Improved Inverse Gap Equation and Quasiparticle Theories of Odd and Even Tin Isotopes*

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The nuclear-structure properties of the odd and even tin isotopes are studied, making use of an improved inverse-gap-equation (IGE) method and the quasiparticle theories. The two-body nuclear force assumed is a realistic potential (or reaction matrix) renormalized for the core polarization, and two cases are considered: (a) the potential of Tabakin, and (b) the reaction matrix of Yale-Shakin. While most types of levels are obtained in good agreement with experiment, others sharply disagree with the data. Only poor agreement with experiment is obtained for the electromagnetic properties of some of the calculated states and for the inelastic electron-scattering form factors. Spectroscopic factors for the one- and two-nucleon transfer reactions are presented and discussed. A warning is given against too much optimism about the indiscriminate applicability of the IGE method. Possible explanations for the difficulties of the method are discussed.

1. INTRODUCTION

ONE of the most disturbing difficulties in detailed shell-model analysis of various regions of nuclei is the uncertainty or our ignorance of the single-particle (s.p.) shell-model energies. Ideally, such energies should be determined as self-consistent solutions of a Hartree-Fock (HF) or a Hartree-Fock-Bogolubov (HFB) calculation based on a realistic two-body potential (or reaction matrix). In practice such calculations are almost prohibitive in some regions of the periodic table. Only very preliminary results of some simplified HF calculations are becoming available (e.g., for the tin isotopes.[1,2]) On the other hand, the experimental information on the so-called s.p. energy levels of the corresponding odd-mass nuclei is hardly ever complete and sufficient. Even in cases when such information is available, the identification of the "empirical" s.p. levels with the "HF s.p. energies" is rather arbitrary and certainly (by definition) at best only very roughly approximate.

Such is the situation with most nuclei in the vibrational region, such as the tin isotopes. One characteristic feature of that nuclear region is the outstanding importance of the BCS superfluidity effect. The predominance of the BCS pairing forces imposes the definition of a s.p. basis in terms of an HFB solution rather than of an HF solution. The s.p. basis is in this case usually expressed in terms of the Bogolubov quasiparticles (qp) [a single-qp basis defining the independent-quasiparticle model (IQM)]. With some reservations, one can attempt to interpret the energy levels of the odd-mass isotopes in terms of such single-qp energies or, better, in terms of the corresponding QTD13 eigenvalues. The QTD13 (or modified Tamm-Dancoff, MTDA) method[3–5] involves diagonalization of energy matrices of one- and three-qp modes in interaction. Similar descriptions of even isotopes in terms of superpositions of two-qp modes [quasiparticle Tamm-Dancoff (QTD)] or of two- and four-qp modes [quasiparticle second Tamm-Dancoff (QSTD)] have proved generally successful.[6–12]

A method known as the inverse gap equation (IGE) has been proposed by Gillet and Rho[13] for determining from the experimental energies of the odd-mass isotopes of single closed-shell nuclei the input parameters for qp

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calculations. The method assumes the first few observed levels with known spin and parity as pure one-qp excitations. The solution of a given IGE problem determines uniquely the HF s.p. energies and the chemical potential λ and thus also the subshell occupation factors (probabilities) v_a^2 of the BCS theory. An IGE solution also yields the coupling (strength) constant V_0 of the BCS pairing force. If one has a purely phenomenological two-body potential, such a unique determination of V_0 is useful, although the obvious question arises of how the solution depends on all the details of such an assumed two-body force. If the result is sensitive to any details of such a force, then one wonders what is the value of the information obtained. If, on the other hand, the pairing force corresponds to a realistic nucleon-nucleon potential (or reaction matrix) renormalized for the core polarization which is always necessary,[10–11] then one is more confident of the utility of the IGE output information on the HF energies and on the v_a^2. However, the departure of V_0 from unity is the measure of the extent to which the simple qp description of a given odd isotope of a single closed-shell nucleus is inappropriate or it implies that the pairing force used is incorrect. An alternative to this conclusion could only be that it is the given potential which is inappropriate. However, if one repeats the calculation with another realistic two-body force and again finds a V_0 drastically different from unity, then the former is a more convincing conclusion. Both in a previous work by Gambhir[14] on the Ni isotopes and in the present work on the Sn isotopes with different realistic two-body forces, the IGE solutions have indeed V_0 very close to unity, which means that the IQM (or the QTD13) interpretation of the corresponding spectra is at least not incompatible with the pairing force assumed.

