Nonstatic Relations between Magnetic Moments in the Quark Model

JERROLD FRANKLIN
Department of Physics, Temple University, Philadelphia, Pennsylvania 19122
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Relations between baryon magnetic moments are derived which are independent of orbital corrections, relativistic corrections, and two-body exchange corrections (and generally any two-body correction) in the quark model, provided that the wave-function contributions leading to these corrections are $SU(3)$-symmetric. Among the relations derived are $\mu(\Xi^-)+3\mu(\Xi^0)=6\mu(\Lambda)+2\mu(\Sigma^+)-3\mu(p)-\mu(n)$ (independent of quark moments) and $\mu(\Xi^0)-\mu(\Xi^-)=\mu(p)+2\mu(n)$ (if $\mu_{\mathcal{P}}=-2\mu_{\mathcal{N}}$).

I. INTRODUCTION

THE quark model has been used to derive the $SU(3)$ and $SU(6)$ magnetic-moment relations[1] and also to investigate symmetry-breaking effects[2,3] on magnetic moments. Symmetry-breaking effects on magnetic moments are difficult to determine unambiguously strictly within a group framework. In the quark model, however, the quark moments can be taken to be independent parameters, which allows more freedom than in the pure symmetry scheme. However, the usual static-quark-model results depend on the neglect of a number of effects which may be important. Although the agreement with experiment, thus far, is good, the situation remains one in which any future disagreement could be explained away by these "other effects." In this paper we derive magnetic-moment relations for which orbital effects, relativistic effects, and exchange effects, the principal other effects, cancel out.[4] The results are relations that are weaker than those of the more restrictive models but would be more difficult to explain away. The new relations are used to predict the Ξ^0 and Ξ^- magnetic moments and the importance of measuring these moments is emphasized.

In Sec. II of this paper we discuss the assumptions of the usual quark model. In Sec. III, we weaken these assumptions and derive new quark-model relations. Finally, in Sec. IV, we discuss our results.

II. QUARK-MODEL ASSUMPTIONS

In the quark model the $SU(3)$ and $SU(6)$ magnetic-moment relations follow from three assumptions:

(1) Baryons are composed of three spin-$\frac{1}{2}$ quarks $\mathcal{P}$, $\mathcal{N}$, and λ, with charges $\frac{2}{3}$, $-\frac{1}{3}$, and $-\frac{1}{3}$, respectively, whose spins couple as if they were bosons.

(2) Baryon magnetic moments arise solely from quark magnetic moments, with orbital effects, relativistic effects, and exchange effects being negligible.

(3) The quark moments are proportional to the quark charges so that $\mu_{\mathcal{P}}=-2\mu_{\mathcal{N}}=-2\mu_\lambda$.

The exact symmetry results can then be broken in the quark model by freeing the quark moments. Assumption (3) can first be relaxed to

(3′) $\mu_{\mathcal{P}}=-2\mu_{\mathcal{N}}$ with no restriction on μ_λ.

This removes the $SU(3)$ prediction[5,6]

$$\mu(\Lambda)=\tfrac{1}{2}\mu(n)=-0.96\,, \tag{1}$$

and replaces the $SU(3)$ prediction[5,6]

$$\mu(\Sigma^+)=\mu(p)=2.79 \tag{2}$$

by[2,7]

$$\mu(\Sigma^+)=\tfrac{1}{9}[8\mu(p)-3\mu(\Lambda)]=2.72\pm0.05\,. \tag{2′}$$

It is interesting that this $SU(3)$ breaking in the quark model does not appreciably change the Σ^+ moment, while one suggestion within the $SU(3)$ framework predicts[8] $\mu(\Sigma^+)=2.20$. Experimentally,[9] $\mu(\Sigma^+)=2.6\pm0.5$, so that the accuracy is not yet sufficient to test these alternatives.

