There also is interesting work by Marshall[17] published during 1963–65 proposing essentially the same view as that arrived at by the author, that electromagnetic zero-point radiation can be regarded as the cause of particle quantum motion. In this sense, quantum motions are experimental evidence for zero-point radiation. However, Marshall does not touch the question treated in this paper, that of deriving Planck's radiation law from electromagnetic zero-point radiation and classical theory. It is interesting that Marshall came upon the Lorentz invariance of the zero-point spectrum as an afterthought. It was this Lorentz invariance of the electromagnetic zero-point spectrum, and the consequent absence of velocity-dependent damping forces, which struck the author as so fundamental that it prompted the line of development carried out in the present paper.

[17] T. W. Marshall, Proc. Roy. Soc. (London) **A276**, 475 (1963); Proc. Cambridge Phil. Soc. **61**, 537 (1965); Nuovo Cimento **38**, 206 (1965). I wish to thank Dr. B. Robertson for bringing this work to my attention.

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Generalization of the Reissner-Nordström Solution to the Einstein Field Equations

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The field equations for coupled gravitational and zero-mass scalar fields in the presence of a point charge are solved in the spherically symmetric static case. The resulting solution is the generalization of the Reissner-Nordström solution in the presence of a zero-mass meson field.

I. INTRODUCTION

The well-known Reissner-Nordström solution of Einstein's equations corresponds to the situation with a charged mass-point at the origin of spherical coordinates. That solution plays a justifiably important role in studies of the interaction of electromagnetic and gravitational fields.

Recently,[1] we had occasion to treat the problem of interacting gravitational and zero-mass meson fields in the axially symmetric, static situation, and obtained the (spherically symmetric) generalization of the Schwarzschild solution in the presence of a zero-mass meson field. The form of that solution was considerably simpler than the form originally obtained by Janis *et al.*[2]

Indeed, the form of the generalized Schwarzschild solution is simple enough to give hope that the Reissner-Nordström solution could also be generalized in the presence of a zero-mass meson field. In this paper we obtain the desired generalization.

The main reason for presenting the solution is that exact solutions of Einstein's gravitational equations are scarce. Thus any such exact solutions may be useful even though somewhat unrealistic.

II. FIELD EQUATIONS

We wish to solve the field equations

$$G_{\mu\nu}=-KT_{\mu\nu}-KE_{\mu\nu}\,, \tag{1}$$

$$T_{\mu\nu}=\varphi_\mu\varphi_\nu-\tfrac{1}{2}g_{\mu\nu}\varphi^\alpha\varphi_\alpha\,, \tag{2}$$

$$E_{\mu\nu}=g^{ab}F_{a\mu}F_{b\nu}-\tfrac{1}{4}g_{\mu\nu}F_{ab}F^{ab}\,, \tag{3}$$

where φ_μ is a gradient and $F_{\mu\nu}$ is the Maxwell tensor.

We are interested in the spherically symmetric, static situation, in which case the line element is[3]

$$ds^2=e^\alpha dR^2+e^\beta d\Omega^2-e^\gamma dt^2\,, \tag{4}$$

and we choose coordinates such that $\alpha+\gamma$ vanishes.

We assume that only φ_1 and F_{14} are nonzero functions of R. We further assume, as in the Reissner-Nordström solution, that J_μ, the current, vanishes except for a point-charge singularity at the origin of coordinates. (Because of our choice of coordinates, the "origin" will not be at $R=0$.)

To proceed, we first look at the Maxwell equation

$$F^{\mu\nu}{}_{|\nu}=0 \tag{5}$$

and obtain immediately that

$$F_{14}=\epsilon e^{-\beta}\,, \tag{6}$$

where ϵ is the charge.

We next look at the wave equation for φ, viz.,

$$\varphi^\mu{}_{|\mu}=0\,, \tag{7}$$

and obtain directly that

$$\varphi_1=ce^{\alpha-\beta} \tag{8}$$

for some constant c.

