Electromagnetic Properties of the Nucleon Resonance $N'(1470)$

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By applying the sidewise dispersion relation to the electromagnetic, π-N, and gravitational vertex functions of the nucleon and $N'(1470)$ while dominating the intermediate states by one-particle states, we show that the magnetic form factor of $N'(1470)$ and the magnetic transition form factor between $N'(1470)$ and the nucleon are proportional to the magnetic form factor of the nucleon with known proportionality constants. We also show that the electromagnetic mass difference of $N'(1470)$ is proportional to the p-n mass difference with the same proportionality constant.

I. INTRODUCTION

IN studying the electromagnetic properties of the nucleon one ordinarily takes the nucleon electromagnetic vertex function with the photon off the mass shell and assumes the dispersion relation in photon momentum squared. In some cases, however, it is more useful to take the nucleon off the mass shell in the vertex function and to study the dispersion relation in the nucleon momentum squared, i.e., the so-called sidewise dispersion relation.[1] The sidewise dispersion relation has two major merits in that it is proved rigorously for the spacelike and lightlike photons and only $I=\frac{1}{2}$ states contribute to the intermediate states. This technique of the sidewise dispersion relation has been used to calculate the anomalous magnetic moments of the electron, muon, and nucleon[2] and the isovector radius of the nucleon.[3] The same technique has also been applied to the π-N and axial-vector vertex functions to obtain the axial-vector coupling constant expressed by the low-energy π-N amplitude;[4] and to the gravitational vertex function of the nucleon to calculate the p-n mass difference.[5]

In the present paper we shall systematically apply this technique to the electromagnetic, π-N, and gravitational vertex functions of the nucleon and $N'(1470)$.[6] Assuming the dominance of $N'(1470)$ in the intermediate states, we shall show that the magnetic form factor of $N'(1470)$ and the magnetic transition form factor between $N'(1470)$ and the nucleon are proportional to the magnetic form factor of the nucleon, and that the proportionality constants can be expressed by experimentally known strong-coupling constants. The magnetic transition form factor can be used to estimate the inelastic contribution to the p-n mass difference in Cottingham's formula in the energy range of the $N'(1470)$ formation.[7] It will also be shown that the electromagnetic mass difference of $N'(1470)$ is proportional to the p-n mass difference with the same proportionality constant as above. The numerical result is $M^+-M^0\simeq-0.2$ MeV. This value is very small and it is very difficult to test experimentally. However, a more detailed phase-shift analysis of the P_{11} π-N scattering may serve to give the experimental measurement of this mass difference.

In Sec. II we shall apply the sidewise-dispersion technique to the electromagnetic vertex functions of the nucleon and $N'(1470)$. Assuming the dominance of the nucleon and $N'(1470)$ poles, we shall derive relations among the form factors of the nucleon and $N'(1470)$ and the transition form factor between the nucleon and $N'(1470)$. This technique will be applied to the gravitational vertex functions of the nucleon and $N'(1470)$ in Sec. III. Again assuming the one-particle dominance, we shall derive a relation between the electromagnetic mass differences of the nucleon and $N'(1470)$. A summary and conclusion will be given in Sec. IV.

II. ELECTROMAGNETIC VERTEX FUNCTIONS

We shall first deal with the electromagnetic vertex function of the nucleon $\langle N|j_\mu(0)|N\rangle$. By the use of the reduction formula, one of the nucleon states can be taken off its mass shell. Thus we define the following vertex function:

$$\Gamma_\mu(p,p')u(p')=i\int d^4x\, e^{ip\cdot x}(-i\gamma\cdot\partial+m)\times\langle 0|T[\psi(x)j_\mu(0)]|N\rangle\left(\frac{(2\pi)^3p_0'}{m}\right)^{1/2},\quad(1)$$

where p_μ and p_μ' are the momenta of the left-hand and right-hand nucleon states, respectively, $\psi(x)$ is the renormalized nucleon field, and $u(p')$ is the free Dirac wave function normalized as $\bar{u}u=1$. Taking into account the Lorentz covariance of Eq. (1) and the Dirac equation for $u(p')$, we can write the most general form of

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[1] A. M. Bincer, Phys. Rev. **118**, 855 (1960).

