High-Resolution Proton Scattering on Zr^{91} and the Core-Excitation Model*

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The reactions $Zr^{90}(p, p')Zr^{90}$ and $Zr^{91}(p, p')Zr^{91}$ have been studied with 12.0-MeV protons. Proton spectra were measured with a broad-range magnetic spectrograph at six reaction angles between 30° and 90°, sufficient to distinguish between $L=2$ and $L=3$ transitions. The Zr^{91} spectrum is interpreted in terms of $d_{5/2}$, $s_{1/2}$, $d_{3/2}$, and $g_{7/2}$ neutron states coupling to the low-lying states of the Zr^{90} core. The predictions of the core-excitation model are compared in detail with the data for the multiplets formed by the $d_{5/2}$ neutron coupling to the 2+ and 3− states in the Zr^{90} core.

I. INTRODUCTION

It has been clear for some time that the structure of nuclei in the vicinity of Zr^{90} can be interpreted in terms of the coupling of the last few particles (holes) with the Zr^{90} core.[1–7] In general, only the ground state of the Zr^{90} core has been considered in this coupling. However, it is expected that all of the states of the core should couple with single-particle states.[8,9] A combination of difficulties (including the complexity of the spectroscopy, mixing of states, and especially the fact that the particle-core coupling often is not weak) has tended to obscure the spectroscopy of this coupling. Recently, however, isobaric analog resonance studies have clearly identified three states in Zr^{91} as single-particle states coupled to the first excited 0+ state at 1.75 MeV in the Zr^{90} core.[10,11]

The lowest-lying states in Zr^{90}, 0+ (g.s.), 0+ (1.75 MeV), 2+ (2.18 MeV), and 5− (2.32 MeV), can be well described in terms of $(p_{1/2})^2$, $(g_{9/2})^2$, and $p_{1/2}g_{9/2}$ two proton configurations.[12–15] However, the ground state and first excited state consist of configurations of the type $a(p_{1/2})^2+b(g_{9/2})^2$, where a^2 and b^2 are approximately 0.67 and 0.33, respectively, for the ground state, and the proportions are interchanged for the 0+ (1.75 MeV) first excited state. This mixing implies a partial collectivity for these states. Furthermore, inelastic scattering experiments [16–21] demonstrate some collectivity for both the 2+ and 5− states. The 3− state at 2.74 MeV has long been recognized as a highly collective octupole vibration, preferentially excited in inelastic scattering reactions.

It is the purpose of this experiment to test the core excitation model for Zr^{91}, using the reaction $Zr^{91}(p, p')Zr^{91}$ at 12.0 MeV and high-resolution spectroscopy. A preliminary report of this research has been given.[22] Combining our data with those of other authors, we attempt to interpret the Zr^{91} spectrum below 4-MeV excitation energy in terms of $d_{5/2}$, $s_{1/2}$, $d_{3/2}$, and $g_{7/2}$ neutron states coupled to Zr^{90} core states. Thus, this paper attempts to explain in some detail the fragmentation of single-particle strength and to invoke a model for the energies and specific states involved in this fragmentation.

II. CORE-EXCITATION MODEL

In inelastic scattering reactions on odd-A nuclei, the core excitation model predicts multiplet spectra formed by the odd particle (hole) in its ground-state orbital (j_p) being coupled to an excited state (j_c) of the even-even core. If the interaction between the core state and the odd particle is assumed to be weak, then the wave function of a multiplet member in the inelastic scattering transition amplitude can be written as a

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vector coupled product of the core and particle wave functions. It follows from the properties of the vector coupling coefficients that the differential cross section for population of a multiplet member of spin J is related by a statistical spin ratio to the core state cross section.[23]

$$\frac{d\sigma}{d\omega}(j_p \rightarrow J) = \frac{2J+1}{(2j_p+1)(2j_c+1)} \frac{d\sigma}{d\omega}(0 \rightarrow j_c). \quad (1)$$

The differential cross section of each multiplet member has the same angular distribution as that of the core state, and the total multiplet strength equals that of the core state.

The particle-core coupling may be treated in a general fashion by writing the interaction as the product of two tensor operators of order k, operating on the coordinates of the particle and core wave functions, respectively.[8] The energy splitting of the particle-core multiplet is given by the diagonal matrix elements of the interaction, assuming weak coupling and no mixing with levels of other configurations.

$$\Delta E(J) = \sum_k \langle j_p j_c JM \mid T^k(c) \cdot T^k(p) \mid j_p j_c JM \rangle, \quad (2a)$$

$$\Delta E(J) = (-)^{j_c+j_p+J} \sum_k C_k(j_c, j_p) \begin{Bmatrix} j_c & j_c & k \\ j_p & j_p & J \end{Bmatrix}, \quad (2b)$$

where $C_k(j_c, j_p)$ is the product of the reduced matrix elements of the tensor operators. It follows from the properties of the 6-j symbols[24] that, insofar as the interaction does not involve $k=0$, the "center of gravity" of the multiplet coincides with the core excitation.

