Some Comments and a Prediction Concerning the Decay $K_2^0 \to \gamma\gamma$†

RONALD ROCKMORE*
Physics Department, Imperial College, London S.W. 7

(Received 4 November 1968; revised manuscript received 23 December 1968)

It is shown how only pseudoscalar (π^0,η) and vector ($K^{*(6)}$) poles need be considered in a pole-model analysis of the decay $K_2^0 \to \gamma\gamma$. The calculation of the decay amplitude is presented in detail, both in an $SU(3)$-symmetric model of VVP couplings with $\phi\omega$ mixing, and in a model (due to Arnowitt *et al.*) of PCAC breakdown with partial $SU(3)$ symmetry, which fits the decay rates of $\pi^0 \to \gamma\gamma$ and $\eta \to \gamma\gamma$ reasonably well. It is found that the pseudoscalar and vector contributions add, instead of partially canceling as claimed by earlier workers, so that theory and experiment are no longer in reasonable agreement. Moreover, a separate calculation of the important vector pole contribution to the decay amplitude in the free-field quark model yields no significant diminution of its $SU(3)$ magnitude.

1. INTRODUCTION

IMPROVEMENT of the earlier theoretical determination of Savoy and Zimerman[1] of the decay $K_2^0 \to \gamma\gamma$ has recently been one of the subjects of an interesting paper by Greenberg.[2] It is noted there[2,3] that "the over-all amplitude [is] obtained by taking the difference of two large numbers, each with substantial theoretical uncertainties." In this paper, we show first (in Sec. 2) how to remove one of the sources of theoretical uncertainty in Greenberg's pole-dominance calculation,[2] and then (in Sec. 3), by careful derivation in a model with $SU(3)$-invariant VVP couplings and with $\phi\omega$ mixing, that the over-all amplitude in the pole model is, on the contrary, the *sum* of two large numbers, its magnitude numerically far from agreement (more than one order of magnitude) with either of the results of the two measurements of the branching ratio $\Gamma(K_2^0 \to \gamma\gamma)/\Gamma(K_2^0 \to \text{all modes})$ performed so far, those of Criegge *et al.*[4] and of Cronin *et al.*[5] This lack of agreement is further worsened in the model of breakdown of partial conservation of axial-vector current (PCAC) with maximal $SU(3)$ symmetry due to Arnowitt *et al.*[6] (see Sec. 4) which fits the $\pi^0 \to \gamma\gamma$ and $\eta \to \gamma\gamma$ rates reasonably well. For completeness, the important $K^{*(6)}$ (vector pole) contribution is separately calculated in the free-field quark model following Young[7]; however, no significant diminution of its $SU(3)$ magnitude is obtained in that crude calculation. It is interesting to note that the restoration of the Cabibbo factor $\sin\theta \cos\theta$ (which, Greenberg[2] argues, must be omitted in the present calculation) would be very useful in bringing theory into better agreement with present experimental data.

2. ELIMINATION OF AXIAL-VECTOR POLES IN $\langle k_1k_2 \text{ out}|H_w|K_2^0\rangle$

In Greenberg's[2] pole-model analysis of

$$\langle k_1k_2 \text{ out}|H_w|K_2^0\rangle,$$

where

$$H_w = (2G/\sqrt{2})d_{6\alpha\beta}(V_\mu{}^\alpha + A_\mu{}^\alpha)(V_\mu{}^\beta + A_\mu{}^\beta), \qquad (1)$$

with $G \simeq 1.1\times10^{-5}/M_p{}^2$, the contributions from pseudoscalar (π^0,η) and axial-vector ($A_1, A_1{}^{(8)}$) poles are dealt with separately (following the work of Savoy and Zimerman[1]). Effectively one makes the replacement[8]

$$A_\mu{}^\alpha = G_{A_1,\alpha}\mathcal{A}_\mu{}^\alpha + C_\alpha\partial_\mu\phi_\alpha \qquad (2)$$

in the relevant terms of H_w,

$$H_w|_{\text{pseudoscalar, axial vector}} = (4G/\sqrt{2})d_{66\alpha}A_\mu{}^\alpha A_\mu{}^6, \quad \alpha=3,8 \qquad (3)$$

so that

$$\langle k_1k_2 \text{ out}|H_w|K_2^0\rangle|_{\text{pseudoscalar and axial-vector poles}} = \frac{4G}{\sqrt{2}}\sum_{\alpha=3,8} d_{66\alpha}\langle 0|A_\mu{}^6(0)|K_2^0\rangle \times\{G_{A_1,\alpha}\langle k_1k_2 \text{ out}|\mathcal{A}_\mu{}^\alpha(0)|0\rangle - ik_\mu C_\alpha\langle k_1k_2 \text{ out}|\phi_\alpha(0)|0\rangle\}. \qquad (4)$$

The two types of pole contribution are then separately given by

$$\langle k_1k_2 \text{ out}|H_w|K_2^0\rangle|_{\text{pseudoscalar}} = \frac{4G}{\sqrt{2}}C_K m_K{}^2 \sum_{\alpha=3,8}\frac{C_\alpha d_{66\alpha}}{m_\alpha{}^2 - m_K{}^2}\langle k_1k_2 \text{ out}|J_\alpha(0)|0\rangle \qquad (5)$$

† Supported in part by the National Science Foundation and by the Rutgers Research Council, and in part by the Air Force Office of Scientific Research OAR through the European Office of Aerospace Research, U. S. Air Force.