[2] S. D. Drell and H. R. Pagels, Phys. Rev. **140**, B397 (1965); R. G. Parsons, *ibid.* **168**, 1562 (1968).

[3] S. D. Drell and D. J. Silverman, Phys. Rev. Letters **20**, 1325 (1968).

[4] H. Suura and L. M. Simmons, Phys. Rev. **148**, 1579 (1966); K. Bardakci, *ibid.* **155**, 1788 (1967); A. Love and R. G. Moorhouse, Glasgow University Report, 1968 (unpublished).

[5] H. R. Pagels, Phys. Rev. **144**, 1261 (1966).

[6] L. D. Roper, R. M. Wright, and B. T. Feld, Phys. Rev. **138**, B190 (1965). For an extensive list of references, see A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **41**, 109 (1969).

[7] T. Muta, Phys. Rev. **171**, 1661 (1968).

$\Gamma_\mu(p,p')u(p')$:

$$\Gamma_\mu(p,p')u(p')=\frac{\gamma\cdot p+m}{2m}[F_1(p^2,k^2)\gamma_\mu+F_2(p^2,k^2)i\sigma_{\mu\nu}k^\nu+F_3(p^2,k^2)ik_\mu]u(p')+\frac{-\gamma\cdot p+m}{2m}[F_4(p^2,k^2)\gamma_\mu+F_5(p^2,k^2)i\sigma_{\mu\nu}k^\nu+F_6(p^2,k^2)ik_\mu]u(p'), \quad (2)$$

where F_1, F_2, $\cdots$, F_6 are the invariant functions of p^2 and k^2 with $k_\mu=p_\mu-p_\mu'$. On the mass shell of the left-hand nucleon ($p^2=m^2$), F_1 and F_2 become the ordinary electric and magnetic form factors and F_3 vanishes due to the time-reversal property of the vertex function.[8] The Ward-Takahashi indentity[9] further restricts the general form of $\Gamma_\mu(p,p')u(p')$ and we see that F_1, F_3, F_4, and F_6 are related to each other:

$$F_1=F_4-\frac{2mk^2}{p^2-m^2}F_3,\quad F_4=e-\frac{k^2}{2m}(F_3-F_6). \quad (3)$$

Hence only four of the F_i's are independent in Eq. (2).

Bincer[1] has already proved that for fixed k^2 ($\leqslant 0$), $F_i(p^2,k^2)$ satisfies a dispersion relation in p^2 with possible subtractions and so we are free to use the dispersion technique. The imaginary part of $F_i(p^2,k^2)$ in p^2 with k^2 fixed ($\leqslant 0$) is given by the relation

$$\mathrm{Abs}\Gamma_\mu(p,p')u(p')=\pi(2\pi)^3\sum_n \delta^4(p_n-p)\times\langle 0|\eta(0)|n\rangle\langle n|j_\mu(0)|N\rangle\left(\frac{(2\pi)^3p_0'}{m}\right)^{1/2}, \quad (4)$$

where $\eta(x)=(-i\gamma\cdot\partial+m)\psi(x)$ and p_n is the four-momentum of the intermediate state $|n\rangle$. The intermediate states are of isospin $\frac{1}{2}$ and nucleon number 1 and so the intermediate state with the lowest possible total mass is the S_{11} or P_{11} π-N state. The photon is not included in the intermediate states, since it gives a higher-order effect in the electromagnetic interaction. In calculating the imaginary part of $F_i(p^2,k^2)$ by Eq. (4) we approximate the intermediate states by the S_{11} and P_{11} π-N states and further dominate these π-N states by resonances. There are four experimentally observed resonances in the S_{11} and P_{11} π-N states, which we have listed in Table I.[10] Among them the so-called Roper resonance $N'(1470)$ has the lowest mass and the largest communication with the elastic channel. Three other resonances are highly inelastic. Hence, it is quite reasonable to dominate the intermediate state by the Roper resonance. In this approximation we obtain[11]

TABLE I. Masses and widths of the $I=J=\frac{1}{2}$ nucleon resonances which may contribute to the intermediate states in Eq. (4).

