown of bilinear unitarity. Linear or weak unitarity still requires factorization for Regge-pole couplings to weak amplitudes as we show in Sec. IV C. One effect of fixed poles is usually to make Regge-pole residues more

[27] M. Jacob and G. F. Chew, *Strong Interaction Physics* (W. A. Benjamin, Inc., New York, 1964).

[28] See the lectures of R. Oehme as reported in *Strong Interactions and High Energy Physics*, edited by R. G. Moorhouse (Plenum Press, Inc., New York, 1964).

[29] C. E. Jones and V. L. Teplitz, Phys. Rev. **159**, 1271 (1967); S. Mandelstam and L. L. Wang, *ibid.* **160**, 1490 (1967); R. Oehme, Phys. Letters **28B**, 122 (1968).

[30] V. D. Mur, Zh. Eksperim. i Teor. Fiz. **44**, 2173 (1963); **45**, 1051 (1963) [English transls.: Soviet Phys.—JETP **17**, 1458 (1963); **18**, 727 (1964)]; H. D. I. Abarbanel and S. Nussinov, Phys. Rev. **158**, 1462 (1967); H. K. Shepard, *ibid.* **159**, 1331 (1967).

[31] S. Fubini and G. Segrè, Nuovo Cimento **45**, 641 (1966).

[32] J. B. Bronzan, I. S. Gerstein, B. W. Lee, and F. E. Low, Phys. Rev. Letters **18**, 32 (1967); Phys. Rev. **157**, 1448 (1967); V. Singh, Phys. Rev. Letters **18**, 36 (1967); see also K. Bardakci, M. B. Halpern, and G. Segré, Phys. Rev. **158**, 1544 (1967).

[33] D. J. Gross and H. Pagels, Phys. Rev. Letters **20**, 961 (1968).

[34] See J. B. Bronzan *et al.*, Phys. Rev. **157**, 1448 (1967), Sec. III.

[35] J. D. Bjorken, Phys. Rev. **148**, 1467 (1966); I. T. Drummond, Nuovo Cimento **53**, 577 (1968).

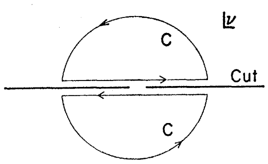


FIG. 1. The contour C of Eq. (22).

singular at nonsense integers than they would otherwise be. This effect will be seen clearly through our sum rules.

B. Sum Rules and Fixed Poles

The preceding arguments motivate us to assume that the typical asymptotic behavior of the $B(\nu,t)$ amplitudes is (with η denoting the crossing phase),

$$B(\nu,t)\sim -\sum_r G_r(t)(e^{-i\pi\alpha_r(t)}+\tau_r)[\sin\pi\alpha_r(t)]^{-1}\nu^{\alpha_r(t)-\lambda} -\sum_{k=1}^{\infty} F_k(t)\nu^{-k}[1+\eta(-1)^k] +\sum_{m=1}^{M} D_m(t)\nu^m[1+\eta(-1)^m], \quad (21)$$

corresponding to Regge poles [of leading signature $\tau=\eta(-)^\lambda$], right-signature fixed poles (at $j=\lambda-k$), and Kronecker deltas (at $j=\lambda+m$). Wrong-signature fixed poles manifest themselves in (21) only in their effect on the $G_r(t)$. We ignore possible Regge cuts because our sum rules are not accurate enough to distinguish between poles and cuts. Nonleading Regge-pole terms (20) can easily be included in spin types 2 and 3.

The sum rules we use can now be derived very easily. The functions $\nu^n B(\nu,t)$ are analytic in the cut ν plane and therefore satisfy

$$\frac{1}{2\pi i}\oint_C d\nu\, \nu^n B(\nu,t)=0\,, \quad (22)$$

where C is the contour of Fig. 1. We evaluate the integral over the semicircular portions approximately by using the asymptotic form (21) and taking ν_c as the radius of the semicircle. We collaspe the contour to the cut, separate out the Born contribution, and obtain the resulting sum rule

$$-\nu_B{}^n C(t)+\frac{1}{\pi}\int_{\nu_0}^{\nu_c} d\nu\, \nu^n\, \mathrm{Im}B(\nu,t) =\frac{1}{\pi}\sum_r G_r(t)\frac{(\nu_c)^{\alpha_r(t)+n-\lambda+1}}{\alpha_r(t)+n-\lambda+1}+F_{n+1}(t) \quad (23)$$

for n satisfying $(-)^{n-\lambda}=-\tau$, and a trivial identity for $(-)^{n-\lambda}=+\tau$. We remind the reader of our notation $\nu=\frac{1}{2}(s-u)$, $\nu_B=\frac{1}{2}t$, and $\nu_0=2m\mu+\mu^2+\frac{1}{2}t$.

Notice that the nth moment sum rule is sensitive only to the fixed pole at $j=\lambda-n-1$, and totally insensitive to possible Kronecker delta terms. The latter, as we shall see, can be tested using the dispersion relations (18) in which experimental values of the real part of the amplitude can be inserted.

Wrong-signature sum rules[17] can be similarly derived by considering an artificial amplitude $\bar{B}(\nu,t)$ with the same right-hand cut and the negative left-hand cut of the corresponding $B(\nu,t)$. Wrong-signature fixed poles manifest themselves in the asymptotic behavior of $\bar{B}(\nu,t)$. The sum rule is derived by considering the integral of $\nu^n\bar{B}(\nu,t)$ over the contour C. For n satisfying $(-)^{n-\lambda}=-\tau$ the result is a trivial identity, and for $(-)^{n-\lambda}=+\tau$ we obtain a sum rule identical in form to (23) with $F_{n+1}(t)$ as the asymptotic coefficient of the wrong-signature fixed-pole term at $j=\lambda-n-1$. Thereforce, we can understand Eq. (23) as valid for all integer n and testing right- (wrong-) signature fixed poles for $(-)^{n-\lambda}=\mp\tau$.

Using an intermediate-state expansion of the retarded product (4), it is easy to see that only the second term of the commutator contributes to the left-hand cut of the amplitudes $B(\nu,t)$. It is therefore amusing to note that the corresponding signatured amplitude $\bar{B}(\nu,t)$ is formally given by an anticommutator expression, and its fixed pole residues are formally determined by equal-time anticommutators. We refer to this situation as antialgebra.

The operation of the fixed-pole mechanisms discussed in Sec. IV A can be clearly seen in Eq. (23). For amplitudes with singular Born term $C(t)$, the left side of the $n=0$ sum rule is singular at $t=0$. This singularity must be matched on the right side either by the fixed-pole residue $F_1(t)$ or by the contribution of a Regge pole satisfying $\alpha(0)=\lambda-1$. In nonvacuum channels, there is no indication of the existence of Regge trajectories with the necessary properties,[36] and we must expect a fixed pole at the highest nonsense point $j=\lambda-1$ with residue singular at $t=0$. In vacuum channels, the Pomeranchuk trajectory has the required intercept and the Born singularity can be matched either by the singular Pomeranchuk term on the right side of (23) or by a wrong-signature fixed pole at $j=1$. The sum rules can be used to distinguish between these alternatives.

We also observe that if a Regge trajectory passes through the nonsense integer $\alpha(t_0)=\lambda-n-1$ for some t_0 and if $G(t_0)\neq 0$, the Regge-pole term in nth moment sum rule has a pole at $t=t_0$. This pole is not present on the left side of Eq. (23), because we are dealing with a nonsense or unphysical point, and it must therefore be cancelled by a similar pole in the fixed-pole residue $F_{n+1}(t)$. Current-algebra amplitudes, where $F_1(t)$ is a

[36] It is instructive to contrast the doubly weak case with the singly weak process $\gamma N\to\pi N$. There the nucleon Born term is singular at $t=m_\pi{}^2$ and the singularity can be matched in the sum rule by the contribution of the π-meson trajectory.

form factor with the ρ-meson pole, are an example of this mechanism.

Curiously enough the fixed-pole residue functions can have poles at spacelike t values. If $G_\rho(t_0)\neq 0$ (or $G_{A_2}(t_1)\neq 0$) at the negative t value t_0 (or t_1) where $\alpha_\rho(t_0)=0$ (or $\alpha_{A_2}(t_1)=0$), the $j=0$ wrong- (or right-) signature fixed-pole residue develops a pole at t_0 (or t_1) corresponding to the nonsense ghost state on the trajectory. In the wrong-signature case, this is clearly a triumph of antialgebra.

C. Unitarity, Factorization, and Fixed Poles

We discuss the implications of the t-channel unitarity condition for the factorization of Regge poles and occurrence of fixed poles in weak amplitudes. The two principal results are essentially known. If the Regge-pole couplings to hadronic channels are factorable,[37] then they are factorable in weak amplitudes as well. Further, unlike the hadronic case, there is no restriction on the occurrence of fixed j-plane singularities in weak amplitudes. These facts are necessary to understand our results for the current-algebra sum rules.

Let Latin subscripts j, $k=1, 2, \cdots, N$ denote some finite set of two-body hadronic channels which may or may not have degenerate thresholds. Define definite parity and signature partial-wave amplitudes by $a_{jk}(t,J)$, symmetric in the channel indices. Let the usual phase-space factors be absorbed in the a_{jk} so that they are related to unitarity S-matrix elements by $S_{jk}(t,J)=\delta_{jk}+2ia_{jk}(t,J)$. If these definitions are summarized in matrix form $S(t,J)=1+2iA(t,J)$, the strong unitarity condition can be written as

$$S^{\rm II}(t,J)=[S(t,J)]^{-1}. \qquad (24)$$

where sheet II is the sheet reached by continuing downward from the physical sheet just above the highest threshold in our set. For each factorable Regge pole there is a trajectory function $\alpha(t)$ and an N-component vector function $|v(t)\rangle$ in channel space, such that near the pole

$$S^{\rm II}(t,J)\approx |v(t)\rangle 1/[J-\alpha(t)]\langle v(t)| . \qquad (25)$$

Let us choose a particular weak channel such as two photons with definite isospin and helicity. We denote the partial-wave transition amplitude between this channel and the strong channel k by $b_k(t,J)$ and join these amplitudes into the vector $|b(t,J)\rangle$. The transition amplitude for the totally weak process $(\gamma+\gamma\rightarrow\gamma+\gamma)$ is denoted by $c(t,J)$. Unitarity for the weak amplitudes, which takes into account the intermediate hadronic states $k=1, 2, \cdots, N$, can be written as

$$|b(t,J)\rangle-|b^{\rm II}(t,J)\rangle=2iA(t,J)|b^{\rm II}(t,J)\rangle , \qquad (26a)$$

$$c(t,J)-c^{\rm II}(t,J)=2i\langle b(t,J)|b^{\rm II}(t,J)\rangle . \qquad (26b)$$

Solving for the second sheet quantities, we get, using (24),

$$|b^{\rm II}(t,J)\rangle=S^{\rm II}(t,J)|b(t,J)\rangle , \qquad (27a)$$

$$c^{\rm II}(t,J)=c(t,J)-2i\langle b(t,J)|S^{\rm II}(t,J)|b(t,J)\rangle . \qquad (27b)$$

It is obvious from Eqs. (24) and (27a) that the Regge-pole residues in doubly weak amplitudes factor in such a way that the quotient of the residues to two different strong channels (e.g., $N\bar{N}$ helicity nonflip and flip) is equal to a quotient of purely hadronic couplings. This is the basis of the factorization tests [see Eq. (45)] used in the study of the sum rules.

It is clear that Eq. (27a) *permits* the presence of fixed poles in doubly weak amplitudes,[38] and we have reviewed in Secs. IV A and IV B several arguments showing that fixed poles actually *are* present at nonsense integers. Equation (27a) suggests that such fixed poles are "multiplicative" and, therefore, make Regge-pole residues singular at the nonsense intersections. This behavior is not at all inconsistent with the unitarity requirement for the fully weak amplitude, and Eq. (27b) indeed indicates that this amplitude will have fixed double poles at nonsense integers and doubly singular Regge residues.

Our interest in this last point arises from our study of the current-algebra sum rules (Sec. VI C) which indicates that the ρ-meson Regge-pole couplings are smooth and nonvanishing near $t=-0.6$ (GeV/c)^2 whereas hadronic amplitudes generally exhibit the well-known nonsense zero (dip) there.[14] This situation is consistent with factorization only because singular $\gamma\gamma$ couplings, corresponding to a fixed double pole at $j=0$ in the $\gamma\gamma\rightarrow\gamma\gamma$ amplitude, are allowed.

D. Conspiracy

We turn our attention now to the conspiracy condition Eq. (15) which relates at $t=0$ the amplitude B_4 containing $\tau P=-1$ trajectories in the t channel to the amplitude B_6 containing $\tau P=+1$. We suppress the isospin superscripts in this discussion. Since Eq. (15) holds identically in s, it imposes constraints on the residues at $t=0$ of these trajectories. Either the couplings $G_4(t)$ and $G_6(t)$ vanish at $t=0$ (evasion) or there exist pairs $\alpha(t)$ and $\alpha_+(t)$ of negative and positive τP trajectories satisfying the conditions (of conspiracy)

$$\begin{aligned}&\alpha_-(0)=\alpha_+(0)\,,\\&G_4(0)=-(1/4m)G_6(0)\,.\end{aligned} \qquad (28)$$

Sum rules for amplitudes B_4 and B_6 can, in principle, be used to investigate possible conspiracies for the π (isospin segment 3) and η (isospin segments 1 and 2).

[37] For discussions of Regge-pole factorization in strong interactions, see M. Gell-Mann, Phys. Rev. Letters **8**, 263 (1962); V. N. Gribov and I. Ya. Pomeranchuk, *ibid.* **8**, 343 (1962); Ref. 28, pp. 175–177.

[38] Fixed poles are normally prohibited in purely hadronic amplitudes because the unitarity condition, Eq. (24), could not be satisfied near them. See Ref. 28, pp. 161–162.

One would simply explore the sum rules as functions of t for several moments to obtain a parametrization of the trajectories and residues. Although this technique has recently been used to investigate π conspiracy in the process $\gamma N \to \pi N$,[39] it does not seem possible to use it for Compton scattering, at least with presently available data. First, the $B_4{}^3$ sum rule has $\lambda = 0$ and $n_{\min} = 1$; it diverges badly asymptotically, emphasizing the most inaccurately known part of the data. Second, the $B_6{}^3$ sum rule, although more accurate, is useful only near $t=0$ for determining the parameters of conspirator trajectories because important contributions from nonconspiring trajectories (such as A_2) mix in away from that point.

Hence, the only number which can be determined from the B_4 and B_6 sum rules and associated with the parameters of a single Regge trajectory with relative confidence is the value of the B_6 sum rule at $t=0$. However, even this number provides an interesting test of conspiracy, through a sum rule of Pagels,[20] which we rederive here to incorporate recent clarification of the questions of conspiracy and of the relation between asymptotic behavior and subtractions.

We start with the $n=0$ sum rule for $B_6{}^3(\nu,t)$ assuming domination by a single Regge pole and a right-signature fixed pole at $j=0$:

$$-C_6{}^3(t)+\frac{1}{\pi}\int_{\nu_0}^{\nu_c} d\nu\, \mathrm{Im}B_6{}^3(\nu,t)=G(t)\frac{(\nu_c)^{\alpha(t)}}{\pi\alpha(t)}+F(t). \quad (29)$$

Now set $t=0$, evaluate the Born term using Table I, and reexpress the continuum contribution using the conspiracy condition (15):

$$-e^2(2\kappa_p+\kappa_p{}^2-\kappa_n{}^2)-\frac{4m}{\pi}\int_{\nu_0}^{\nu_c}\frac{d\nu}{\nu}\,\mathrm{Im}B_4{}^3(\nu,0)=G(0)\frac{\nu_c{}^{\alpha(0)}}{\pi\alpha(0)}+F(0). \quad (30)$$

We proceed with the derivation under two different assumptions.

1. Pure Regge Approach

We assume that the π meson lies on a Regge tratrajectory $\alpha_\pi(t)$ which couples to the $B_4{}^3$ amplitude with strength $G_\pi(t)$. If $G_\pi(0)\neq 0$ then there is a conspirator $\alpha_c(t)$ which couples to $B_6{}^3$ with strength $G_c(t)$, and these functions may be identified with the Regge functions in Eq. (30). We set $F(t)\equiv 0$.

The amplitude $B_4{}^3$ has a pole at $t=m_\pi{}^2$ corresponding to the π^0 intermediate state in the t channel. The residue of the pole is closely related to the lifetime of the π^0. Using $B_4(\nu,t)=A_3(\nu,t)$ where A_3 is the Hearn-Leader amplitude, and comparing the residue of the pole in the π^0 Regge-pole term defined in (21) with Eqs. (2.8) and (2.12) of Ref. 20, we find

$$-2G_\pi(m_\pi{}^2)/\pi\alpha_\pi'(m_\pi{}^2)=g_{\pi N}m_\pi{}^2F_\pi(m_\pi{}^2),\qquad F_\pi{}^2(m_\pi{}^2)=64\pi/m_\pi{}^3\tau, \quad (31)$$

where τ is the π^0 lifetime and $g_{\pi N}$ is the $\pi N\bar{N}$ coupling constant. We assume that the π Regge-pole coupling $G_\pi(t)$ varies slowly with t so that

$$G_\pi(m_\pi{}^2)\approx G_\pi(0). \quad (32)$$

We use (24) to rewrite (30) [with $F(t)=0$] as

left-hand side of (30)

$$=-G_\pi(0)[4m\pi\alpha_\pi(0)]^{-1}\nu_c{}^{\alpha_\pi(0)} \approx G_\pi(m_\pi{}^2)[4m\pi m_\pi{}^2\alpha_\pi'(m_\pi{}^2)]^{-1}\nu_c{}^{\alpha_\pi(0)}. \quad (33)$$

Using (31), we then obtain

$$\frac{e^2}{4m}(2\kappa_p+\kappa_p{}^2-\kappa_n{}^2)+\frac{1}{\pi}\int_{\nu_0}^{\nu_c}\frac{d\nu}{\nu}\,\mathrm{Im}B_4{}^3(\nu,0)=\frac{4g_{\pi N}}{m_\pi{}^2}\left(\frac{\pi m_\pi}{\tau}\right)^{1/2}\nu_c{}^{\alpha_\pi(0)}, \quad (34)$$

which is the form of Pagels's sum rule appropriate for pure Regge behavior.

