$K^{*+} \to K^+\pi\pi$ Decay in Broken $SU(3)$†

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An effective Lagrangian for pseudoscalar, vector, and axial-vector mesons is used to calculate the $K^{*+} \to K\pi\pi$ decay rate. It is shown that the octet broken-$SU(3)$ PVV interaction generally increases the calculated $K^{*+} \to K\pi\pi$ rate over that of the symmetric limit. Experimental determination of the exact rate would allow fixing of parameters in this broken $SU(3)$ scheme, which as yet have been determined by measured decay rates only to be within a range.

It has been shown by Brown, Munczek, and Singer[1] that broken $SU(3)$ in the PVV interaction is in quantitative agreement with available experimental data. They have also shown that this scheme gives a significant contribution to the K mass difference.[2] Fitting the broken $SU(3)$ model to known decay rates allows establishment of the various parameters used; however, the determination is not complete, because some of the parameters may take more than one value, and some may vary over a range of values. It is shown in this paper that the $K^{*+} \to K\pi\pi$ rate will effectively fix these parameters.

The Lagrangian density employed is

$$\mathcal{L} = \mathcal{L}_V + \mathcal{L}_{AP} + \mathcal{L}_{PVV}. \quad (1)$$

Here $\mathcal{L}_V$ is the generalized Yang-Mills Lagrangian for the vector mesons, $V_\mu{}^a$, $a=1, \cdots, 8$, with $SU(3)$-breaking introduced by current mixing.[3–5]

$$\mathcal{L}_V = -\tfrac{1}{4}K^{ab}V_{\mu\nu}{}^aV_{\mu\nu}{}^b + \tfrac{1}{2}M^2V_\mu{}^aV_\mu{}^a - \tfrac{1}{4}K^{00}V_{\mu\nu}{}^0V_{\mu\nu}{}^0 + \tfrac{1}{2}M^2V_\mu{}^0V_\mu{}^0 - \tfrac{1}{2}K^{80}V_{\mu\nu}{}^8V_{\mu\nu}{}^0. \quad (2)$$

The K^{ab} factor gives the mass splitting of the octet, where

$$K^{ab} = \delta^{ab} + \sqrt{3}\epsilon_0 d^{ab8}, \qquad V_{\mu\nu}{}^a = \partial_\mu V_\nu{}^a - \partial_\nu V_\mu{}^a - gf^{abc}V_\mu{}^bV_\nu{}^c. \quad (3)$$

Diagonalization of (2) with respect to the physical ω and φ yields the relation of the $V_\mu{}^a$ to the physical fields:

$$V_\mu{}^{1,2,3} = \frac{1}{\sqrt{K_\rho}}\rho_\mu{}^{1,2,3}, \qquad V_\mu{}^{4,5,6,7} = \frac{1}{\sqrt{K_{K^*}}}K^*{}_\mu{}^{4,5,6,7}, \quad (4a)$$

$$V_\mu{}^8 = \frac{-\sin\theta}{\sqrt{K_\omega}}\omega_\mu + \frac{\cos\theta}{\sqrt{K_\varphi}}\varphi_\mu, \quad V_\mu{}^0 = \frac{\cos\theta}{\sqrt{K_\omega}}\omega_\mu + \frac{\sin\theta}{\sqrt{K_\varphi}}\varphi_\mu,$$

with

$$K_i = M^2/M_i{}^2 \ \ (i=\rho, K^*, \omega, \varphi), \qquad M = 847 \text{ MeV}, \ \theta = 27.5°. \quad (4b)$$

The PVV interaction term contains the $SU(3)$-breaking parameters, and is given by

$$\mathcal{L}_{PVV} = \tfrac{1}{4}\epsilon_{\alpha\beta\mu\nu}(hD^{abc}V_{\alpha\beta}{}^aV_{\mu\nu}{}^bP^c + \lambda D^{ab}V_{\alpha\beta}{}^aP^bV_{\mu\nu}{}^0), \quad (5)$$

where $V_\mu{}^0$ is the $SU(3)$ singlet vector meson and $V_\mu{}^a$, P^a, $a=1, 2, \cdots, 8$, are the vector and pseudoscalar octets, respectively.