It is possible to remove assumptions (3) and (3′) completely so that there is no restriction on quark moments.[3] The only prediction that is dropped is the $SU(6)$ result,[10]

$$-\mu(p)/\mu(n)=\tfrac{3}{2}\,. \tag{3}$$

All other quark-model predictions change but little because of the close agreement of Eq. (3) with the experimental value of $-\mu(p)/\mu(n)=1.46$. For instance, Eq. (2′) is changed to[3]

$$\mu(\Sigma^+)=(1/15)[16\mu(p)+4\mu(n)-5\mu(\Lambda)]=2.71\pm0.05\,. \tag{2″}$$

[1] G. Morpurgo, Physics 2, 95 (1965); W. Thirring, Notes from the Internationale Universitaetswochen für Kernphysik, Austria, March 1966 (unpublished).
[2] H. R. Rubinstein, F. Scheck, and R. H. Socolow, Phys. Rev. 154, 1608 (1967).
[3] J. Franklin, Phys. Rev. 172, 1807 (1968). We refer to this paper as I.
[4] Orbital effects and relativistic effects are discussed in I. For a discussion of exchange effects in the equivalent nuclear case, see R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Co., Inc., Cambridge, Mass., 1953), pp. 241ff.
[5] S. Coleman and S. L. Glashow, Phys. Rev. Letters 6, 423 (1961).
[6] The numerical values are in units of proton magnetons, $e/2m_p$, and come from the experimental values $\mu(p)=2.79$, $\mu(n)=-1.91$.
[7] The Λ moment used here is from the latest data compilation of A. H. Rosenfeld *et al.*, Rev. Mod. Phys. 40, 77 (1968).
[8] M. A. B. Bég and A. Pais, Phys. Rev. 137, B1514 (1965).
[9] The Σ^+ moment is that quoted as a world average by T. S. Mast *et al.*, Phys. Rev. Letters 20, 1312 (1968).
[10] M. A. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters 13, 514 (1964).

Actually, there are theoretical reasons for believing that $\mu_{\mathcal{P}}=2\mu_{\mathfrak{N}}$ and the close agreement of $-\mu(p)/\mu(n)$ with $\frac{3}{2}$ suggests this. This point is discussed more fully in I, where it is also shown that relativistic effects are a likely cause for the deviation from $\frac{3}{2}$.

In the next section, we further weaken the quark-model assumptions by dropping most of assumption (2) and part of assumption (1). It is then still possible to derive some relations among baryon moments. These relations are contained in those using the stronger assumptions, but they would be harder to break.

III. MAGNETIC-MOMENT RELATIONS UNDER WEAKER ASSUMPTIONS

In I, we considered baryon wave functions formed by taking the first two quarks in a three-quark wave function to be identical and to satisfy particular statistics (either Bose or Fermi, depending on what other assumptions are made). This led, uniquely, to all the baryons except the Σ^0 and Λ^0. For these we took the first two quarks to be the $\mathcal{P}$ and $\mathfrak{N}$ quarks in a state corresponding to identical particles satisfying the same (for Σ^0) and the opposite (for Λ) statistics as for identical particles in the other baryon wave functions. When charge independence is assumed for quark interactions, these wave functions put the Σ^0 into an isotopic-spin triplet and the Λ into an isotropic singlet.

It is difficult with these wave functions, however, to relate the Λ to the other baryons by a symmetry assumption, because its wave function is intrinsically different from those of the other baryons. Therefore, instead of using the Λ as one of our basic states, we introduce the mathematical baryon states

$$U=(\mathfrak{N}\lambda,\mathcal{P}) \tag{4}$$

and

$$V=(\mathcal{P}\lambda,\mathfrak{N}), \tag{5}$$

where the wave-function notation (q_1q_2,q_3) means that the first two quarks are in a state corresponding to identical quarks with the same statistics as for the other baryons. The third quark then couples to them to form a state of total spin $\frac{1}{2}$. With the U and the V replacing the Λ baryon, there are now nine baryon states, but only eight of them are independent since there will be some relation between the U, V, and Σ^0 states. For $SU(3)$ symmetry, we have

$$U=-\tfrac{1}{2}(\Sigma^0+\sqrt{3}\Lambda), \tag{6}$$

$$V=-\tfrac{1}{2}(\Sigma^0-\sqrt{3}\Lambda), \tag{7}$$

and the relation

$$U+V+\Sigma^0=0. \tag{8}$$

With $SU(3)$ symmetry, the U and the V are the $U_3=0$ and $V_3=0$ members, respectively, of U-spin and V-spin triplets.