2. Elementary π

Here we assume that Regge-pole terms are unimportant on the right side of Eq. (30), and that the sum rules evaluates the residue of the $j=0$ right-signature fixed pole. If $F(0)\neq 0$, as our numerical result shows, then the conspiracy condition requires a Kronecker δ_{j0} term[33] in the amplitude $B_4{}^3$ with asymptotic coefficient $D_0(0) = -4mF(0)$ at $t=0$. We assume that the Kronecker δ_{j0} coefficient has a pole at $t=m_\pi{}^2$ corresponding to the elementary π^0 meson and that this pole term dominates at $t=0$. We then can write

$$-8mF(0)\approx g_{\pi N}F_\pi(m_\pi{}^2), \quad (35)$$

and the sum rule (30) becomes

$$\frac{e^2}{4m}(2\kappa_p+\kappa_p{}^2-\kappa_n{}^2)+\frac{1}{\pi}\int_{\nu_0}^{\nu_c}\frac{d\nu}{\nu}\,\mathrm{Im}B_4{}^3(\nu,0)=\frac{4g_{\pi N}}{m_\pi{}^2}\left(\frac{\pi m_\pi}{\tau}\right)^{1/2}. \quad (36)$$

At presently practicable cutoff energies one cannot distinguish between (34) and (36), and, therefore, one cannot directly probe the Regge-pole nature of the pion in Compton scattering. The sum rule does provide a check on the over-all strength of the asymptotic structure corresponding to the π meson and on the assumption of smooth variation of the effective π-pole residue. The sum rule for amplitude $B_7{}^5$ in which the π

trajectory can be exchanged (although $j=0$ is a nonsense point) also provides some information on conspiracy.

Sum rules similar to (34) and (36) can be written for the η meson. We refer the reader to Sec. VI for further discussion of our results on conspiracy.

E. Polarizability and Kronecker Deltas

We finally discuss a possible test for the presence of Kronecker delta terms in physical Compton scattering amplitudes.

The amplitude $B_1{}^i(\nu,t=0)$, in isospin segments 1–3, satisfies the dispersion relation

$$B_1{}^i(\nu,0)=-\frac{a^i me^2}{\nu^2}+\frac{1}{\pi}\int_{\nu_0}^{\infty}d\nu'\frac{2\nu'\,\mathrm{Im}B_1{}^i(\nu,0)}{\nu'^2-\nu^2}+c^i+d^i\nu^2\,, \quad (37)$$

where we have included contributions of Kronecker deltas at $j=2$ and $j=4$. The nucleon-pole coefficient is $a_1=a_2=\frac{1}{2}a_3=2$. Using the crossing relations (10) at $t=0$, we find

$$\begin{aligned}\nu^2 B_1{}^i(\nu,0)&=m(M_{\frac{1}{2}1;\frac{1}{2}1}+M_{\frac{1}{2}-1;\frac{1}{2}-1})\\&=4m^2 f_1{}^i(\nu)\,,\end{aligned} \quad (38)$$

where $f_1(\nu)$ is the forward spin-averaged Compton scattering amplitude of the classical era of dispersion relations.[40] A power-series expansion about $\nu=0$ gives

$$f_1{}^i(\nu)=-(a^ie^2/4m)+b^i\nu^2+O(\nu^4)\,. \quad (39)$$

The parameter b^i is related quite simply to the energy derivative at threshold of the forward unpolarized Compton scattering differential cross section,[41,42] and to the sum of electric and magnetic polarizabilities of the nucleon[43] by $4m^2b^i=4\pi(\alpha^i+\beta^i)$. Combining (37)–(39) and using the optical theorem, we find for the polarizability sum

$$(\alpha+\beta)^i=\frac{m}{\pi^2}\int_{\nu_0}^{\infty}\frac{d\nu}{\nu^2}\sigma_t{}^i(\nu)+\frac{c^i}{4\pi}\,. \quad (40)$$

This sum rule has long been known in the form with $c^i=0$ (no Kronecker delta) and has been used to constrain a two-parameter fit to low-energy Compton scattering.[44] Drell[42] has recently emphasized the importance of using Eq. (40) to test for the presence of the δ_{j2} term in the asymptotic behavior of proton Compton scattering.[45] In this case, the total photoabsorption cross section is known up to 6 GeV, and the rapidly convergent integral term can be quite accurately estimated from the data. Unfortunately, it is the polarizability sum, which could be determined in low-energy (20–80 MeV) Compton scattering experiments, which is unknown. Thus, we have here a situation in which measurement of a single low-energy parameter can answer an important question in high-energy physics, and we join Drell in urging active consideration of low-energy proton Compton scattering experiments.

Our contribution to the question of the Kronecker delta term consists of the evaluation of the integral term in Eq. (40) in isospin segments 1–3.

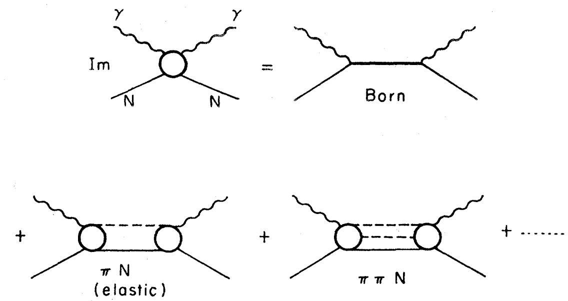


FIG. 2. Unitarity condition in Compton scattering.

V. TREATMENT OF EXPERIMENTAL DATA

The most conspicuous feature of the nucleon Compton process is the lack of direct experimental data. Since the sum rules (23) involve only the imaginary parts of Compton amplitudes, we use unitarity to express the integrands bilinearly in terms of hadronic photoproduction amplitudes.

The unitarity condition is shown schematically in Fig. 2. One must sum the contributions from all intermediate states that are energetically allowed. Experimentally the quasielastic (πN) intermediate state dominates[46] up to photon lab energies (E_{lab}) of 0.5 GeV, and between 0.5 and 1.1 GeV the inelastic contribution is dominated by the $\pi\pi N$ state in the configuration $\pi\Delta$ (see Fig. 10).

In studies of sum rules for the processes $\pi N\to\pi N$, $KN\to KN$,[7] and $\gamma N\to\pi N$,[39,47] there are "experimen-

[39] A. Bietti, P. Di Vecchia, F. Drago, and M. L. Paciello, Phys. Letters **26B**, 457 (1968); **26B**, 736 (1968); P. Di Vecchia, F. Drago, C Ferro Fontán, R. Odorico, and M. L. Paciello, Phys. Letters **27B**, 296 (1968).

[40] M. Gell-Mann, M. L. Goldberger, and W. E. Thirring, Phys. Rev. **95**, 1612 (1954).

[41] M. Cini and R. Stroffolini, Nucl. Phys. **5**, 684 (1958).

[42] S. D. Drell, Comments Nucl. Particle Phys. **1**, 196 (1967).

[43] A. M. Baldin, Nucl. Phys. **18**, 310 (1960); V. A. Petrunkin, Zh. Eksperim. i Teor. Fiz. **40**, 1148 (1961) [English transl.: Soviet Phys.—JETP **13**, 808 (1961)]; S. R. Choudhury and D. Z. Freedman, Phys. Rev. **168**, 1739 (1968).

[44] V. I. Goldansky, O. A. Karpukhin, A. V. Kutsenko, and V. V. Pavlovskaya, Nucl. Phys. **18**, 473 (1960).

[45] J. K. Walker [Phys. Rev. Letters **21**, 1618 (1968)] has given an impressive bound on the size of a possible $j=2$ Kronecker delta by using Compton scattering data at 18 GeV.

[46] Aachen-Berlin-Bonn-Hamburg-Heidelberg-München Collaboration, Phys. Rev. **175**, 1669 (1968). This paper contains references to earlier work by the same group. Cambridge Bubble Chamber Group, Phys. Rev. **155**, 1477 (1967).

[47] S. Y. Chu and D. P. Roy, Phys. Rev. Letters **20**, 958(1968); **21**, 57 (1968); K. V. Vasavada and K. Raman, *ibid.* **21**, 577 (1968).

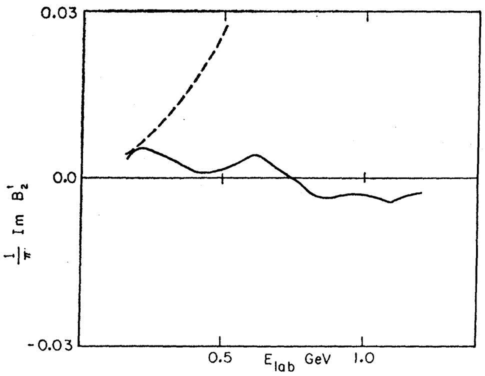


FIG. 3. The value of $(1/\pi)\,\mathrm{Im}B_2^1$ plotted against photon lab energy. The dashed line is the prediction of BDW (Ref. 1) and the solid line that of Walker (Ref. 2).

tal data" available for both real and imaginary parts of the amplitudes. This leads to two advantages which we do not enjoy. First, continuous-moment sum rules, involving real parts, can be used. Second, inelasticity is automatically incorporated, and one need not treat individually the contributions of different intermediate states.

A. πN Intermediate State

There have been many theoretical and phenomenological attempts to describe low-energy photoproduction,[1,2,48–50] $\gamma_N \to \pi N$. Only two of these are sufficiently complete for our purposes, since we require a description of photoproduction amplitudes which is accurate as to phase, helicity, and isospin dependence. The multipole analysis of Walker[2] is a direct fit to the experimental data, up to photon energies of 1.2 GeV, using a Born term, Breit-Wigner terms for known resonances, plus correction terms. Berends, Donnachie, and Weaver[1] (BDW) have given a more theoretical treatment, based on dispersion relations, which extends only to $E_{\mathrm{lab}} = 0.5$ GeV. Their results do not fit the data as well as Walker but probably contain a better estimate of the helicity and isospin structure of the background.

In our estimate of the πN contribution to ImB, we calculate the integral up to 0.5 GeV using both BDW and Walker and compare the two evaluations. Above this energy we use Walker's analysis. In the low-energy region there are often serious discrepancies between the BDW and Walker multipoles, particularly for isoscalar photons. When one calculates the experimental $d\sigma/dt$ for photoproduction this difference shows up most clearly in $\gamma n \to \pi^- p$, where BDW predicts a much flatter t distribution than Walker for the energy range $0.4 \leq E_{\mathrm{lab}} \leq 0.5$. The data used by Walker would appear to agree with his own analysis[2] and not BDW.[1]

To illustrate the importance of this difference, we plot in Figs. 3–5 the values of $(1/\pi)\,\mathrm{Im}B$ at $t=0$ for three sum rules of particular interest. One (B_3^4), the helicity flip current-algebra amplitude, has a small discontinuity at 0.5 GeV between the BDW and Walker analyses. The Drell-Hearn amplitude involving two isoscalar photons (B_2^1) is badly discontinuous while the discontinuity of B_2^3, the Drell-Hearn amplitude in which isoscalar and isovector photons interfere, is intermediate between these two extremes.

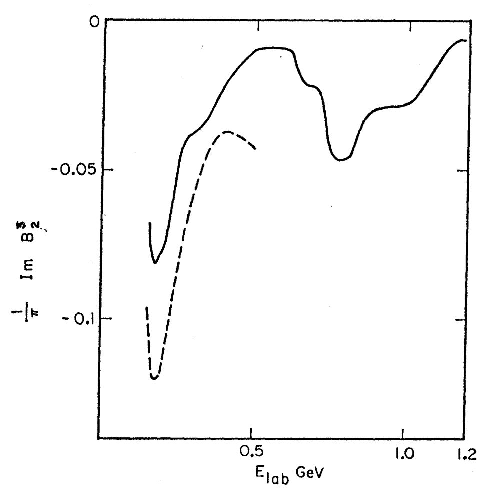


FIG. 4. The value of $(1/\pi)\,\mathrm{Im}B_2^3$ plotted against photon lab energy. The dashed line is the prediction of BDW (Ref. 1) and the solid line that of Walker (Ref. 2).

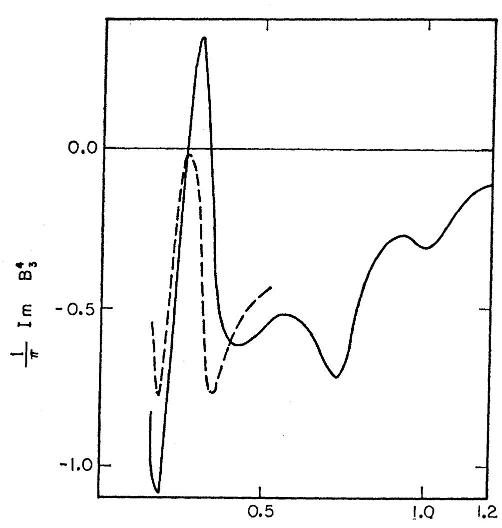


FIG. 5. The value of $(1/\pi)\,\mathrm{Im}B_3^4$ plotted against photon lab energy. The dashed line is the prediction of BDW (Ref. 1) and the solid line that of Walker (Ref. 2).

[48] H. Joos, in *The Heidelberg International Conference on Elementary Particles*, edited by H. Filthuth (North-Holland Publishing Company, Amsterdam, 1968). This contains a useful review on some of the attempts to understand low-energy photoproduction.

[49] Y. C. Chau, N. Dombey, and R. G. Moorhouse, Phys. Rev. **163**, 1632 (1967).

[50] J. Engels, A. Müllensiefen, and W. Schmidt, SLAC Report No. SLAC-PUB-415 (unpublished).

Both the BDW and Walker data are essentially given directly in terms of multipoles. To calculate ImB for our sum rules, we use Eq. (A2) expressing the B_j^i in terms of s-channel helicity amplitudes and then decompose into partial waves. Then the partial-wave unitarity equation[22,25] enables us to express the Compton scattering partial-wave amplitudes in terms of photoproduction multipoles.

There is, unfortunately, a technical difficulty in this approach in that the box diagram of Fig. 6, leads to a divergence of the partial-wave series for $t \lesssim -0.28$. This was countered by calculating (in a way too inelegant to reveal) the divergent part of Fig. 6 and subtracting its partial-wave decomposition from the divergent series produced by the photoproduction multipoles.

B. Inelastic Intermediate States

We must now turn to the insertion of inelastic intermediate states in our unitarity sum. In the energy range of interest $\pi\pi N$ is the most important inelastic state and this is predominantly produced in the quasi-two-body state $\pi\Delta$.[3,46] Thus at 0.7 GeV, $\pi\Delta$ is essentially 100% of the inelasticity while at $E_{\text{lab}}=1.1$ GeV it is more like $50 \to 70\%$.

In order to describe $\pi N \to \pi\Delta$, we use the Stichel-Scholz[3] model, which approximates[51] the amplitude by the s-channel nucleon Born term and the u channel Δ pole of Fig. 7(a). We chose to calculate these graphs by fixed t-dispersion relations utilizing the known residues at the poles. Then by gauge invariance the t-channel one-π exchange term [Fig. 7(b)] is automatically included. This model fits the data well near $t=0$ both in $d\sigma/dt$ and the density matrix elements ρ_{33}, ρ_{31}, ρ_{3-1} describing the decay of the Δ.

This calculation ignores the magnetic moments of the N and the Δ which are important away from $t=0$. Other obvious omissions are the higher s-channel resonances, which can be estimated, and the u-channel resonances, which cannot, due to the unknown $\gamma\Delta \to N^{**}$ coupling. One effect of these omitted terms is to destructively interfere with the Born terms of Fig. 7, and reduce the calculated cross section. They also introduce nonzero values into helicity and isospin states not populated in the model of Fig. 7. The relative size of these effects may be estimated by examining $\gamma N \to \pi N$ at large $|t|$ where the mass difference of N and Δ becomes negligible and we have similar kinematics. However, we contented ourselves with taking the amplitude of Fig. 7 and multiplying it by a form factor $F(t)$ determined so as to fit the experimental values of $d\sigma/dt$ for $\pi N \to \pi\Delta$. This simulates the destructive interference at large $|t|$ of the omitted terms but not the population of new helicity and isospin states. The helicity structure thus obtained is essentially the same as that given by an absorption-model calculation based on the one-pion exchange graph [Fig. 7(b)]. Thus, Fig. 7, with form factor, already contains the most important effects given by absorptive corrections. A typical $F(t)$ at $E_{\text{lab}}=0.85$ GeV was given by

$$F^2(t)=0.66\exp(-2.9t-12t^2),$$

whose value at $t=0$ reflects the ambiguity in extracting the coupling parameters of the unstable Δ resonance.

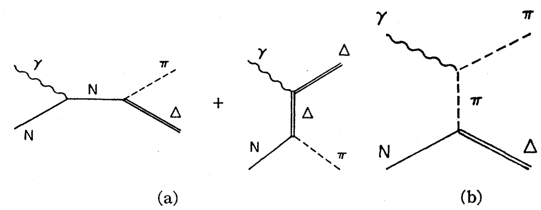


FIG. 7. (a) The diagrams considered in the Stichel-Scholz model (Ref. 3) of $\gamma N \to \pi\Delta$. (b) The one-pion-exchange contribution to $\gamma N \to \pi\Delta$.

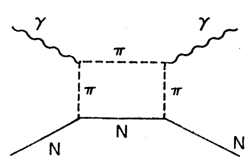
It may be worth noting that in our modified Stichel-Scholz model for $\gamma N \to \pi\Delta$, the amplitudes involving isoscalar photons vanish. We expect the isoscalar photon contribution to be small (because there is no π exchange pole) and of the same order as many omitted effects in the isovector part. Such effects are difficult to estimate.