$$\sum_J (2J+1)\Delta E(J) = 0, \qquad k \neq 0 \quad (3a)$$

$$\sum_J E(J) \frac{2J+1}{(2j_p+1)(2j_c+1)} = E_{\text{core}}. \quad (3b)$$

A monopole interaction term shifts the multiplet as a whole. The multipole interaction strengths $C_k(j_c, j_p)$ may be obtained in terms of observed energy splittings $\Delta E(J)$ by using the orthogonality relation for the 6-j symbols.

$$C_k = (2k+1) \sum_J (-)^{j_c+j_p+J} \Delta E(J)(2J+1) \begin{Bmatrix} j_c & j_c & k \\ j_p & j_p & J \end{Bmatrix}. \quad (4)$$

III. EXPERIMENTAL METHOD

Zr^{90} and Zr^{91} targets were bombarded by a 12.0-MeV proton beam from the Florida State University EN tandem Van de Graaff accelerator. The proton beam was collimated by $\frac{1}{4}\times3$-mm slits in front of the target and finally collected in a Faraday cup. Beam current on target was generally about 0.5 μA. The scattered protons were analyzed in a single-gap Browne-Buechner magnetic spectrograph[25] and recorded on an array of three 5×25-cm, 50-μ-thick nuclear emulsion plates clamped along the focal plane of the spectrograph. After plate development, the proton tracks were counted in $\frac{1}{2}$-mm strips under microscopes equipped with calibrated stages.

Zirconium targets were prepared by evaporating isotopically enriched ZrO_2 [26] upon thin carbon backings mounted upon aluminum frames. The Zr^{90} and Zr^{91} targets were approximately 98 and 92% isotopically pure, respectively. After the first proton bombardment, the beam impact spot was visible on each target. This target spot was reproduced on subsequent runs by careful alignment with a telescope previously fixed on the position of a beam spot burned in a paper target. The variation in target thickness in the immediate vicinity of the beam spot was assumed to be negligible. For the purpose of target thickness measurement, short 12.7-MeV proton bombardments of each target were taken at 70°. Target thicknesses were then calculated from elastic cross-section data measured for Zr^{90} at 12.7 MeV.[27] Zirconium target thicknesses ranged from 40–70 μg/cm^2, with the carbon backings being about the same thickness. The calculated zirconium target thicknesses were used to determine absolute differential cross sections.

Peak positions and areas were extracted by a computer routine which made a least-squares fit of the proton spectra to Gaussian functions. Where necessary, a linear or quadratic background function was included in the minimum χ^2 fit. A kinematic code was used to compute excitation energies and absolute cross sections and to identify impurity peaks. Short exposures were taken before each experimental run to determine the position of the intense $Zr^{90,91}$, O^{16}, and C^{12} elastic scattering peaks, which were used for energy calibrations for kinematic computations.

IV. RESULTS

$Zr^{90,91}(p, p')$ spectra were measured at six reaction angles from 30°–90° in order to distinguish between $L=2$ and $L=3$ transitions. The reaction spectra at 55° are shown in Fig. 1. The energy resolution at 55° is 11 KeV full width at half-maximum (FWHM). At other angles, the energy resolution varied from 9 to 15 keV FWHM. The spectra in Fig. 1 extend from just above the position of the intense C^{12} elastic peak up to about 3.3-MeV excitation energy. The rapidly increasing background observed in the 55° spectra results from proton scattering from various defining slit edges. There is no background under the 90° data up to 3.5-MeV excitation energy, but the background is kinematically shifted

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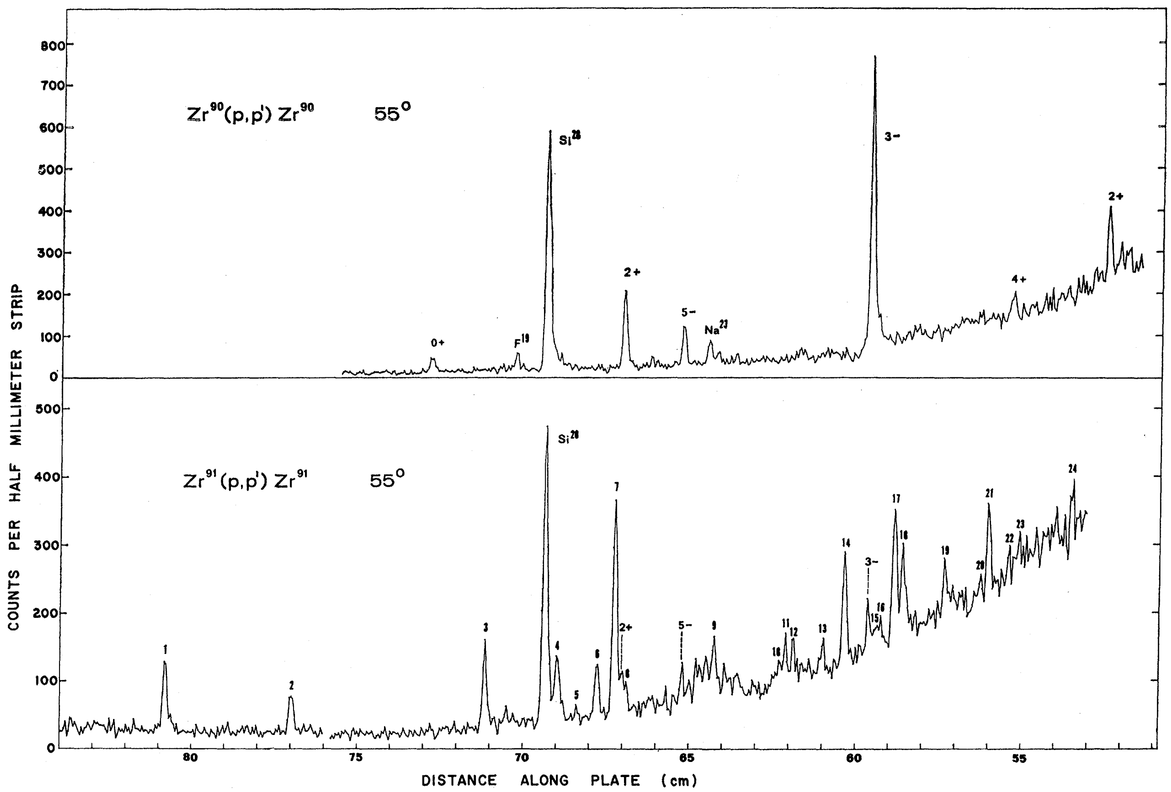

