is identically zero. We begin by observing that $I(a)$ is analytic in the a plane, apart from a cut along the real axis from 0 to $-\infty$. The discontinuity across this cut at $a=-A$ is proportional to

$$\rho(A)\equiv\int_0^1 \frac{zdz}{1-z}\int_0^\infty \frac{udu}{(u+1)^3}\times[1-u(1-z)]\delta(uz+z/(1-z)-A)$$
$$=\int_0^{A/(A+1)} dz\frac{z(1-z)[A-(A+1)z][(2+A)z-A]}{[-z^2+A(1-z)]^3}$$
$$=-\frac{1}{2A}\int_0^{A/(A+1)} dz\frac{d}{dz}\left\{\frac{z^2[A-(A+1)z]^2}{[-z^2+A(1-z)]^2}\right\}=0, \qquad \text{(C2)}$$

which means that $I(a)$ is an entire function. Furthermore, for $\mathrm{Re}a\geq 0$,

$$|I(a)|\leq\int_0^1 dz\int_0^\infty \frac{udu}{(u+1)^3}\left|\frac{z+uz(1-z)}{uz(1-z)+z+(1-z)a}\right|$$
$$\leq\int_0^1 dz\int_0^\infty \frac{udu}{(u+1)^3}=\frac{1}{2} \qquad \text{(C3)}$$

and

$$I(0)=2\int_0^\infty \frac{udu}{(u+1)^3}\int_0^1\frac{dz}{u(1-z)+1}-\int_0^\infty \frac{udu}{(u+1)^3}=\frac{1}{2}-\frac{1}{2}=0. \qquad \text{(C4)}$$

Since Feynman integrals like Eq. (C1) never lead to functions of exponential type, Eqs. (C1)–(C4) show that $I(a)\equiv 0$.

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Deck Effect with K Exchange*

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(Received 27 January 1969)

It is shown that the nature of the vertex function for $\bar{K}N\rightarrow\Lambda\pi$ favors a kinematical configuration of the $\rho\pi\Lambda$ in $\bar{K}N\rightarrow\Lambda\pi\rho$ which effectively competes with the kinematical configuration responsible for the Deck effect, and substantially flattens the low-mass enhancement. This reaction is thus a far more suitable one in which to verify and study the A_1 ($\rho\pi$) enhancement. These results should also be applicable to a study of the $K^*(1300)$ in the reaction $\pi N\rightarrow\Lambda K^*(890)\pi$.

A MAJOR difficulty in the experimental analysis of low-mass enhancements such as the $A_1(1080)$ in the reaction[1] $\pi N\rightarrow\rho\pi N$ and the $K^*(1300)$ in the reaction $KN\rightarrow K^*(890)\pi N$ is the possible presence of strong kinematic background, from the Deck-type mechanism, centered about the low-mass region of interest. Intrinsic ambiguities in the calculations of these kinematic effects have further prevented a firm experimental answer to the question of the existence of these low-mass resonances. In this paper we examine the conjecture[2] that the Deck-type mechanism plays a substantially weaker role in the reactions $\bar{K}N\rightarrow\Lambda\rho\pi$ and $\pi p\rightarrow\Lambda K^*(890)\pi$. Our analysis provides strong support for the correctness of this conjecture, and hence these reactions are more suitable for studies of low-mass resonances such as the A_1 and the $K^*(1300)$.

In Fig. 1 particles 3 and 5 are baryons and particle 1 is either a K or π, while particle 4 is a vector particle, $\rho(760)$ or $K^*(890)$. The original calculation[3] by Deck computed the 3+4 mass spectrum assuming that the virtual scattering $2+3\rightarrow 5+6$ (πN elastic scattering) is sharply peaked forward and essentially constant in energy. The result of this calculation is that the 4+5 mass distribution is greatly enhanced at low mass values, corresponding to the sometimes observed A_1. Stodolsky[4] has presented an explanation of this enhancement based on the observation that the $\Delta^2+m_2^2$ denominator, together with the diffraction nature of

* Work performed at Brookhaven National Laboratory was under the auspices of the U. S. Atomic Energy Commission. Work performed at City College was supported in part by a City University Summer Research Grant.

[1] The A_1 was first reported by Bellini *et al.*, Nuovo Cimento **29**, 896 (1963). Additional references may be found in Particle Data Group, Rev. Mod. Phys. **41**, 109 (1969).