$$D^{abc} = d^{abc} + \sqrt{3}\epsilon_1 d^{abd}d^{d8c} + \tfrac{1}{2}\sqrt{3}\epsilon_2(d^{acd}d^{d8b} + d^{bcd}d^{d8a}) + (\sqrt{\tfrac{1}{3}})\epsilon_3\delta^{ab}\delta^{c8}, \quad (6)$$

$$D^{ab} = \delta^{ab} + \sqrt{3}\epsilon_4 d^{ab8}. \quad (7)$$

This type of breaking is only along the λ_8 axis; i.e., $\mathcal{L}_{PVV}$ transforms under $SU(3)$ as a scalar plus the eighth component of a vector.

The term $\mathcal{L}_{AP}$ is an $SU(3)$ generalization of the Lagrangian used by Brown and Munczek[6] in calculating the pion mass difference. This Lagrangian provides mixing of the π and A_1 fields through the parameter α.

$$\mathcal{L}_{AP} = \mathcal{L}_\Phi + \alpha\mathcal{L}_P + \mathcal{L}_{P\Phi}, \quad (8a)$$

$$\mathcal{L}_\Phi = -\tfrac{1}{4}S^{ab}\Phi_{\mu\nu}{}^a\Phi_{\mu\nu}{}^b + M_A{}^2\Phi_\mu{}^a\Phi_\mu{}^a, \quad (8b)$$

$$\mathcal{L}_P = \tfrac{1}{2}D_\mu P^aD_\mu P^a - \tfrac{1}{2}(\mu_0)_{ab}{}^2P^aP^b, \quad (8c)$$

$$\mathcal{L}_{P\Phi} = M_0 D_\mu P^a\Phi_\mu{}^a, \quad (8d)$$

$$S^{ab} = \delta^{ab} + 0.18\sqrt{3}d^{ab8}, \quad (\mu_0)_{ab}{}^2 = (\mu_0)_a{}^2\delta_{ab}, \quad (8e)$$

where $\Phi_\mu{}^a$ and P^a are pseudovector and pseudoscalar fields, respectively, $a=1, 2, \cdots, 8$, with $\Phi_{\mu\nu}{}^a = D_\mu\Phi_\nu{}^a - D_\nu\Phi_\mu{}^a$, and $D_\mu(\ )^a = \partial_\mu(\ )^a - gf^{abc}V_\mu{}^b(\ )^c$. Solutions to the equations of motion for (8b)–(8d) gives the relation

$$\Phi_\mu{}^a = \frac{1}{\sqrt{S^{aa}}}\mathcal{A}_\mu{}^a - \frac{M_0}{M_A{}^2}\partial_\mu P^a, \quad (9)$$

where $\mathcal{A}_\mu{}^a$ is a spin-1 axial-vector field of mass $M_a{}^2 = M_A{}^2/S_{aa}{}^2$, and P^a is the pseudoscalar field of mass

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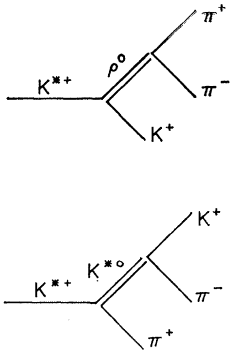

FIG. 1. Diagrams for $K^{*+}\to K^+\pi^+\pi^-$ decay.

$\mu_a=\alpha^{1/2}(\mu_0)_a$. Clearly, $\mathcal{A}_\mu{}^{1,2,3}$ is $A_{1\mu}{}^{1,2,3}$. α is given by

$$\alpha=1+M_0^2/M_A^2. \tag{10}$$

The specific problem to which this paper is addressed is the calculation of the $K^{*+}\to K\pi\pi$ decay rate. This is accomplished by assuming an intermediate vector meson; i.e., the K^{*+} decays into a vector meson and a pseudoscalar meson, and the vector meson in turn decays into two pseudoscalar mesons. Thus the pertinent interaction is $\mathcal{L}_{PVV}+\mathcal{L}_{PPV}$. From Eqs. (8c) and (10), one term of the PPV interaction is obtained,