The improvement of the IGE procedure first proposed and applied by one of us[15] consists in interpreting the observed energies of the odd isotopes and the odd-even mass differences in terms of the QTD13 eigenvalues (superpositions of one- and three-qp modes) rather than in terms of pure (independent) quasiparticles (IQM). The QTD13 definition and equations are now coupled to IGE. One solves the problem by an iteration procedure consisting of successive IGE solutions and QTD13 matrix diagonalizations. The procedure, although far from being free of criticism, is by definition more realistic than the simple IGE method because it includes contributions from the residual qp interactions. In the first step one sets up and solves the ordinary IGE problem and the output set (the BCS E_a, u_a, v_a) is fed into the corresponding QTD13 energy matrix. The differences between the input energies (E_a=the observed energies) and the output energies (QTD13 eigenvalues) δE_a are then added to E_a and used as input in the second IGE problem, etc. The final solution is obtained when the QTD13 eigenvalues of the nth iteration almost coincide with the observed energies (the original E_a). The corresponding nth qp-input set is the final set, and the corresponding IGE solution gives the final HF energies. The iteration procedure converges very rapidly ($n=2$, 3 in practice in cases where the first δE_a are not too large). The spectra of the corresponding even isotopes are then calculated with the QSTD method by feeding in the qp energies and occupation probabilities obtained with the use of the final qp sets of the above method.

The above method has, for a number of interactions, given a remarkable agreement between the observed and the calculated low-lying levels of the odd and the even isotopes of nickel.[14,15] This agreement is striking because the qp description should not be particularly reliable in the case of nickel, where one has only three active subshells and all of them are of small degeneracy. Moreover, a small number of valence neutrons does not favor a statistical-mechanical theory such as that of BCS.

It was therefore interesting to examine the case of the Sn isotopes, for which the qp description is considered particularly appropriate, in order to see if good agreement with experiment can be obtained and whether the success of the work of Refs. 14 and 15 is not somewhat accidental.

Since the wave functions are generally a much more critical test of a model than are the energy levels, we have examined, in addition to the spectra, some of the electromagnetic reduced transition rates $B(E\lambda)$, the static moments, the inelastic electron-scattering form factor, and the spectroscopic (parentage) factors for the one- and two-neutron transfer reactions.

2. ENERGY SPECTRA OF THE ODD ISOTOPES OF TIN

By definition of the improved inverse-gap-equation (IIGE) method described in Sec. 1, the first (lowest-lying) energy level of each value of spin and parity is made almost to coincide with the corresponding observed level. The merit of the method in determining the spectra of the odd isotopes, if any, could possibly lie in the correct or reasonably good prediction of the positions of the second or higher-lying levels of each spin-parity assignment J^π. Clearly, the structure of the wave functions is most important and this, if successfully predicted both for the first and for the other low-lying states, could be a merit of the method; the question of the wave functions (the QTD13 eigenvectors of IIGE) concerns our results of Sec. 4.

As commonly accepted, we describe the low-lying nuclear states of Sn in terms of the five valence neutron subshells: $2d_{5/2}$, $1g_{7/2}$, $3s_{1/2}$, $2d_{3/2}$, and $1h_{11/2}$. However, we include the effect of the neutron and proton core subshells through the renormalization for the "core

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TABLE I. QTD13 energy eigenvalues of the first few levels of Sn^{115} obtained by the IIGE (second iteration) using the core renormalized (i) Tabakin and (ii) Yale-Shakin force with $V_0=1$ and $V_0\neq1$. The experimental energies are of Ref. 20, in which half of the odd-even mass difference calculated from the observed binding energies [Eqs. (11) and (12) of Ref. 15] are added.