[2] D. J. Crennell, G. R. Kalbfleisch, K. W. Lai, J. M. Scarr, and T. G. Schumann, Phys. Rev. Letters **19**, 44 (1967).

[3] R. T. Deck, Phys. Rev. Letters **13**, 169 (1964); U. Maor and T. A. O'Halloran, Jr., Phys. Letters **15**, 281 (1965). For a more recent analysis see M. Ross and Y. Yam, Phys. Rev. Letters **19**, 546 (1967); for an alternative explanation of the A_1, see N. P. Chang, *ibid.* **14**, 806 (1965).

[4] L. Stodolsky, Phys. Rev. Letters **18**, 973 (1967).

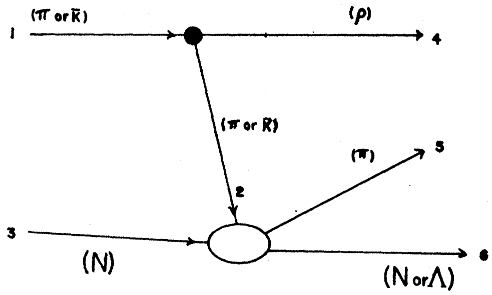

FIG. 1. Feynman diagram for the Deck effect.

the virtual scattering, strongly favors a kinematical configuration in which particles 4 and 5 are emitted parallel in the laboratory system. The preference of the $\rho\pi$ system to be parallel provides the enhancement in the low-energy region, where the ρ and π have lower relative velocity.

The difference between the kinematic enhancement in

$$\bar{K}N \to \Lambda\rho\pi \tag{1a}$$

and the kinematic enhancement in

$$\pi N \to N\rho\pi \tag{2a}$$

arises predominantly from the difference in the lower vertex of Fig. 1, where one has the reactions

$$\bar{K}N \to \Lambda\pi \tag{1b}$$

and

$$\pi N \to \pi N, \tag{2b}$$

respectively. The inelastic reaction (1b) falls off rapidly with increasing energy:

$$\sigma(\bar{K}N \to \Lambda\pi) \sim s^{-2},$$

where s is the $\Lambda\pi$ c.m. energy, while $\sigma(\pi N \to \pi N)$ is constant at high energy.[5] This falloff in energy produces a preference for small s, where reaction (2b) contains a substantial nondiffractive part. Furthermore, there is some evidence[6] that the diffraction part itself is less sharply peaked. In addition the s^{-2} energy dependence favors a kinematic configuration where the π and Λ are parallel, which is quite different from the kinematic configuration which produces the Deck effect.

In order to confirm these ideas, we have made numerical calculations for reaction 1. The differential cross section for $\bar{K}N \to \Lambda\rho\pi$ is given by

$$\frac{\partial\sigma}{\partial s_{4,5}} = \frac{1}{8(Wp_1)^2}\int dt \int \left[\frac{q_5}{\sqrt{s_{4,5}}} d(\cos\theta_5) d\varphi_5\right]_{4,5\text{ c.m.}} \times \Phi(\Delta^2)|M_2|^2, \tag{3}$$

where $s_{4,5}$ is the square of the invariant mass of particles 4+5 (the ρ and π, respectively); W is the total energy in the over-all c.m. system, p_1 is the magnitude of the 3-momentum of particle 1 in the over-all c.m. frame; $t=(p_6-p_3)^2$ is the (negative) 4-momentum transfer between 6 and 3, Δ^2 is the square[7] of the four-momentum of the virtual particle, and

$$\Phi(\Delta^2) = \frac{f_\rho^2[\Delta^2+(m_4-m_1)^2][\Delta^2+(m_4+m_1)^2]}{4m_4^2(\Delta^2+m_2^2)^2} \tag{4}$$

is the spin-summed square of the vertex function for

$$\bar{K} \to \rho^0 + \text{virtual } K^-$$

divided by the square of the propagator, and SU_3 has been used in writing the $\rho K\bar{K}$ coupling constant in terms of the $\rho\pi\pi$ coupling constant f_ρ. For a width $\Gamma(\rho^0 \to \pi^+\pi^-)$ of 100 MeV, one has $f_\rho^2/4\pi \approx 2.0$. The quantity $|M_2|^2$ is related to the scattering of $2+3 \to 5+6$ with all particles on the mass shell by