Mass	Wave	Total width Γ (MeV)	Γ_{el}/Γ
$N'(1470)$	P_{11}	210	0.65
$N(1530)$	S_{11}	120	0.35
$N'(1710)$	S_{11}	300	0.40
$N'(1750)$	P_{11}	330	0.30

$$\mathrm{Im}F_i(p^2,k^2)=\pi\delta(p^2-M^2)(m^2-M^2)g_{NR}F_i^{RN}(k^2),\quad k^2\leqslant 0 \quad (5)$$

where m and M are the masses of the nucleon and the Roper resonance, respectively, and g_{NR} and $F_i^{RN}(k^2)$ are defined as

$$\langle 0|\eta(0)|R\rangle=(m-M)g_{NR}\left(\frac{m}{(2\pi)^3p_0}\right)^{1/2}u(p), \quad (6)$$

$$\langle R|j_\mu(0)|N\rangle=\frac{1}{(2\pi)^3}\left(\frac{mM}{p_0p_0'}\right)^{1/2}\bar{u}(p)[F_1^{RN}(k^2)\gamma_\mu+F_2^{RN}(k^2)i\sigma_{\mu\nu}k^\nu+F_3^{RN}(k^2)ik_\mu]\times u(p'). \quad (7)$$

Here $|R\rangle$ is the Roper-resonance state with the momentum p_μ, and the transition form factor $F_1^{RN}(k^2)$ is related to $F_3^{RN}(k^2)$ by current conservation such that

$$F_1^{RN}(k^2)=\frac{k^2}{m-M}F_3^{RN}(k^2). \quad (8)$$

Provided that one of the linear combinations of F_i's, say, $F(p^2,k^2)$, vanishes at $p^2=\infty$, then we can write down an unsubtracted dispersion relation in p^2 with k^2 fixed ($\leqslant 0$):

$$F(p^2,k^2)=\frac{m^2-M^2}{M^2-p^2}g_{NR}F^{RN}(k^2), \quad (9)$$

where $F^{RN}(k^2)$ is a linear combination of F_i^{RN}'s with the same coefficients as those of $F(p^2,k^2)$. As we have mentioned, $F(p^2,k^2)$ reduces to the ordinary electromagnetic form factor of the nucleon on the mass shell $p^2=m^2$, i.e., $F(m^2,k^2)=F^N(k^2)$. Therefore, we easily obtain

$$F^N(k^2)=-g_{NR}F^{RN}(k^2),\quad k^2\leqslant 0. \quad (10)$$

Thus we obtain a simple relation between the nucleon form factor and the R-N transition form factor under the assumption of the unsubtracted sidewise dispersion relation. One may wonder if this relation becomes complicated when we take into account the contribution of other resonances. It is easily seen, however, that $F^{RN}(k^2)$ and $F^N(k^2)$ are still proportional to each other

[8] F. J. Ernst, R. G. Sachs, and K. C. Wali, Phys. Rev. **119**, 1105 (1960).

[9] J. C. Ward, Phys. Rev. **78**, 182 (1950); Y. Takahashi, Nuovo Cimento **6**, 371 (1957). In our vertex function this identity reads $k^\mu\Gamma_\mu(p,p')u(p')=e\gamma\cdot ku(p')$.

[10] C. Lovelace, in *Proceedings of the Heidelberg International Conference on Elementary Particles*, edited by H. Filthuth (North-Holland Publishing Co., Amsterdam, 1968), p. 79.

[11] If the Roper resonance were stable, the matrix element of the nucleon current $\langle 0|\eta(0)|R\rangle$ would vanish, i.e., $g_{NR}=0$, as can be easily seen by the renormalization condition in the presence of the particle mixing. See, e.g., T. Muta, Progr. Theoret. Phys. (Kyoto) **35**, 1099 (1966).

even if these resonances are introduced in the intermediate states. It is the proportionality constant which is slightly changed by the introduction of the other resonances.