We are now in a position to derive magnetic-moment relations by making the assumption that different baryons have the same wave functions. This is equivalent to assuming $SU(3)$ symmetry for the wave functions.[11] This assumption is already implicit in the quark model if assumption (2) is made, because assumption (2) removes any possible effects of different wave functions. Now we drop assumption (2) in favor of

(2′) Baryon magnetic moments arise from quark contributions that are independent of which baryon the quarks are in.

With this assumption, baryon magnetic-moment relations follow by simply equating magnetic-moment linear combinations that include the same numbers of each quark in each position of the (q_1q_2,q_3) wave function on each side of the equation. Given this prescription, the manner in which quark spins add up does not enter, so that we can relax assumption (1) to read

(1′) Baryons are composed of three quarks $\mathcal{P}$, $\mathfrak{N}$, and λ, with charges $\frac{2}{3}$, $-\frac{1}{3}$, and $-\frac{1}{3}$, respectively.

We have also, at this point, made no assumption about quark moments.

We first list the relations, Eqs. (9)–(12a), that follow from assumptions (1′) and (2′) and then show how they follow from the prescription suggested by assumption (2′). We get the magnetic-moment relations

$$\mu(\Sigma^+)+\mu(\Sigma^-)=2\mu(\Sigma^0), \tag{9}$$

$$\mu(n)+\mu(\Xi^0)=2\mu(U)=\tfrac{1}{2}[3\mu(\Lambda)+\mu(\Sigma^0)+2\sqrt{3}\mu(\Lambda,\Sigma^0)] \tag{10}$$

and

$$\mu(p)+\mu(\Xi^-)=2\mu(V)=\tfrac{1}{2}[3\mu(\Lambda)+\mu(\Sigma^0)-2\sqrt{3}\mu(\Lambda,\Sigma^0)], \tag{11}$$

for the I-spin, U-spin, and V-spin triplets, respectively. These formulas have been shown previously[12,13] to follow from conservation of the appropriate internal spin and an assumption about the form of the magnetic-moment operator. Here this assumption is that the magnetic moment comes from quarks, independent of any assumption about quark moments themselves. One other relation that follows from assuming the same wave function for each baryon is

$$\mu(\Sigma^+)+\mu(n)+\mu(\Xi^-)=\mu(\Sigma^-)+\mu(p)+\mu(\Xi^0). \tag{12a}$$

To illustrate how assumption (2′) leads to a complete cancellation of the quark contributions, we rewrite Eq.

[11] The explicit assumption of $SU(3)$ symmetry need only be made for those aspects of the wave functions that lead to nonstatic magnetic-moment corrections. For instance, for the orbital corrections, we assume that the constant coefficients α_i of Eq. (A9) of I are the same for each baryon. This is the minimal assumption that can be made and still keep the quark model relevant for baryon moment predictions.

[12] R. Marshak, S. Okubo, and G. Sudarshan, Phys. Rev. **106**, 599 (1957).

[13] H. Lipkin (unpublished).

(12a) in terms of the quark wave functions:

$$\mu(\mathcal{PP},\lambda)+\mu(\mathcal{NN},\mathcal{P})+\mu(\lambda\lambda,\mathcal{N}) =\mu(\mathcal{NN},\lambda)+\mu(\mathcal{PP},\mathcal{N})+\mu(\lambda\lambda,\mathcal{P}). \quad (12b)$$

Inspection of Eq. (12b) shows that each type of quark appears the same number of times (once) in each position of the quark wave function on each side of the equation. Therefore, the orbital and relativistic effects discussed in I would be the same for each side and would cancel out.[14] Exchange effects occur for pairs of quarks having different charges and depend on the relative charges of the quarks in each pair.[4] Since the $\mathcal{N}$ and λ quarks have the same charge, they can be treated as equivalent for exchange effects and inspection of Eq. (12b) shows that exchange effects cancel out as well. Exchange effects also cancel when the quark pair considered is in a pure symmetry state.[4] Therefore, in a state like $(\mathcal{PN},\lambda)$, the $\mathcal{P}$-$\mathcal{N}$ pair has no exchange effect. Then rewriting Eq. (9) as

$$(\mathcal{PP},\lambda)+(\mathcal{NN},\lambda)=2(\mathcal{PN},\lambda), \quad (9')$$

we see that exchange effects, as well as orbital and relativistic effects, cancel for this type of relation.