* Rutgers Faculty Fellow, on leave of absence from the Physics Department, Rutgers, The State University, New Brunswick, N.J.

[1] C. A. Savoy and A. H. Zimerman, Nuovo Cimento **57A**, 201 (1968).

[2] D. F. Greenberg, Nuovo Cimento **56A**, 597 (1968).

[3] This is also the case in Ref. 1.

[4] L. Criegge, J. D. Fox, H. Frauenfelder, A. O. Hanson, G. Moscati, C. F. Perdrisat, and J. Todoroff, Phys. Rev. Letters **17**, 150 (1966).

[5] J. W. Cronin, P. F. Kunz, W. S. Ris, and P. C. Wheeler, Phys. Rev. Letters **18**, 25 (1967).

[6] R. Arnowitt, M. H. Friedman, and P. Nath, Phys. Letters **27B**, 657 (1968). We find their theory predicts $\Gamma(\pi^0 \to \gamma\gamma) = 6.6$ eV, in closer agreement with the experimental value, 7.3 eV, than the value they quote.

[7] B.-L. Young, Phys. Rev. **161**, 1620 (1967); **161**, 1615 (1967).

[8] In our notation, $C_3 = C_\pi$, $C_8 = C_\eta$, $C_6 = C_K$ and, correspondingly, $m_3 = m_\pi$, $m_8 = m_\eta$, $m_6 = m_K$. Note that Greenberg (Ref. 2) takes $C_\alpha = C_\pi$.

and

$$\langle k_1k_2\text{ out}|H_w|K_2{}^0\rangle|_{\text{axial vector}}$$
$$=-\frac{4G}{\sqrt{2}}C_K\sum_{\alpha=3,8}\frac{C_\alpha d_{66\alpha}ik_\mu m_{A_1,\alpha}}{m_{A_1,\alpha}{}^2-m_K{}^2}\times\langle k_1k_2\text{ out}|J_\mu{}^{(A_1,\alpha)}(0)|0\rangle. \quad (6a)$$

According to Ref. 2, application of PCAC to Eq. (6a) then yields

$$\langle k_1k_2\text{ out}|H_w|K_2{}^0\rangle|_{\text{axial vector}}$$
$$=\frac{4G}{\sqrt{2}}C_K\sum_{\alpha=?,8}\frac{C_\alpha d_{66\alpha}m_{A_1,\alpha}{}^2}{m_{A_1,\alpha}{}^2-m_K{}^2}\langle k_1k_2\text{ out}|J_\alpha(0)|0\rangle \quad (6b)$$

for the latter contribution. We wish to point out that one can alternatively write

$$\langle k_1k_2\text{ out}|H_w|K_2{}^0\rangle|_{\text{pseudoscalar and axial-vector poles}}$$
$$=\frac{4G}{\sqrt{2}}C_K\sum_{\alpha=3,8}d_{66\alpha}ik_\mu\langle k_1k_2\text{ out}|A_\mu{}^\alpha(0)|0\rangle$$
$$=-\frac{4G}{\sqrt{2}}C_K\sum_{\alpha=3,8}d_{66\alpha}\langle k_1k_2\text{ out}|\partial_\mu A_\mu{}^\alpha(0)|0\rangle$$
$$=-\frac{4G}{\sqrt{2}}C_K\sum_{\alpha=3,8}d_{66\alpha}m_\alpha{}^2C_\alpha\langle k_1k_2\text{ out}|\phi_\alpha(0)|0\rangle, \quad (7)$$

by strict application of PCAC, so that

$$\langle k_1k_2\text{ out}|H_w|K_2{}^0\rangle|_{\text{pseudoscalar and axial-vector poles}}$$
$$=\frac{4G}{\sqrt{2}}C_K\sum_{\alpha=3,8}\frac{C_\alpha d_{66\alpha}m_\alpha{}^2}{m_\alpha{}^2-m_K{}^2}\langle k_1k_2\text{ out}|J_\alpha(0)|0\rangle. \quad (8)$$

Thus

$$\langle k_1k_2\text{ out}|H_w|K_2{}^0\rangle|_{\text{pseudoscalar and axial-vector poles}}$$

may be expressed in terms of pseudoscalar pole contributions only. Further, since from Eqs. (5), (6a), and (8) we have

$$\sum_{\alpha=3,8}C_\alpha d_{66\alpha}\left(\frac{m_K{}^2}{m_\alpha{}^2-m_K{}^2}\langle k_1k_2\text{ out}|J_\alpha(0)|0\rangle-ik_\mu\frac{m_{A_1,\alpha}}{m_{A_1,\alpha}{}^2-m_K{}^2}\langle k_1k_2\text{ out}|J_\mu{}^{(A_1,\alpha)}(0)|0\rangle\right)$$
$$=\sum_{\alpha=3,8}C_\alpha d_{66\alpha}\frac{m_\alpha{}^2}{m_\alpha{}^2-m_K{}^2}\langle k_1k_2\text{ out}|J_\alpha(0)|0\rangle, \quad (9)$$

it follows that

$$\langle k_1k_2\text{ out}|J_\alpha(0)|0\rangle$$
$$=\frac{-ik_\mu m_{A_1,\alpha}}{m_{A_1,\alpha}{}^2-m_K{}^2}\langle k_1k_2\text{ out}|J_\mu{}^{(A_1,\alpha)}(0)|0\rangle. \quad (10)$$