FIG. 1. Proton spectra of the reactions $Zr^{90}(p, p')Zr^{90}$ and $Zr^{91}(p, p')$ Zr^{91} at 55°.

down into the region below 3-MeV excitation energy at forward angles. The only serious impurity in these spectra is the 2+ first excited state of Si^{28} which was inadvertently deposited on the targets from the evaporator diffusion pump fluid. This impurity unfortunately obscured some of the zirconium data at 90° and 75°.

Differential cross sections (μb/sr) measured for the $Zr^{90}(p, p')$ and $Zr^{91}(p, p')$ reactions are given in Tables I and II, respectively. Only those spectral peaks which clearly reproduce from one angle to another are listed. Below each differential cross section is given the probable error, which includes both statistical error and an estimated 10% error due to target-thickness determination. For simplicity, neither these tables nor any plotted angular distributions show error due to background fitting. It should be noted, however, that the probable error due to background fitting is not negligible and is a function of angle. This error is about 100 μb/sr for all the 30° data, about 30 μb/sr for all the 45° data, and about 15 μb/sr for the 55° and 75° data above 2.5-MeV excitation energy. Reliable data above 3.1 MeV was obtained only at 90°. Energy levels above 3.1 MeV shown in Fig. 2 represent peaks having differential cross section greater than 10 μb/sr at 90°.

The angular distributions of differential cross sections measured for the 2+, 5−, and 3− states in Zr^{90} are plotted in Fig. 3. These data were obtained for comparison with core strength observed in Zr^{91} and for correction for about 5% Zr^{90} isotopic abundance in the Zr^{91} target. The curves plotted in Fig. 3 represent the experimental angular distributions for the reaction $Zr^{90}(p, p')$ at 12.7-MeV bombarding energy taken from the Oak Ridge National Laboratory report.[27] Distorted-wave Born-approximation (DWBA) fits to these data may be found in a recent paper by Dickens, Eichler, and Satchler.[21] The experimental curves are employed here to guide the eye and to compare our data with that of the ORNL group.

V. DISCUSSION

In the interest of pressing the core excitation model as far as possible, we have considered all states in Zr^{91} from the point of view of the interaction between a few shell-model neutrons and the various states of the Zr^{90} core. Although we believe there is remarkable qualitative agreement, much additional experimental work is clearly required. We have often allowed ourselves considerable license in interpreting the data because of insufficient experimental information.

A. Neutron States Coupled with the Zr^{90} Ground State

Single-particle states built on the Zr^{90} ground state should be especially strongly populated by the reaction

TABLE I. Level energies (MeV) and differential cross sections (μb/sr) for the reaction $Zr^{90}(p, p')Zr^{90}$. The probable error is given beneath each cross section.

Spin parity of level	Level energy (MeV)	Differential cross sections (μb/sr) 30°	45°	55°	75°	82°	90°
0+	0.0	265×10^4	454×10^3	111×10^3	846×10^2	629×10^2	364×10^2
		(34×10^4)	(59×10^3)	(14×10^3)	(110×10^2)	(75×10^2)	(47×10^2)
0+	1.75		130	98	83	49	
			(23)	(19)	(16)	(9)	
2+	2.18	899	669	404		481	457
		(135)	(94)	(57)		(63)	(59)
5−	2.32	174	207	190	218	116	
		(38)	(33)	(30)	(35)	(19)	
3−	2.74	1240	1350	1480	786	632	669
		(174)	(189)	(178)	(102)	(76)	(87)
4+	3.08			75	116		91
				(14)	(20)		(15)
2+	3.31			320	244	192	104
				(45)	(37)	(27)	(17)

