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Experimental Determination of the Transition Effect in Electromagnetic Cascade Showers*

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The transition effect that occurs when an electromagnetic cascade crosses the boundary between different materials has been measured. This transition effect is a rapid change in the numbers and spectra of electrons and photons which constitute the cascade, and in the resultant energy deposition. The magnitude of this effect depends on the age of the shower at the discontinuity in the critical energies. In the present experiment, 1-GeV cascade showers were developed in glass, iron, and lead, and 5-GeV showers in lead, each followed by a second medium of Plexiglas. The energy deposited as a function of position in the Plexiglas has been measured for a variety of shower ages and for thicknesses of the transition region between 0 and 1 radiation length. The results show a smaller transition effect than predicted by Approximation B of the cascade theory developed by Rossi and Greisen, but the measured transition effect is sufficiently rapid to cause systematic errors on the order of 20% in cascade measurements in some cases.

1. INTRODUCTION

THE properties of electromagnetic showers have been studied experimentally by means of detectors located in and around the media in which the showers develop.[1] If the critical energy of a detector differs from the critical energy of the primary medium, a transition will take place in the numbers and energy spectra of electrons and photons as the shower propagates from the primary medium into the detector material. The magnitude of this transition effect has been calculated[2] using the formalism of Approximation B developed by Rossi and Greisen.[3] Significant effects on the resultant energy deposition were predicted for a detector layer as thin as 0.01 radiation length.

These results indicate complications in the use of traditional shower counters as accurate spectrometers for cosmic-ray particles. Moreover, the limitations inherent in Approximation B make corrections based on the calculations doubtful. Thus, it became desirable to check these predictions experimentally.

Theoretical and experimental[4–6] work on this effect has been performed before. Fetisov[6] has measured the transition effect from lead to copper for incident electron energies between 100 and 550 MeV, using a copper-walled ionization chamber. From his results, Fetisov concludes that the integral energy deposition measured with lead absorbers is decreased by a factor of 1.9±0.1 by the transition effect in the 2.5-mm-thick copper walls of the ionization chamber. This result was independent of the energy of the incident electrons. Clearly, more extensive experimental evidence was needed to determine the magnitude of the transition effect for the commonly used shower-detecting materials.

In the present experiment, the Cambridge Electron Accelerator and the Stanford Mark III linear accelerator were used as the source of 5-GeV and 1-GeV electrons, respectively. These electrons induced electromagnetic cascades in glass, iron, and lead, followed by

TABLE I. Numerical parameters associated with the target elements.

Target	Density (g cm^{-3})	Radiation length (g cm^{-2})	Critical energy (MeV)
Lead	11.35	6.4	7.4
Iron (cold-rolled steel)	7.92	13.9	20.7
Glass	2.54	27.4	47.3
Plexiglas	1.05	44.4	88.0

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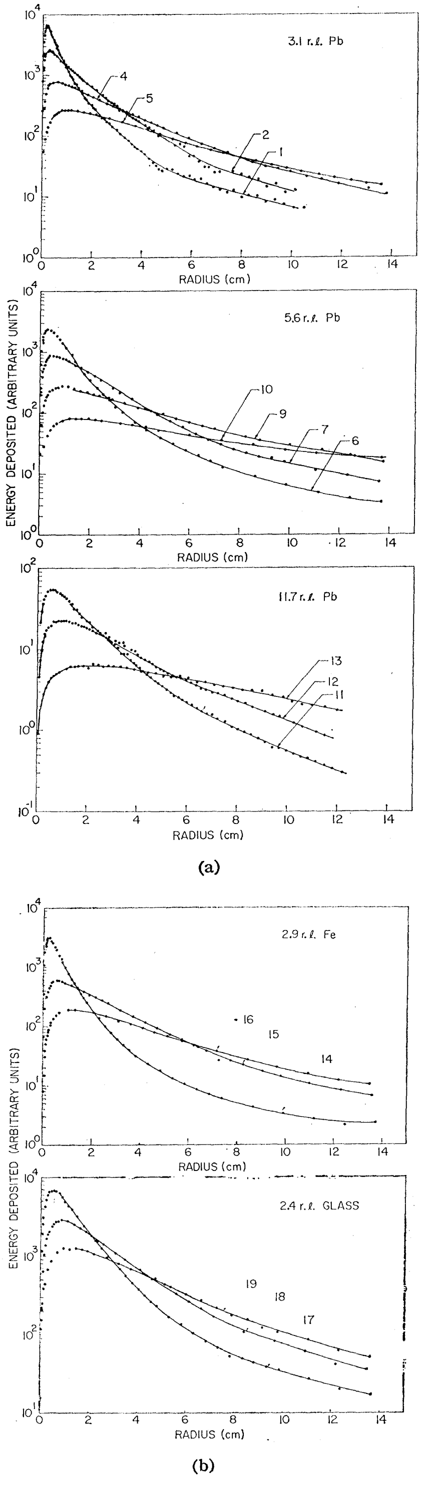