It is unlikely that $\mu(\Sigma^0)$, $\mu(\Sigma^0,\Lambda)$, or $\mu(\Sigma^-)$ (the Σ^- decay shows little asymmetry) will be measured in the near future. However, the Ξ^0 and Ξ^- baryons show large asymmetry in their decay and $\mu(\Xi^0)$ and $\mu(\Xi^-)$ should be measurable. Therefore, we eliminate $\mu(\Sigma^0)$, $\mu(\Sigma^0,\Lambda)$, and $\mu(\Sigma^-)$ from Eqs. (9)–(12a), leaving

$$3\mu(\Xi^0)+\mu(\Xi^-)=6\mu(\Lambda)+2\mu(\Sigma^+)-3\mu(p)-\mu(n) =-5.7\pm1.4, \quad (13)$$

where we have used experimental values to evaluate the right-hand side. Equation (13) is a quark-model prediction that is independent of quark moments and of the corrections listed in assumption (2) to the extent that these corrections come from wave-function components that are $SU(3)$-symmetric. It relies only on assumptions (1′) and (2′).

[14] It can, in fact, be shown by explicit calculation using Eq. (A9) of I and a generalization of Eq. (15) of I that orbital and relativistic contributions do cancel out of the linear combinations in Eqs. (9)–(12a). One purpose of the present paper, however, is to show that such explicit calculation is unnecessary.

TABLE I. Predictions for Ξ^0 and Ξ^- magnetic moments (in units of proton magnetons) for the assumptions discussed in the text.

Assumption	$\mu(\Xi^0)$	$\mu(\Xi^-)$
$SU(3)$ or (1), (2), (3)	-1.91	-0.88
(1), (2)	-1.59 ± 0.20	-0.65 ± 0.20
(1′), (2′), (3′)	-1.7 ± 0.4	-0.7 ± 0.4
	$\mu(\Xi^0)-\mu(\Xi^-)=-1.03$	
(1′), (2′)	$3\mu(\Xi^0)+\mu(\Xi^-)=-5.7\pm1.4$	

One further relation can be derived with the additional assumption (3′) that $\mu_{\mathcal{P}}=-2\mu_{\mathcal{N}}$. Then the contribution for two $\mathcal{N}$ quarks will cancel the contribution for one $\mathcal{P}$ quark in the same location in the (q_1q_2,q_3) wave function. This leads to

$$\mu(\Xi^0)-\mu(\Xi^-)=\mu(p)+2\mu(n)=-1.03. \quad (14)$$

If we now went back to assumption (3) that $\mu_{\mathcal{P}}=-2\mu_{\mathcal{N}}=-2\mu_\lambda$, all the $SU(3)$ magnetic-moment results would, of course, follow.

IV. DISCUSSION

The magnetic-moment relations represented by Eqs. (13) and (14) are not new. They are already included in the $SU(3)$ relations or the static-quark-model relations [using assumptions (1), (2), and (3′)]. However, Eqs. (13) and (14) require less restrictive assumptions. They indicate relations that would hold if $SU(3)$ symmetry is broken by allowing arbitrary quark moments or if the quark-model assumptions are weakened as indicated. If Eq. (13) turns out not to agree with experiment, it would be very difficult to explain away in the quark model. Equation (14) serves as a test of the assumption that $\mu_{\mathcal{P}}=-2\mu_{\mathcal{N}}$ for the quark moments. For these reasons, good experimental determinations of the Ξ^0 and Ξ^- magnetic moments are particularly critical tests of the quark model.

In Table I, we list the predictions for the Ξ^0 and Ξ^- magnetic moments under various assumptions. We note that there is very little difference among the various results. This would make it difficult to observe symmetry breaking in the Ξ^0 and Ξ^- moments, but their measurement remains a strict test of the quark model.