$Zr^{90}(d, p)Zr^{91}$. Cohen and Chubinsky[28] and Forster *et al.*[29] have studied this reaction and measured angular distributions. Their results, including angular momentum assignments and normalized spectroscopic factors, are shown together with the results of this experiment in Fig. 2. Also included are data from the reaction $Zr^{92}(p, d)Zr^{91}$ studied by Ball and Fulmer.[30] For simplicity, we assign only the $d_{5/2}$(g.s.), $s_{1/2}$(1.21 MeV), $d_{3/2}$(2.06 MeV), and $g_{7/2}$(2.21 MeV) states. These assignments represent the lowest energy states having proper l value and high spectroscopic factor for the (d, p) reaction. The respective excitation energies observed in this experiment are 1.21, 2.08, and 2.19 MeV The 2.08-MeV state is very weak; so this identification is tentative. The analogs of these states have been observed in the excitation function of elastic proton scattering from Zr^{90}.[10,31] In the following, we attempt to relate all other states observed in the Zr^{91} spectrum to the coupling of these neutron states with excited states in the Zr^{90} core. The proposed coupling scheme is shown in Fig. 6.

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B. Neutron States Coupled with the 0+ (1.75 MeV) State in Zr^{90}

According to the core excitation model, we should expect $\frac{5}{2}+$, $\frac{1}{2}+$, and $\frac{3}{2}+$ states in Zr^{91} at the energy of the 0+ (1.75 MeV) first excited state in Zr^{90} plus the single-particle neutron energies in Zr^{91}. The predicted energies would be 1.75, 2.96, and 3.81 MeV, respectively. States of the proper l value have been observed at 1.48, 2.58, and 3.70 MeV in the (d, p) experiment.[28] These states have low (d, p) spectroscopic factors, and the analogs of these states have been oberved to resonate strongly with the 0+ (1.75 MeV) state in Zr^{90}, the respective resonances being at 6.28, 7.34, and 8.50 MeV proton bombarding energy.[10,31] Therefore, their interpretation as $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$ states built on the 0+ first excited state of Zr^{90} is strongly suggested in spite of the initial assignment[28] of the 1.48-MeV state as $d_{3/2}$.

The 0+ first excited state of Zr^{90} is weakly excited in this proton scattering experiment (see Fig. 1). In the Zr^{91} spectrum, we observe the $d_{5/2}$ state with moderate strength at 1.46 MeV. Our high-resolution spectroscopy reveals three closely spaced states at 2.524, 2.549, and 2.566 MeV. We believe one of these is the expected $s_{1/2}$ state, but a definite assignment is not possible. A similar situation holds for the $d_{3/2}$ state in the vicinity of 3.70 MeV. The spin assignments shown in Fig. 6 are based on the experimental data of Ref. 28 as modified

TABLE II. Level energies (MeV) and differential cross sections (μb/sr) for the reaction $Zr^{91}(p, p')Zr^{91}$. The probable error is given beneath each cross section.

Level No.	Level energy	l	Differential cross sections (μb/sr) 30°	45°	55°	75°	82°	90°
g.s.	0.0		309×10^4 (37×10^4)	386×10^3 (46×10^3)	110×10^3 (13×10^3)	860×10^2 (100×10^2)	755×10^2 (91×10^2)	412×10^2 (49×10^2)
1	1.21		278 (44)	152 (23)	135 (22)			
2	1.46		167 (28)	125 (20)	90 (15)	101 (14)		
3	1.876	2	610 (85)	244 (34)	158 (25)	165 (23)	161 (21)	120 (14)
4	2.035	2	278 (42)	109 (17)	144 (26)	85 (13)	79 (11)	72 (9)
5	2.08			16 (4)	25 (6)		7 (2)	
6	2.123	2	369 (55)	145 (25)	107 (17)	109 (15)	102 (13)	96 (12)
7	2.162	3	233 (37)	351 (53)	408 (57)		202 (26)	236 (28)
8	2.19			49 (9)	59 (11)		35 (5)	31 (4)
9	2.385			70 (12)	119 (19)	48 (9)		40 (6)
10	2.524			90 (14)	48 (9)	25 (5)		35 (5)
11	2.549			103 (22)	78 (13)	51 (10)		20 (3)
12	2.566			94 (15)	80 (14)	37 (6)		43 (6)
13	2.630	3		55 (9)	61 (12)	41 (8)		36 (5)
14	2.683	3		162 (26)	197 (28)	105 (16)		94 (12)
15	2.75			70 (11)	61 (13)	38 (6)		27 (4)
16	2.77			47 (8)	39 (8)	24 (4)		21 (3)

Table II (*Continued*)

Level No.	Level energy	l	30°	45°	55°	75°	82°	90°
			Differential cross sections (μb/sr)					
17	2.800	3		256	248	99	90	99
				(33)	(35)	(15)	(12)	(12)
18	2.821	3		162	151	64	55	66
				(24)	(24)	(11)	(8)	(9)
19	2.92	(2)		99	71	74	64	60
				(15)	(12)	(14)	(12)	(8)
20	3.00	(4)		45	32	27	18	13
				(8)	(7)	(5)	(3)	(2)
21	3.022	3		150	138	67	68	71
				(26)	(21)	(10)	(10)	(9)
22	3.07			60	43	37	24	25
				(10)	(8)	(6)	(4)	(4)
23	3.09	(2)		106	56	70	57	53
				(16)	(10)	(14)	(9)	(7)
24	3.22				58			18
					(10)			(3)