In order to find the contribution to ImB of the $\pi\Delta$ state, we follow the same procedure as for πN. Namely, we decompose $\pi N \to \pi\Delta$ into partial waves and use partial-wave unitarity.[22] We note that the diagram of Fig. 8 does not cause a divergence of the partial-wave series until $t \approx -1.2$ and so we need no special action like that necessary for Fig. 6.

In order to describe the inelasticity not produced in the $\pi\Delta$ intermediate state, we add incoherently the contributions of higher resonances as in Fig. 9 multiplied by the factor

$$(\Gamma_{\text{inel}}-\Gamma_{\pi\Delta})/\Gamma_{\text{tot}},$$

FIG. 6. A diagram causing a divergence of the partial-wave series in the (s,t) region of interest.



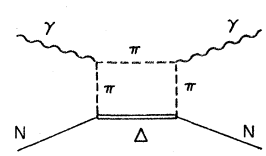


FIG. 8. A diagram *not* causing a divergence of the partial-wave series in the (s,t) region of interest.

[51] H. Harari, in *Proceedings of the 1967 International Symposium on Electron and Photon Interactions at High Energy* (U. S. Atomic Energy Commission, Vienna, 1967)). This review talk contains a good criticism of the Stichel-Scholz and other models for $\gamma N \to \pi\Delta$.

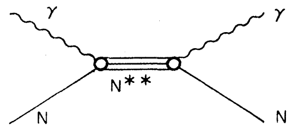


FIG. 9. A diagram representing our treatment of inelasticity *not* due to the $\pi\Delta$ state.

so as to get the fraction not already included in the πN and $\pi\Delta$ states.

Since we must use both the $\gamma N \to \pi N$ multipole analyses[2] and the $\pi N \to \pi N$ phase shifts[8] in order to extract the $\gamma N \to N^{**}$ coupling by factorization, the incoherent resonance contribution is ambiguous because of differences in the resonance mass and width parameters in Refs. 2 and 8. There are further ambiguities due to our inaccurate knowledge[52] of the $\pi\Delta$ partial widths $\Gamma_{\pi\Delta}$ and because of defects in the treatment of resonances in our model for $\gamma N \to \pi\Delta$. These ambiguities are taken into account in our error analysis.

Finally, we would like to record a possibly more fundamental objection to the simulation of inelastic effects in weak amplitudes using a resonance-dominance model. In hadronic amplitudes, large t-channel contributions (such as our π exchange in $\gamma N \to \pi\Delta$) violate the unitarity bound in the s channel and usually lead to an s-channel resonance which can give an alternate description of the t-channel phenomenon. In weak processes such as photoproduction and Compton scattering, there is no unitarity bound and there is less reason to believe that t-channel exchanges can be reasonably described by s-channel resonances. We realize that vector dominance relates Compton scattering to strong processes (e.g., $\rho N \to \rho N$) but this only deepens the mystery.[53]

C. Errors in Evaluation of Sum Rule

We assigned errors to our sum rules by the following method. Divide the contribution to the sum rule into ten pieces. Seven of these coming from the πN intermediate state (namely, Walker's 6 resonant partial waves S_{11}, P_{11}, P_{33}, D_{13}, D_{15}, F_{15} plus the sum of nonresonant partial waves) plus one piece each for the $\pi\Delta$ and non-$\pi\Delta$ inelastic contributions. The last contribution is the nucleon form factor needed for the fixed pole in the current-albebra sum rules $B_{1\to 3}{}^4$. The error in the last is estimated from the dispersion in the various fits to the form factors.[54] The first nine quantities were assigned preset errors ranging from 10% for well-determined isovector photon couplings to 100% for some isoscalar couplings. The size of the discontinuity between BDW and Walker at 0.5 GeV was a help in judging these errors. The total error is found by adding the above as uncorrelated errors to an error estimated from assuming the discontinuity at 0.5 GeV propagated over an s range chosen as 0.3 GeV^2.

Although this arbitrary method cannot be trusted to give more than a rough indication of the error in any given sum rule, we might hope that it does give an accurate picture of the relative errors of the sum rules for different isospins, helicities, and t values.

D. Fit to σ_{total} (γp)

Recent measurements of the total hadron photoproduction cross section on protons[55] permit an independent test of our treatment of the photoproduction data. The experimental situation is shown in Fig. 10. The single-pion $(\gamma p \to \pi^+ n + \gamma p \to \pi^0 p)$ photoproduction cross sections dominate up to lab energies just below 0.5 GeV, where the two-pion cross section begins to rise rapidly. The latter is clearly dominated by the $\gamma p \to \pi^- \Delta^{++}$ process. The shaded band is the "prediction," errors included, of our model for $\sigma_{\text{total}}(\gamma p)$. It is the sum of single-π photoproduction multipoles plus the Stichel-Scholz parametrization of $\gamma p \to \pi^- \Delta^{++}$ plus the incoherent resonance sum. The good fit of the shaded band to the two directly measured $\sigma_{\text{total}}(\gamma p)$ points indicates that the neglected processes, such as $\gamma p \to \eta p$, give contributions smaller than errors, a fact which is substantiated by individual cross-section measurements.[46]

Both the single-pion and $\pi\Delta$ photoproduction cross sections, which are π exchange dominated, begin to fall rapidly at $E_{\text{lab}} = 1.1$ GeV, and our model, based on the sum of these processes, also falls. Photoproduction processes which can proceed diffractively, such as $\gamma p \to \rho p$, begin to dominate $\sigma_{\text{total}}(\gamma p)$ above 1.1 GeV. In the absence of accurate dynamical models for these processes, we are forced to cut off all sum rules at the 1.1-GeV value.

VI. ANALYSIS OF SUM RULES

A. General Properties

We finally come to a description of our evaluation of the sum rules (23). We have calculated the left-hand side of (23) for t varying between 0 and -0.9 and for all 26 sum rules corresponding to the various spin and isospin states. We have also taken different values of n in the range 0–3, thus obtaining information about both right- and wrong-signature fixed poles in (23). We have selected from these the most interesting sum rules and present our results graphically in Figs. 11–28. Before commenting on the significance of these results, we will describe the meaning of the sundry quantities plotted in the figures.

[52] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).

[53] We recall the well-known dilemma that $\rho N \to \rho N$ has no right-signature fixed poles while Compton scattering certainly has them.

[54] L. H. Chan, K. W. Chen, J. R. Dunning, Jr., N. F. Ramsey, J. K. Walker, and R. Wilson, Phys. Rev. **141**, 1298 (1966).

[55] H. G. Hilbert *et al.*, Phys. Letters **27B**, 474 (1968); J. Ballam *et al.*, Phys. Rev. Letters **21**, 1544 (1968); J. T. Beale *et al.*, Cal Tech Report No. CTSL-42 (unpublished).

The integrals $I_j^i(n)$ are defined to be the left-hand side of (23) evaluated in units such that $\hbar=c=\mathrm{GeV}=1$. Thus,

$$I_j^i(n)=-(t/2)^n C_j^i(t)+\frac{1}{\pi}\int_{\nu_0}^{\nu_c} d\nu\, \nu^n\, \mathrm{Im}B_j^i(\nu,t)\,, \quad (41)$$

where the first term is the Born contribution.

Here the cutoff ν_c corresponds to a photon lab energy of 1.12 above which the published data[54] on $\sigma_{\mathrm{total}}(\gamma p)$ shows our model for $\mathrm{Im}B_j^i$ to be undoubtedly wrong.

In the graphs, ◇ represents the integrals $I_j^i(n)$ with errors estimated as described in Sec. V C. The integrals are evaluated using the BDW multipole analysis[1] from the threshold to 0.5 GeV and Walker's analysis[2] above that energy. All the sum rules have also been evaluated with Walker's multipoles for the whole energy range, eliminating BDW. Usually, the difference between these evaluations is smaller than our estimated errors but where they differ significantly we also graph the pure Walker evaluation of $I_j^i(n)$ which we denote by □.

The Born-term contribution to $I_j^i(n)$ is represented by a solid line where in the current-algebra sum rules $I_{1,2,3}{}^4$ this also includes the fixed-pole contribution. In $I_2^4(1)$, the dashed line indicates the Born term without the fixed pole.

The lowest value $n=n_{\min}$ (0 or 1) such that (23) is a right-signature sum rule is given in Table I. In theory, one may use the value of $I_j^i(n_{\min})/I_j^i(n_{\min}+2)$ to estimate a value for the intercept α of the Regge pole assumed to saturate both sum rules. However, the presence of unknown fixed poles in $I_j^i(n_{\min}+2)$ renders this dubious in our case. Instead, for sum rules $I_j^i(n\neq n_{\min})$ we plot the quantity (denoted by Δ on the graph)

$$Q_j^i(n)=\frac{\alpha-\lambda+n_{\min}+1}{\alpha-\lambda+n+1}\nu_c^{\,n-n_{\min}} I_j^i(n_{\min})\,, \quad (42)$$

where for α we put the values already known from the analysis of strong interactions. We include generous errors in our knowledge of α in the plotted errors of $Q_j^i(n)$. If $Q_j^i(n)$ and $I_j^i(n)$ differ significantly, it may indicate the presence of a fixed pole.

In spin type 2, we indicate with X an estimate of the nonasymptotic parts of P, P', ρ, and A_2 exchange calculated from (19), (20) and Appendix C as

$$N_2^i(n)=-\frac{(2-\alpha)t}{2\alpha}\frac{(\alpha+n_3-1)}{(\alpha+n-2)}\nu_c^{\,n-n_3-1} I_3^i(n_3)\,, \quad (43)$$

where n_3 is the value of $n_{\min}$ for spin type 3 and the same isospin i.

Finally, in the conspiracy sum rules (spin type 6) we indicate with a ∇ symbol an estimate of the nonconspiring contribution calculated from factorization as

$$I_6^i(0)=\frac{-t}{2\nu_c^2}\frac{\alpha+2}{\alpha} I_5^i(1) I_3^{i'}(1)/I_1^{i'}(1)\,, \quad (44)$$

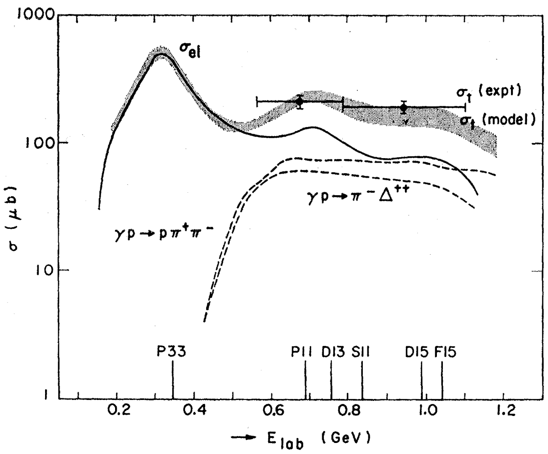


FIG. 10. Contributions to $\sigma_{\mathrm{total}}(\gamma p)$ for lab energies $E_{\mathrm{lab}}<1.2$ GeV. The solid line σ_{el} is the sum of the single-pion photoproduction cross sections $\gamma p\to\pi^+ n$ and $\gamma p\to\pi^0 p$ as compiled by Beale *et al.* (Ref. 54). The experimental points with error bars are direct measurements of the total cross section by the DESY group (Ref. 54). Also plotted as dashed lines are the experimental (Ref. 46) $\gamma p\to p\pi^+\pi^-$ cross section and the part of this due to the $\pi^-\Delta^{++}$ state. The shaded band is the "prediction" for $\sigma_{\mathrm{total}}(\gamma p)$ of the model described in Sec. V.

and we restrict to $i=1, 2, 3$ as $i=4$ has a (known) fixed pole. For $i=3$ we have $i'=3$, while for $i=1, 2$ we take $i'=2$ as being more reliable than $i'=1$ (because isovector photon couplings are more accurately determined than isoscalar).

The main tools in the analysis of our results are the sum rule graphs just described. Perhaps the most important thing we are interested in is to discriminate between Regge-pole and fixed-pole contributions to the sum rules. For higher-moment sum rules, this can be done through the quantity $Q_j^i(n)$ of Eq. (42). For some lowest-moment ($n=n_{\min}$) sum rules, we exploit the factorization property of Regge residues [this has already been used in obtaining (44)]. For example, the amplitudes $\mathrm{Im}B_1^i$ and $\mathrm{Im}B_3^i$ are dominated at high energy by, respectively, the nucleon helicity nonflip and nucleon helicity flip couplings of the *same* Regge pole. If there are no fixed-pole contributions to the sum rules I_1^i and I_3^i, then factorization (see Appendices B and C) requires

$$I_1^i/I_3^i=N_n/2N_f=(4m^2-t)A'/2\nu_c B\,, \quad (45)$$

where A' and B are the conventional nonflip and flip residues used to describe πN and KN scattering.[56] If the sum-rule ratio agrees with the value calculated from hadronic processes, then we have evidence suggesting that the fixed-pole contribution to these sum rules is unimportant.

Another quantity which is sensitive to fixed-pole contributions to the sum rules is the effective trajectory $\alpha_j^i(n)$ (which is also a function of t) defined numerically

[56] W. Rarita, R. J. Riddell, Jr., C. B. Chiu, and R. J. N. Phillips, Phys. Rev. **165**, 1615 (1968).

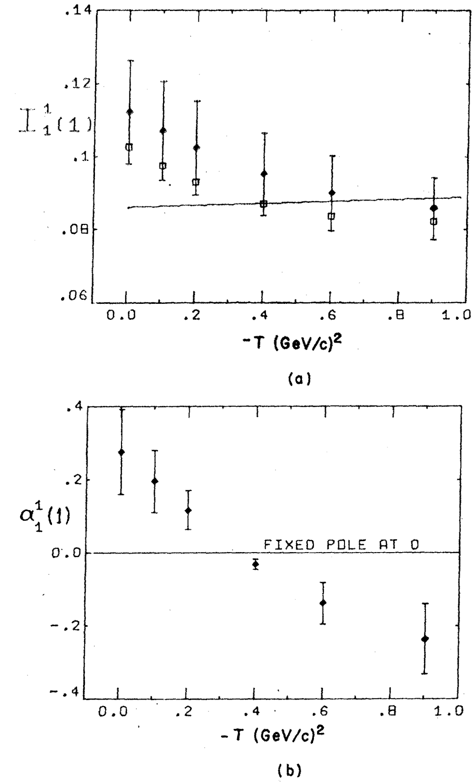


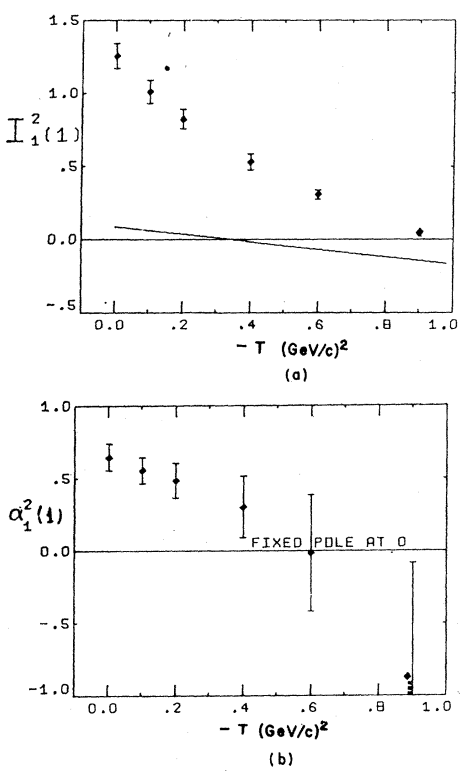
FIG. 11. Pomeranchuk-exchange nonflip sum rule (isoscalar photons). See VI A for the graphical notation and VI B for comments. (a) The $n=1$ sum rule $I_1^1(1)$. (b) The corresponding effective α.

by

$$\alpha_j^i(n) = (\lambda - n - 1) + \nu^{n+1}\, \mathrm{Im} B_j^i(\nu,t)/\pi I_j^i(n)\,, \qquad (46)$$

where we average the numerator over energies E_{lab} between 0.88 and 1.12 GeV. This quantity is the effective trajectory $\alpha(t)$ whose Regge term [as in (21)] both saturates the sum rule $I_j^i(n)$ and fits the imaginary part data averaged over the upper end of our integration range. By examining Eqs. (21) and (23), one can see the following. If $\alpha_j^i(n)$ comes out reasonably close in shape to the trajectory known to couple to the amplitude B_j^i, then this indicates that the fixed pole in that sum rule is weak. However, if $\alpha_j^i(n)$ turns out closer to the fixed-pole value $(\lambda - n - 1)$ to which the sum rule $I_j^i(n)$ is sensitive, then we have evidence for a strong fixed pole which contributes to the denominator in (46) but not to the numerator since a fixed-pole term is purely real.

Graphs of the quantity $\alpha_j^i(n)$ are used whenever their accuracy allows useful information to be extracted. The plotted errors in the graphs include those of $I_j^i(n)$ and the dispersion obtained by varying the numerator in (46) over the energy range 0.88–1.12 GeV. Unfortunately, $\alpha_j^i(n)$ is rather sensitive to errors in the parametrization of the data near 1 GeV and depends on the dubious assumption of the validity of Regge behavior at this low energy. For this reason, evidence from the effective α graphs must be taken with a grain of salt.