FIG. 1. Plot of the energy deposited in an annular section as a function of the radius for cascades initiated by 1-GeV electrons, (a) in lead and (b) in iron and glass. The units for the energy deposited are arbitrary, but the curves at any one depth can be compared directly. The figures attached to the curves are the numbers of the measurement points, and they are listed in Table II. The curves drawn through the experimental points are visual fits to the data.

a second medium of Plexiglas. The transition effect was studied as a function of the thickness of the transition region from 0 to 1 radiation length and as a function of the shower age or thickness of the initial shower material.

The experimental techniques employed in this work are described in Sec. 2. In Sec. 3, the results of this experiment are presented and the interpretation of these measurements in terms of typical shower counters is discussed.

2. EXPERIMENTAL TECHNIQUE

Preliminary measurements of the transition effect were performed using 5-GeV electrons from the Cambridge Electron Accelerator (CEA). The target consisted of plates of lead, 10 cm on an edge, followed by plates of Plexiglas. A plate of plastic scintillator, 3 cm on an edge, and 2 mm thick was used as the detector. The light output from the entire plastic scintillator was observed by two photomultiplier tubes, which produce a signal proportional to the energy deposited in the detector. The ratio of the output of the detector to the incident beam intensity provides a measure of the total energy deposited at a given longitudinal position. The detector assembly could be placed at various positions in the stack of Plexiglas plates to sample the distribution of energy deposition as a function of the thickness of Plexiglas between the detector and the target material.

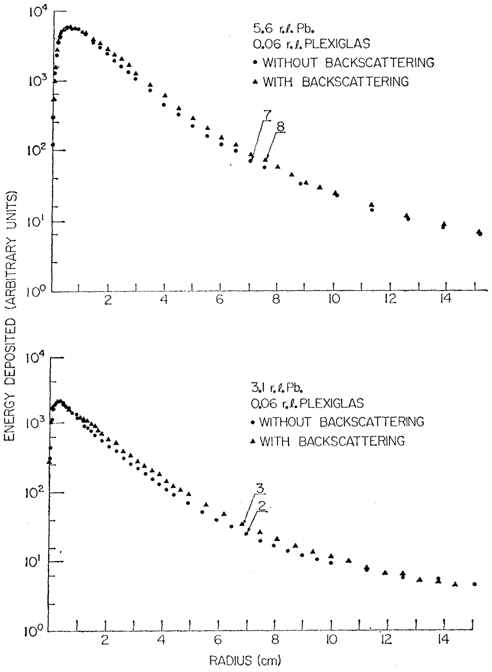

FIG. 2. Comparison of the energy deposited in an annular section as a function of the radius, with and without a backscattering layer of 5 cm of lead behind the counter for cascades initiated by 1-GeV electrons. Figures attached to the curves are the numbers of the measurement points from Table II.

TABLE II. List of experimental setups used at the Stanford linear accelerator. 1-GeV electrons.

Measurement point No.	Target material	Absorber thickness		Transition into Plexiglas		Energy deposited (normalized)	Backscattering with 5 cm Pb
		(cm)	(rad. lengths)	(cm)	(rad. lengths)		
1	Pb	1.75	3.1	0.3	0.01	1.00±0.03	no
2	Pb	1.75	3.1	2.9	0.07	0.86±0.03	no
3	Pb	1.75	3.1	2.9	0.07	0.98±0.04	yes
4	Pb	1.75	3.1	12.8	0.30	0.55±0.02	no
5	Pb	1.75	3.1	44.8	1.06	0.30±0.01	no
6	Pb	3.17	5.6	0.3	0.01	1.00±0.03	no
7	Pb	3.17	5.6	2.9	0.07	0.72±0.03	no
8	Pb	3.17	5.6	2.9	0.07	0.80±0.03	yes
9	Pb	3.17	5.6	12.8	0.30	0.43±0.02	no
10	Pb	3.17	5.6	44.8	1.06	0.22±0.02	no
11	Pb	6.60	11.7	0.3	0.01	1.00±0.03	no
12	Pb	6.60	11.7	2.9	0.07	0.82±0.03	no
13	Pb	6.60	11.7	12.8	0.30	0.56±0.03	no
14	Fe	5.08	2.9	0.3	0.01	1.00±0.03	no
15	Fe	5.08	2.9	12.8	0.30	0.64±0.03	no
16	Fe	5.08	2.9	44.8	1.06	0.37±0.04	no
17	Glass	26.0	2.41	0.3	0.01	1.00±0.03	no
18	Glass	26.0	2.41	12.8	0.30	0.76±0.03	no
19	Glass	26.0	2.41	44.8	1.06	0.52±0.03	no