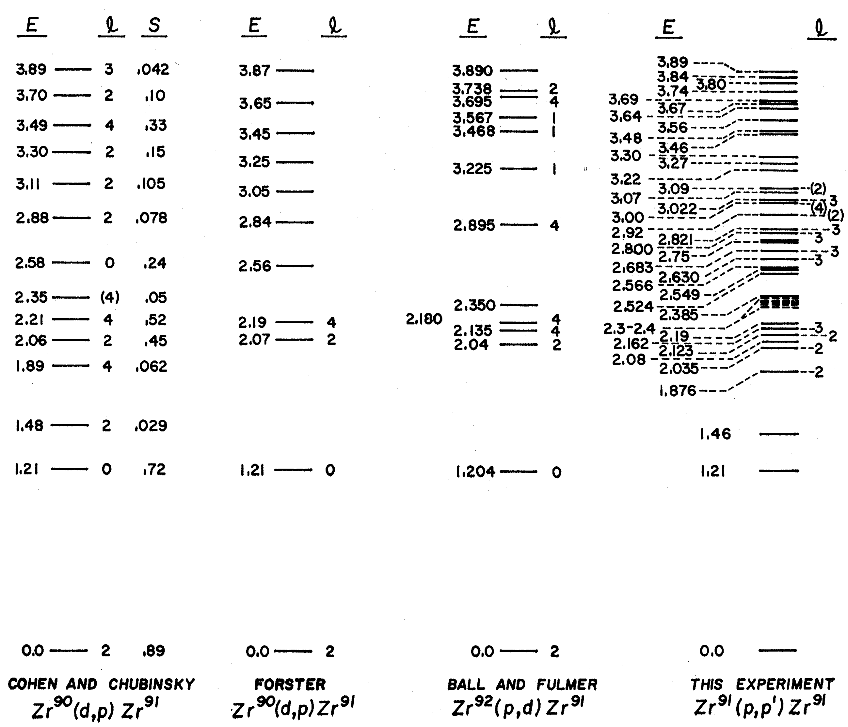

Fig. 2. Comparison of the Zr^{91} energy levels observed in this experiment with the results of Zr^{90} (d, p) Zr^{91} and Zr^{92} (p, d) Zr^{91} measurements (see Refs. 28–30).

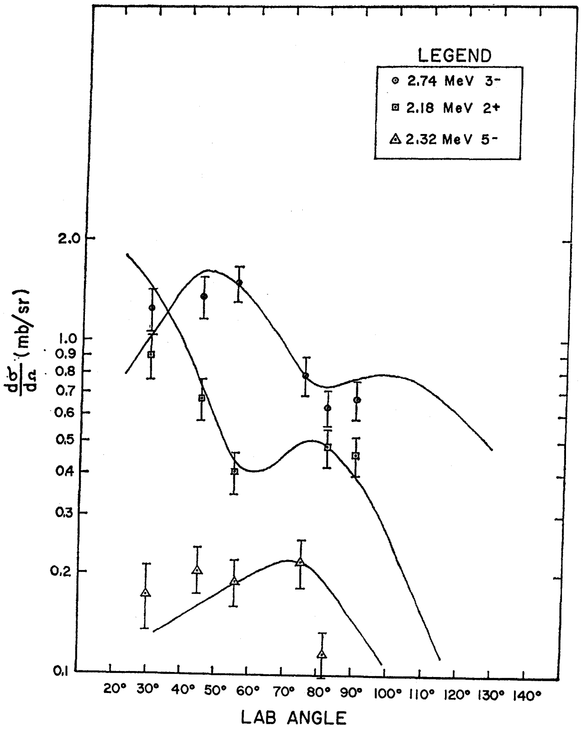

FIG. 3. Differential cross-section angular distributions for the 2+ (2.18 MeV), 5− (2.32 MeV), and 3− (2.74 MeV) states in Zr^{90}. The curves represent the corresponding ORNL experimental angular distributions (see Ref. 27).

by the analog-state experiment.[10,31] It should be noted, however, that if the core coupling assignments to the $d_{5/2}$ and $s_{1/2}$ states were assumed to be pure, we should have an $L=0$ transition to the 1.46-MeV state, and the transition to the $s_{1/2}$ state should be forbidden in first order. In fact, the transition to the 1.46-MeV state has a mixed $L=0$ plus $L=2$ character, as discussed further in the next section, and the transition to the $s_{1/2}$ state is probably observed. This information, together with the spectroscopic factor of the 2.58-MeV state observed in the (d, p) reaction,[28] indicates that there is considerable mixing between single-particle states and core excitation states.

