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Calculation of the Transition Effect in Electromagnetic Cascades for Depths beyond Shower Maximum*

CAROL JO CRANNELL
Department of Physics, The Catholic University of America, Washington, D. C. 20017

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A model of the propagation of electromagnetic cascade showers for depths beyond shower maximum is used to calculate the magnitude of the transition effect from lead to Plexiglas. The results of the calculation compare favorably with previously determined experimental results.

THE manner in which high-energy electrons and photons transport and deposit energy in matter is through the production of electromagnetic cascade showers. The energy is propagated both laterally and longitudinally by an increasing number of diminishing energy particles. This multiplicative process continues until the average particle energy falls below the value known as the critical energy. At the region in the shower known as shower maximum, the average particle energy is approximately equal to the critical energy, and the number of cascade particles exceeds that at any other depth in the shower. At longitudinal depths in the shower well beyond shower maximum, the energy deposition is known to decrease exponentially as a function of depth.

The lateral and longitudinal distribution of energy deposition in electromagnetic cascades has been studied both experimentally and theoretically. It was suggested by Greisen[1] that at depths beyond shower maximum, minimum-attenuation γ rays are the primary energy-transporting component of the shower. This was verified by subsequent experimental determinations[2–7] of the longitudinal distribution of energy deposition in materials with a wide range of atomic number.

In lead, with an atomic number of 83, the minimum in the γ-ray attenuation curve is sufficiently narrow so that all the shower components of energy less than the energy of minimum-attenuation γ rays are much more rapidly absorbed. That this is so is corroborated by the experimentally determined behavior of showers in lead at depths well beyond shower maximum. The purely exponential decrease of the longitudinal energy-distribution curve with a rate approximately equal to the minimum-attenuation coefficient for γ rays in lead indicates that an equilibrium spectral-distribution of shower particles is ultimately achieved with the highest-energy component being minimum-attenuation γ rays. The "daughter" products of absorbed γ rays, either from the Compton or from the photoelectric process, have a negligible effect in the energy-transport process.

In the present work, a model based on the preceding observations is used to investigate the transition effect at a depth well beyond shower maximum. This transition occurs in the numbers and spectra of particles when a shower passes from one medium to another medium of different atomic number. The energy-transport–deposition process is treated as a parent-daughter decay in which the minimum-attenuation γ ray is the "parent" and the "daughter" is the interaction product. To simplify this calculation, the interaction daughter products are treated as electrons which carry off all the energy lost by the minimum-attenuation γ rays.[8] The rate of energy absorption then is determined by the rate of change of the number of electrons with depth at any depth in the shower. The rate of change of the number of electrons is given by

$$d\epsilon/dt=\gamma\lambda_\gamma-\epsilon\lambda_\epsilon\,, \tag{1}$$

where ϵ is the number of electrons, γ is the number of γ rays, λ_γ is the γ-ray attenuation coefficient, and λ_ϵ is the electron energy-absorption coefficient.

As described previously, the number of minimum-attenuation γ rays decreases exponentially with depth, so that Eq. (1) has a solution of the form

$$\epsilon=\frac{\lambda_\gamma}{\lambda_\epsilon-\lambda_\gamma}\gamma_0\left[e^{-\lambda_\gamma(t-t_0)}-e^{-\lambda_\epsilon(t-t_0)}\right]+\epsilon_0 e^{-\lambda_\epsilon(t-t_0)}\,, \tag{2}$$

where γ_0 and ϵ_0 are the numbers of γ rays and electrons at the depth t_0, which can be any depth in the shower beyond which the average particle energy is less than the energy of the minimum-attenuation γ rays.

At a depth in lead, well beyond shower maximum, such that there is an equilibrium spectrum of electrons

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[8] This is admittedly a poor assumption since it underestimates the fraction of the energy transported by γ rays and overestimates the fraction transported by electrons. However, the magnitude of the transition effect depends on the ratio of the fraction of energy carried by electrons in Plexiglas to the fraction carried by electrons in lead. The ratio calculated in this way is expected to give a more realistic estimate of the transition effect than the previous model which had a cutoff energy greater than the energy of minimum-attenuation γ rays.