Additional measurements of the transition effect were performed using 1-GeV electrons from the Stanford Mark III linear accelerator. Checks of the system were performed using the same detector and target which had been used at CEA. These checks indicated that some energy was escaping undetected through the sides of the detector. This technique proved inadequate, therefore, since it allowed no means of estimating the fraction of the energy escaping.

The subsequent data for the primary media of glass, iron, and lead were measured using a different detector. The target and detector assemblies were similar to those described previously[1] for the measurement of the three-dimensional distribution of energy deposition for electron-induced showers.

In Table I, the numerical parameters associated with the target media are presented.

The detector assembly consisted of a shielded probe inserted into a hole drilled in one of the Plexiglas plates. The probe used to measure the energy deposition as a function of position in the Plexiglas consisted of a pellet of anthracene, a polished aluminum and Lucite light pipe, and an EMI-US 6094 B photomultiplier tube. The probe could be raised and lowered remotely to sample the distribution of energy deposition as a function of radial distance from the beam axis. The entire detector assembly could be placed at various positions in the stack of Plexiglas plates to sample the distribution of energy deposition as a function of the thickness of Plexiglas between the detector and the target material. For these data, the radial distribution curves were integrated and an extrapolation to infinite radius was used to estimate the amount of energy escaping beyond a radius of 10 cm.

The intensity of the electron beam was measured by means of a gas Čerenkov monitor[7] which produced a signal proportional to the charge in the incident beam. To find the radial distribution of energy deposition, the ratio of the signal from the probe to the signal from the monitor was obtained as a function of the radial distance of the detector from the beam axis. These radial distribution curves were measured as a function of the thickness of Plexiglas between the target and the detector for a glass target, an iron target, and for three different thicknesses of the lead target. For each of these measurements, Plexiglas plates were placed after the detector assembly so that the detector was contained as nearly as possible in a Plexiglas medium. To determine the effect of alternating layers of target and detector materials, as are found in many typical shower counters, two of the radial distribution curves were repeated but with lead plates replacing the Plexiglas immediately after the detector assembly. A measurable amount of backscattering was produced.

Each of the radial distribution curves was integrated to find the total energy deposited at the specific longitudinal position. For each value of the target thickness studied, the data were normalized so that the energy deposited after the smallest thickness of Plexiglas (approximately 0.32 cm) was set equal to 1.0. This normalization was necessary because neither the beam monitor nor the probe was calibrated absolutely. However, the absolute values of energy deposition in copper, tin, and lead absorbers have been measured and published[1] before, using essentially the same equipment.

3. RESULTS AND DISCUSSION

Figures 1 and 2 show the energy deposited in an annulus of radius r for various experimental configurations at the Stanford accelerator. These configurations

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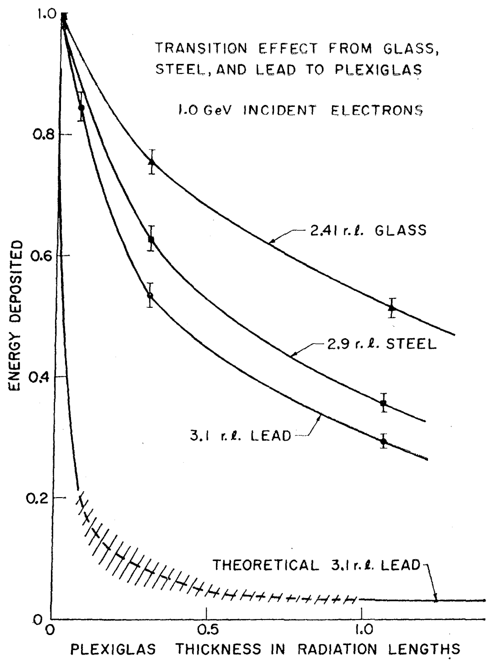