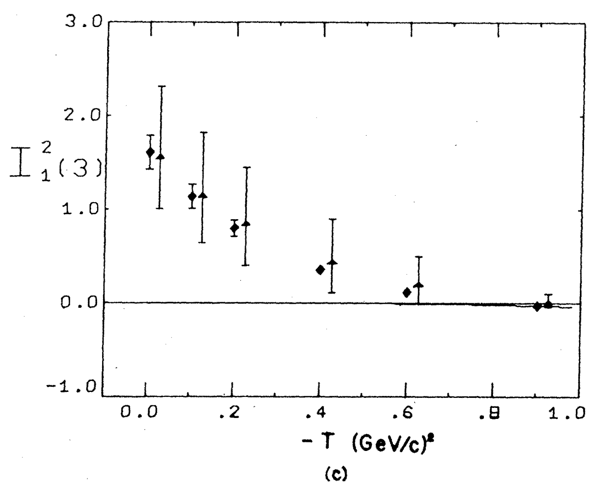


FIG. 12. Pomeranchuk-exchange nonflip sum rule (isovector photons). See VI B for comments. (a) The $n=1$ sum rule $I_1^2(1)$. (b) The effective α corresponding to (a). (c) The $n=3$ sum rule $I_1^2(3)$.

B. Regge-Pole Sum Rules: $I_{1,3}{}^{1,2,3}$

Although right-signature fixed poles can be present in the amplitudes $B_{1,3}{}^{1,2,3}$, there is no compelling theoretical reason, such as would follow from the mechanisms discussed in Sec. IV A, for them to be present. Therefore, we might expect the right-signature sum rules ($n=1$, 3) for these amplitudes to be dominated by the P, P', and A_2 Regge poles. Further, we should expect reasonable answers from these sum rules, because they are at least as convergent as the corresponding low-moment sum rules in πN and KN scattering.[9]

If there are no $j=0$ fixed poles, then the $n=1$ sum rules $I_{1,3}{}^{1,2,3}(1)$ should directly measure the photon (helicity flip) couplings of the P, P', and A_2, and the quotient $I_1{}^i(1)/I_3{}^i(1)$ should reveal, through Eq. (45), the same nonflip/flip nucleon coupling ratio obtained by analyzing πN, KN, and NN elastic scattering. The current models[7,56,57] for these amplitudes would lead us to believe that near $t=0$

$$\begin{aligned} &A'/\nu B \quad \text{for } P \text{ and } P'\sim\tfrac{1}{2}\,, \\ &A'/\nu B \quad \text{for } \rho \text{ and } A_2\sim 1/20\,, \end{aligned} \tag{47}$$

remembering that our definition of ν is $2m$ larger than the usual $(s-u)/4m$.

There is also some evidence that the amplitude A' has an additional zero for P' and A_2 near $t\sim-0.5$ over and above that needed to erase the ghost. The evidence for this zero comes from a photoproduction FESR[47] for the A_2 while for P' the zero is indicated by πN FESR's[7] and also by the structure in $p\bar{p}$ elastic scattering near $t\sim-0.5$.[58] The work of Refs. 47 and 58 was claimed to be evidence for the so-called no-compensation mechanism for the P' and A_2. This has an extra zero in both the flip and nonflip couplings but in fact their analysis was most sensitive to the nonflip zero and for the A_2, at least, one can rule out the flip zero from high-energy data for $\pi N\to\eta N$ and $\pi N\to\eta\Delta$. If this zero is a true effect of the leading Regge trajectory, and not due to interference with secondary trajectories, our sum rules should reproduce it.

1. *P and P′ Exchange Sum rules*: $I_{1,3}{}^{1,2}(1,3)$

Here there are two possible isospin states, 1 and 2, corresponding to isoscalar and isovector photons and one may expect the latter to be more reliable. Thus, in general, the amplitudes involving isoscalar photons will have rather small ImB because the resonance couplings of Walker are larger for isovector than isoscalar photons and because our model for the inelasticity has a very small isoscalar part. Thus isospin-1 sum rules

[57] R. J. N. Phillips and W. Rarita, Phys. Rev. **139**, B1336 (1965); Guy Plaut, Orsay Report (unpublished). The high-energy data are insufficient to determine the Regge-pole parameters for $K^{\pm}p$ elastic scattering. The FESR's of Ref. 7 have clarified the situation to some extent.

[58] C. B. Chiu, S. Y. Chu, and L. L. Wang, Phys. Rev. **161**, 1563 (1967).

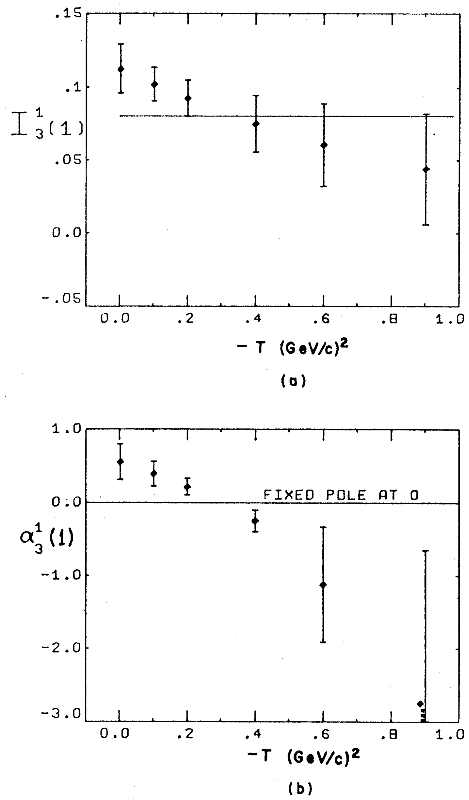


FIG. 13. Pomeranchuk-exchange flip sum rule (isoscalar photons). See VI B for comments. (a) The $n=1$ sum rule $I_3{}^1(1)$. (b) The corresponding effective α.

tend to be dominated by their Born terms which are not always small. Under such circumstances Eq. (46) predicts that the effective α will be nearer the fixed-pole value $\lambda-n-1$ than the intercept of the hoped for Regge pole. One should, however, note that BDW and Walker are not in quantitative agreement (cf. Fig. 3) and such sum rules have a large discontinuity at $E_{\text{lab}}=0.5$ GeV. In Fig. 11(a), we have plotted the results of using Walker from threshold rather than BDW and, as expected, this leads to results showing a smaller deviation of $I_j{}^i(n)$ from its Born value.

The nicest sum rule of this section is $I_1{}^2(1)$ shown in Fig. 12(a). The corresponding α [Fig. 12(b)] estimated as in (46) is in agreement with an expected average $P+P'$ intercept while even the higher-moment sum rule $I_1{}^2(3)$ [Fig. 12(c)] shows agreement with $I_1{}^2(1)$. Both results suggest that there is no important $j=0$ fixed pole.

The corresponding flip sum rule $I_3{}^2(1)$ [Fig. 13(a)] is not so spectacular with both $\alpha_3{}^2(1)$ [Fig. 13(b)] and $I_3{}^2(3)$ (not shown) showing less agreement with the $P+P'$ and preferring a lower intercept.

The isoscalar photon sum rules $I_1{}^1(1)$ and $I_3{}^1(1)$ (Figs. 11 and 14) do not provide striking evidence for or against a fixed pole at $j=0$.

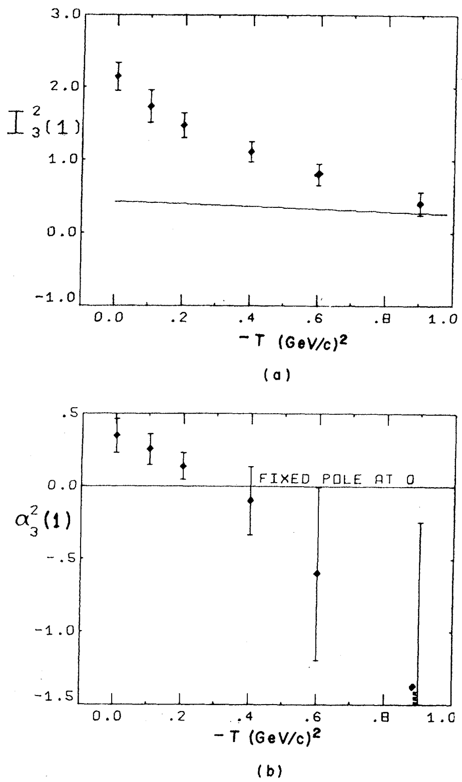


FIG. 14. Pomeranchuk-exchange flip sum rule (isovector photons). See VI B for comments. (a) The $n=1$ sum rule $I_3^2(1)$. (b) The corresponding effective α.

From Eq. (45), we find at $t=0$

$$A'/\nu B \text{ for } P+P' \text{ from isospin } 1\sim 0.6\,, \quad \text{from isospin } 2\sim 0.3\,, \tag{48}$$

which agree reasonably with the πN result of 0.5. Of course it is quite possible that the ratio of P and P' is very different in πN and Compton scattering (and again it may differ here in the two isospin states). However, this does not affect the above argument too much as high-energy data on $\pi^{\pm}p$ polarization suggest[56] $A'/\nu B$ is similar for both P and P'.

In fact,[59,60] one may attempt to calculate the relative amount of P and P' in our amplitudes by using at $t=0$ the linear combination $\frac{1}{2}[I_1^1(1)+I_1^2(1)+I_1^3(1)]\sim\frac{1}{2}I_1^2(1)$ which only involves σ_{total} data for the γp state, and to combine it with the σ_{total} data known up to 7.5 GeV.[55] If one fits the latter to $A\nu^{\alpha_P-1}(1+c\nu^{\alpha_{P'}-\alpha_P})$ subject to the constraint provided by the finite-energy sum rule, one finds

$$\alpha_P=1\,,\quad \alpha_{P'}=0.65 \quad \text{gives} \quad c=5.7\pm5.0\,,$$
$$\alpha_P=1\,,\quad \alpha_{P'}=0.6 \quad \text{gives} \quad c=2.0\pm0.9\,.$$

Thus the closeness of α_P and $\alpha_{P'}$ makes it difficult to disentangle their separate contributions but in any case there is a good simultaneous fit to the FESR and the σ_{total} data. This is in agreement with our rougher estimates $\alpha_1^2(1)$, $I_1^2(3)$ which also indicate there is no necessity for a large $j=0$ right-signature fixed pole.

Our work also agrees with that of Costa *et al.*[60] and Creutz *et al.*[59] The latter authors stress the importance of looking for a $j=0$ fixed pole but it is strange that they should use a sum rule [namely, $I_1^1(3)+I_1^2(3)+I_1^3(3)$], sensitive to $j=-2$ fixed poles, as part of their investigation.

2. A_2 Exchange Sum Rules: $I_{1,3}^3(1)$

Our results are given in Figs. 15 and 16 and both the sum rules and the effective α plots appear to be consistent with A_2 exchange. At $t\approx-0.5$ we expect a zero in $I_1^3(1)$ and none in $I_3^3(1)$ which is not inconsistent with our graphs. At $t=0$ we find from (45)

$$A'/\nu B=1/(7\text{–}15)\,,$$

which is not ridiculous compared with (47). (However, see our comment in VI E.)

On the basis of an argument involving F/D ratios, factorization and a crude evaluation (Born term only) of the $I_1^3(1)$ sum rule for the nucleon and its $SU(3)$ partners Σ and Ξ, Gross and Pagels[10] have suggested that there is an important $j=0$ fixed pole in this sum

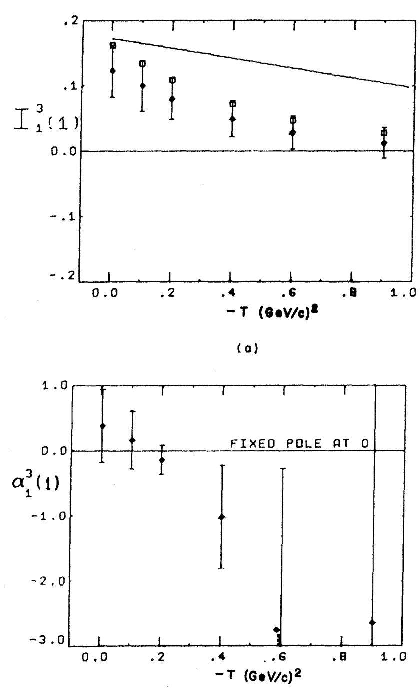


FIG. 15. A_2-exchange nonflip sum rule. See VI B for comments. (a) The $n=1$ sum rule $I_1^3(1)$. (b) The corresponding effective α.

[59] M. J. Creutz, S. D. Drell, and E. A. Paschos, SLAC Report No. SLAC-PUB-499 (unpublished).

[60] G. Costa, C. A. Savoy, and G. Shaw, Nuovo Cimento 57, 890 (1968).

rule. From our more complete saturation of the nucleon sum rule and the associated effective α plot [Fig. 15(b)], we find no evidence for a large fixed pole (particularly if the BDW isoscalar photon multipoles are correct). However, our method is not very sensitive to this because of the closeness of the A_2 intercept to zero. If our findings are to be compatible with Gross and Pagels, then their fixed pole must couple predominantly to the strange baryons.

C. Current-Algebra Sum Rules: $I_{1,2,3}{}^4$

1. Time-Time Sum Rules: $I_{1,3}{}^4$

Here we study the sum rules obtained by taking matrix elements of the equal-time commutator of time components of the isovector current between nculeon states with helicity nonflip $I_1{}^4(0)$ and helicity flip $I_3{}^4(0)$. Although these sum rules are well known,[4,12,13] previous evaluations[61] seem to have been solely concerned with $I_1{}^4(0)$ at $t=0$, where it coincides with the Cabibbo-Radicati sum rule.[12]

These sum rules have Born contributions which are infinite at $t=0$ and require the existence of a $j=1$ fixed pole to produce a finite answer. (See Sec. IV B.) Current algebra, after the usual technical assumptions,[4] predicts that the fixed-pole residues [as defined in Eq. (23)] are

$$\begin{aligned} F_1(t) &= -(2me^2/t)G_E{}^V(t) \quad \text{in} \quad I_1{}^4(0)\,, \\ F_1(t) &= -(2e^2/t)G_M{}^V(t) \quad \text{in} \quad I_3{}^4(0)\,, \end{aligned} \tag{49}$$

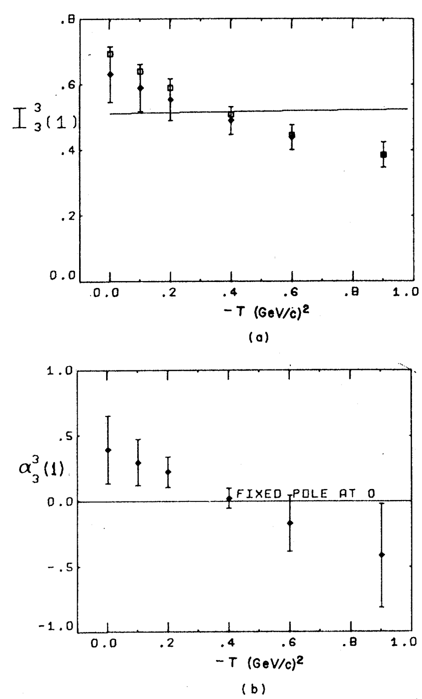


FIG. 16. A_2-exchange flip sum rule. See VI B for comments. (a) The $n=1$ sum rule $I_3{}^3(1)$. (b) The corresponding effective α.

[61] F. J. Gilman and H. J. Schnitzer, Phys. Rev. **150**, 1362 (1966).

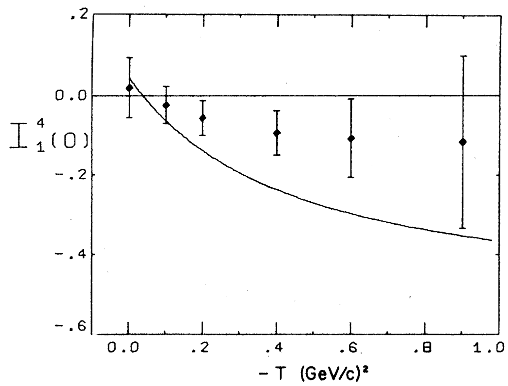


FIG. 17. Nonflip current-algebra sum rule $I_1{}^4(0)$. See VI C 1 for comments.

where $G_E{}^V(t)$ and $G_M{}^V(t)$ are the usual electric and magnetic isovector form factors of the nucleon normalized to $G_E{}^V(0)=1$ and $G_M{}^V(0)=1+\kappa_p-\kappa_n$.

For our test of these sum rules we first note that the ratio of couplings of the ρ Regge pole at $t=0$ can be estimated from πN scattering as $A'/\nu B\approx 1/20$, a number which is reduced by a factor of $2\rightarrow 3$ from its value at the ρ pole $t=m_\rho{}^2$. If factorization holds, we must have for all t [see Appendix B and Eqs. (23), (45), and (49)]

$$\frac{A'}{\nu_c B}=\frac{2}{4m^2-t}\,\frac{I_1{}^4(0)+(2me^2/t)G_E{}^V(t)}{I_3{}^4(0)+(2e^2/t)G_M{}^V(t)}\,. \tag{50}$$

In Figs. 17 and 18(a), the sum rules $I_1{}^4(0)$ and $I_3{}^4(0)$ are plotted with the fixed poles of Eq. (49) subtracted off. If current algebra has supplied us with the correct value of the fixed poles, then the resulting sum rules are superconvergent[5] and for high-energy cutoffs the data points should lie very near to the zero line of the figures. Thus, one is somewhat comforted that the data points lie in between their generalized Born terms and zero.

Since the form factors have been subtracted off, the plotted points of Figs. 17 and 18(a) correspond exactly to the numerator and denominator of the last factor of

TABLE II. $\alpha=0$ sense-nonsense factors. Dependence on α near $\alpha=0$ of Regge-pole vertex functions for various sense-nonsense mechanisms. The helicity nonflip (n) and flip (f) vertex functions N_n, N_f for the hadronic $N\bar{N}$ vertex and P_n, P_f for the weak $\gamma\gamma$ vertex are defined in Appendix B. The α dependence given assumes that possible fixed poles are "additive" in hadronic amplitudes and "multiplicative" in doubly weak amplitudes. The latter corresponds to the case of "singular $\gamma\gamma$ couplings" discussed in the text. "Regular" $\gamma\gamma$ vertex functions would be a factor of α smoother than those listed here.