Patently, it is a "weaker" form (soft kaon) of PCAC, i.e.,

$$\langle k_1k_2|J_\alpha(0)|0\rangle\simeq-(ik_\mu/m_{A_1,\alpha})\times\langle k_1k_2\text{ out}|J_\mu{}^{(A_1,\alpha)}(0)|0\rangle, \quad (11)$$

with attendant non-negligible errors, that is being applied in Ref. 2. We remark that the contribution to $g_{K_2{}^0\gamma\gamma}$ calculated from Eq. (8) is about 10% smaller than the sum of Eqs. (5) and (6b), the sum of pseudoscalar and axial-vector pole contributions, with the "induced" axial-meson-$\gamma\gamma$ coupling given in Eq. (11).[9]

3. SIMPLIFIED POLE-MODEL ANALYSIS OF $\langle k_1k_2\text{ out}|H_w|K_2{}^0\rangle$; $SU(3)$-SYMMETRIC MODEL WITH $\phi\omega$ MIXING

The reasonable agreement between theory and experiment claimed by Refs. 1 and 2 depends rather crucially on the partial cancellation of the vector and the sum of pseudoscalar and axial-vector contributions to $\langle k_1k_2\text{ out}|H_w|K_2{}^0\rangle$, a result we are unable to reproduce. Since both the work of Savoy and Zimmerman[1] and that of Greenberg[2] are somewhat deficient in the calculational details to check this point, we present this analysis fully here, now somewhat simplified because of the results of Sec. 2. Thus we write

$$\langle k_1k_2\text{ out}|H_w|K_2{}^0\rangle=\langle k_1k_2\text{ out}|H_w|K_2{}^0\rangle_{\text{pseudoscalar}}+\langle k_1k_2\text{ out}|H_w|K_2{}^0\rangle_{\text{vector}}, \quad (12)$$

with[10]

$$\langle k_1k_2\text{ out}|H_w|K_2{}^0\rangle_{\text{pseudoscalar}}$$
$$=\frac{4G}{\sqrt{2}}C_K\sum_{\alpha=3,8}\frac{C_\alpha d_{66\alpha}}{m_\alpha{}^2-m_K{}^2}\langle k_1k_2\text{ out}|J_\alpha(0)|0\rangle \quad (13)$$

and

$$\langle k_1k_2\text{ out}|H_w|K_2{}^0\rangle_{\text{vector}}$$
$$=\frac{4G}{\sqrt{2}}\sum_{\alpha=3,8}d_{66\alpha}\langle k_1k_2\text{ out}|V_\mu{}^\alpha V_\mu{}^6|0\rangle. \quad (14)$$

To elucidate the matter of the relative sign of the pseudoscalar (13) and vector (14) contributions, we work for the remainder of this section in terms of the conventional $SU(3)$-symmetric model of VVP couplings with $\phi\omega$ mixing. (However, we take into account symmetry-breaking effects provided by spectral-function sum rules of the Weinberg type.[11]) Thus,

$$\mathcal{H}'=\tfrac{1}{2}\sqrt{3}fd_{ijk}\epsilon_{\lambda\rho\tau\sigma}\partial_\lambda\phi_\rho{}^i\partial_\tau\phi_\sigma{}^j\phi^k+g\epsilon_{\lambda\rho\tau\sigma}\partial_\lambda\phi_\rho{}^0\partial_\tau\phi_\sigma{}^i\phi^i, \quad (15)$$

[9] Numerically we find $8.8\times10^{-9}/m_K$ compared with the sum of $5.2\times10^{-9}/m_K$ (pseudoscalar poles) and $4.4\times10^{-9}/m_K$ (axial-vector poles). We take $C_\pi=C_K=C_\eta=97$ MeV as in Ref. 2 for purposes of comparison.

[10] At the risk of being overexplicit, we note that $\langle A|\partial_\mu L|B\rangle=i(k_B-k_A)_\mu\langle A|L|B\rangle$, $J_\alpha(0)=-(-\Box+m_\alpha{}^2)\phi_\alpha(0)$, and $a^2=a_\nu a_\nu=\mathbf{a}^2-a_0{}^2$.