FIG. 3. Transition effect in glass at 2.41 radiation lengths (r.l.), in iron at 2.9 r.l., and lead at 3.1 r.l. for primary electrons with energy of 1 GeV. At these depths, showers created by these electrons are close to maximum development. In this figure, as well as in Figs. 4, 5, and 6, the ordinate represents the energy deposited in the Plexiglas, normalized to unity at the transition boundary; the abscissa gives the thickness of the secondary medium of Plexiglas in radiation lengths. In this figure, as in Fig. 5, account has been taken of the fact that the first measurement was carried out at a depth of 0.32 cm (0.01 r.l.) in the Plexiglas. Also indicated is the theoretical prediction of the transition effect at 3.1 r.l. of lead. The hashed regions in Figs. 3, 4, and 5 indicate the region of interpolation between the two regions in which the theory is applicable. In Figs. 3 and 5, the curves drawn through the experimental points are a visual fit to the data. The indicated errors include estimates of systematic uncertainties.

are listed in Table II. The data presented in Table II show the type and thickness of the absorber in which the cascade is developed. The cascade then passes into Plexiglas and is measured after various thicknesses of this transition medium. The last column of Table II gives the energy deposited at each measurement point, relative to a normalization of unity for the smallest Plexiglas thickness. This value for the energy deposited is obtained by summing over the "ring" distribution curves of Figs. 1 and 2 out to a radius of 10 cm, and by extrapolating the measured curve to infinitely large radii to determine the energy deposited beyond 10 cm. In the worst case, the extrapolated energy was found to be 30% of the total energy deposited. This was estimated to be responsible for an uncertainty of ±10% in the energy deposited. A measurement uncertainty (reproducibility) of less than ±3% is inherent in all the data, and any uncertainties reported to be larger than ±3% are due to the estimated uncertainties in the extrapolations to infinity.

The "ring" distribution curves shown in Figs. 1 and 2 display a rapid lateral dilution of the energy flux, thus indicating the difficulty of covering the entire range of the energy flux with the detector. From a comparison of Figs. 1 and 2, one may also see that the angular distribution of the energy flux becomes wider with increasing shower age. It appears reasonable that the change in the slope of the "ring" distribution curves beyond a radius of 4 cm is another manifestation of the transition effect. At a radius outside the core of the shower, the slope of the ring distribution curve is determined by interactions in the Plexiglas. The resultant attenuation is different from the attenuation which was dominated by the primary medium.

Figure 3 shows the transition effect from glass, iron, and lead into Plexiglas at 2.41, 2.90, and 3.1 radiation lengths, respectively, in the 1-GeV Stanford measurements. At those depths, the 1-GeV electron-induced cascades are close to shower maximum. The transition effect for initial media of glass, iron, and lead with a secondary medium of Plexiglas was calculated according to the formalism developed previously.[2] The results are shown in Fig. 4. The depths of the initial media were chosen to correspond to the depths for which the experimental data were measured. Qualitatively, the results are in good agreement with the experimental data. The predicted transition effect is seen to be greater the larger the ratios of the critical energies of the primary and

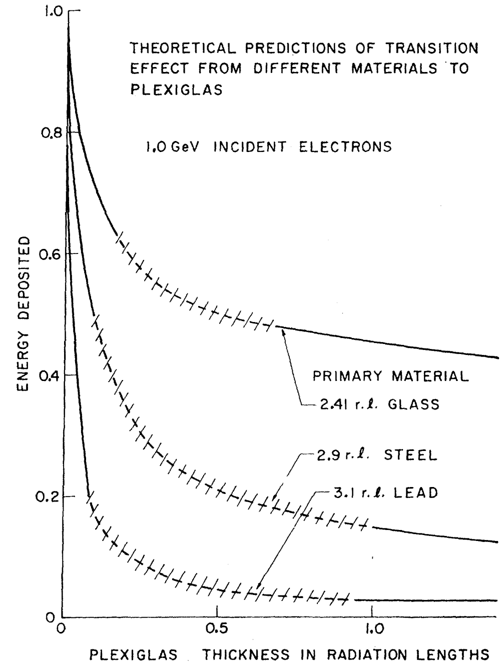

FIG. 4. Theoretical predictions of the transition effect from primary materials of 2.41 r.l. of glass, 2.9 r.l. of steel, and 3.1 r.l. of lead, to Plexiglas for showers produced by 1-GeV electrons. Comparison of the figure with Fig. 3 shows that there is qualitative agreement between theory and experiment for primary materials with different atomic numbers. See caption of Fig. 3 for further explanations.

secondary media. Quantitatively, however, the predicted transition effect is too large. This discrepancy has a straghtforward explanation. It has been pointed out before[2] that the steep change predicted by theory if the cascade penetrates into a medium with different critical energy is due to the fact that the differential spectra for both electrons and photons diverge for small-particle energies under Approximation B. This cannot be true in nature, and it is therefore expected that the real effect cannot be as steep as predicted by theory.