Mechanism	Signature	N_n	N_f	P_n	P_f
Choosing sense	$-$	1	α	1	1
Choosing nonsense	$+$ or $-$	$\sqrt{\alpha}$	$\sqrt{\alpha}$	$1/\sqrt{\alpha}$	$1/\sqrt{\alpha}$
Chew's mechanism	$+$	$\sqrt{\alpha}$	$\alpha\sqrt{\alpha}$	$1/\sqrt{\alpha}$	$1/\sqrt{\alpha}$
No-compensation mechanism	$+$	α	α	1	1

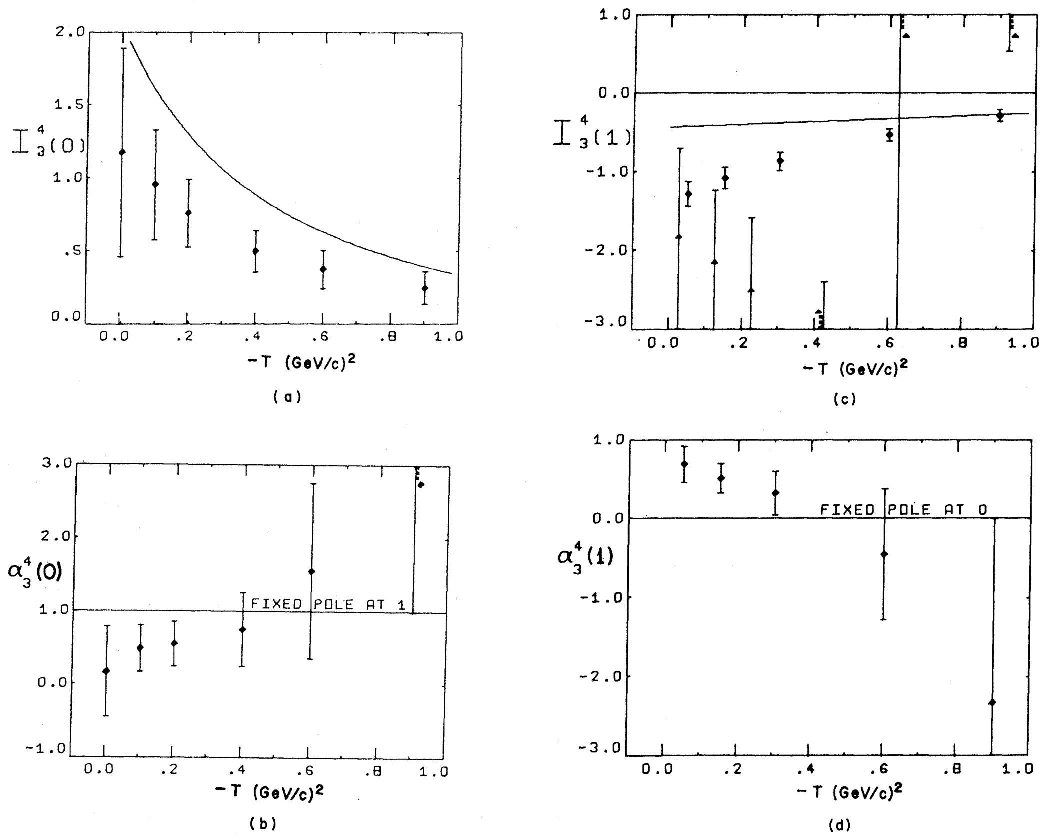


FIG. 18. Spin-flip current-algebra sum rule. See VI C 1 for comments. (a) The $n=0$ sum rule $I_3^4(0)$. (b) The effective α corresponding to (a). (c) The $n=1$ wrong-signature sum rule $I_3^4(1)$. (d) The effective α corresponding to (c).

Eq. (50) and determine the ρ couplings through Eq. (23). We see from the figures that the general character of the sum rules is given by the Born minus fixed-pole contributions. At $t=0$ we have for the finite part of these contributions

$$\begin{aligned}&\text{Born minus fixed pole of } I_1^4(0)\\ &\quad= -0.0244[2.11-7.05(d/dt)G_E{}^V(0)],\\ &\text{Born minus fixed pole of } I_3^4(0)\\ &\quad= -0.026[13.7-7.05(d/dt)G_M{}^V(0)].\end{aligned} \tag{51}$$

Since $(d/dt)G_E{}^V(0)\approx 3.3$ and $(d/dt)G_M{}^V(0)\approx 13.0$, $I_1^4(0)$ exhibits a large cancellation between the finite part of the Born term and the derivative of the form factor. In $I_3^4(0)$, this cancellation does not occur. Therefore, the smallness of the nonflip/flip ratio of the ρ Regge couplings at $t=0$ is qualitatively realized by the Born minus fixed-pole contributions to the sum rules. Note that in the ρ dominance model for the form factors the ratio of the fixed-pole contributions at $t=0$ is essentially the value $A'/\nu B$ at the ρ *pole*. The ratio

TABLE III. Breakup of $I_j^i(n)$ [defined in (41)], at $t=0$, into the contributions of various intermediate states as defined in Sec. VI I.

	Total	Born	Inelastic	πN intermediate state P_{33}	D_{13}	D_{15}	F_{15}	Rest
$I_1^2(1)$	1.25	0.086	0.27	0.39	0.1	0.01	0.04	0.36
$I_1^3(0)$	0.12	0.17	0.008	0.0	−0.01	0.002	0.02	−0.07
$I_1^4(0)$	0.02	0.044	−0.004	0.31	−0.09	−0.01	−0.02	−0.20
$I_2^2(0)$	0.06	0.358	−0.05	−0.43	−0.09	−0.006	−0.02	0.31
$I_2^3(0)$	−0.09	−0.024	−0.005	0.0	0.01	−0.002	−0.01	−0.06
$I_3^4(0)$	1.2	2.04	−0.18	0.45	−0.32	−0.1	−0.06	−0.65
$I_6^2(0)$	0.08	−0.97	−0.15	0.53	−0.02	0.01	−0.002	0.69
$I_7^5(1)$	−0.04	−0.094	0.005	0.0	0.02	−0.002	−0.002	0.02

of the complete contribution to the sum rules (Born minus fixed pole plus continuum) at $t=0$ is in numerical agreement, through Eq. (50), with the πN result if the correct value of $I_1{}^4(0)$ lies in the upper half of the error bar in Fig. (17). Of course, one expects factorization to hold only to the extent that a ρ' contribution[62] is unimportant.

The agreement at $t=0$ extends to nonzero t for $B_1{}^4$ as $I_1{}^4(0)$ remains small for all t. In this sum rule, we expect the Höhler zero[14] at $t\approx -0.2$, and this zero is exhibited in Fig. 17. Indeed, this statement is strengthened by the fact that factorization requires that $I_1{}^4(0)$ lies in the upper part of the error bar at $t=0$. At $\alpha_\rho(t)=0$, we expect a zero in the sum rule (for ρ choosing either sense or nonsense) if the $\rho\rightarrow\gamma\gamma$ coupling is regular and no zero if it is singular (see Sec. IV C, Appendices B and C, and Table II). In our opinion, the data (slightly) favor the absence of a zero. Unfortunately, the effective α calculation [Eq. (46)] for this sum rule is of no use, because the sum rule is so small. We would be dividing by a small number with large errors in (46). Incidentally, at $t=0$, $I_1{}^4(0)$ is in agreement with earlier work[61] both as to the value of the sum rule and the relative size of individual multipole contributions (see Table III).

In $I_3{}^4(0)$, the situation is not so good at large t. The effective trajectory $\alpha_3{}^4(0)$ [Fig. 18(b)] shows little agreement with the expected ρ shape and the large value $\alpha\gtrsim 1$ for $t<-0.5$ would seem to indicate that we should have subtracted off a form factor of larger modulus than $(-2e^2/t)G_M{}^V(t)$. Taken at face value, this is a violation of current algebra. However, it hinges on a rather delicate feature of the data. Thus, $\mathrm{Im}B_3{}^4(\nu,t)$, for $E_{\mathrm{lab}}\approx 1$ GeV, changes sign near $t=-0.5$ because the dominant resonant contribution $[\frac{5}{2}^+(1688)]$ vanishes,[63] and this sign change forces $\alpha_3{}^4(0)$, calculated from Eq. (46), above the fixed-pole value. Although the vanishing of the resonance contribution is perhaps expected,[64] it does mean that the resultant amplitude depends delicately on the more uncertain parameters of Walker's analysis,[2] as well as our own dubious analysis of the inelastic contribution. This, together with our theoretical bias, makes us prefer to ignore this apparent violation of current algebra.

Therefore, assuming that the current-algebra prediction of the fixed pole is correct, we note the interesting point that $I_3{}^4(0)$ has no zero near $\alpha_\rho(t)=0$. If the ρ chooses sense at $\alpha=0$ we expect a double zero if $\rho\rightarrow\gamma\gamma$ is nonsingular and a single zero if it is singular. The ρ choosing nonsense predicts one less zero than the above. Thus our sum rule predicts ρ choosing nonsense with a singular $\rho\rightarrow\gamma\gamma$ coupling. If current algebra were wrong, the larger fixed pole necessary to produce a better $\alpha_3{}^4(0)$ could also produce a zero in the ρ coupling at $\alpha_\rho(t)=0$.

Finally we show the sum rule $I_3{}^4(1)$ and its associated $\alpha_3{}^4(1)$ in Figs. 18(c) and 18(d). The sum rule is sensitive to a wrong-signature fixed pole at $j=0$ which is needed, if our interpretation of the ρ in $I_3{}^4(0)$ is correct, with a singular residue at $\alpha_\rho(t)=0$ in order to cancel the pole of the Regge term. It is evident from Fig. 18(c) that something, presumably the fixed pole, has nicely cancelled the singularity in the Δ contribution, Eq. (42), and has produced a sum rule with a smooth variation in t. The effective $\alpha_3{}^4(1)$ suggests ρ exchange at small $|t|$ and, somewhat dubiously, since the sign change mentioned in connection with $I_3{}^4(0)$ also occurs here, suggests the fixed-pole value at large $|t|$. Therefore, $I_3{}^4(1)$ is certainly not inconsistent with an interpretation that current algebra is correct for $I_3{}^4(0)$, but one must admit $I_3{}^4(1)$ is hardly a stringent test of that interpretation. We do favor the interpretation that current algebra is correct. However, it is rather remarkable, although hopefully coincidental, that $I_3{}^4(1)$ and $\alpha_3{}^4(1)$ are consistent with *no* $j=0$ wrong-signature fixed pole and a ρ with a single zero in its residue function. Unfortunately, as we have seen, such a ρ is inconsistent with the $n=0$ sum rule unless you increase the $j=1$ fixed pole from its current-algebra value (49).

We cannot claim on the basis of this work to have definitely confirmed or refuted current algebra although we do favor the former alternative. First, both the sum rules appear to be converging and second, we obtain agreement near $t=0$ with the hypothesis of ρ dominance of the sum rules once the form-factor terms are subtracted off.

At large t, assuming current algebra is right, we obtain the interesting prediction that ρ chooses nonsense with a singular $\rho\rightarrow\gamma\gamma$ coupling which eliminates the zeros found in ρ couplings to hadronic processes. In this picture of the ρ couplings, the wrong-signature fixed pole at $j=0$ plays very different roles in weak and strong processes. In the strong case, this fixed pole seems to be purely "additive,"[29] giving zeros in the ρ-Regge term but spoiling the Schwarz[17] sum rules. In the weak case, it is "multiplicative" and fills in the zeros.

2. *Time-Space Sum Rules*: $I_2{}^4(1)$

Using low-energy theorems and the assumption of an unsubtracted dispersion relation, Bég[16] obtained a sum rule for the amplitude $B_2{}^4(\nu,t)$ at $t=0$. This sum rule was rederived and extended to all t by Adler and Dashen[4] using the equal-time commutator of the time and space components of the isovector current and the infinite momentum limit. One interesting property of

[62] W. Rarita and B. M. Schwarzschild, Phys. Rev. **162**, 1378 (1967); J. Beaupre, R. Logan, and L. Sertorio, Phys. Rev. Letters **18**, 259 (1967).

[63] To be exact the resonance vanishes at $t\sim -0.3$ while interference with the background moves the zero to $t\sim -0.5$. Note that in the πN scattering the position of such zeros is fixed by the mass and spin of the resonance. In Compton scattering, the location of zeros depends on the relative size of the two possible couplings of the resonance to the γN system.

[64] Thus, it would be natural to associate the zero of $\mathrm{Im}B_3{}^4$ with a zero of the ρ residue function as was done in the πN case (Ref. 6). But then this ρ zero should manifest itself in the sum rule and it does not.

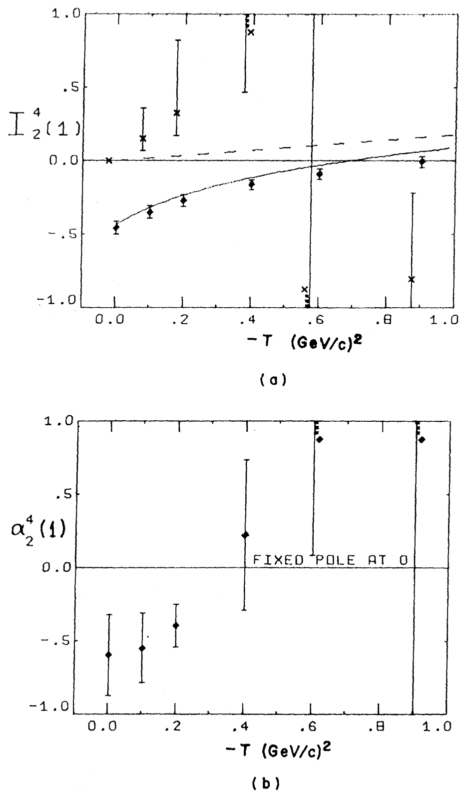


FIG. 19. The time-space current-algebra sum rule. See VI C 2 for comments. (a) The $n=1$ sum rule $I_2{}^4(1)$. (b) The corresponding effective α.

this sum rule is that it is invalid in a field theory of free nucleons, because the infinite momentum damping assumptions fail in that theory. On the basis of Regge theory (Appendix D) the fixed pole (effectively at $j=0$) of $B_2{}^4(\nu,t)$ can be calculated to be

$$F_2(t)=e^2G_M{}^V(t)+H(t)\,, \tag{52}$$

where the first term is the nonasymptotic contribution of the $J^{PG}=1^{-+}$ fixed pole of $B_3{}^4$, and the second term is the contribution of a possible $J^{PG}=0^{-+}$ fixed pole. If the current-algebra derivations of the sum rule are correct, then $H(t)\equiv 0$.

We show $I_2{}^4(1)$ in Fig. 19(a) and $\alpha_2{}^4(1)$ in Fig. 19(b). The X's denote the nonasymptotic contributions of the ρ trajectory which Regge theory permits us to calculate from $I_3{}^4(0)$ (see Appendices C and D). This contribution is meaningless near $\alpha_\rho(t)=0$ because its singularity there must be cancelled by a compensating trajectory.[65] The current-algebra fixed-pole residue $e^2G_M{}^V(t)$ is subtracted off and the combined Born minus fixed pole is plotted as the solid line in Fig. 19(a). The Born term alone is plotted as the dashed line to show the dominant effect of the $e^2G_M{}^V(t)$ term.

If the current-algebra fixed pole was correct then, at least for the mythical high-energy cutoff, the data points ◇ would be expected to lie near the zero line in Fig. 19(a). Since the data points have a sign opposite to the ρ nonasymptotic term (near $t=0$ where the latter might be trusted) and even lie on the wrong side of the generalized Born term, Fig. 19(a) suggests that the current-algebra prediction is wrong and that $H(t)\approx -e^2G_M{}^V(t)$.

However, $\alpha_2{}^4(1)$ does not support this interpretation near $t=0$ and indicates an effective intercept consistent with an X trajectory[4] $[\tau^{PG}=(+)^{-+}]$ with $\alpha_X(0)\approx -0.5$, instead of the fixed pole value of zero. Although the sum rule results are presumably more reliable than the effective α determination at our low cutoff energy, we speculate further on the X trajectory. If $\alpha_X(t)$ stays one unit below the ρ up to $t\approx -0.6$, it could well be the necessary compensator, a possibility which is supported by the fact that the X coupling apparently has opposite sign to the ρ nonasymptotic term. The wild behavior of $\alpha_2{}^4(1)$ for $-t>0.4$ could be due to a complicated cancellation between the ρ and its compensator. On the timelike side, if $\alpha_X(t)$ were roughly parallel to α_ρ one would expect a 0^{-+} meson at reasonably low mass, for which the lowest threshold decay channels are 4π and $K\bar{K}\pi$. Further, if $I_2{}^4(1)$ is satisfied by an X trajectory, not a 0^{-+} fixed pole, this Regge pole will contribute via its nonasymptotic term (see Appendix D) to $I_3{}^4(0)$. This effect is quite large $[\sim 25\%$ of $I_3{}^4(0)]$ at $t=0$ but negligible at the crucial larger $|t|$ values.

In summary, although the sum rule $I_2{}^4(1)$ seems to show that the current-algebra prediction is incorrect, and that the fixed-pole value is much nearer the free-field-theory value of zero, the effective $\alpha_2{}^4(1)$ plot allows us to explain this on the basis of a large X trajectory contribution.

D. Antialgebra Sum Rules: $I_{1,3}{}^{1-3}(0)$, $I_7{}^5(0)$

Current algebra purports to associate right-signature $j=1$ fixed poles with the equal-time commutators of currents satisfying pretty algebraic properties. In Sec. IV D, we anticipated the proposal of a fundamental algebra of anticommutators to describe wrong-signature fixed poles at $j=1$. Of particular interest are those sum rules which share with $I_{1,3}{}^4(0)$ the property of having Born terms which are singular at $t=0$. In the current-algebra case, this normalization condition on the fixed pole, in terms of the Born singularity, corresponds to current conservation.