[11] R. J. Oakes and J. J. Sakurai, Phys. Rev. Letters **19**, 1266 (1967).

with

$$J^k=\tfrac{1}{2}\sqrt{3}fd_{ijk}\epsilon_{\lambda\rho\tau\sigma}\partial_\lambda\phi_\rho{}^i\partial_\tau\phi_\sigma{}^j+g\epsilon_{\lambda\rho\tau\sigma}\partial_\lambda\phi_\rho{}^0\partial_\tau\phi_\sigma{}^k. \quad (16)$$

Using the conventions[12]

$$\begin{aligned}\phi_\nu{}^0&=(\sqrt{\tfrac{1}{3}})\phi_\nu+(\sqrt{\tfrac{2}{3}})\omega_\nu,\\ \phi_\nu{}^8&=-(\sqrt{\tfrac{1}{3}})\omega_\nu+(\sqrt{\tfrac{2}{3}})\phi_\nu,\end{aligned} \quad (17)$$

one finds

$$\begin{aligned}g_{\omega\rho\pi}&=-(\sqrt{\tfrac{1}{3}})f+(\sqrt{\tfrac{2}{3}})g,\\ g_{\phi\rho\pi}&=(\sqrt{\tfrac{2}{3}})f+(\sqrt{\tfrac{1}{3}})g,\end{aligned} \quad (18)$$

so that the requirement $g_{\phi\rho\pi}=0$ leads to

$$g=-\sqrt{2}f=(\sqrt{\tfrac{2}{3}})g_{\omega\rho\pi}.$$

From

$$\langle k_1k_2\text{ out}|J^\alpha(0)|0\rangle=-ie\epsilon_\nu(k_2)\int d^4x\,e^{-ik_2\cdot x}\langle k_1|T\{G_\rho\phi_\nu{}^3(x)+\tfrac{1}{3}\sqrt{3}[G_\omega\omega_\nu(x)+G_\phi\phi_\nu(x)],J^\alpha(0)\}|0\rangle, \quad (19)$$

$$\langle k_1|\{\phi_\nu{}^3(0),\omega_\nu(0),\phi_\nu(0)\}|0\rangle=-e\epsilon_\nu(k_1)\left\{\frac{G_\rho}{m_\rho{}^2},\frac{G_\omega}{\sqrt{3}m_\omega{}^2},\frac{G_\phi}{\sqrt{3}m_\phi{}^2}\right\}, \quad (20)$$

one finds straightforwardly

$$\begin{aligned}\langle k_1k_2\text{ out}|J^3(0)|0\rangle&=\frac{2e^2G_\rho G_\omega}{3m_\rho{}^2m_\omega{}^2}(\sqrt{3}fd_{833}-\sqrt{2}g)\epsilon_{\mu\nu\tau\sigma}k_{1\mu}\epsilon_\nu(k_1)k_{2\tau}\epsilon_\sigma(k_2)\\ &=-\frac{2e^2G_\rho G_\omega}{\sqrt{3}m_\rho{}^2m_\omega{}^2}g_{\omega\rho\pi}\epsilon_{\mu\nu\tau\sigma}k_{1\mu}\epsilon_\nu(k_1)k_{2\tau}\epsilon_\sigma(k_2),\end{aligned} \quad (21)$$

$$\begin{aligned}\langle k_1k_2\text{ out}|J^8(0)|0\rangle&=\bigg(-\frac{e^2G_\rho{}^2}{m_\rho{}^4}\sqrt{3}fd_{833}-\frac{2e^2G_\phi{}^2}{3\sqrt{3}m_\phi{}^4}[fd_{888}+(\sqrt{\tfrac{2}{3}})g]\\ &\quad-\frac{2e^2G_\omega{}^2}{3\sqrt{3}m_\omega{}^4}[\tfrac{1}{2}fd_{888}-(\sqrt{\tfrac{2}{3}})g]+\frac{2e^2G_\phi G_\omega}{3\sqrt{3}m_\phi{}^2m_\omega{}^2}\\ &\quad\times[\sqrt{2}fd_{888}-(\sqrt{\tfrac{1}{3}})g]\bigg)\epsilon_{\mu\nu\tau\sigma}k_{1\mu}\epsilon_\nu(k_1)k_{2\tau}\epsilon_\sigma(k_2)\\ &=\frac{e^2}{\sqrt{3}}\left(\frac{G_\rho{}^2}{m_\rho{}^4}+\frac{G_\omega{}^2}{3m_\omega{}^4}-\frac{2G^2}{3m_\phi{}^4}\right)\\ &\quad\times g_{\omega\rho\pi}\epsilon_{\mu\nu\tau\sigma}k_{1\mu}\epsilon_\nu(k_1)k_{2\tau}\epsilon_\sigma(k_2).\end{aligned} \quad (22)$$

Similarly, contracting out either photon in the vector contribution given by Eq. (14) yields

$$\begin{aligned}\langle k_1k_2\text{ out}|H_w|K_2{}^0\rangle_{\text{vector}}&=\frac{4G}{\sqrt{2}}\bigg[d_{663}(-ie)\epsilon_\nu(k_2)\langle k_1|\int d^4x\,e^{-ik_2\cdot x}\\ &\quad\times T\{V_\nu{}^{(\text{em})}(x),G_\rho\phi_\mu{}^3(0)\}V_\mu{}^6(0)|K_2{}^0\rangle+d_{668}(-ie)\epsilon_\nu(k_2)\\ &\quad\times\langle k_1|\int d^4x\,e^{-ik_2\cdot x}T\{V_\nu{}^{(\text{em})}(x),-(\sqrt{\tfrac{1}{3}})G_\omega\omega_\mu(0)\\ &\quad+(\sqrt{\tfrac{2}{3}})G_\phi\phi_\mu(0)\}V_\mu{}^6(0)|K_2{}^0\rangle\bigg]+(k_1\leftrightarrow k_2)\\ &=-\frac{4G}{\sqrt{2}}\left[d_{663}\frac{G_\rho{}^2}{m_\rho{}^2}+\frac{d_{668}}{\sqrt{3}}\left(\frac{G_\omega{}^2}{m_\omega{}^2}+\frac{G_\phi{}^2}{m_\phi{}^2}\right)\right]e\epsilon_\nu(k_2)\\ &\quad\times\langle k_1|V_\nu{}^6(0)|K_2{}^0\rangle+(k_1\leftrightarrow k_2)\\ &=\frac{8G}{3\sqrt{2}}\frac{eG_\rho{}^2}{m_\rho{}^2}\epsilon_\nu(k_2)\langle k_1|G_6\phi_\nu{}^6(0)|K_2{}^0\rangle+(k_1\leftrightarrow k_2).\end{aligned} \quad (23)$$