C. Neutron States Coupled with the 2+ (2.18 MeV) State in Zr^{90}

The core excitation model predicts a quintuplet of states in Zr^{91}, having spins ranging from $\frac{1}{2}$ to $\frac{9}{2}$ and $L=2$ angular distributions, formed by the $d_{5/2}$ neutron coupling to the 2+ (2.18 MeV) state in Zr^{90}. These states should be populated according to the $2J+1$ rule in the proportions 0.067, 0.133, 0.200, 0.267, and 0.333, respectively. We observe states at 1.876, 2.035, and 2.123 MeV which are identified as the $\frac{9}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$ states, respectively. The measured angular distributions for these states are plotted in Fig. 4. The curves plotted in Fig. 4 represent the ORNL experimental angular distribution for the Zr^{90} 2+ state[27] multiplied by the $2J+1$ proportionality factors 0.333, 0.200, and 0.267. Different scaling factors are employed for the three states for illustrative purposes. We emphasize that these curves serve to test both the angular distribution and the absolute magnitude of the cross sections measured for these three states against the predictions of the core excitation model. That the calculated cross section for the 2.035-MeV state is much too large at 55° can probably be explained by the fact that this peak sits on the tail of the strong Si^{28} impurity (see Fig. 1).

The $\frac{9}{2}+$ state at 1.876 MeV appears quite pure, as expected; for there should be no other like spin-parity states nearby. The $\frac{5}{2}+$ (2.035 MeV) and $\frac{7}{2}+$ (2.123 MeV) states appear to show the effect of some mixing with the like spin-parity states at 1.46 and 2.19 MeV, respectively (see Fig. 6). In particular, we note that mixing of the 2.035-MeV and 1.46-MeV states probably causes the surprisingly strong excitation of the 1.46-MeV state. In fact, our data and the data of Blok *et al.*[32] suggest that the 1.46-MeV state has predominantly $L=2$ character. We have been unable to positively identify the weaker $\frac{3}{2}$ and $\frac{1}{2}$ members of the $L=2$ multiplet, but we believe them to be present, either unresolved from the strong peak at 2.162 MeV or contained in the poorly resolved structure around 2.3–2.4

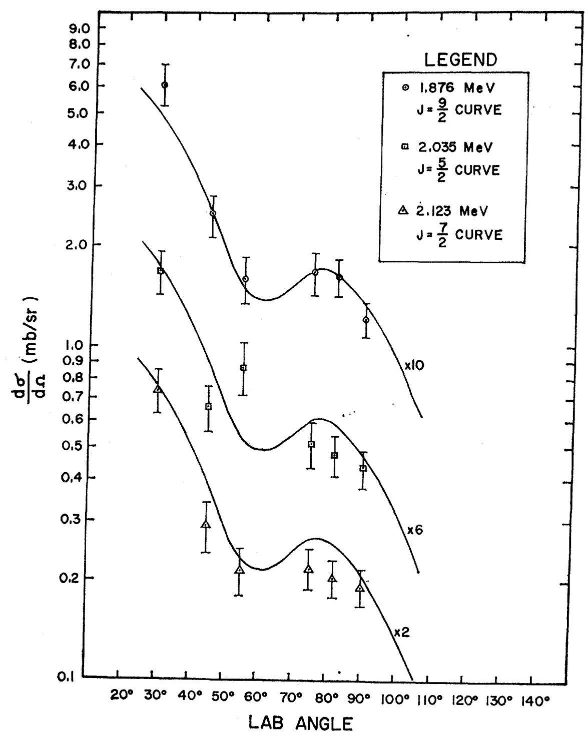

FIG. 4. Differential cross-section angular distributions for the 1.876-, 2.035-, and 2.123-MeV states in Zr^{91}. The curves represent the ORNL experimental angular distribution for the Zr^{90} 2+ (2.18 MeV) state (see Ref. 27) multiplied by $2J+1$ proportionality factors.

[32] H. P. Blok, G. D. Thijs, J. J. Kraushaar, and M. M. Stautberg (to be published).

MeV. Our limited data (see Table II) suggest that the clearly resolved peak at 2.385 MeV is a likely candidate for the $\frac{3}{2}+$ member, with about one-fourth of its expected strength dissipated in mixing with the nearby $\frac{3}{2}+$ member of the ground-state multiplet. With this tentative assignment and the position of the $\frac{1}{2}$ member assumed nearby, the center of gravity of the $L=2$ multiplet is calculated to lie at about 2.07 MeV, which coincides with a strong resonance at 6.85-MeV proton bombarding energy in the p_2 (2.18 MeV 2+) channel of the $Zr^{90}(p, p')Zr^{90}$ reaction.[31] It should be noted again that the (d, p) reaction can populate the positive parity states being discussed as core excitation states only because of mixing with single-particle states.

States at 2.88, 3.11, and 3.30 MeV, having $l=2$ angular distributions, are reported from the (d, p) experiment.[28] These states are possible candidates for the $\frac{5}{2}+$ and $\frac{3}{2}+$ doublet expected from coupling the $s_{1/2}$ neutron with the Zr^{90} 2+ core state. We observe states at 2.92 and 3.09 MeV with probable $L=2$ angular distributions which coincide with strong resonances at 7.70- and 7.89-MeV proton bombarding energy in the p_2 (2.18 MeV 2+) channel of the $Zr^{90}(p, p')Zr^{90}$ reaction.[31] However, these two states have about equal strength (see Table II), which is about three times that of the three states at 3.22, 3.27, and 3.30 MeV. Our data are insufficient to make any identification of spin for the expected doublet on the basis of angular distributions and $2J+1$ intensity relationships. However, all the data are consistent with the assignment of the 2.92- and 3.09-MeV states as $\frac{3}{2}$ and $\frac{5}{2}$ members of the doublet expected from coupling the $s_{1/2}$ neutron with the 2+ core state. The spectroscopy of this doublet is expected to be complicated by mixing with states of like spin parity formed by the $d_{5/2}$ neutron coupling to the 2+ (3.31 MeV) and 4+ (3.08 MeV) states in Zr^{90}.