As the linear combination $F(p^2,k^2)$ we may choose $F_i(p^2,k^2)$ itself or the electric and magnetic combinations of F_i's:

$$G_E(p^2,k^2)=F_1(p^2,k^2)+(k^2/2m)F_2(p^2,k^2), \quad (11)$$

$$G_M(p^2,k^2)=F_1(p^2,k^2)+2mF_2(p^2,k^2). \quad (12)$$

Let us first try $F_1(p^2,k^2)$ as $F(p^2,k^2)$; then we have the relation (10) between $F_1{}^N(k^2)$ and $F_1{}^{RN}(k^2)$. Now, because of the current conservation [see Eq. (8)], $F_1{}^{RN}(k^2)$ should vanish at $k^2=0$ while $F_1{}^N(0)=e$ for the proton. Hence, this relation for the proton is inconsistent. This means that we need at least one subtraction in the sidewise dispersion relation for $F_1(p^2,k^2)$ of the proton. Therefore, we cannot obtain any meaningful relation between $F_1{}^p(k^2)$ and $F_1{}^{R^+p}(k^2)$, where the superscript p and R^+ mean the proton and the positively charged Roper resonance, respectively. There is no inconsistency in the case of the neutron and so one may impose the relation (10) on $F_1{}^n(k^2)$ and $F_1{}^{R^0n}(k^2)$. On the other hand, we have no restriction on the magnetic form factor $F_2(p^2,k^2)$ and/or $G_M(p^2,k^2)$. Hence the relation (9) for $F_2(p^2,k^2)$ and/or $G_M(p^2,k^2)$ seems to hold. In the case of $G_M(p^2,k^2)$ we have a bit more complicated relation,

$$G_M{}^N(k^2)=-\frac{g_{NR}}{1-k^2/(m+M)^2}\left[\frac{M-m}{M+m}G_E{}^{RN}(k^2)+\left(\frac{2m}{M+m}-\frac{k^2}{(M+m)^2}\right)G_M{}^{RN}(k^2)\right], \quad (13)$$

where[7]

$$G_E{}^{RN}(k^2)=F_1{}^{RN}(k^2)+\frac{k^2}{M+m}F_2{}^{RN}(k^2), \quad (14)$$

$$G_M{}^{RN}(k^2)=F_1{}^{RN}(k^2)+(M+m)F_2{}^{RN}(k^2). \quad (15)$$

If we take into account that $(M-m)/(M+m)\ll 1$ and $G_E{}^{RN}(0)=0$, we can safely neglect the electric term. In our crude approximation we can also put $2m/(M+m)\simeq 1$ and we have

$$G_M{}^N(k^2)\simeq -g_{NR}G_M{}^{RN}(k^2). \quad (16)$$

In order to express the constant g_{NR} in terms of the experimentally known constants, we have to apply the sidewise-dispersion technique to the π-N vertex function and dominate the intermediate states by the Roper resonance.[12] In this manner we obtain the following relation:

$$g_{NR}=-g_{NN\pi}/g_{RN\pi}, \quad (17)$$

where $g_{NN\pi}$ and $g_{RN\pi}$ are the π-N and π-R-N renormalized coupling constants, respectively[13]:

$$g_{NN\pi}{}^2/4\pi=14.6, \quad g_{RN\pi}{}^2/4\pi\simeq 2.5. \quad (18)$$

Thus we finally obtain the following relations, each of which comes from the different assumption on the asymptotic behavior of the form factor in the limit $p^2\to\infty$:

$$F_2{}^{RN}(k^2)=\frac{g_{RN\pi}}{g_{NN\pi}}F_2{}^N(k^2), \quad (19)$$

$$G_M{}^{RN}(k^2)=\frac{g_{RN\pi}}{g_{NN\pi}}G_M{}^N(k^2). \quad (20)$$

We already know the functional form of $F_2{}^N(k^2)$ and $G_M{}^N(k^2)$ from the analysis of the e-N scattering experiment. In order to test Eqs. (19) and (20) experimentally, we need to know the functional form of $F_2{}^{RN}(k^2)$ and $G_M{}^{RN}(k^2)$. This can be done by analyzing the experiment of the electroproduction of the Roper resonance. There has been, however, no experimental evidence on the electroproduction of the Roper resonance.[14] At $k^2=0$, Eqs. (19) and (20) reduce to the relations between the nucleon magnetic moment and the magnetic transition moment. The magnetic transition moment between the proton and R^+ can be measured by analyzing the photopion production processes in the energy range of the Roper resonance. The analysis of the experimental data by Chau, Dombey, and Moorhouse[15] seems to prefer Eq. (20) to Eq. (19).[16] Equation (20) can be used to calculate the inelastic contribution from the Roper resonance to the p-n mass difference in Cottingham's formula and it gives the magnetic contribution of about -0.09 MeV to the difference m_p-m_n.[7]