$$\frac{d\sigma(2+3 \to 5+6)}{d\Omega} = \frac{4\pi^2}{s_{5,6}}\left(\left|\frac{\mathbf{p}_6}{\mathbf{p}_3}\right|\right)|M_2|^2, \tag{5}$$

where s_{56} is the square of the invariant mass of 5 and 6 [$s_{56}=-(p_5+p_6)^2$], and $|\mathbf{p}_6|$ and $|\mathbf{p}_3|$ are the magnitudes of the 3-momenta of particles 6 and 3, respectively, in the 5+6 c.m. system. In Eq. (3), the angular integration over $q_5 d\cos\theta_5 d\varphi_5$ is to be performed in the 4+5 c.m. system, where the 3-momentum of particle 5 is q_5. We have, for the purposes of numerical computation, taken the numerator in $\Phi(\Delta^2)$ at $\Delta^2=0$, as opposed to taking it on the mass shell, since for the masses of $K^+K^-\rho$ the mass-shell values would yield a negative contribution to the cross section. We have also taken the mass-shell values for the scattering at the lower vertex. This method of dealing with the Δ^2 dependence at the vertices is used in conjunction with the neglect of the propagator form factors of the Ferrari-Selleri type. The ambiguities associated with the off-mass-shell effects preclude the possibility[8] of obtaining a reliable estimate of the absolute magnitude of the contribution to the cross section of the Deck effect.

In Fig. 2 we show the results of our calculation for the $\rho\pi$ mass distribution for $\bar{K}N \to \rho\pi\Lambda$ at 3.52 BeV c.m. energy corresponding to 6.0 BeV/c incident lab

[5] D. R. O. Morrison, in Proceedings of the Stony Brook Conference on High-Energy Two-Body Collisions, 1966 (unpublished).

[6] See *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967). M. Ferro-Luzzi, Rapporteur, presents the energy dependence of the coefficients in a Legendre-polynomial expansion of the differential cross section from 0.5 to 2.2 BeV/c incident K^- lab momentum of reaction 2. The angular distribution at 2.24 BeV/c is presented in G. W. London, R. R. Rau, N. P. Samios, S. S. Yamamoto, M. Goldberg, S. Lichtman, M. Primer, and J. Leitner, Phys. Rev. **143**, 1034 (1966). A reasonably good fit to their histogram may be obtained by taking 40% diffraction scattering of the form $d\sigma/dt = s^{-2}\exp(5.5t)$, where t is in (BeV/c)^2 and 60% of a second-order Legendre-polynomial expansion. The histogram also is fitted by assuming as little as 25% diffraction scattering with an exponent of 2.5t.

[7] Our metric is such that $p^2=p_\mu p_\mu=-m^2$ for a real particle of mass m.

[8] J. M. Shpiz, K. W. Lai, and M. S. Webster, Phys. Rev. **153**, 1722 (1967), have shown that the shape of a mass distribution similar to one considered here is insensitive to wide variations of the Δ^2 dependence of the matrix element while the total integrated cross section can vary greatly for different choices of the Δ^2 dependence.

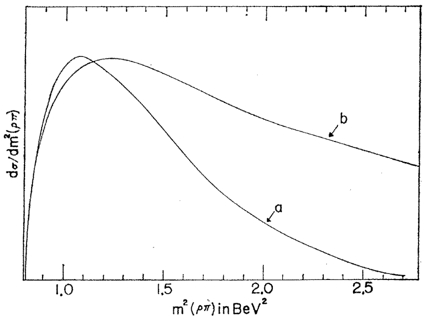

FIG. 2. Curve a shows mass-squared distribution for $\rho\pi$ in $\pi N \rightarrow \rho\pi N$ at $W=2.8$ BeV using $d\sigma(\pi N \rightarrow \pi N)/dt = ce^{9t}$, where t is in $(\mathrm{BeV}/c)^2$ (original Deck effect). Curve b shows mass-squared distribution for $\rho\pi$ in $KN \rightarrow \rho\pi\Lambda$ at $W=3.52$ BeV using for $d\sigma(KN \rightarrow \pi\Lambda)/dt$ the fit discussed in Ref. 6. Both (a) and (b) have been arbitrarily normalized to the same peak.

momentum. The vertex function for $KN \rightarrow \pi\Lambda$ has been chosen to fit the observed angular distribution as discussed in Ref. 6. For comparison we have shown the curve corresponding to Deck's original calculation[3] for $\pi N \rightarrow \rho\pi N$ at 2.8 BeV c.m. energy. The curves show clearly that the kinematic enhancement in the K reaction varies much more slowly over the low-energy region of interest than does that from the π reaction.