Since

$$\begin{aligned}\langle k_1|\phi_\nu{}^6(0)|K_2{}^0\rangle&=-i\int d^4x\,e^{ik\cdot x}\langle k_1|T\{\phi_\nu{}^6(0),J^6(x)\}|0\rangle\\ &=-\frac{e}{m_{K^*}{}^2}\bigg[\sqrt{3}fd_{663}\frac{G_\rho}{m_\rho{}^2}+\frac{1}{\sqrt{3}}fd_{668}\left(-\frac{G_\omega}{m_\omega{}^2}+\frac{\sqrt{2}G_\phi}{m_\phi{}^2}\right)\\ &\quad+\tfrac{1}{3}g\left(\sqrt{2}\frac{G_\omega}{m_\omega{}^2}+\frac{G_\phi}{m_\phi{}^2}\right)\bigg]\epsilon_{\mu\nu\tau\sigma}k_{2\mu}k_{1\tau}\epsilon_\sigma(k_1),\end{aligned} \quad (24)$$

we have

$$\begin{aligned}\langle k_1k_2\text{ out}|H_w|K_2{}^0\rangle_{\text{vector}}&=-\frac{8G}{3\sqrt{2}}\frac{e^2G_\rho{}^2G_6}{m_\rho{}^2m_{K^*}{}^2}\left[\frac{G_\rho}{m_\rho{}^2}+\frac{G_\omega}{\sqrt{3}m_\omega{}^2}+(\sqrt{\tfrac{2}{3}})\frac{G_\phi}{m_\phi{}^2}\right]g_{\omega\rho\pi}\\ &\quad\times\epsilon_{\mu\nu\tau\sigma}k_{2\mu}\epsilon_\nu(k_2)k_{1\tau}\epsilon_\sigma(k_1).\end{aligned} \quad (25)$$

Collecting the results, Eqs. (13), (21), (22), and (25), in compact form, we find

$$\begin{aligned}\langle k_1k_2\text{ out}|H_w|K_2{}^0\rangle&=\epsilon_{\mu\nu\tau\sigma}k_{1\mu}\epsilon_\nu(k_1)k_{2\tau}\epsilon_\sigma(k_2)\frac{2Ge^2}{\sqrt{2}}g_{\omega\rho\pi}\\ &\quad\times\bigg\{-\frac{C_6C_3m_\pi{}^2}{m_K{}^2-m_\pi{}^2}\frac{2G_\rho G_\omega}{\sqrt{3}m_\rho{}^2m_\omega{}^2}+\frac{C_6C_8m_\eta{}^2}{m_K{}^2-m_\eta{}^2}\\ &\quad\times\frac{1}{3}\left(\frac{G_\rho{}^2}{m_\rho{}^4}+\frac{G_\omega{}^2}{3m_\omega{}^4}-\frac{2}{3}\frac{G_\phi{}^2}{m_\phi{}^4}\right)-\frac{4}{3}\frac{G_\rho{}^2G_6}{m_\rho{}^2m_{K^*}{}^2}\\ &\quad\times\left[\frac{G_\rho}{m_\rho{}^2}+\frac{G_\omega}{\sqrt{3}m_\omega{}^2}+(\sqrt{\tfrac{2}{3}})\frac{G_\phi}{m_\phi{}^2}\right]\bigg\}.\end{aligned} \quad (26)$$

[12] R. F. Dashen and D. H. Sharp, Phys. Rev. **133**, B1585 (1964).

Note that the η-pole contribution (which dominates the pseudoscalar-pole contribution) and the K^*-pole contribution have the *same* sign. Evaluation of the coefficient of $\epsilon_{\mu\nu\tau\sigma}k_{1\mu}\epsilon_\nu(k_1)k_{2\tau}\epsilon_\sigma(k_2)$ proceeds much as in Ref. 2; we find, using $G_6/m_{K^*}=G_\rho/m_\rho$, $G_\omega/m_\omega{}^2\simeq-0.083$, and $G_\phi/m_\phi{}^2\simeq0.118$,

$$(g_{K_2{}^0\gamma\gamma})_{\pi^0,\eta}\simeq-5.2\times10^{-9}/m_K$$

and

$$(g_{K_2{}^0\gamma\gamma})_{K^*}\simeq-11.3\times10^{-9}/m_K.$$

Since $|g_{K_2{}^0\gamma\gamma}|_{SU(3)}\simeq16.5\times10^{-9}/m_K$ as compared to

$$|g_{K_2{}^0\gamma\gamma}|_{\text{Criegge}}\simeq0.81\times10^{-9}/m_K$$

and

$$|g_{K_2{}^0\gamma\gamma}|_{\text{Cronin}}\simeq1.94\times10^{-9}/m_K,$$

it cannot be said that[1] theory and experiment are in "reasonable agreement." Moreover, this situation persists in the other viable models discussed in the next section.