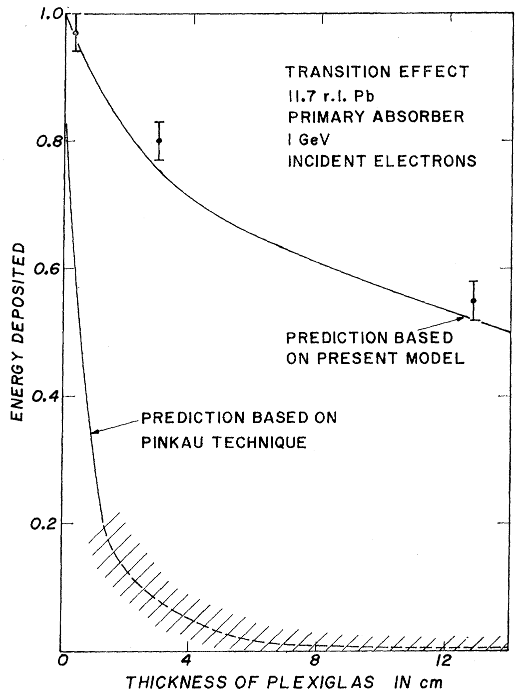

FIG. 1. The energy deposited per unit depth as a function of the thickness of the Plexiglas transition region is shown for a thickness of the primary absorber of 11.7 r.l. (radiation lengths) Pb. The experimental points are from the data of Crannell *et al.* (Ref. 9). The experimental data and the predicted curves are normalized to unity at zero depth.

and γ rays, the number of electrons will be approximately given by

$$\epsilon \simeq \frac{\lambda_{\gamma\,\mathrm{Pb}}}{\lambda_{\epsilon\,\mathrm{Pb}}-\lambda_{\gamma\,\mathrm{Pb}}}\,\gamma\,, \tag{3}$$

where the subscript Pb denotes the coefficients characteristic of lead. If the lead medium is not continuous, but instead has a boundary followed by a Plexiglas medium, the number of electrons as a function of depth in the Plexiglas can be calculated in a similar fashion. Since the energy of minimum-attenuation γ rays in lead is much less than the energy of minimum-attenuation γ rays in Plexiglas, Eq. (2) can be used to calculate the number of electrons at any depth x beyond the lead-Plexiglas boundary. Using Eq. (3) to calculate the number of electrons approaching the boundary, the ratio of the number of electrons at any depth x in the Plexiglas to ϵ_0, the number crossing the boundary, is given by

$$\epsilon/\epsilon_0=\frac{(\lambda_{\epsilon\,\mathrm{Pb}}-\lambda_{\gamma\,\mathrm{Pb}})}{\lambda_{\gamma\,\mathrm{Pb}}}\left(\frac{\lambda_\gamma}{\lambda_\epsilon-\lambda_\gamma}\right)(e^{-\lambda_\gamma x}-e^{-\lambda_\epsilon x})+e^{-\lambda_\epsilon x}\,, \tag{4}$$

where λ_γ and λ_ϵ correspond to the attenuation coefficient for γ rays and the energy-absorption coefficient for electrons of the appropriate energy in Plexiglas.

In the numerical computation, the following parameters have been used:

$\lambda_{\gamma\,\mathrm{Pb}}=0.041$ cm^2 g^{-1} for 3.5-MeV γ rays (minimum-attenuation energy) in lead;
$\lambda_\gamma=0.032$ cm^2 g^{-1} for 3.5-MeV γ rays in Plexiglas;
$\lambda_{\epsilon\,\mathrm{Pb}}=\lambda_\epsilon=0.57$ cm^2 g^{-1} for 3.5-MeV electrons in both lead and Plexiglas;
$\rho=1.05$ g cm^{-3}, the density of Plexiglas.

Thus (4) reduces to

$$\epsilon/\epsilon_0=0.8e^{-0.034x}+0.2e^{-0.6x}\,, \tag{5}$$

where x is measured in cm. The first term, $0.8e^{-0.034x}$, represents electrons due to converted γ rays, and the second term, $0.2e^{-0.6x}$, represents the electrons from the lead, which are more preferentially absorbed in Plexiglas. The ratio of the number of γ rays at any depth in Plexiglas, γ, to the number of γ rays crossing the lead Plexiglas boundary, γ_0, is similarly given by

$$\gamma/\gamma_0=e^{-0.034x}. \tag{6}$$

One quantitative measure of the transition effect is that the relative proportion of electrons to γ rays drops from unity to 0.8 after approximately 2 cm of Plexiglas and remains 0.8 for all subsequent depths.

In Fig. 1, ϵ/ϵ_0 as calculated from Eq. (5) is shown as a function of x. The computed values are drawn as a solid curve and the data measured by Crannell *et al.*[9] are shown as experimental points. For comparison, the curve calculated using the method of Pinkau[10] is also shown. The method of Pinkau uses the spectra from Approximations A and B of Rossi and Greisen,[11,12] which have an inherent cutoff energy larger than the energy of minimum-attenuation γ rays.

The present model gives reasonable agreement with the experimental data and provides a simplified method for estimating the size of the transition effect for depths beyond shower maximum in electromagnetic cascades. To the extent that the model is valid, it would predict that the transition effect is not cumulative, but that the original equilibrium spectra of both electrons and γ rays would be restored in a layer of several g cm^{-2} of the primary medium. Furthermore, it would predict that the important quantity is not the number of radiation lengths of the "transition depth," but the number of g cm^{-2}.

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