Because the singular Born-term mechanism (discussed in Ref. 18 and our Sec. IV A) applies, the sum rules $I_{1,3}{}^3(0)$ and $I_7{}^5(0)$ are guaranteed to exhibit wrong-signature $j=1$ fixed poles with singular coupling strength at $t=0$ fixed by the Born term. Since isoscalar photons with small continuum contributions are involved, we also expect that the fixed-pole couplings at large $|t|$ follows the shape of the Born term. In the

[65] M. Gell-Mann, M. L. Goldberger, F. E. Low, E. Marx, and F. Zachariasen, Phys. Rev. 133, B145 (1964).

case of right-signature $j=1$ fixed poles (if current algebra is correct), this is not true because the fixed-pole couplings display the marked t dependence of the form factors, Eq. (49).

As a typical example we show $I_1{}^3(0)$ in Fig. 20. It is clear that the data points $\diamondsuit$ follow the Born term (solid line) and lie far from the Δ points calculated, Eq. (42), assuming no wrong-signature fixed pole. Because the continuum contribution is small, $\alpha_1{}^3(0)$ would clearly support the fixed-pole interpretation.

Because the Pomerànchuk pole [with $\alpha_P(t)=1$ at $t=0$] is present, the sum rules $I_{1,3}{}^{1,2}(0)$ need not have a wrong-signature $j=1$ fixed pole but can be satisfied by the Pomeranchuk Regge-pole term with singular coupling at $t=0$. $I_1{}^2(0)$ is presented in Fig. 21. The lack of correspondence between the sum-rule points $\diamondsuit$ and the $P+P'$ contribution Δ calculated from $I_1{}^2(1)$ definitely shows the existence of a strong $j=1$ fixed pole, and this interpretation is supported by $\alpha_1{}^2(0)$ (not shown).

The interesting behavior of $I_1{}^2(0)$ at large $|t|$ should be noted. Comparison of the data points $\diamondsuit$ with the Regge contribution Δ shows that the sum rule is dominated by the fixed-pole term even for $|t|\geq 0.6$. The fact that the wrong-signature fixed-pole couplings do not decrease rapidly with increasing $-t$ may be related to the presence of left-hand cuts in the wrong-signature couplings not present in the right-signature case.

The formula

$$\sigma_T=2\pi e^2\alpha_P'(0)[\tfrac{1}{4}Y^2+\tfrac{1}{3}I(I+1)], \qquad (53)$$

for the total photon cross section on hadron targets of hypercharge Y and isospin I, was derived in Ref. 18 assuming pure Pomeranchuk pole dominance. Existence of the wrong-signature $j=1$ fixed pole invalidates this formula, at least for nucleons. Equation (53) is very dubious on other grounds, since, using factorization, one can derive from it clearly erroneous results for the ratio of asymptotic total cross sections for any strongly interacting system. Neither our sum rules nor the factorization argument directly invalidates the weaker hypothesis—namely, absence of the $j=1$ fixed pole in $\gamma\pi\to\gamma\pi$ only—used by Mueller and Trueman.[66]

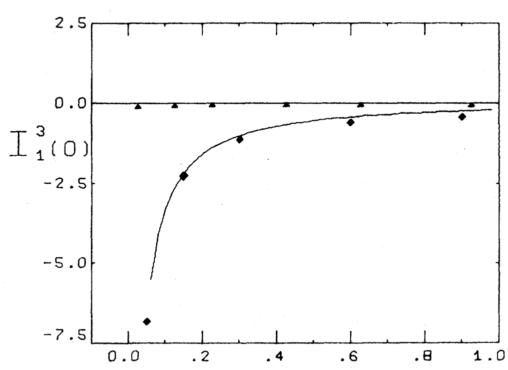


FIG. 20. A_2-exchange nonflip wrong-signature sum rule $I_1{}^3(0)$. See VI D for comments.

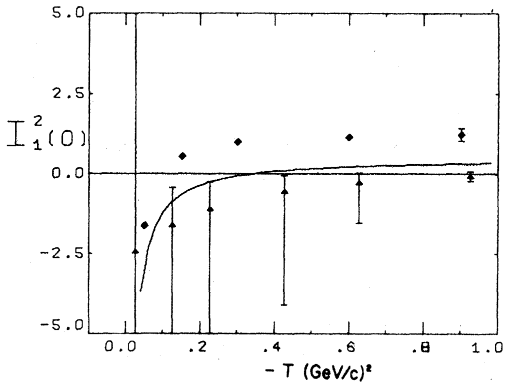


FIG. 21. Pomeranchuk-exchange nonflip wrong-signature sum rule $I_1{}^2(0)$. See VI D for comments.

E. Drell-Hearn Sum Rules: $I_2{}^{1-3}(0)$

These sum rules are sensitive to right-signature $j^P=1^+$ fixed poles in the amplitudes $B_2{}^{1-3}(\nu,t)$. If conventional theory is correct, the fixed poles are absent and, since we have helicity flip $\lambda=2$, the amplitudes satisfy superconvergence relations.[5,67] In explanation of the phrase "conventional theory," we cite two facts. First, the assumption of superconvergence for $B_2(\nu,t)$ is, at $t=0$, equivalent[68] to the assumption, used in the original derivation[19] of the Drell-Hearn sum rule, of low-energy theorem plus unsubtracted dispersion relation for the forward spin-flip Compton amplitude $f_2(\nu)$. Second, it would seem that the superconvergent sum rules follow from the conventional algebra of the time component of the appropriate isospin part of the electromagnetic current plus the usual technical assumptions of the infinite-momentum method.[4]

Drell and Hearn considered only the proton sum rule obtained by adding $\frac{1}{2}[I_2{}^1(0)+I_2{}^2(0)+I_2{}^3(0)]$, but, at the cost of using the more uncertain isoscalar photon data, we investigate all three sum rules. Normal parity contributions to $B_2(\nu,t)$ are suppressed by one power of energy,[69] and we therefore consider the abnormal-parity trajectories D and E as well as the normal P and P' in isospins 1 and 2 ($I=0$ exchange) and the abnormal A_1 and normal A_2 in isospin 3

[66] A. H. Mueller and T. L. Trueman, Phys. Rev. **160**, 1306 (1967).

[67] T. L. Trueman, Phys. Rev. Letters **17**, 1198 (1966); **18**, 822 (1967).

[68] For a discussion of this point see the Appendix of S. R. Choudhury and D. Z. Freedman, Phys. Rev. **168**, 1739 (1968).

[69] We have plotted our estimates of these nonasymptotic terms in the figures and they are always small.

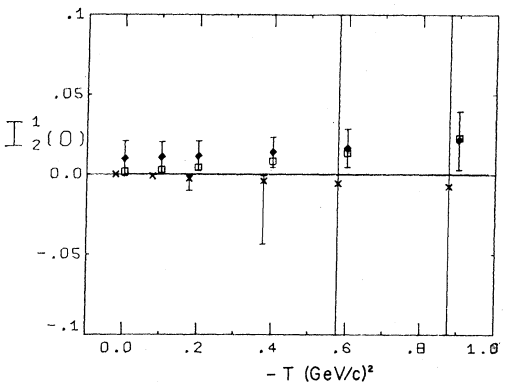


FIG. 22. Drell-Hearn sum rule $I_2^1(0)$ (isoscalar photons). See VI E for comments.

($I=1$ exchange). We write schematically

$$I_2^{1,2}(0) \sim \nu_c^{\alpha_{D,E}(t)-1} + \nu_c^{\alpha_{P,P'}(t)-2}, \quad I_2^3(0) \sim \nu_c^{\alpha_{A1}(t)-1} + \nu_c^{\alpha_{A2}(t)-2}, \tag{54}$$

indicating the asymptotic powers of the Regge-pole contributions.

On the basis of the expected intercepts of these Regge poles, all three sum rules should superconverge at large cutoff energy. However, some doubt has been expressed[66] concerning the convergence of the $I=0$ exchange sum rules on the basis of Regge-cut theory. If there are important abnormal-parity components of the two-Pomeranchuk Regge cut, then to within logarithms we would expect $\mathrm{Im}B_2^{1,2}(\nu,t) \sim \nu^{-1}$ and the corresponding sum rules would diverge. Note that a fixed pole would make $\mathrm{Re}B_2 \sim \nu^{-1}$ and the sum-rule integral would still converge.

Our results are presented in Figs. 22–24. If the superconvergence assumptions (rapid falloff of $\mathrm{Im}B_2$, absence of fixed pole in $\mathrm{Re}B_2$) are satisfied, then at sufficiently high cutoff the data points should lie right on the zero line in the graphs. The value of $I_2^1(0)$ (Fig. 22) is very small and seems quite satisfactory[20] within the large errors (see Fig. 3 for the disturbing picture of

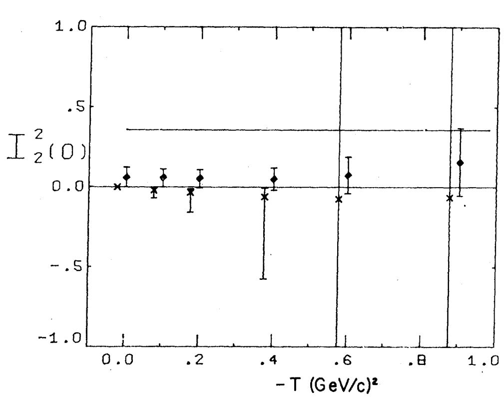


FIG. 23. Drell-Hearn sum rule $I_2^2(0)$ (isovector photons). See VI E for comments.

$\mathrm{Im}B_2^1$). The sum rule $I_2^2(0)$ (Fig. 23) shows an impressive cancellation[19,20] between the Born term and the continuum for all t. The data points are consistent with zero (within errors) even at our low cutoff energy, and the sum rule must be deemed a success.

In $I_2^3(0)$, Fig. 24(a), on the other hand, continuum and Born-term reinforce, for both the pure Walker and the BDW plus Walker evaluations, and produce a sum rule which gives no hint of the expected superconvergence. This judgment is based on relative size of sum rule and Born contribution rather than on the absolute size of the former. Although the rule of thumb that the scale of a convergent sum rule is set by its Born term has proven quite reasonable, it is not clear *a priori* that it should be true, and it therefore becomes important to compute $\alpha_2^3(0)$.

In the context of this sum rule, the question answered by the effective α calculation can be rephrased as follows. What is the trajectory shape $\alpha(t)$ whose Regge term fits our observed sum-rule result at cutoff 1.12 GeV, but would hopefully make the sum-rule superconverge to the zero line at higher energies? It is clear from Fig. 24(b) that $\alpha_2^3(0)$ exceeds even the Froissart bound for small t (it could not produce superconvergence) and lies much higher than the expected α_{A_1} or $\alpha_{A_2}-1$ trajectories. Therefore, the only way we can interpret these results is to say that there is an important axial-vector ($J^{PG}=1^{+-}$) fixed-pole contribution.

This is our most surprising result. The Drell-Hearn sum rule fails in the isospin segment where one would have least expected failure. Such a fixed pole would invalidate either the usual current algebra or the technical assumptions necessary to derive the covariant sum rule $I_2^3(0)$ from the antecedent equal-time commutator.

Although this fixed pole seriously challenges our theoretical ideas, it seems to have one beneficial effect on our sum-rule results as follows. As shown in Appendix D, an axial-vector fixed pole with coupling $A(t)$ to the amplitude B_2^3 also contributes nonasymptotically to B_3^3. We take $A(t)$ from $I_2^3(0)$, and assume that its nonasymptotic effect in B_3^3 is not modified by a possible 0^+ fixed pole there [$S(t)$ in Eq. (D14)]. We then recalculate at $t=0$ the nonflip/flip ratio [Eq. (45)] for the A_2 Regge pole [assuming domination of $I_3^3(1)$ by the A_2 and the fixed pole]. This gives a decreased value in better agreement, with the expected $A'/\nu B$ of strong interactions, than the previous value [calculated assuming $A(t)=S(t)=0$].

We close this section by reminding any remaining readers that the Drell-Hearn proton sum rule, obtained by adding our three isotopic components, agrees with the original analysis[19] within errors.

F. Conspiracy Sum Rules: $I_6^{1\to3}(0)$, $I_7^5(1)$

We have discussed the theory of these sum rules in Sec. IV D.

As pointed out by Pagels[20] there is cancellation in $I_6^2(0)$ between the continuum and the Born terms, with the result that both $I_6^{1,2}(0)$ (Figs. 25 and 26) are consistent with zero at $t=0$. Thus, we have evidence against a large conspiring pole with vacuum quantum numbers. Correspondingly, there is no hope of using these sum rules to obtain information on the $\eta \to 2\gamma$ coupling.

For the pion conspirator sum rule $I_6^3(0)$ [Fig. 27(a)] we confirm Pagels's result[20] at $t=0$ but the flatness in t of $\alpha_6^3(0)$ [Fig. 27(b)] bears more resemblance to a right-signature $j=0$ fixed pole than a pion conspirator Regge trajectory. In fairness, it must be said there is little reliable information from purely strong interactions on the slope of the conspirator and recent[70] photoproduction data suggest that the intercept is essentially zero up to $-t=2$ GeV^2.

We note that determination of the $\pi^0 \to 2\gamma$ coupling through the Pagels sum rule critically involves the assumption of smooth extrapolation to $t=0$ of the π-pole term. In similar kinematic configurations involving π exchange (e.g., $\gamma p \to \pi^+ n$, $np \to pn$), the π exchange amplitude is more consistent with the rapidly varying form $(2m_\pi^2)^{-1}(t+m_\pi^2)(t-m_\pi^2)^{-1}$ near $t=0$ rather than the smooth pole form $(t-m_\pi^2)^{-1}$ taken by Pagels. It is not clear whether the rapidly varying form should apply to doubly weak Compton scattering since the success of the absorptive model for π exchange suggests that the rapid variation is connected with the strong interaction unitarity condition.

From our numerical result for $I_6^3(0)$ at $t=0$, we obtain through Eq. (36) the prediction $\tau_{\pi_0}=2.5\times 10^{-16}$ sec on the basis of a smooth π-pole residue which would become a factor of 4 smaller if the rapidly varying term above were used. These two values[20,52] quite closely enclose the possible range of experimental values, although the second possibility, rapidly varying pole term, would seem to be preferred on the basis of the wallet-card value.[52]

In principle, we can test whether the zero at $t=-m_\pi^2$ of the rapidly varying term is the factorable zero of a π-Regge-pole residue by studying the sum rule $I_7^5(1)$, Fig. 28(a), to which the π-Regge trajectory should couple although there is no π pole at $t=m_\pi^2$ because of photon helicity flip. The sum rule shows no hint of a zero. However, any attempt to use this fact to speculate about π meson Reggeization would be thwarted because $\alpha_7^5(1)$, Fig. 28(b), suggests an effective trajectory somewhat lower than π. Although the zero in question is suggested by simple π conspiracy models for $np \to pn$ and $\gamma p \to \pi^+ n$,[71] there is ample evidence from strong processes that[72] the zero does not factorize.

[70] B. Richter, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, 1968* (CERN, Geneva, 1968).
[71] R. Brower and J. W. Dash, Phys. Rev. **175**, 2014 (1968).
[72] G. C. Fox and L. Sertorio, Phys. Rev. **176**, 739 (1968); G. V. Dass and C. D. Froggatt, Rutherford Laboratory Report No. RPP/A 46 (unpublished).

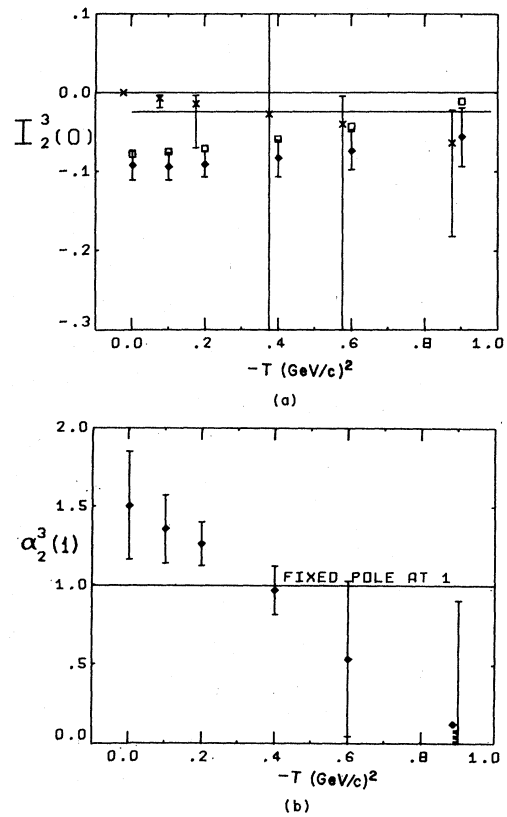


FIG. 24. Drell-Hearn sum rule (isovector exchange). See VI E for comments. (a) The $n=0$ sum rule $I_2^3(0)$. (b) The corresponding effective α.

G. Other Sum Rules (Spin Segments 4, 5, and 8)

Spin types 4 and 5 are too divergent for useful information to be obtained from our low cutoff. We tried to use spin type 5 to predict the nonconspiring contribution to spin type 6, through Eq. (44), and obtained only untrustworthy and useless results. The sum rule $I_5^4(0)$ has an unknown fixed pole at $j=-1$ necessary to cancel the singular Born term.

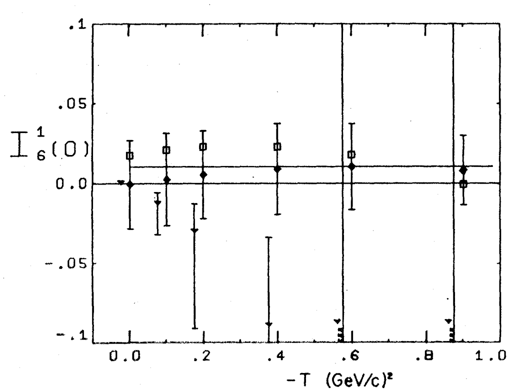


FIG. 25. η conspirator sum rule $I_6^1(0)$ (isoscalar photons). See VI F for comments.