From the above calculations, we conclude that a more suitable reaction for searching for low-mass resonances such as the $A_1(1080)$ is $\bar{K}N \rightarrow \Lambda\rho\pi$ (not $\pi p \rightarrow p\rho\pi$), where the usual Deck-type background is greatly reduced. In addition the possibility of interference between the kinematic background and the A_1 enhancement[9] is also reduced in the $\bar{K}N$ reactions since the competing kinematic configuration will, in general, favor an angular distribution different from that of the A_1. Similar conclusions can also be drawn for the $K^*(1300)$ from the reaction $\pi p \rightarrow \Lambda K^*(890)\pi$.

We wish to express our gratitude to Dr. Ralph P. Shutt for encouragement and support, and one of us (JMS) thanks Dr. R. Ronald Rau for the hospitality shown him at Brookhaven during the summer of 1967 when this investigation was undertaken. Thanks are due also to Dr. R. M. Lea and Dr. N. P. Chang for helpful discussions.

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Single- and Double-Pion Production by One-Pion Exchange and a Comparison with Experimental Data between 1.6 and 20 GeV/c†

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A detailed comparison has been made between predictions of elementary one-pion exchange (OPE) and existing experimental data. The Benecke-Dürr (BD) parametrization was used to describe the vertex functions. The BD parametrization has one free parameter R for each vertex. The momentum transfer (t) distributions as measured between 1.6 and 10 GeV/c for the reactions $\bar{p}p \rightarrow \bar{\Delta}^{--}\Delta^{++}$ [$\Delta \equiv \Delta(1236)$], $pp \rightarrow \Delta^{++}n$, $\pi^+p \rightarrow \Delta^{++}\rho^0$, and $\pi^-p \rightarrow n\rho^0$ were used to fit the parameters $R_{\Delta N\pi}$, $R_{NN\pi}$, and $R_{\rho\pi\pi}$ which describe the $NN\pi$, $\Delta N\pi$, and $\rho\pi\pi$ vertices. With the three-parameter fit an excellent description of the data is achieved for $|t|<1$ GeV^2 at all energies, a result which independently of any model demonstrates that the energy dependence of these reactions is that of elementary OPE. From the R parameters, values for various pionic rms radii were deduced: $\langle r_{NN\pi}^2\rangle^{1/2} = 1.06\pm0.04$ F, $\langle r_{\Delta N\pi}^2\rangle^{1/2} = 0.86\pm0.02$ F, and $\langle r_{\rho\pi\pi}^2\rangle^{1/2} = 0.65\pm0.05$ F. The $NN\pi$ and $\Delta N\pi$ values agree with results from πN and ep scattering. As a further consistency check, the BD parametrization was used to describe the $(\frac{3}{2}, \frac{3}{2})$ pion nucleon phase shift δ_{33} in the neighborhood of the Δ. A good fit to the δ_{33} data is found. The value of $R_{\Delta N\pi}$ agrees within 20% with that from the fit to the t distributions. The OPE predictions were calculated for the reactions $\pi^{\pm}p \rightarrow p\pi^+\pi^{\pm}\pi^-$ in absolute magnitude and compared with available experimental results on effective-mass and momentum-transfer distributions at beam momenta between 2.1 and 20 GeV/c. In general, the shape of the distributions is quite well reproduced. Bumps which are present in the $p2\pi$ mass distributions, and which may be taken as evidence for the production of nucleon isobars, can be understood as reflections of the OPE process. The OPE contributions are substantial at all energies; they amount to $\sim$40% near threshold and increase to $\sim$90% at 20 GeV/c, in contrast to the naive expectation that at higher energies the exchange of particles with higher spin will dominate.

I. INTRODUCTION

SINCE the idea of the dominance of one-pion exchange (OPE) was developed a decade ago,[1] numerous OPE calculations have been carried out using different techniques to calculate off-mass-shell corrections for the vertex functions involved. For some reac-

† Work supported by the U. S. Atomic Energy Commission and the Bundesministerium für Wissenschaftliche Forschung, Germany.

* On leave of absence from the Deutsches Elektronen Synchrotron (DESY).

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