The same technique as above can be applied to the electromagnetic vertex function of the Roper resonance $\langle R|j_\mu(0)|R\rangle$. In calculating the absorptive part we dominate the intermediate states by the one-nucleon state and obtain

$$\mathrm{Im}F_i{}^R(p^2,k^2)=\pi\delta(p^2-m^2)(M^2-m^2)g_{RN}F_i{}^{NR}(k^2), \quad (21)$$

where $F_i{}^R(p^2,k^2)$ is the form factor of the Roper resonance similar to that of the nucleon, and g_{RN} and $F_i{}^{NR}(k^2)$ are defined in a similar manner as before. They are related to g_{RN} and $F_i{}^{RN}(k^2)$ in the following way:

$$g_{RN}=1/g_{NR}, \quad F_i{}^{RN}(k^2)=[F_i{}^{NR}(k^2)]^*, \quad (22)$$

where the second relation comes from the Hermiticity of the electromagnetic current $j_\mu(x)$. Again we observe that $F_1{}^{R^+}(p^2,k^2)$ needs at least one subtraction in the sidewise dispersion relation with k^2 fixed ($\leqslant 0$), while $F_2{}^R(p^2,k^2)$ and/or $G_M{}^R(p^2,k^2)$ may satisfy an unsub-

[12] For details of the calculation, see T. Muta, Nuovo Cimento **51A**, 1154 (1967); D. H. Lyth, Phys. Rev. **165**, 1786 (1968).

[13] The first paper cited in Ref. 12.

[14] W. K. H. Panofsky, in *Proceedings of the Heidelberg International Conference on Elementary Particles*, edited by H. Filthuth (North-Holland Publishing Co., Amsterdam, 1968), p. 371.

[15] Y. C. Chau, N. Dombey, and R. G. Moorhouse, Phys. Rev. **163**, 1632 (1967).

[16] The second paper cited in Ref. 12.

tracted sidewise dispersion relation. Taking into account Eq. (22) and using Eqs. (19), we can eliminate $F_2^{NR}(k^2)$ from Eq. (21). We obtain

$$F_2^R(k^2)=\left(\frac{g_{RN\pi}}{g_{NN\pi}}\right)^2 F_2^N(k^2). \tag{23}$$

In the same way we have

$$G_M^R(k^2)=\left(\frac{g_{RN\pi}}{g_{NN\pi}}\right)^2 G_M^N(k^2). \tag{24}$$

III. GRAVITATIONAL VERTEX FUNCTION

As is well known, the physical mass of the nucleon, m, is expressed in terms of the trace of the gravitational vertex function with the nucleon at rest[17]:

$$m=(2\pi)^3\langle N|g^{\mu\nu}\theta_{\mu\nu}(0)|N\rangle_{\mathbf{p}=0}, \tag{25}$$

where $\mathbf{p}$ is the three-momentum of the nucleon, and $\theta_{\mu\nu}(x)$ the energy-momentum tensor. In the same manner the physical mass of the Roper resonance, M, is given by

$$M=(2\pi)^3\langle R|g^{\mu\nu}\theta_{\mu\nu}(0)|R\rangle_{\mathbf{p}=0}. \tag{26}$$

Here we have treated the Roper resonance as if it were stable. In the following, however, we shall frequently take its unstable nature into account, and then the transition matrix element of $g^{\mu\nu}\theta_{\mu\nu}(0)$ between the nucleon and the Roper resonance will not vanish.