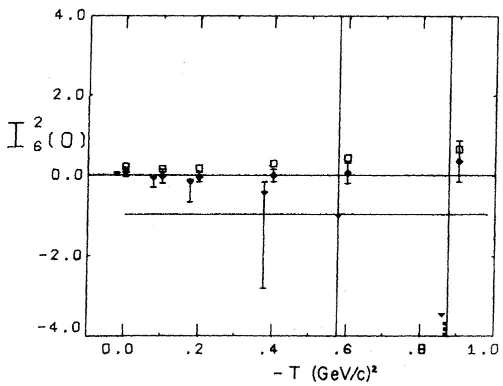


FIG. 26. η conspirator sum rule $I_6^2(0)$ (isovector photons). See VI F for comments.

Unfortunately $I_4^4(0)$ and $I_6^4(1)$ have the same continuum but different Born terms. Thus we need a fixed pole in one or both of them. It is presumably in $I_6^4(1)$ because this has $\tau P=+$ and it would then be the spinflip analog of the $I_5^4(0)$ fixed pole. However, the sum rules (not shown), if anything, prefer the assignment of a fixed pole to $I_4^4(0)$.

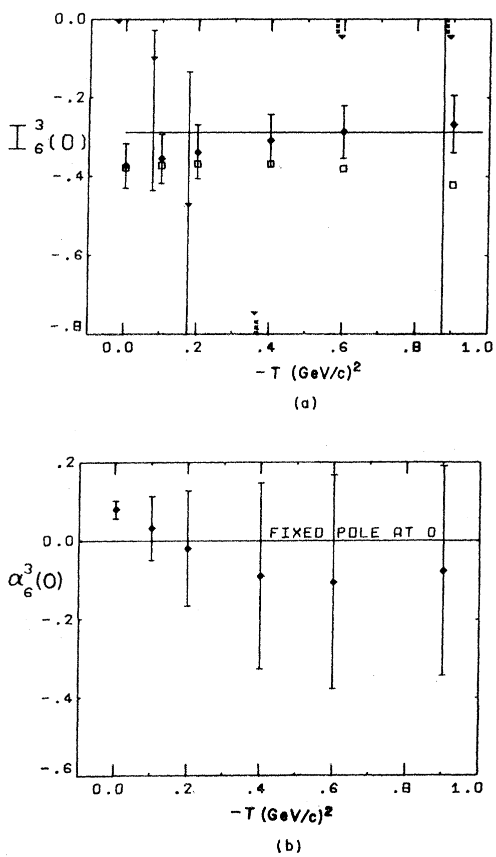


FIG. 27. π conspirator sum rule. See VI F for comments. (a) The $n=0$ sum rule $I_6^3(0)$. (b) The corresponding effective α.

Finally, $I_8^5(1)$ (not shown) appears to exhibit a fixed pole at $j=-1$ rather than the hoped for A_1 Regge pole. We remember the A_1 was also somewhat elusive in $I_2^3(0)$.

H. Polarizabilities

On integrating (40) up to $E_{\text{lab}}=1.12$ GeV, we find (assuming $c_i=0$) the results given in Table IV. Here the column headed Walker uses his analysis from threshold onwards while that headed BDW uses the analysis of Ref. 1 from 0.15 to 0.5 GeV and Walker thereon. The last row contains the proton's polarizability and is half the sum of the first 3 lines. As described in Sec. IV E, this and row 3 (isospin 3 which is the difference between the proton and neutron) may be hoped to be measured experimentally.

From the published data[54] on σ_{total} for photons on protons we may estimate the contribution of the integral from 1.12 to ∞ for the proton as follows. From 1.12 to 5.5, we get 0.9×10^{-43} cm^3 (error $\sim20\%$) and from 5.5 onwards, $\lesssim0.2\times10^{-43}$ cm^3. The former comes from direct integration and the latter from assuming σ_{total} does not increase after 5.5 GeV.

One may try to estimate the integral from 1.12 to ∞ for isospin 1 and 3 by assuming it to be dominated by the Regge pole saturating $I_1^1(1)$ and $I_1^3(1)$, respectively. The result obtained is an order of magnitude smaller than the difference between the two determinations of the integral up to 1.12.

I. Relative Importance of Different Intermediate States

In our graphical results, we have only given the total integral over ImB in (41). So that one may judge the relative importance of the contributions of various intermediate states, we give in Table III the breakup of

$$\frac{1}{\pi}\int_{\nu_0}^{\nu_c} d\nu\, \nu^n\, \mathrm{Im}B$$

for various sum rules. The columns headed P_{33}, D_{13}, D_{15}, and F_{15} give the separate contributions of the πN intermediate state in these spin and isospin quantum numbers. This isolates the important resonances in our energy range. The remaining contribution of the πN state is in the rest column while further columns give the inelastic and Born contributions to (41). The resonant S_{11} and P_{11} contributions to the rest column are small and this column thus represents nonresonant background which near threshold gets large contributions from the photoproduction Born terms. Both the total and πN columns are evaluated using the BDW analysis up to 0.5 GeV, and Walker's thereafter.

We would like to warn the reader that the first four πN columns include the total contribution of these states integrated over the whole energy range and not just the resonant portion. Thus, in $I_3^4(0)$, the resonant

TABLE IV. The polarizability in the various isospin states (see Sec. VI H). The units are 10^{-43} cm^3.

Isospin state	Walker	BDW
1	0.2	0.4
2	25.6	25.5
3	−1.0	−2.0
proton	12.4	12.0

F_{15} is much bigger than the resonant D_{15} state but this latter entry is large in Table III due to low-energy contributions of these quantum numbers.

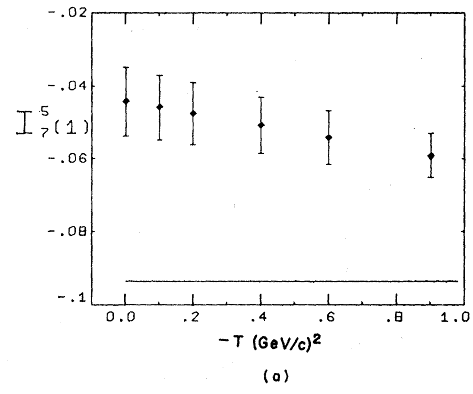
VII. METHODOLOGICAL COMMENTS

We discuss here some of the features, both desirable and undesirable, of our analysis and make suggestions for possible improvements and related future work.

For tests of the Drell-Hearn and current-algebra sum rules, which derive from theoretical features particular to Compton amplitudes (e.g., algebraic properties of conserved currents), it would be desirable to relax the close dependence of our analysis on the Regge-pole model of high-energy behavior. Although model-independent statements concerning the validity of the sum rules could presumably be easily obtained if the cutoff were sufficiently high, at the present cutoff we can say only the following. Adopting the phenomenological criterion that the scale of a convergent[73] sum rule is set by its generalized Born term (Born minus theoretically predicted fixed pole) it is clear from the figures that the $I=0$ exchange Drell-Hearn sum rule $I_2^2(0)$ and the time-component current-algebra sum rules $I_1^4(0)$ and $I_3^4(0)$ must be regarded as successful, while the $I=1$ Drell-Hearn sum rule $I_2^3(0)$ and the Bég sum rule $I_2^4(1)$ seem to be failures. To strengthen these statements we have been forced, at this low cutoff energy, to explore the consistency of our results with the Regge-pole parameters which have been obtained from high-energy data and FESR calculations on hadronic processes. Actually the exploration of the Regge-pole model enriches our understanding of high-energy behavior. For example, we regard our results concerning the lack of nonsense zeros in ρ Regge coupling to the Compton amplitude as one of the more interesting facts which this analysis has revealed.

Our study has been handicapped by the lack of generally accurate estimates of the imaginary parts of Compton amplitudes. In this situation, it becomes crucial to study as many sum rules as possible in order to obtain some feeling for the reliability of the results. For example, if one studies five equally convergent sum rules and finds that four of them go according to theoretical expectations and the fifth contains a surprise, it is then rather difficult to explain away the surprise on the basis of poor data.

[73] To determine whether a sum rule is convergent we use Regge theory to suggest the effective power ν^α of the integrand at high energy. This least-controversial and best-established property of Regge theory has also been used for Compton amplitudes by H. Harari, Phys. Rev. Letters 17, 1303 (1966).



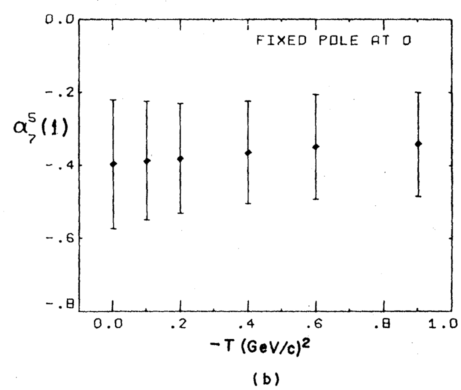


FIG. 28. π spin-flip sum rule. See VI F for comments. (a) The $n=1$ sum rule $I_7^5(1)$. (b) The corresponding effective α.

It is, of course, distressing that we were forced to cut off our integrals at the dubiously asymptotic value of $E_{\text{lab}}=1.12$ GeV. In spin segments 2 and 3, this low cutoff was reflected in the quantitative importance of the nonasymptotic terms in the Regge formalism, suppressed by a factor $1/\nu$ from the leading terms.

Unfortunately, it appears very hard to extend our integrals beyond $E_{\text{lab}}=1.12$ GeV as long as we use unitarity to estimate the imaginary part. Thus, above our cutoff, a multitude of inelastic states become important and one would have to make models of the spin and isospin structure of all of these to find the imaginary part of the general Compton amplitude. Hence, to extend our cutoff we would need data on Compton scattering itself but even this would not allow us to probe the general isospin state.

It follows that in the foreseeable future the main improvement in the evaluation of our sum rules must come from an improved treatment of the region up to 1.12 GeV, and here the elastic (πN) intermediate state is dominant (see Sec. VI I).

It is rather disconcerting that different multipole analyses of low-energy photoproduction experiments, and perhaps even different experiments, are inconsistent. An obvious approach which would hopefully lead to an improved multipole analysis would be to combine the theoretical treatment of BDW and the phenomenological method of Walker. Thus, one could formulate the dispersion theory with parameters, representing its weakest points, to be determined from a fit to the data. Such a treatment would at least have the virtue of incorporating elementary theoretical constraints such as Watson's theorem[74] on the phase of multipole amplitudes, which is not obeyed in purely phenomenological analyses. It is also possible that the use of theoretical models for the inelastic reactions $\gamma N \to \pi\Delta$ and $\pi N \to \pi\Delta$ would permit an approximate incorporation of unitarity for photoproduction above the BDW cutoff energy of $E_{\rm lab}=0.5$ GeV.

The multipole parameters of the $\gamma N \to \pi N$ process are very fundamental physical data. Improved data and multipole analyses for this process would be vital in attempts to confirm (or refute) our finding of a fixed pole in the isovector Drell-Hearn sum rule and to decide the open and important question of the validity of current algebra at large $-t$ in the $I_1{}^4(0)$ and $I_3{}^4(0)$ sum rules. We urge that experimental and theoretical effort not be relaxed until the photoproduction multipoles are known to an accuracy comparable to the πN phase shifts, although we realize that a much larger effort is required.

Since the greatest discrepancy getween BDW and Walker is in the isoscalar photon multipoles, it would be very useful to study the FESR's for isoscalar photoproduction to determine whether the size of the predicted Regge-pole terms is compatible with the isoscalar component of high-energy photoproduction which can be estimated from recent data.[70] Such an analysis could determine whether isoscalar photon multipoles were underestimated in Walker's analysis.

Since there is an experiment underway at the Cambridge Electron Accelerator to measure the proton Compton scattering differential cross section in the 4–5-GeV energy range, it would be interesting to use the sum rules to work up a Regge-pole prediction for this quantity. This could be done very easily with our existing computer programs.

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[74] K. M. Watson, Phys. Rev. **88**, 1163 (1952).

APPENDIX A

We give here the relation of our amplitudes defined by Eqs. (14) and (17) to the invariant amplitudes A_k of Hearn and Leader[25] and reduced s-channel amplitudes defined analogously to (13) by

$$\hat{M}_{\lambda_3\lambda_4;\lambda_1\lambda_2}=(\cos\tfrac{1}{2}\theta_s)^{-|\lambda_{34}+\lambda_{12}|}(\sin\tfrac{1}{2}\theta_s)^{-|\lambda_{34}-\lambda_{12}|}M_{\lambda_3\lambda_4;\lambda_1\lambda_2}. \qquad \text{(A1)}$$

We now list the expressions for the amplitudes $B_j{}^i$ in which for clarity we have omitted the isospin index i.

$$\begin{aligned}
B_1&=-\tfrac{1}{2}(us-m^4)^{-1}[(A_2-A_1)(4m^2-t)+(A_4-A_5)m(s-u)]\\
&=(s-m^2)^{-2}\{s^{-1/2}(s+m^2)\sin^2(\tfrac{1}{2}\theta_s)\hat{M}_{\frac{1}{2}1;-\frac{1}{2}1}+m[\hat{M}_{\frac{1}{2}1;\frac{1}{2}1}+\cos^2(\tfrac{1}{2}\theta_s)\hat{M}_{-\frac{1}{2}1;-\frac{1}{2}1}]\},\\
B_2&=(us-m^4)^{-1}[A_6(s-u)+\tfrac{1}{2}t(A_5-A_4)]\\
&=(s-m^2)^{-2}\{\hat{M}_{\frac{1}{2}1;\frac{1}{2}1}+2ms^{-1/2}\sin^2(\tfrac{1}{2}\theta_s)\hat{M}_{\frac{1}{2}1;-\frac{1}{2}1}-[1-m^2t(s-m^2)^{-2}]\hat{M}_{-\frac{1}{2}1;-\frac{1}{2}1}\},\\
B_3&=(m^4-us)^{-1}[A_6(4m^2-t)+\tfrac{1}{2}(s-u)(A_4-A_5)]\\
&=(s-m^2)^{-4}\{(s-m^2)^2\hat{M}_{\frac{1}{2}1;\frac{1}{2}1}-2ms^{1/2}(s-u)\hat{M}_{\frac{1}{2}1;-\frac{1}{2}1}+[m^2t+(s-m^2)(s+3m^2)]\hat{M}_{-\frac{1}{2}1;-\frac{1}{2}1}\}, \qquad \text{(A2)}\\
B_4&=A_3\\
&=+\tfrac{1}{2}s^{1/2}(s-m^2)^{-1}[\hat{M}_{-\frac{1}{2}-1;\frac{1}{2}1}+\sin^2(\tfrac{1}{2}\theta_s)\hat{M}_{\frac{1}{2}-1;-\frac{1}{2}1}],\\
B_5&=t^{-1}[(A_1+A_2)(4m^2-t)-(A_4+A_5)m(s-u)]\\
&=+s(s-m^2)^{-2}\{s^{-1/2}(s+m^2)\times[\hat{M}_{-\frac{1}{2}-1;\frac{1}{2}1}-\sin^2(\tfrac{1}{2}\theta_s)\hat{M}_{\frac{1}{2}-1;-\frac{1}{2}1}]-4m\cos^2(\tfrac{1}{2}\theta_s)\hat{M}_{\frac{1}{2}-1;\frac{1}{2}1}\},\\
B_6&=A_4+A_5\\
&=-(s-m^2)^{-2}\{2ms^{1/2}[\hat{M}_{-\frac{1}{2}-1;\frac{1}{2}1}-\sin^2(\tfrac{1}{2}\theta_s)\hat{M}_{\frac{1}{2}-1;-\frac{1}{2}1}]+2(s+m^2)\sin^2(\tfrac{1}{2}\theta_s)\hat{M}_{\frac{1}{2}-1;\frac{1}{2}1}\}.\\
B_7&=-s^{1/2}(s-m^2)^{-3}\hat{M}_{\frac{1}{2}1;-\frac{1}{2}1},\\
B_8&=s(s-m^2)^{-3}\hat{M}_{\frac{1}{2}-1;\frac{1}{2}1}.
\end{aligned}$$

APPENDIX B

In our study of the sum rules in Sec. VI, we will need to know the exact predictions that factorization of the Regge couplings makes for our singularity-free amplitudes $B_1 \to B_8$. In this appendix, we outline a derivation of these conditions while in Appendix C we give the resultant expressions for $G_j(t)$, $H_j(t)$ [defined in (19) and (20)] in terms of singularity-free vertex functions. These latter we will denote by P_n, P_f for photon-photon coupling in nonflip (n) and spin-flip (f) states and N_n, N_f for the corresponding nucleon-antinucleon couplings. We will add a superscript c if the pole conspires.[75]

First, we write our t-channel helicity amplitudes

$$A_{\lambda_3\lambda_1:\lambda_4\lambda_2}=-\frac{(e^{-i\pi\alpha}+\tau)}{2\sin\pi\alpha}e^{i\pi(\lambda_{13}-\lambda_{24})/2}\gamma_{\lambda_1\lambda_3}{}'\gamma_{\lambda_2\lambda_4}{}'\nu^{\alpha}; \qquad \text{(B1)}$$

[75] D. Z. Freedman and J. M. Wang, Phys. Rev. **160**, 1560 (1967); E. Leader *ibid.* **166**, 1599 (1968). Also see the discussion in Sec. IV D.

while to include terms of order $\alpha-1$ it is necessary to multiply the resultant form that (B1) gives to the reduced amplitudes (13) by

$$1-[\lambda_{\min}(\lambda-\alpha)/2\alpha\nu][-t(4m^2-t)]^{1/2}, \tag{B2}$$

where

$$\lambda=\max(|\lambda_{13}|,|\lambda_{24}|),\quad \lambda_{\min}=\min(|\lambda_{13}|,|\lambda_{24}|)\times\mathrm{sgn}(\lambda_{13}\lambda_{24}).$$

We will need (B2) to derive the form of $H(t)$ defined in Eq. (20). (This is considered in greater detail in Appendix D.)