$$-(1+M_0^2/M_A^2)gf^{abc}\partial_\mu P^a V_\mu{}^b P^c, \tag{11}$$

and the other from (8d),

$$(M_0^2/M_A^2)gf^{abc}\partial_\mu P^a V_\mu{}^b P^c. \tag{12}$$

Thus the PPV vertex may be written

$$-gf^{abc}\partial_\mu P^a V_\mu{}^b P^c, \tag{13}$$

which is the usual antisymmetric $SU(3)$ coupling of two pseudoscalar octets to the vector octet. It is relevant to note here that the total Lagrangian contains no direct interactions of the $VPPP$ type.

Thus the interaction term is written

$$\mathcal{L}^{\text{int}}=\tfrac{1}{4}\epsilon_{\alpha\beta\mu\nu}\{hD^{abc}V_{\alpha\beta}{}^a V_{\mu\nu}{}^b P^c+\lambda D^{ab}V_{\mu\nu}{}^a P^b V_{\alpha\beta}{}^0\} -gf^{abc}\partial_\mu P^a V_\mu{}^b P^c. \tag{14}$$

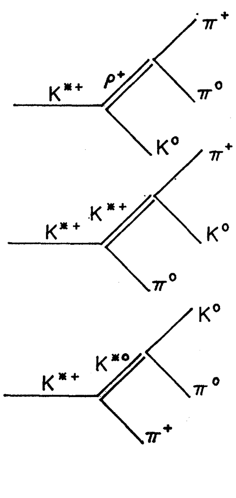

FIG. 2. Diagrams for $K^{*+}\to K^0\pi^0\pi^+$ decay.

The terms $\lambda D^{ab}V_{\mu\nu}{}^a P^b V_{\alpha\beta}{}^0$ and $\mathcal{L}_\Phi$ do not contribute in the calculation of the $K^{*+}\to K\pi\pi$ decay rate.

The possible modes for the $K^{*+}\to K\pi\pi$ decay are as follows:

$$K^{*+}\to K^+\pi^+\pi^-,$$
$$K^{*+}\to K^0\pi^+\pi^0,$$
$$K^{*+}\to K^+\pi^0\pi^0.$$

The diagrams for these processes are shown in Figs. 1–3, respectively. The $K^{*+}\to K^+\pi^+\pi^-$ decay has two possible intermediate vector mesons, the ρ^0 and the K^{*0}. The amplitudes for these two processes are labeled A_ρ and A_{K^*}.

$$A_\rho=\epsilon_{\alpha\beta\mu\nu}K^{*+}{}_\mu\varepsilon_\nu\frac{\pi^-{}_\alpha\pi^+{}_\beta}{k_\rho{}^2-m_\rho{}^2}F_\rho, \tag{15a}$$

$$A_{K^*}=\epsilon_{\alpha\beta\mu\nu}K^{*+}{}_\mu\varepsilon_\nu\frac{K^+{}_\alpha\pi^-{}_\beta}{k_{K^*}{}^2-m_{K^*}{}^2}F_{K^*}, \tag{15b}$$

where $K^{*+}{}_\mu$ is the four-momentum of the decaying K^{*+}, and ε_ν is its polarization. $\pi^+{}_\mu$, $\pi^-{}_\mu$, and $K^+{}_\mu$ are

TABLE I. $SU(3)$-breaking parameters and calculated $K^{*+}\to K\pi\pi$ decay rates.

$\epsilon_1=0.77$

Y	$\chi=\epsilon_1-\frac{1}{2}\epsilon_2$	$\Gamma(K^{*+}\to K^+\pi^+\pi^-)$ (10^{-2} MeV)	$\Gamma(K^{*+}\to K\pi\pi)$ (10^{-2} MeV)	% of total
2.1[a]	−5.44	1.749	4.872	0.097
−1.45[a]	7.13	11.243	32.195	0.644
3.3	−9.69	8.559	24.041	0.481
−2.65	11.38	25.358	72.401	1.448

[a] Indicates value of Y for which the tadpole contribution was used in calculation of the K mass difference.

the four-momenta of the decay products, and $k_\rho{}^2$ and $k_{K^*}{}^2$ are the invariants $(\pi^-+\pi^+)^2$ and $(K^++\pi^-)^2$, respectively. F_ρ and F_{K^*} are the symmetry-breaking parts of the amplitudes and are given as

$$F_\rho=(2gh/K_\rho\sqrt{K_{K^*}})(1-\tfrac{1}{2}\chi), \tag{16a}$$

$$F_{K^*}=[2gh/(K_{K^*})^{3/2}](1+\chi), \tag{16b}$$

with $\chi=\epsilon_1-\frac{1}{2}\epsilon_2$.