D. Neutron States Coupled with the 5− (2.32 MeV) State in Zr^{90}

The third excited state in Zr^{90} is the 5− state at 2.32 MeV. This state is usually considered to be a pure two-proton $p_{1/2}g_{9/2}$ state. Indeed, the lifetime and γ transition probability ratios can be interpreted assuming this pure shell-model configuration.[12,14,15] However, inelastic scattering to this state[16–21] suggests a small amount of collective character of a 32nd pole vibration. To the extent that this vibration is present in the 5− state in the Zr^{90} core, we may expect the $Zr^{91}(p, p')Zr^{91}$ reaction to excite an $L=5$ multiplet formed by the $d_{5/2}$ neutron coupling to the Zr^{90} 5− state. There are a large number of weak, poorly resolved states populated in the vicinity of 2.3–2.4 MeV, including the relatively prominent peak at 2.385 MeV discussed in the previous section. However, even in the few cases where peak energies reproduce from one angle to another, we have not been able to make any interpretation from the calculated cross sections of the fitted data. Therefore, we can only speculate that this structure represents an $L=5$ multiplet plus probably the $\frac{3}{2}$ and $\frac{1}{2}$ members of the $L=2$ multiplet. An $L=5$ assignment for structure seen in this energy region has been suggested by the 18.7-MeV proton scattering data of Blok *et al.*[32]

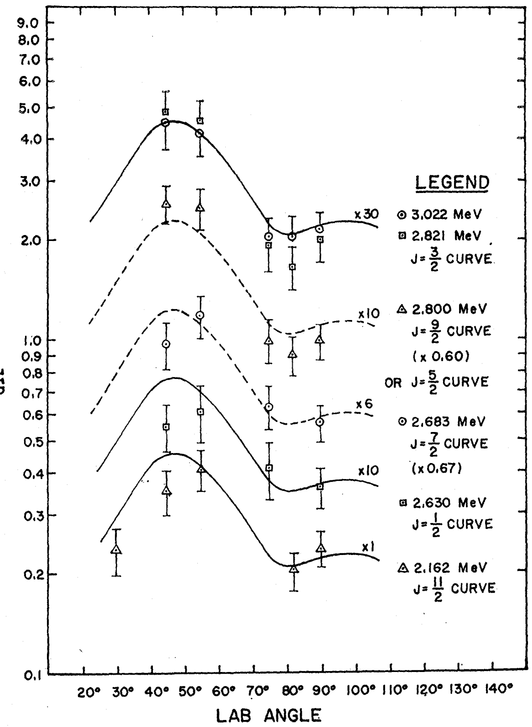

FIG. 5. Differential cross-section angular distributions for the 2.162-, 2.630-, 2.683-, 2.800-, 2.821-, and 3.022-MeV states in Zr^{91}. The solid curves represent the ORNL experimental angular distribution for the Zr^{90} 3− (2.74 MeV) state (see Ref. 27) multiplied by $2J+1$ proportionality factors. The dashed curves have been adjusted by the factor shown to fit the data.

E. Neutron States Coupled with the 3− (2.74 MeV) State in Zr^{90}

The fourth excited state in Zr^{90} has spin parity 4− and is not expected to have collective nature. Almost degenerate with it, at 2.74 MeV, is the well-known, highly collective 3− octupole vibration. This state is expected to couple with the single-particle neutron states to form strongly populated particle core multiplets. In particular, the $d_{5/2}$ neutron state should couple with the 3− state to form a sextuplet of states having spins ranging from $\frac{1}{2}$ to $\frac{11}{2}$ and $L=3$ angular distributions. The core excitation model predicts that the multiplet members will be populated in the proportions 0.047, 0.095, 0.143, 0.191, 0.238, and 0.286, respectively. However, we may expect appreciable mixing of the higher spin states with like spin-parity states formed by the $d_{5/2}$ neutron coupling to the 5− and 4− core states in Zr^{90}.