4. OTHER MODELS

A. Model of PCAC Breakdown of Arnowitt *et al.*

Here we examine the prediction for $K_2{}^0\to\gamma\gamma$ decay in the model of PCAC breakdown with maximal $SU(3)$ symmetry due to Arnowitt *et al.*[6] Since this model fits the $\pi^0\to\gamma\gamma$ and $\eta\to\gamma\gamma$ rates reasonably well, unlike the preceding model, it might furnish a more realistic prediction for the decay $K_2{}^0\to\gamma\gamma$. In the model of Arnowitt *et al.*,[6] modification of PCAC leads to the meson current

$$J^i(0)=(4/C_i)\epsilon_{\mu\nu\alpha\beta}(\lambda d_{ijk}\partial_\mu\phi_\nu{}^i\partial_\alpha\phi_\beta{}^k+\lambda'\partial_\mu\phi_\nu{}^i\partial_\alpha\phi_\beta),\quad \phi_\nu{}^8\equiv\omega_\nu \tag{27}$$

with $\lambda=0.348$ and $\lambda'=0.041$. Since

$$\langle k_1k_2\text{ out}|J^3(0)|0\rangle=-\frac{e^2G_\rho8\lambda}{\sqrt3m_\rho{}^2C_3}\left(\frac{2G_\omega}{\sqrt3m_\omega{}^2}+\frac{\lambda'G_\phi}{\lambda m_\phi{}^2}\right)\times\epsilon_{\mu\nu\tau\sigma}k_{1\mu}\epsilon_\nu(k_1)k_{2\tau}\epsilon_\sigma(k_2), \tag{28}$$

$$\langle k_1k_2\text{ out}|J^8(0)|0\rangle=-\frac{e^28\lambda}{C_8}\left(\frac{G_\rho{}^2}{m_\rho{}^4}d_{833}+\frac{G_\omega{}^2}{3m_\omega{}^4}d_{888}+\frac{\lambda'G_\phi G_\omega}{\lambda3m_\phi{}^2m_\omega{}^2}\right)\times\epsilon_{\mu\nu\tau\sigma}k_{1\mu}\epsilon_\nu(k_1)k_{2\tau}\epsilon_\sigma(k_2), \tag{29}$$

and

$$\langle k_1|V_\nu{}^6(0)|K_2{}^0\rangle=-\frac{8e\lambda G_6}{C_6m_K{}^{*2}}\left(\frac{G_\rho}{m_\rho{}^2}d_{663}+\frac{G_\omega}{\sqrt3m_\omega{}^2}d_{668}+\frac{\lambda'G_\phi}{\lambda\sqrt3m_\phi{}^2}\right)\times\epsilon_{\mu\nu\tau\sigma}k_{2\mu}k_{1\tau}\epsilon_\sigma(k_1), \tag{30}$$

one has

$$\begin{aligned}\langle k_1k_2\text{ out}|H_w|K_2{}^0\rangle&=\epsilon_{\mu\nu\tau\sigma}k_{1\mu}\epsilon_\nu(k_1)k_{2\tau}\epsilon_\sigma(k_2)\\&\times\left\{\frac{32e^2GC_K\lambda}{\sqrt2}\left[\frac{d_{663}m_\pi{}^2}{m_K{}^2-m_\pi{}^2}\frac{2G_\rho G_\omega}{3m_\rho{}^2m_\omega{}^2}+\frac{d_{668}m_\eta{}^2}{m_K{}^2-m_\eta{}^2}\left(\frac{G_\rho{}^2}{m_\rho{}^4}d_{833}+\frac{G_\omega{}^2}{3m_\omega{}^4}d_{888}\right)\right]+\frac{32e^2GC_K\lambda'}{\sqrt2}\right.\\&\times\left(\frac{d_{663}m_\pi{}^2}{m_K{}^2-m_\pi{}^2}\frac{G_\rho G_\phi}{\sqrt3m_\rho{}^2m_\phi{}^2}+\frac{d_{668}m_\eta{}^2}{m_K{}^2-m_\eta{}^2}\frac{G_\phi G_\omega}{3m_\phi{}^2m_\omega{}^2}\right)-\left.\frac{64e^2GG_6G_\rho{}^2}{3\sqrt2C_Km_K{}^{*2}m_\rho{}^2}\left[2\lambda\left(d_{663}\frac{G_\rho}{m_\rho{}^2}+d_{668}\frac{G_\omega}{\sqrt3m_\omega{}^2}\right)+\frac{\lambda'G_\phi}{\sqrt3m_\phi{}^2}\right]\right\}.\end{aligned} \tag{31}$$