In both Figs. 3 and 5, allowance has also been made for the fact that the smallest depth measured using the Stanford arrangement was about 0.32 cm inside the Plexiglas associated with the detector assembly. The data have been renormalized so that the smooth curve drawn through the experimental points indicates a value for the ratio of 1.0 at the transition boundary. Figure 5 shows the transition effect occurring from lead to Plexiglas at 3.1, 5.6, and 11.7 radiation lengths of lead in the 1-GeV Stanford measurements. Also shown are the results of the 5-GeV Cambridge measurements after 5.2 radiation lengths of lead. The measurements performed at Stanford indicate that while at depths beyond shower maximum the effect of undetected radially escaping energy is large, the effect was expected to be less than 20% in the CEA measurements shown in Fig. 5. Hence, they have been included here.

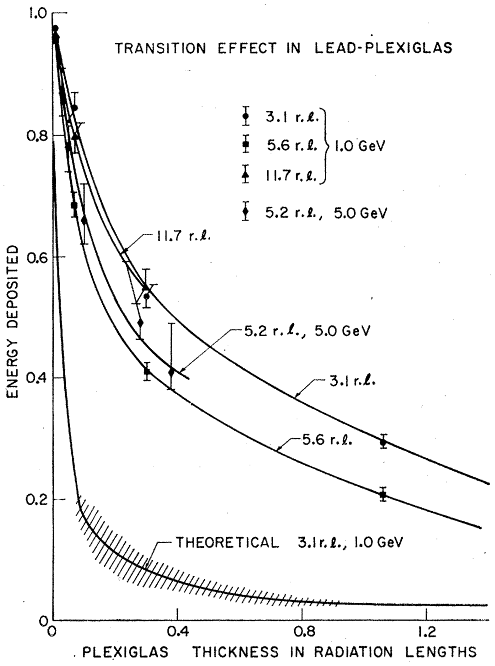

FIG. 5. Transition effect from lead to Plexiglas at 3.1, 5.6, and 11.7 r.l. for 1-GeV primary electrons, and at 5.2 r.l. for 5-GeV primary electrons. Also indicated is the theoretical prediction of the transition effect at 3.1 r.l. of lead. See caption of Fig. 3 for further explanations.

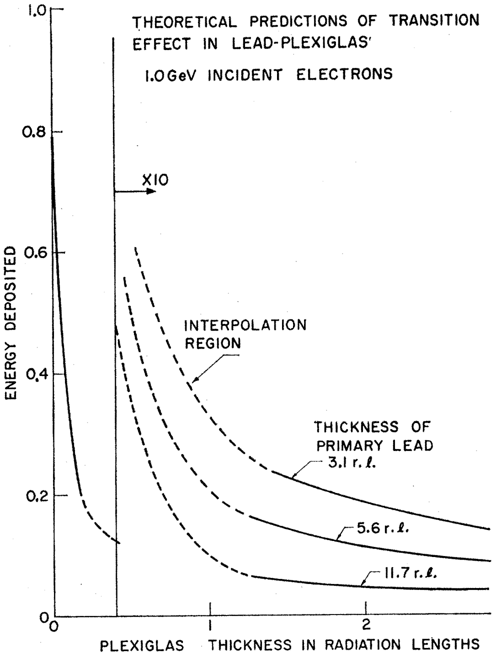

FIG. 6. Theoretical predictions of the transition effect for different thickness of lead to Plexiglas. In this plot all three curves are essentially the same for thicknesses of Plexiglas less than 0.4 r.l. For thicknesses of Plexiglas greater than 0.4 r.l. the ordinate is expanded by a factor of 10 for illustrative purposes. A comparison of this figure with Fig. 5 shows that there is not even qualitative agreement between the theory and experiment. Possible explanations of this effect are discussed in the text. See caption of Fig. 3 for further explanations.