It is then a simple matter to calculate the components of $T_{\mu\nu}$ and $E_{\mu\nu}$, and one finds that the nonzero

[1] R. Penney, Phys. Rev. **174**, 1578 (1968).
[2] A. I. Janis, E. T. Newman, and J. Winicour, Phys. Rev. Letters **20**, 878 (1968).
[3] J. L. Synge, *Relativity: The General Theory* (Wiley-Interscience, Inc., New York, 1960), p. 270.

ones are

$$T_1{}^1=-T_2{}^2=-T_3{}^3=-T_4{}^4,$$
$$E_1{}^1=-E_2{}^2=-E_3{}^3=+E_4{}^4, \tag{9}$$

and hence realizes that $G_1{}^1+G_2{}^2$ must vanish. The resulting equation is directly integrable, and one obtains $\alpha-\beta$ easily.

Finally, one finds that $G_1{}^1-G_4{}^4$ gives an integrable combination, and the problem is complete save for checking Eq. (3) and identifying parameters.

III. SOLUTION

After considerable juggling, the result of solving the field equations may be placed in the following form, best suited to recognition as a generalization of the Reissner-Nordström solution:

$$e^{\alpha}=\left(R^2-2mR+\frac{K\epsilon^2}{2A^2}\right)^{-A}\left(\frac{b(R-a)^A-a(R-b)^A}{b-a}\right)^2, \tag{10}$$

$$e^{\beta}=(R^2-2mR+K\epsilon^2/2A^2)e^{\alpha}. \tag{11}$$

The parameters are connected by the relations

$$2A^2ab=K\epsilon^2,\quad a+b=2m, \tag{12}$$

$$A^2Kc^2=(1-A^2)(2A^2m^2-K\epsilon^2), \tag{13}$$

and φ_1 and F_{14} are as previously expressed, and $1\geqslant A\geqslant 0$.

By taking $A=1$ one obtains the usual solution corresponding to a point charge ϵ with mass m at $R=0$. Similarly, by taking $\epsilon^2=0$, one obtains the generalized Schwarzschild solution.[1]

IV. CONCLUSION

We have obtained a spherically symmetric, static solution of Einstein's field equations corresponding to a charged mass-point at the origin, surrounded by the field of a zero-mass meson. However unrealistic such a situation may appear, it is heartening to realize that one is able to obtain analytic solutions of a problem with three fields coupled nonlinearly.

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Twofold Joint Photocount Statistics for Mixed Thermal and Coherent Radiation*

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The conventional model of the photodetection process and the short-time approximation are used to obtain the factorial moment generating function for the N-fold detection of mixed thermal and coherent radiation. This generating function is then used to obtain the photocount distribution for twofold joint photodetection. The photocount distribution is expressed as a threefold summation over Laguerre polynomials. The distribution is examined for the limiting cases of purely thermal radiation and uncorrelated fields.

THE photon counting technique has been found to be an extremely useful tool for studying the statistics of optical fields and in recent years a considerable amount of literature has been devoted to it. Most of this work has involved a single photodetector. However, since joint photocount statistics provide information about the field which cannot be obtained from the output statistics of a single photodetector, the joint photodetection process has received some attention.

Arecchi *et al.*[1] have investigated the twofold joint photocount distribution of a Gaussian-Markovian radiation field. Bedard[2] has obtained the N-fold joint photocount distribution for a thermal radiation field. This paper presents the factorial moment generating function for an N-fold joint photocount distribution of mixed thermal and coherent radiation. This generating function is then used to obtain the twofold joint photocount distribution of mixed thermal and coherent radiation.

The probability of recording m counts in the interval $[t_0, t_0+T]$ with a single photodetector is[3-5]

$$p(m,t_0,T)=\frac{(-1)^m}{m!}\frac{\partial^m}{\partial\lambda^m}Q(\lambda,t_0,T)\bigg|_{\lambda=1}, \tag{1}$$

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[1] F. T. Arecchi, A. Berne, and A. Sona, Phys. Rev. Letters **17**, 260 (1966).

[2] G. Bedard, Phys. Rev. **161**, 1304 (1967).

[3] L. Mandel, in *Progress in Optics*, edited by E. Wolf (North-Holland Publishing Co., Amsterdam, 1963), p. 181.

[4] P. L. Kelley and W. H. Kleiner, Phys. Rev. **136**, A316 (1964).

[5] R. J. Glauber, in *Quantum Optics and Electronics*, edited by C. deWitt, A. Blandin, and C. Cohen-Tannoudji (Gordon and Breach, Science Publishers, Inc., New York, 1965), p. 63.