Before attempting a numerical evaluation of expression (31), we note that in fitting the rates $\pi^0\to\gamma\gamma$ and $\eta\to\gamma\gamma$, which are separately invariant under the sign reversal $G_\omega\to-G_\omega$, $G_\phi\to-G_\phi$, Arnowitt *et al.*[6] have assumed $G_\varphi=\frac12\sqrt3m_\phi{}^2f_Y{}^{-1}\cos\theta_Y$ and $G_\omega=-G_\phi(m_\omega{}^2/m_\phi{}^2)\times\tan\theta_Y$; however, the conventional assumption[7] [based on the $SU(3)$ result[13] $\text{amp}(\pi^0\to\gamma\gamma)=\sqrt3\ \text{amp}(\eta\to\gamma\gamma)$] that $\text{sgn}(g_{\pi^0\gamma\gamma}/g_{\eta\gamma\gamma})=+$, *requires* $G_\phi=-\frac12\sqrt3m_\phi{}^2f_Y{}^{-1}\times\cos\theta_Y$ and $G_\omega=-G_\phi(m_\omega{}^2/m_\phi{}^2)\tan\theta_Y$ instead. Thus, in its application to the decay $K_2{}^0\to\gamma\gamma$, the model of PCAC breakdown proposed by Arnowitt *et al.*[6] is not completely determined until, say, the phase $\text{sgn}(g_{\pi^0\gamma\gamma}/g_{\eta\gamma\gamma})$ is specified. In the conventional case, we find $(g_{K_2{}^0\gamma\gamma})_{\pi^0,\eta}\simeq11.3\times10^{-9}/m_K$ and $(g_{K_2{}^0\gamma\gamma})_{K^*}\simeq17.5\times10^{-9}/m_K$, so that $|g_{K_2{}^0\gamma\gamma}|\simeq28.8\times10^{-9}/m_K$. Thus "theory" and experiment are even farther apart when we make use of the experimental results for $\pi^0\to\gamma\gamma$ and $\eta\to\gamma\gamma$.

B. Free-Field Quark Model and K^* Contribution to $K_2{}^0\to\gamma\gamma$

Although arguments[14] have recently been advanced for favoring the commutation relations of the gauge-field algebra[15] over those of the free-field quark model,[16] or $U(6)\times U(6)$, we present here an analysis of the apparently sizable K^* contribution to $K_2{}^0\to\gamma\gamma$ in terms of the latter model for completeness. It should be noted that numerically (in the case of $\pi^0\to\gamma\gamma$ decay) the quark model is certainly in fair agreement with experiment. In any event our crude analysis may serve to indicate the sort of model dependence to be expected in the case of the vector contribution where experimental information regarding the couplings G_{K^*KV} ($V=\rho^0,\phi,\omega$)

[13] This can be obtained from Eqs. (21) and (22) in the appropriate limits.

[14] G. W. Barry and J. J. Sakurai, Nuovo Cimento **50A**, 326 (1968).

[15] T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 1029 (1967).

[16] R. P. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters **13**, 678 (1964).

is lacking. We write

$$-i\int d^4x\, e^{-ik_1\cdot x}e\langle 0|T\{V_\mu^{(\mathrm{em})}(x),V_\nu^6(0)\}|K_2^0\rangle=\epsilon_{\mu\sigma\nu\tau}k_{1\sigma}k_{2\tau}M(k_1^2,k_2^2)\,, \tag{32}$$

where

$$M(k_1^2,k_2^2)_{\text{vector dom.}}=-\tfrac{1}{2}eG_6g_{\omega\rho\pi}\frac{1}{k_2^2+m_{K^*}^2}\left\{\frac{G_\rho}{k_1^2+m_\rho^2}+\frac{G_\omega/\sqrt{3}}{k_1^2+m_\omega^2}+\frac{\sqrt{2}G_\phi/\sqrt{3}}{k_1^2+m_\varphi^2}\right\}$$

$$=M(0,0)_{\text{V.D.}}+\frac{k_2^2eG_6g_{\omega\rho\pi}}{2m_{K^*}^2(k_2^2+m_{K^*}^2)}\left[\frac{G_\rho}{m_\rho^2}+\frac{G_\omega}{\sqrt{3}m_\omega^2}+(\sqrt{\tfrac{2}{3}})\frac{G_\omega}{m_\phi^2}\right]$$

$$+\frac{k_1^2eG_6g_{\omega\rho\pi}}{2m_{K^*}^2}\left[\frac{G_\rho}{m_\rho^2(k_1^2+m_\rho^2)}+\frac{G_\omega}{\sqrt{3}m_\omega^2(k_1^2+m_\omega^2)}+(\sqrt{\tfrac{2}{3}})\frac{G_\phi}{m_\phi^2(k_1^2+m_\phi^2)}\right]$$

$$-\frac{k_1^2k_2^2eG_6g_{\omega\pi\rho}}{2m_{K^*}^2(k_2^2+m_{K^*}^2)}\left[\frac{G_\rho}{m_\rho^2(k_1^2+m_\rho^2)}+\frac{G_\omega}{\sqrt{3}m_\omega^2(k_1^2+m_\omega^2)}+(\sqrt{\tfrac{2}{3}})\frac{G_\phi}{m_\phi^2(k_1^2+m_\phi^2)}\right] \tag{33}$$