We find almost all of the expected $L=3$ strength in Zr^{91} distributed among the six states at 2.162, 2.630,

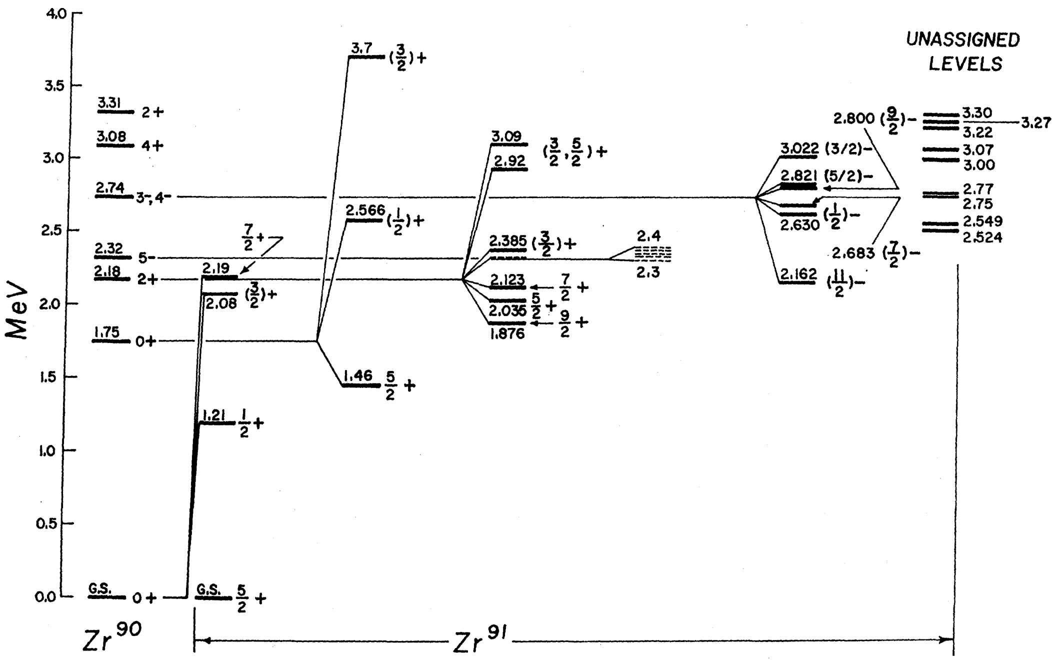

FIG 6. Proposed particle-core coupling scheme of the Zr^{91} energy levels.

2.683, 2.800, 2.821, and 3.022 MeV. This result is in qualitative agreement with the recent results of Blok *et al.*[32] and with the results of several low-resolution He^3 and He^4 scattering experiments.[11,20,33] The experimental angular distributions of these six states are plotted in Fig. 5. Various scaling factors have been used for illustrative purposes. We have attempted to make tentative identification of the multiplet member spins according to the $2J+1$ intensity rule. This interpretation is illustrated by the curves plotted in Fig. 5. Solid curves represent the ORNL experimental angular distribution for the Zr^{90} $3-$ state[27] multiplied by $2J+1$ proportionality factors. Dashed curves have been adjusted by the stated factor to fit the data.

The 3.022- and 2.821-MeV states have almost equal cross sections, very close to the prediction for the $\frac{3}{2}-$ state, and it is impossible to determine from our data which is the $\frac{3}{2}-$ state. We assign one of these states spin $\frac{3}{2}$ and the other spin $\frac{5}{2}$, with 95/143 or $\frac{2}{3}$ of the predicted $\frac{5}{2}-$ strength present. Both the $\frac{9}{2}-$ and $\frac{7}{2}-$ states, tentatively assigned at 2.800 and 2.683 MeV, respectively, also exhibit only about $\frac{2}{3}$ of their predicted strength (as shown by the dashed curves in Fig. 5) probably due to mixing with other like spin-parity states. As shown in Fig. 5, the cross section of the 2.800-MeV state is actually very close to that predicted for spin $\frac{5}{2}$. However, considering the probability of configuration mixing and the lack of any other state approximating the spin $\frac{9}{2}$ strength, the most reasonable assignment for the 2.800-MeV state seems to be $\frac{9}{2}-$ with one-third of its strength missing. The $\frac{1}{2}-$ state assigned at 2.630 MeV appears to have its strength intact, as expected. The $\frac{11}{2}-$ state, assigned at 2.162 MeV, is surprising in that it lies so far below the position of the Zr^{90} $3-$ (2.74 MeV) state with its strength apparently intact. This may be understood in a qualitative sense, however, in terms of the mixing of this $\frac{11}{2}-$ state with the $h_{11/2}$ single-particle state which is expected in the vicinity of 5 MeV in Zr^{91}. The center of gravity of this core multiplet, using the tentative assignments shown in Fig. 6, is 2.61 MeV, which is to be compared with 2.74-MeV excitation energy of the $3-$ octupole state in Zr^{90}.

A very tentative assignment has been made for the 3.89-MeV state as one of the members of the $\frac{5}{2}-$, $\frac{7}{2}-$ doublet expected from coupling the $s_{1/2}$ neutron with the $3-$ core state.

IV. CONCLUSION

With the exception of a few relatively weak states, it is possible to describe all of the states below 3 MeV in Zr^{91} in terms of the coupling of single-particle neutron states with various states of the Zr^{90} core. The relationship between the Zr^{90} core and the levels in Zr^{91} is shown in Fig. 6. Although configuration mixing complicates the spectroscopy and is especially serious for the positive parity states, there is in most cases remarkable agreement between experiment and assignments based on the simple core excitation model. Above 3 MeV, the interpretation is at best qualitative, for lack of complete experimental data.

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[33] H. W. Broek and J. L. Yntema, Phys. Rev. **138**, B334 (1965).