The transition effect for three different thicknesses of the initial lead medium with a secondary medium of Plexiglas were also calculated. The results are shown in Fig. 6. The depths of the initial media were chosen to correspond to the depths for which the Stanford data were measured. They occur at approximately shower maximum, just beyond shower maximum, and in the region of the shower in which the shower is exponentially decaying. These results are not even in qualitative agreement with the experimental data which are shown in Fig. 5. To understand why the experimental results indicate that the transition effect is less in the deepest position in the shower than in the region just beyond shower maximum, one must consider the spectral composition of the shower at these respective depths and in what way it differs from the spectral distribution inherently assumed in Approximation B.

The theory given by Approximation B characterizes the particles at shower maximum as having energies near the critical energy, and at shower maximum this approximation is known to be good. Approximation B does not treat adequately the interactions of particles with less than the critical energy. Previous measurements indicate, however, that deep in the shower, well past shower maximum, the shower is propagated primarily by minimum-attenuation γ rays. The energy

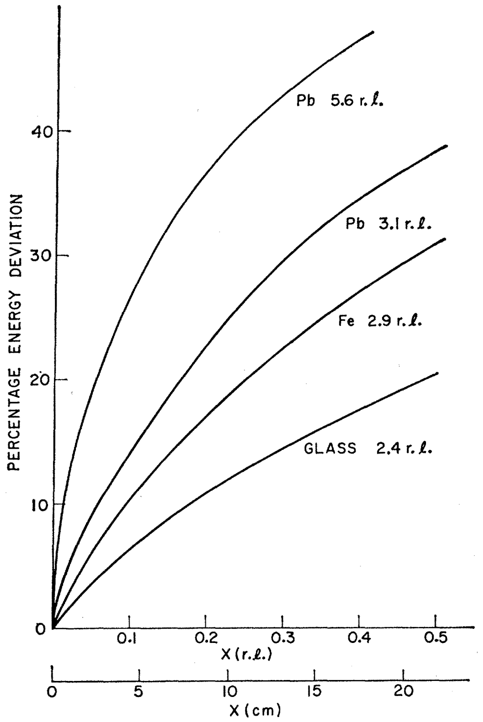

Fig. 7. The average deviation from 100% of the energy deposited within a layer of depth X of Plexiglas is plotted versus X. Curves are given for lead, iron, and glass absorbers. This deviation indicates the systematic error which would be made if one assumed that a scintillator would correctly sample the energy deposition within the absorbers. Results are from the 1-GeV Stanford measurements.

of the minimum attenuation γ rays is less than the critical energy. For high-Z materials such as lead, the energy of the minimum attenuation γ rays is significantly lower than in low-Z materials, such as Plexiglas. The relatively low-energy γ rays, leaving the primary lead media, are more preferentially converted in the Plexiglas. This effect tends to reduce the magnitude of the transition effect at large depths.

One should not expect the transition effect to change drastically with increasing primary energy of the cascade, if previous theoretical[2] calculations are considered. Rather, the effect should be a function of the shower age. In this respect, the Cambridge results taken at 5-GeV primary energy are interesting in that they seem to show a change in the transition effect with respect to the Stanford measurements at the same shower age, namely, a depth of 3.1 radiation lengths and 1-GeV primary energy. However, as can be seen in Fig. 5, the disagreement is not much greater than the estimated experimental uncertainty, of which ±5% is due to the inherent resolution of the measuring technique and the remainder is due to estimated systematic uncertainties in the amount of energy escaping the detector.

Finally, the practical applications of the measurements reported here are presented in Fig. 7. The error associated with a cascade measurement using a scintillator of thickness X is plotted as a function of X. Individual curves are shown for various thicknesses and materials of the primary absorber employed in this experiment. These curves have been calculated in the following way: The measured transition curves presented in Figs. 3 and 5 were integrated out to a thickness X, and each value of that integral was divided by the integral which would have been obtained if the energy deposited in the plastic material had shown no transition effect (i.e., had remained at the level 1.0 in Figs. 3 and 5). The difference of the value thus obtained from 100% has been plotted in Fig. 7. The results are presented in this form in order that they might be related readily to typical conditions for experimental measurements. Usually the experimenter assumes that the light signal received from a plastic scintillator is proportional to the scintillator thickness, and, moreover, that the signal is a measure of the particle number and energy deposited inside the primary absorber just prior to entering the scintillator. That these assumptions are not valid has been demonstrated by this experiment, and the associated errors due to the transition effect are shown in Fig. 7.

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