and

$$M(k_1^2,k_2^2)_{\text{quark model}}=M(0,0)_{\text{Q.M.}}+\frac{k_2^2G_6f_{K^*K\gamma}}{m_{K^*}^2(k_2^2+m_{K^*}^2)}+k_1^2\sum_{V=\rho^0,\phi,\omega}\frac{e\beta_{K^*(0)KV}}{m_V^2(k_1^2+m_V^2)}$$

$$-\frac{k_1^2k_2^2eG_6g_{\omega\rho\pi}}{2m_{K^*}^2(k_2^2+m_{K^*}^2)}\left[\frac{G_\rho}{m_\rho^2(k_1^2+m_\rho^2)}+\frac{G_\omega}{\sqrt{3}m_\omega^2(k_1^2+m_\omega^2)}+(\sqrt{\tfrac{2}{3}})\frac{G_\phi}{m_\phi^2(k_1^2+m_\phi^2)}\right], \tag{34}$$

with $f_{K^*K\gamma}=\frac{2}{3}f_{\omega\pi\gamma}$. The requirement that $M(k_1^2,k_2^2)_{\text{Q.M.}}$ vanish asymptotically yields

$$0=M(0,0)_{\text{Q.M.}}+\frac{2G_6}{3m_{K^*}^2}f_{\omega\pi\gamma}+\sum_{V=\rho^0,\omega,\phi}\frac{e\beta_{K^*(0)KV}}{\hat{V}m^2}$$

$$-\frac{eG_6g_{\omega\rho\pi}}{2m_{K^*}^2}\left[\frac{G_\rho}{m_\rho^2}+\frac{G_\omega}{\sqrt{3}m_\omega^2}+(\sqrt{\tfrac{2}{3}})\frac{G_\phi}{m_\phi^2}\right], \tag{35a}$$

while the application of the Bjorken limit[17] to expression (32),

$$-ie\int d^4x\, e^{-ik_1\cdot x}$$

$$+\langle 0|T\{V_\mu^{(\mathrm{em})}(x),V_\nu^6(0)\}|K_2^0\rangle \xrightarrow[|k_{10}|\to\infty]{} \frac{e}{k_{10}}$$

$$\times\int d\mathbf{x}\, e^{-i\mathbf{k}_1\cdot\mathbf{x}}\langle 0|[V_\mu^{(\mathrm{em})}(\mathbf{x},0),V_\nu^6(0)]|K_2^0\rangle\,,$$

with the quark-model commutation relations

$$e[V_\mu^3(\mathbf{x})+\tfrac{1}{3}\sqrt{3}V_\mu^8(\mathbf{x}),V_\nu^6(0)]$$

$$=-ie(d_{636}+\tfrac{1}{3}\sqrt{3}d_{686})\epsilon_{4\mu\nu\tau}\delta(\mathbf{x})A_\tau^6(0)+\cdots,$$

yields

$$\tfrac{2}{3}eC_\pi=\tfrac{2}{3}G_6f_{\omega\pi\gamma}+\sum_V e\beta_{K^*(0)KV}$$

$$-\tfrac{1}{2}eG_6g_{\omega\rho\pi}\left\{\frac{1}{m_{K^*}^2}\left[G_\rho+\frac{G_\omega}{\sqrt{3}}+(\sqrt{\tfrac{2}{3}})G_\phi\right]\right.$$

$$\left.+\left[\frac{G_\rho}{m^2}+\frac{G_\omega}{\sqrt{3}m_\omega^2}+(\sqrt{\tfrac{2}{3}})\frac{G_\phi}{m_\phi^2}\right]\right\}. \tag{35b}$$

For computational purposes we make the crude approximation $m_V^2\cong m_{K^*}^2$, and find, dropping terms

$$O((m_{V'}^2-m_{V''}^2)/m_{K^*}^2)\,,$$

$$M(0,0)_{\text{Q.M.}}\simeq-\frac{eG_6g_{\omega\rho\pi}}{2m_{K^*}^2}$$

$$\times\left[\frac{G_\rho}{m_\rho^2}+\frac{G_\omega}{\sqrt{3}m_\omega^2}+(\sqrt{\tfrac{2}{3}})\frac{G_\phi}{m_\phi^2}\right]-\frac{2eC_\pi}{3m_{K^*}^2}. \tag{36}$$

For the case[7,14] $\mathrm{sgn}(f_{\omega\pi\gamma}G_\rho/C_\pi)=-$, for which[14] $g_{\omega\rho\pi}^2/4\pi=0.58/m_\pi^2$, we find $(g_{K_2^0\gamma\gamma})_{K^*,\text{Q.M.}}\simeq10.9\times10^{-9}/m_K$, which, in spite of the crudeness of our approximations, does not seem to indicate a significant diminution of the $SU(3)$ magnitude.

ACKNOWLEDGMENTS

It is a pleasure to thank Dr. L. A. P. Balázs and K. J. Barnes for their interest and C. Isham for a discussion. I also wish to thank Professor A. Salam and Professor P. T. Matthews for their hospitality at Imperial College.

[17] J. D. Bjorken, Phys. Rev. **148**, 1467 (1966).