For the $K^0\pi^+\pi^0$ mode, there are three possible intermediate vector mesons, the ρ^+, K^{*+}, and the K^{*0}. The corresponding amplitudes, invariants, and symmetry-breaking factors are

$$A_{\rho^+}=\epsilon_{\alpha\beta\mu\nu}K^{*+}{}_\mu\varepsilon_\nu\frac{\pi^+{}_\alpha\pi^0{}_\beta}{k_{\rho^+}{}^2-m_\rho{}^2}F_{\rho^+},\quad k_{\rho^+}{}^2=(\pi^++\pi^0)^2, \tag{17a}$$

$$A_{K^{*+}}=\epsilon_{\alpha\beta\mu\nu}K^{*+}{}_\mu\varepsilon_\nu\frac{K^0{}_\alpha\pi^+{}_\beta}{k_{K^{*+}}{}^2-m_{K^{*+}}{}^2}F_{K^{*+}},\quad k_{K^{*+}}{}^2=(K^0+\pi^+)^2, \tag{17b}$$

$$A_{K^{*0}}=\epsilon_{\alpha\beta\mu\nu}K^{*+}{}_{\mu}\frac{\pi^0{}_{\alpha}K^0{}_{\beta}}{k_{K^{*0}}{}^2-m_{K^*}{}^2}F_{K^{*0}},$$

$$k_{K^{*0}}{}^2=(\pi^0+K^0)^2,\quad (17c)$$

$$F_{\rho^+}=2\sqrt{2}(gh/K_{\rho}\sqrt{K_{K^*}})(1+\chi),\quad (18a)$$

$$F_{K^{*+}}=\sqrt{2}[gh/(K_{K^*})^{3/2}](1-\tfrac{1}{2}\chi),\quad (18b)$$

$$F_{K^{*0}}=\sqrt{2}[gh/(K_{K^*})^{3/2}](1+\chi).\quad (18c)$$

The $K^+\pi^0\pi^0$ mode has only one allowed intermediate state; however, there are two diagrams (Fig. 3) due to the indistinguishability of the two product π^0's. The corresponding amplitudes, A_1 and A_2, are

$$A_1=\epsilon_{\alpha\beta\mu\nu}K^{*+}{}_{\mu}\varepsilon_{\nu}\frac{\pi^0{}_{1\alpha}K^+{}_{\beta}}{k_1{}^2-m_{K^*}{}^2}F,\quad k_1{}^2=(\pi^0{}_1+K^+)^2,\quad (19a)$$

$$A_2=\epsilon_{\alpha\beta\mu\nu}K^{*+}{}_{\mu}\varepsilon_{\nu}\frac{\pi^0{}_{2\alpha}K^+{}_{\beta}}{k_2{}^2-m_{K^*}{}^2}F,\quad k_2{}^2=(\pi^0{}_2+K^+)^2,\quad (19b)$$

with the single symmetry-breaking factor

$$F=[gh/(K_{K^*})^{3/2}](1-\tfrac{1}{2}\chi).\quad (20)$$

TABLE II. $SU(3)$-breaking parameters and calculated $K^{*+}\to K\pi\pi$ decay rates.

$\epsilon_1=1.3$ Y	$\chi=\epsilon_1-\frac{1}{2}\epsilon_2$	$\Gamma(K^{*+}\to K^+\pi^+\pi^-)$ (10^{-2} MeV)	$\Gamma(K^{*+}\to K\pi\pi)$ (10^{-2} MeV)	% of total
1.93[a]	−6.89	2.066	5.777	0.116
−1.55[a]	9.13	10.387	29.678	0.594
3.14	−12.45	9.274	26.11	0.522
−2.76	14.7	23.424	66.788	1.336

[a] Indicates value of Y for which the tadpole contribution was used in calculation of the K mass difference.