First we shall apply the sidewise-dispersion technique to the gravitational vertex function of the nucleon.[5] By contracting the nucleon from the left-hand state by the reduction technique, we can define the vertex function with the graviton and one of the nucleons off the mass shell:

$$\Gamma_N(p,p')u(p')=i\int d^4x\, e^{ip\cdot x}(-i\gamma\cdot\partial+m)\times\langle 0|T[\psi_N(x)g^{\mu\nu}\theta_{\mu\nu}(0)]|N\rangle\left(\frac{(2\pi)^3p_0'}{m}\right)^{1/2}, \tag{27}$$

where $\psi_N(x)$ is the renormalized nucleon field. The general form of this vertex function is

$$\Gamma_N(p,p')u(p')=\left(\frac{\gamma\cdot p+m}{2m}G^N(p^2,k^2)+\frac{-\gamma\cdot p+m}{2m}\bar{G}^N(p^2,k^2)\right)u(p'), \tag{28}$$

where $k_\mu=p_\mu-p_\mu'$, and $G^N(p^2,k^2)$ and $\bar{G}^N(p^2,k^2)$ are two invariant amplitudes. From the condition (25) we obtain

$$m=G^N(m^2,0). \tag{29}$$

Thus, we may call $G^N(p^2,k^2)$ the mass form factor in analogy with the charge and magnetic form factors in the electromagnetic vertex function. The imaginary part of the mass form factor $G^N(p^2,k^2)$ in the variable p^2 is easily calculated by assuming the dominance of the Roper resonance in the intermediate states,[18]

$$\mathrm{Im}G^N(p^2,k^2)=\pi\delta(p^2-M^2)(m^2-M^2)g_{NR}G^{RN}(k^2), \tag{30}$$

where g_{NR} is the same constant as the one in Sec. II and the transition form factor $G^{RN}(k^2)$ is defined by

$$\langle R|g^{\mu\nu}\theta_{\mu\nu}(0)|N\rangle=\frac{1}{(2\pi)^3}\left(\frac{mM}{p_0p_0'}\right)^{1/2}\bar{u}(p)u(p')G^{RN}(k^2). \tag{31}$$

The mass form factor $G^N(p^2,k^2)$ does not vanish in the limit $p^2\to\infty$ unless the bare mass of the nucleon vanishes. Hence we need a subtraction in the sidewise dispersion relation. We make a subtraction at $p^2=m^2$ and obtain

$$G^N(p^2,k^2)=G^N(m^2,k^2)+\frac{m^2-p^2}{M^2-p^2}g_{NR}G^{RN}(k^2). \tag{32}$$

The same procedure can also be applied to the gravitational vertex function of the Roper resonance. We define the vertex function for the Roper resonance similar to Eq. (27) from Eq. (26) by using the reduction technique, and assume the dominance of the one-nucleon state in the intermediate states to calculate the imaginary part of the mass form factor of the Roper resonance $G^R(p^2,k^2)$. Assuming the once-subtracted sidewise dispersion relation, we obtain

$$G^R(p^2,k^2)=G^R(M^2,k^2)+\frac{M^2-p^2}{m^2-p^2}g_{RN}G^{NR}(k^2), \tag{33}$$

where the gravitational transition form factor $G^{NR}(k^2)$ is defined by $\langle N|g^{\mu\nu}\theta_{\mu\nu}(0)|R\rangle$ and is related to $G^{RN}(k^2)$ such that

$$G^{NR}(k^2)=[G^{RN}(k^2)]^*. \tag{34}$$

On account of Eq. (34) we can eliminate $G^{RN}(k^2)$ and $G^{NR}(k^2)$ from Eqs. (32) and (33) to get a relation between $G^N(p^2,k^2)$ and $G^R(p^2,k^2)$. If we let p^2 go to infinity and put $k^2=0$ in this relation, we obtain a relation between the masses of the nucleon and Roper resonance:

$$M-G^R(\infty,0)=\left(\frac{g_{RN\pi}}{g_{NN\pi}}\right)^2[m-G^N(\infty,0)]. \tag{35}$$

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[17] H. Umezawa, *Quantum Field Theory* (North-Holland Publishing Co., Amsterdam, 1956), p. 257. The dispersion-theoretical treatment of the gravitational vertex function has been first applied by H. Miyazawa, Y. Oi, and M. Suzuki, Progr. Theoret. Phys. (Kyoto) Suppl. Extra No. 436 (1965). See also H. R. Pagels, Phys. Rev. **144**, 1250 (1966).

[18] In the case of the gravitational vertex function we cannot neglect the γ-N intermediate state. But in our approximation of the Roper dominance the contribution of this state is partially included in the Roper-resonance state, since the Roper resonance communicates with the γ-N state.