We must now remove the kinematic singularities from γ' which we do first for the $\gamma-\gamma$ coupling by defining

$$\gamma_{11}'=tP_n,\quad \gamma_{1-1}'=P_f, \tag{B3}$$

if the particle evades at $t=0$, while if it conspires we put

$$\gamma_{11}'=i(-t)^{1/2}P_n^c,\quad \gamma_{1-1}'=i(-t)^{1/2}P_f^c. \tag{B4}$$

For the $N\bar{N}$ coupling we must consider separately $\tau P=+$ and $\tau P=-$.

(a) $\tau P+$ (*nonconspiring*):

$$\gamma_{\frac12\frac12}'=iN_n/(4m^2-t)^{1/2},\quad \gamma_{\frac12-\frac12}'=-(-t)^{1/2}N_f/(4m^2-t)^{1/2}, \tag{B5}$$

(b) $\tau P+$ (*conspiring*):

$$\gamma_{\frac12\frac12}'=-(-t)^{1/2}N_n^c/(4m^2-t)^{1/2},\quad \gamma_{\frac12-\frac12}'=iN_f^c/(4m^2-t)^{1/2}; \tag{B6}$$

(c) $\tau P-$ (*nonconspiring*):

$$(\pi)\quad \gamma_{\frac12\frac12}'=i(-t)^{1/2}N_n,\qquad (A_1)\quad \gamma_{\frac12-\frac12}'=N_f, \tag{B7}$$

(d) $\tau P-$ (*conspiring*):

$$(\pi)\quad \gamma_{\frac12\frac12}'=N_n^c. \tag{B8}$$

Substituting $(B1\rightarrow 8)$ into Eqs. (14) and (17) we get the results given in Appendix C.

We will wish to compare our ratio of spin-nonflip to spin-flip couplings for P, P', ρ, and A_2 exchange with those obtained from analyzing strong interactions. However, it is conventional[56] to analyze πN and KN elastic scattering in terms of invariant amplitudes A' and B which are related to our formalism by

$$N_f/N_n=\nu B/(4m^2-t)A'. \tag{B9}$$

The behavior of $N_{n,f}$ and $P_{n,f}$ near $\alpha=0$ for various sense-nonsense mechanisms is given in Table II.

APPENDIX C

Here we give the expansion of the functions $G_j^i(t)$ and $H_j^i(t)$ of Eqs. (19) and (20) in terms of the factorized vertex functions of Appendix B. We omit the isospin index i in all these results.

(i) $\tau P=+$ *contributions*:

$$\begin{aligned}
&G_2=G_4=G_7=G_8=0,\\
&H_1=H_3=H_4=H_5=H_6=H_7=H_8=0,\\
&G_1=\tfrac12(N_nP_f+tN_n^cP_f^c),\\
&H_2=-[(2-\alpha)/2\alpha]t(N_fP_f+N_f^cP_f^c),\\
&G_3=(N_fP_f+N_f^cP_f^c),\\
&G_5=-(N_nP_n+N_n^cP_n^c),\\
&G_6=(tN_fP_n+N_f^cP_n^c).
\end{aligned} \tag{C1}$$

(ii) $\tau P=-$ *contributions*:

$$\begin{aligned}
&G_1=G_3=G_5=G_6=0,\\
&H_1=H_2=H_4=H_5=H_6=H_7=H_8=0,\\
&G_2=N_fP_f,\\
&H_3=(4m^2-t)[(2-\alpha)/2\alpha]N_fP_f,\\
&G_4=-\tfrac12(tN_nP_n+N_n^cP_n^c),\\
&G_7=-\tfrac12(N_nP_f+N_n^cP_f^c),\\
&G_8=\tfrac12N_fP_n.
\end{aligned} \tag{C2}$$

APPENDIX D

Although the direct connection established in Sec. III B between asymptotic terms of the amplitudes $B(\nu,t)$ [Eq. (21)] and contributions to the sum rules [Eq. (23)] is sufficient to understand most of the physics contained in the sum rules, for some features it is necessary to go farther into the Reggeization of parity-conserving helicity amplitudes. This is especially necessary for spins 2 and 3 because Regge poles of both parities contribute and because we have the additional complication of a large nonsense interval in the j plane.

Since the imposing but straightforward details of Reggeization are known[65,76,77] for hadronic amplitudes, we concentrate here on effects of fixed poles and on matters directly connected with the interpretation of our sum rules such as the nonasymptotic Regge contributions [Eq. (20)] and compensators.

We study the amplitudes

$$A_\pm(\nu,t)=\hat{A}_{\frac12-\frac12;1-1}\pm\hat{A}_{-\frac12\frac12;1-1}, \tag{D1}$$

which differ from $B_{2,3}$ by the kinematic factors of Eq. (14), and the definite-parity partial-wave amplitudes

$$a_\pm^j(t)=a_{\frac12-\frac12;1-1}^j(t)\pm a_{-\frac12\frac12;1-1}^j(t) \tag{D2}$$

defined in the usual way.[22] After defining signatured partial-wave amplitudes, introducing rotation functions of the second kind,[78] and performing the Mandel-

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[76] S. Mandelstam, Ann. Phys. (N. Y.) **21**, 8 (1963).
[77] W. Drechsler, Nuovo Cimento **53A**, 115 (1968).
[78] M. Andrews and J. Gunson, J. Math. Phys. **5**, 1391 (1964).

stam-Sommerfeld-Watson contour shift, we obtain the representation

$$A_{\pm}(\nu,t)=\frac{1}{8\pi i}\sum_{\tau=\pm}\int_{\frac{3}{2}-i\infty}^{\frac{3}{2}+i\infty}dj\frac{(2j+1)}{\cos\pi j}(\tau+e^{-i\pi j})\times[a_{\pm}{}^{j\tau}(t)E_{21+}{}^{j}(z)+a_{\mp}{}^{j\tau}(t)E_{21-}{}^{j}(z)]. \quad \text{(D3)}$$

We take $t\lesssim 0$ so that Regge poles satisfy $\text{Re}\alpha(t)<\frac{3}{2}$ and do not explicitly appear in (D3). We have ignored a discrete sum over half-integral j values because its terms are asymptotically (in ν) weaker than those we are interested in and because they cancel out when further shifts of the integration contour are made. The angular functions appearing in (D3) are given by

$$E_{\lambda\mu\pm}{}^{j}(z)=\{[(1-z)/2]^{1/2}\}^{-|\lambda-\mu|}\{[(1+z)/2]^{1/2}\}^{-|\lambda+\mu|}\times e_{-\lambda-\mu}{}^{-j-1}(z)\pm\{[(1-z)/2]^{1/2}\}^{-|\lambda+\mu|}\times\{[(1+z)/2]^{1/2}\}^{-|\lambda-\mu|}e_{-\lambda\mu}{}^{-j-1}(z), \quad \text{(D4)}$$

and the e functions differ from those of Andrews and Gunson[78] by the factor $(-)^{\lambda-\mu}$. The scattering cosine z is given by

$$z=-\frac{2\nu}{[-t(4m^2-t)]^{1/2}}. \quad \text{(D5)}$$

For Compton amplitudes with definite crossing, the signature, parity, and isospin are all correlated. See Table I. For given τ and P from the Table the $a_{\pm}{}^{j\tau}$ with subscript $(-\tau P)$ vanish.

The E functions have the asymptotic behavior (for $\lambda\geq|\mu|\geq 0$)

$$E_{\lambda\mu+}{}^{j}(z)\sim f(j)z^{j-\lambda}\{1+[g(j)/j]z^{-2}+O(z^{-4})\},$$
$$E_{\lambda\mu-}{}^{j}(z)\sim f(j)[\mu(\lambda-j)/j]z^{j-\lambda-1}\times\{1+[h(j)/(j-1)]z^{-2}+O(z^{-4})\}, \quad \text{(D6)}$$

where $g(j)$ and $h(j)$ are regular (albeit zero for some λ and μ) at integer values and $f(j)$ has the following behavior:

$$\begin{aligned} f(j)&\sim(j-j_0)^{-1}, && \text{near } j_0=\lambda,\lambda+1,\lambda+2,\cdots\\ &\sim(j-j_0)^{-1/2}, && \text{near } j_0=|\mu|,|\mu|+1,\cdots,\lambda-1\\ &\sim\text{regular}, && \text{near } j_0=0,1,\cdots,|\mu|-1 \quad \text{(D7)}\\ &\sim(j-j_0)^{-1}, && \text{near } j_0=-|\mu|,-|\mu|+1,\cdots,-1\\ &\sim(j-j_0)^{-1/2}, && \text{near } j_0=-\lambda,-\lambda+1,\cdots,-|\mu|-1\\ &\sim\text{regular}, && \text{near } j_0=-\lambda-1,-\lambda-2,\cdots. \end{aligned}$$

Although the leading term in the asymptotic series is regular near a positive nonsense-nonsense integer, subsidiary terms may be singular, as is crudely shown in (D6). The exact relation between the singular parts of the E functions at reflected integers in the nonsense-nonsense interval is

$$\lim_{j\to j_0}(j-j_0)E_{\lambda\mu\pm}{}^{j}(z)=-(-)^{\lambda-\mu}\text{sgn}(\lambda\mu)\lim_{j\to j_0}(j-j_0)E_{\lambda\mu\mp}{}^{-j-1}(z). \quad \text{(D8)}$$

If fixed poles are present, then the partial-wave amplitudes $a_{\pm}{}^{j\tau}(t)$ are expected to have the j-plane behavior of their Born terms, namely,

$$\begin{aligned} a_{\pm}{}^{j\tau}(t)&\ \text{regular}, && \text{near } j_0=2,3,4\\ &\sim(j-j_0)^{-1/2}, && \text{near } j_0=1\\ &\sim(j-j_0)^{-1}, && \text{near } {}^{3}{}_{0}=0,-1 \quad \text{(D9)}\\ &\sim(j-j_0)^{-1/2}, && \text{near } j_0=-2\\ &\sim(j-j_0)^{-1}, && \text{near } j_0=-3,-4,\cdots \end{aligned}$$

where we have again specialized to the particular helicity values, $\lambda=2$, $\mu=1$, that we are interested in. In the absence of fixed poles, the expected behavior is a factor of $(j-j_0)$ smoother at all nonsense points $(j_0\leq 1)$.

The singular parts of the partial-wave amplitudes at the reflected nonsense-nonsense integers $j_0=0$ and $j_0=-1$ are related by

$$\lim_{j\to j_0}(j-j_0)a_{\pm}{}^{j\tau}(t)=\lim_{j\to j_0}(j-j_0)a_{\mp}{}^{-j-1(-\tau)}(t). \quad \text{(D10)}$$

This condition expresses the absence of fixed double poles at $j_0=-1$ and follows formally from the Froissart-Gribov definition, and a mathematical relation, similar to (D8), for the rotation functions. Equation (D10) implies that fixed poles occur in pairs at $j=0$ and $j=-1$ with residues satisfying (D10) and that for every Regge trajectory passing through $\alpha(t)=0$ with nonvanishing residue, there is a compensating trajectory[79] of opposite parity and signature passing through $\alpha'(t)=-1$.

All of this technicality is necessary to understand what happens in (D3) when the vertical contour of integration is shifted to the line $\text{Re}j=-\frac{3}{2}$. The double poles encountered do not contribute asymptotically and obnoxious terms such as fixed powers in the imaginary part of the amplitude cancel between the $j=0$ and $j=-1$ contributions because of the phenomenon of compensation expressed by (D8) and (D10). The net result is a set of relatively simple expressions for the asymptotic terms of the amplitudes $A_{\pm}(\nu,t)$ or $B_{2,3}(\nu,t)$, which we proceed to give.

The current-algebra amplitudes $B_{2,3}{}^{4}(\nu,t)$ have asymptotic contributions from isovector right-signature fixed poles at $J^{PG}=1^{-+}$ and 0^{-+} and from the ρ-Regge trajectory and a mythical X trajectory[80] with

[79] We are puzzled by the following aspect of compensator theory for hadronic amplitudes at right-signature nonsense points. Here partial-wave unitarity requires $a_{\pm}{}^{j\tau}(t)$ to be regular and Eq. (D10) reduces to a trivial identity. Further, although explicit fixed poles have been eliminated in this way, the amplitude still contains the corresponding fixed integer power unless we require the stronger condition $a_{\pm}{}^{j_0\tau}(t)=-a_{\mp}{}^{-j_0-1(-\tau)}(t)$. It seems to be this condition that leads to Regge-pole compensators. It is curious that absence of fixed powers does not follow from partial-wave unitarity and must be assumed independently.

[80] See Ref. 4. We thank Professor Roger Dashen, who has independently worked out the j-plane analysis given in this Appendix, for helpful discussions concerning the Bég sum rule.

$\tau^{PG}=(+)^{-+}$. We find

$$B_3{}^4(\nu,t)\approx\frac{2e^2}{t}G_M{}^V(t)\nu^{-1}-H'(t)\nu^{-3}-G_\rho(t)\times\frac{(-1+e^{-i\pi\alpha_\rho(t)})}{\sin\pi\alpha_\rho(t)}\nu^{\alpha_\rho(t)-2}-G_X(t)(4m^2-t)\times\frac{2-\alpha_X(t)}{2\alpha_X(t)}\frac{(1+e^{-i\pi\alpha_X(t)})}{\sin\pi\alpha_X(t)}\nu^{\alpha_X(t)-3},\quad\text{(D11)}$$

$$B_2{}^4(\nu,t)\approx-e^2G_M{}^V(t)\nu^{-2}-H(t)\nu^{-2}+tG_\rho(t)\frac{2-\alpha_\rho(t)}{2\alpha_\rho(t)}\frac{(-1+e^{-i\pi\alpha_\rho(t)})}{\sin\pi\alpha_\rho(t)}\nu^{\alpha_\rho(t)-3}-G_X(t)\frac{1+e^{-i\pi\alpha_Xt}}{\sin\pi\alpha_X(t)}\nu^{\alpha_X(t)-2}.\quad\text{(D12)}$$

We have used current algebra to relate the residue of the 1^{-+} fixed pole to the isovector magnetic form factor. Here $H(t)$ is the coupling of a hypothetical 0^{-+} pole, and $H'(t)$ is a kinematic singularity-free function which expresses the net contribution of the non-asymptotic term of the 1^{-+} fixed pole, and the 0^{-+} fixed pole and its compensator at $J^P=(-1)^-$.

We have not included explicitly the effects of compensating Regge trajectories near $\alpha=-1$ which are necessary to cancel the singularity at $\alpha_\rho(t)=0$ in $B_2{}^4$ and the possible singularity at $\alpha_X(t)=0$ in $B_3{}^4$.

Notice that the $I_3{}^4(0)$ sum rule is sensitive only to the 1^{-+} fixed pole, while the $I_2{}^4(1)$ sum rule has contributions from both $G_M{}^V(t)$ and $H(t)$. If current algebra is correct and the infinite-momentum method is valid for the commutator of one time and one space component, then the resulting Bég sum rule predicts[80] that $H(t)\equiv 0$. From the standpoint of current algebraists, failure of the Bég sum rule would mean that either current algebra or the infinite-momentum method is wrong.[4] However, from the standpoint of Regge theorists, success of the $I_3{}^4(0)$ sum rule and failure of the Bég sum rule would indicate the existence of a 0^{-+} fixed pole. But in assessing the $I_2{}^4(1)$ sum rule, one must be careful to take into account the possible effect of an X trajectory contribution.

The isospin-symmetric amplitudes $B_{2,3}{}^i(\nu,t)$, $i=1$, 2, 3, have asymptotic contributions from possible fixed poles at $J^P=1^+$ and 0^+. We explicitly treat $B_{2,3}{}^3(\nu,t)$, to which the A_2 and A_1 Regge trajectories contribute. Lettering $A(t)$ and $S(t)$ denote the couplings of the 1^+ and 0^+ fixed poles, we find the asymptotic expressions

$$B_2{}^3(\nu,t)\approx-2A(t)\nu^{-1}+2S'(t)\nu^{-3}+tG_{A_2}(t)\times\frac{2-\alpha_{A_2}(t)}{2\alpha_{A_2}(t)}\frac{1+\exp[-i\pi\alpha_{A_2}(t)]}{\sin\pi\alpha_{A_2}(t)}\nu^{\alpha_{A2}(t)-3}-G_{A_1}(t)\frac{-1+\exp[-i\pi\alpha_{A_1}(t)]}{\sin\pi\alpha_{A_1}(t)}\nu^{\alpha_{A1}(t)-2},\quad\text{(D13)}$$

$$B_3{}^3(\nu,t)\approx-(4m^2-t)A(t)\nu^{-2}-2S(t)\nu^{-2}-G_{A_2}(t)\frac{1+\exp[-i\pi\alpha_{A_2}(t)]}{\sin\pi\alpha_{A_2}(t)}\times\nu^{\alpha_{A2}(t)-2}-(4m^2-t)G_{A_1}(t)\frac{2-\alpha_{A_1}(t)}{2\alpha_{A_1}(t)}\times\frac{-1+\exp[-i\pi\alpha_{A_1}(t)]}{\sin\pi\alpha_{A_1}(t)}\nu^{\alpha_{A1}(t)-3}.\quad\text{(D14)}$$

Here we have a situation opposite to that of the current-algebra segment. The Drell-Hearn sum rule $I_2{}^3(0)$ is sensitive to the axial-vector fixed pole only, while the sum rule $I_3{}^3(1)$ detects the combined effect of the axial-vector and scalar fixed poles. In a derivation of these sum rules based on quark-model current algebra and the infinite-momentum limit, both fixed poles are absent. See Secs. VI B (2) and VI E for our experimental results on this question.