The decay rate is proportional to the square of the amplitude, and thus for the $K^{*+}\to K^+\pi^+\pi^-$ mode

$$\Gamma(K^{*+}\to K^+\pi^+\pi^-)=C[|A_\rho|^2+|A_{K^*}|^2+2\,\mathrm{Re}(A_\rho A_{K^*})].$$

The symmetry-breaking terms appear in combinations: $(1+\chi)^2$, $(1-\frac{1}{2}\chi)^2$, and $(1+\chi)(1-\frac{1}{2}\chi)$. It is convenient at this point to define two parameters, Y and W.

$$Y=(1-\tfrac{1}{2}\chi)/(1+\epsilon_1),\quad (21)$$

$$W=(1+\chi)/(1+\epsilon_1).\quad (22)$$

The decay rates are now given in terms of these two parameters, using the values of $g^2/4\pi$ and $(h^2m_\pi{}^2/4\pi)\times(1+\epsilon_1)^2$ obtained by Brown, Muczek, and Singer.[1]

$$g^2/4\pi=3.35,\quad (h^2m_\pi{}^2/4\pi)(1+\epsilon_1)^2=0.10.$$

This gives the rates for the three modes as

$$\Gamma(K^{*+}\to K^+\pi^+\pi^-)=(1.32)(1.378Y^2+5.642WY+5.859W^2)\times10^{-3}\ \mathrm{MeV},\quad (23)$$

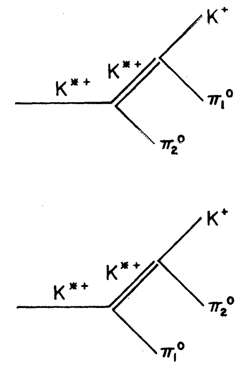

FIG. 3. Diagrams for $K^{*+}\to K^+\pi^0\pi^0$ decay.

$$\Gamma(K^{*+}\to K^0\pi^+\pi^0)=(1.32)(3.029Y^2+11.479WY+11.172W^2)\times10^{-3}\ \mathrm{MeV},\quad (24)$$

$$\Gamma(K^{*+}\to K^+\pi^0\pi^0)=(1.32)(9.618Y^2)\times10^{-5}\ \mathrm{MeV},\quad (25)$$

with

$$(m_{K^*}/2K_{K^*})g^2h^2(1+\epsilon_1)^2=1.32\ \mathrm{MeV}^{-1}.\quad (26)$$

In Ref. 1 the parameter ϵ_1 was determined to have one of the two values $\epsilon_1=0.77$ or 1.3. These values were used by Brown, Munczek, and Singer[2] in fitting the K mass difference. With these values of ϵ_1 the fit to the K mass difference allows eight values of the parameter Y, four of which correspond to the use of a tadpole contribution in the mass-difference calculation. Tables I and II list these values of Y, the corresponding value of χ, together with the calculated decay rate for the $K^{*+}\to K^+\pi^+\pi^-$ mode. The rate for the total $K^{*+}\to K\pi\pi$ decay is given both in MeV and percent of the total K^* decay rate (49.5 MeV).

It is noted that in the symmetric limit, i.e., $\epsilon_1=\epsilon_2=0$, $Y=W=1$, the total $K^{*+}\to K\pi\pi$ decay rate is calculated to be 5.10×10^{-2} MeV or 0.102%. Clearly, there is an alteration in the rate due to the $SU(3)$ breaking. The current experimental upper limit on the $K\pi\pi$ mode of K^* decay is 0.2%. This indicates that only two values of Y are permissible: $Y=2.1$ for $\epsilon_1=0.77$, and $Y=1.93$ for $\epsilon_1=1.3$. It is noted that both these values demand a tadpole contribution to the K mass difference when calculated using the broken-$SU(3)$ scheme of Brown, Munczek, and Singer.[2] Although the symmetry breaking generally produces large effects on meson

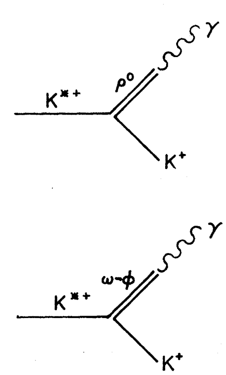

FIG. 4. Diagrams for radiative decay of K^{*+}.

decay rates, the differences predicted in this case for the experimentally allowed rates are rather small.