Since $G^N(\infty,0)$ and $G^R(\infty,0)$ are related to the bare mass of the nucleon, it is plausible to assume that as $p^2\to\infty$,

$$G^p(p^2,0)-G^n(p^2,0)\to 0\,,\qquad G^{R^+}(p^2,0)-G^{R^0}(p^2,0)\to 0\,. \tag{36}$$

This was the essential assumption made by Pagels[5] to obtain any sensible result on the electromagnetic mass difference of the nucleon from the sidewise dispersion relation of $G^N(p^2,0)$. The electromagnetic shift of the ratio of the strong-coupling constants $g_{NN\pi}{}^2$ and $g_{RN\pi}{}^2$ is expected to be small compared with the ratio of the mass shift,[19] and so, in our crude approximation, we can safely neglect the electromagnetic shift of the strong-coupling constants. With the assumption (36) we finally obtain the following relation:

$$M_{R^+}-M_{R^0}=\left(\frac{g_{RN\pi}}{g_{NN\pi}}\right)^2(m_p-m_n)\,, \tag{37}$$

where we have used the charge-independent values for $g_{RN\pi}$ and $g_{NN\pi}$. Inserting the experimental values, $m_p-m_n=-1.3$ MeV, and Eq. (18), we can estimate the right-hand side of Eq. (37) and find $M_{R^+}-M_{R^0}\simeq-0.2$ MeV. This value is very small to be observed experimentally. We hope, however, that the further development of the π-N phase-shift analysis will give an experimental determination of this mass difference.

IV. SUMMARY AND CONCLUDING REMARK

We have systematically derived the relations between the electromagnetic quantities of the nucleon and the Roper resonance by applying the sidewise dispersion relations to the electromagnetic, π-N, and gravitational vertex functions and by assuming one-particle dominance in the intermediate states. With the special assumption on the subtraction in the sidewise dispersion relation, we have obtained the following relations:

$$F_2{}^{RN}(k^2)=\frac{g_{RN\pi}}{g_{NN\pi}}F_2{}^N(k^2)\,, \tag{38}$$

$$F_2{}^R(k^2)=\left(\frac{g_{RN\pi}}{g_{NN\pi}}\right)^2F_2{}^N(k^2)\,, \tag{39}$$

$$G_M{}^{RN}(k^2)=\frac{g_{RN\pi}}{g_{NN\pi}}G_M{}^N(k^2)\,, \tag{40}$$

$$G_M{}^R(k^2)=\left(\frac{g_{RN\pi}}{g_{NN\pi}}\right)^2G_M{}^N(k^2)\,, \tag{41}$$

$$M_{R^+}-M_{R^0}=\left(\frac{g_{RN\pi}}{g_{NN\pi}}\right)^2(m_p-m_n)\,. \tag{42}$$

On the mass shell of the photon ($k^2=0$), Eqs. (38) and (40) reduce to the relations between the magnetic transition moment and the nucleon magnetic moment. We can then compare them with the experimental data on photopion production from the proton. It seems that the experimental data fit Eq. (40) better.[16] Once experimental data on the electroproduction of the pion from the nucleon are available in the energy range of Roper-resonance production, we shall be able to test the k^2 dependence of Eqs. (38) and (40). The relation (40) can be employed to calculate the contribution of the Roper resonance to the p-n mass difference in Cottingham's formula. The estimation has been done in a separate paper,[7] in which was obtained $(m_p-m_n)_R=-0.09$ MeV as the magnetic contribution of the Roper resonance.

Finally, we would like to make a remark about the quark-model prediction on the photoproduction of the Roper resonance from the nucleon.[20] It has been shown that, if the P_{11} nucleon resonance is assigned to an octet, the photoproduction amplitude of this resonance should vanish. Even if we assign it to a decuplet, the process $\gamma+p\to P_{11}$ is forbidden. Thus the quark model seems to contradict our result. Our result fits rather well to the experimental data on the photoproduction of the Roper resonance, while the fact that so far there is no clear evidence of the $N'(1470)$ formation in the electropion production on the nucleon seems to prefer the quark-model prediction. The future experiment will solve the problem.

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