Fixing of ϵ_2 allows determination of ϵ_3 since the relations have been established[2]:

$$\epsilon_1 = 0.77\,, \quad \epsilon_2 + \epsilon_3 = 2.1\,,$$
$$\epsilon_1 = 1.3\,, \quad \epsilon_2 + \epsilon_3 = -2.7\,.$$

Experimental determination of the exact $K\pi\pi$ decay rate to within 10^{-4} MeV would fix ϵ_1, ϵ_2, and ϵ_3. Consideration of the radiative decays of the ω and φ is required for fixing ϵ_4.

It is perhaps useful to note here that the radiative decay of the K^{*+} ($K^{*+} \to K^+\gamma$) contains a different symmetry-breaking factor from that of the $K^{*+} \to K^+\pi^+\pi^-$ decay: $(1-\frac{1}{2}\epsilon_1+\frac{3}{4}\epsilon_2)$ as opposed to $(1-\frac{1}{2}\epsilon_1+\frac{1}{4}\epsilon_2)$ for the intermediate ρ^0. The reason for this is that the electromagnetic field couples to the isospin singlet ($V_\mu{}^8$), as well as the neutral component of the isospin triplet ($\rho_\mu{}^0$) of the vector meson octet.

$$\mathcal{L}_{\mathrm{EM}} = (em^2/g)(V_\mu{}^3 + \tfrac{1}{3}\sqrt{3}V_\mu{}^8)A_\mu\,.$$

It is the presence of the ω-φ term which accounts for the discrepancy (Fig. 4).

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Duality and Secondary Trajectories

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We show analytically that the slope of the dual or output trajectories is equal to the input slope for the partial-wave projections of the Regge amplitude. This result is independent of the special kinematics, and seems not to require a crossing-symmetric amplitude. We find an infinite set of secondary trajectories, and show that their spacing depends on the slope and approaches $\Delta s = 2/\alpha'$ GeV^2 for large s. The detailed high-energy structure of some Argand diagrams is shown.

I. INTRODUCTION

It has been shown by Schmid[1] that the partial-wave projection of a crossed-channel Regge-pole contribution can result in partial-wave amplitudes which produce circlelike traces in the Argand diagram which rotate counterclockwise with increasing energy. In particular, he analyzed the ρ Regge-exchange amplitude for πN scattering. This produces loops in the Argand diagrams, the tops of which correspond closely to the prominent N^* resonances.

This mechanism for producing Argand loops has been examined by various authors[2] in a variety of situations with different degree of success. One reason is that Schmid has shown only a small sector of the Argand loops which suggested a circlelike structure. Another question is whether the maxima or, for that matter, any point of the loops can be identified as a resonance. First, the resulting partial-wave amplitude does not have resonance poles. Then it can be shown easily that two resonances in the same channel which have a mass difference which is less than their widths create a single loop only and none of the two resonances will be at the top. In addition, many functions can be given which show a phase increasing through $\frac{1}{2}\pi$ but which have nothing to do with a resonance. We are well aware that this leads to a complex set of questions on which we do not enter here but hope to return to later. On the other hand, in the region of interest, the lower part of the N^* and Δ trajectories, one finds experimentally that the resonances are well separated so that it seems a not unreasonable working hypothesis to identify each top of an Argand loop as a theoretical resonance position. While the Regge input is a high-energy representation in the t channel only, we have taken the partial-wave projections for wider range of energies. One of the interesting findings is that for each partial wave one obtains an entire family of spiraling loops. With increasing energy these loops get smaller and smaller but, since they remain well within the unitarity circle, the question arises whether the maxima of these secondary loops should again be interpreted as resonances. While these additional loops represent a

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