$A=115$	Tabakin		Yale-Shakin		
J^π	$V_0=1$	$V_0=1.114$	$V_0=1$	$V_0=1.099$	Expt.
$1/2^+$	1.041(94)	1.000(93)	1.061(93)	0.997(91)	1.000
	4.117(4)	4.024(5)	4.009(4)	3.869(5)	2.970
$3/2^+$	1.511(95)	1.478(94)	1.517(95)	1.472(93)	1.490
	3.204(∼0)	3.061(∼0)	3.110(∼0)	2.935(∼0)	
$5/2^+$	1.997(96)	1.972(95)	2.008(94)	1.967(93)	1.980
	3.420(∼0)	3.280(∼0)	3.321(∼0)	3.156(∼0)	(2.280)
	3.714(0.5)	3.591(0.8)	3.596(0.7)	3.444(1)	(2.630)
$7/2^+$	1.595(97)	1.570(96)	1.599(96)	1.564(95)	1.600
	3.549(∼0)	3.452(∼0)	3.557(∼0)	3.422(0.1)	
$11/2^-$	1.727(97)	1.705(96)	1.725(95)	1.682(94)	1.730
	3.785(∼0)	3.709(0.2)	3.852(1)	3.730(2)	

polarization" of the effective nuclear force (and of the effective operators of electromagnetic interactions). The s.p. wave functions are those of the harmonic oscillator with the spring constant $\sqrt{\nu}=0.454$ F^{-1}.

The two-body "bare" nuclear force is taken to be (a) the nonlocal potential of Tabakin[16] and (b) the Yale-Shakin reaction matrix.[17] The second-order corrections for the core polarization (the three-particle one-hole diagrams) are those of the variant "*S*1" of Ref. 10. The neutron and proton core subshells included are four: $1g_{9/2}$, $2p_{1/2}$, $1f_{5/2}$, and $2p_{3/2}$. The separations of the proton levels from the level $2d_{5/2}$ are of -4.0 MeV ($1g_{9/2}$) and -12.0 MeV (the remaining three); this assumption is taken from the work of Bando.[18] The open subshells for the protons are the same as the five valence neutron levels (between the magic numbers 50 and 82). Again, the s.p. proton energies of these levels are those of Ref. 18. For simplicity, we assume only one open ("particle") subshell for the core neutrons: the $1h_{11/2}$ level, distant 6.4 MeV from the $1g_{9/2}$ level and 14.4 MeV from the other core levels.[18] This simplification for the core neutrons is justified by the fact that the entire effect of the polarization of the core neutrons is, anyway, practically negligible as compared with that of the core protons; another prescription leads to almost identical results (cf. Refs. 10 and 11).

We present below our IIGE results for the Sn isotopes 115, 117, 119, and 121. The IGE and the QTD13 FORTRAN codes employed in our work are those of Refs. 14 and 15 and of Refs. 5 and 19, respectively. All the pertinent defining equations are given in all these references and in Kuo *et al.*[3] Our numerical results corresponding to the second iteration of IIGE are reproduced in Tables I–IV. For each A and nuclear force considered we give two series of results: one for $V_0=1$ (the actual nuclear force given), the BCS input set (E_a, u_a, v_a) for QTD13 being that of the last (second) IGE solution; and another one for $V_0\neq1$ determined from the second IGE solution. The first few (two or three) QTD13 eigenvalues are given for each case and the percentage of the one-qp component of the corresponding eigenvector is given in parentheses. The V_0 of the second IGE is always about 1.1, which means that the qp interpretation of the lowest-lying (J^π) levels with the given force as the BCS pairing force may be a reasonable approximation. Moreover, the differences between the corresponding QTD13 eigenvalues and eigenvectors calculated with $V_0=1$ and $V_0\neq1$ are very small indeed. The results for both the forces (Tabakin and Yale-Shakin) are generally very similar for all the cases. The only characteristic difference is slightly wider spacings between the first and the second (higher) levels in the case of the Tabakin potential. No results are given for the Yale-Shakin force for $A=121$ because in this case the IGE conditions cannot be satisfied and no solution is obtained [note the unstable (singular) behavior of the HF energies already at $A=119$ as indicated in Table V].

The observed energies are given in the last columns of Tables I–IV. The data are taken from Refs. 20–22. We see that the calculated second and third levels ($\frac{5}{2}^+_{2,3}$) lie considerably too high when compared

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TABLE II. QTD13 energy eigenvalues of the first few levels of Sn^{117}. For details see caption of Table I.

$A=117$	Tabakin		Yale-Shakin		
J^π	$V_0=1$	$V_0=1.146$	$V_0=1$	$V_0=1.137$	Expt.
$1/2^+$	1.224(95)	1.193(93)	1.254(94)	1.201(92)	1.180
	3.884(2)	3.773(3)	3.817(3)	3.667(4.8)	
$3/2^+$	1.368(96)	1.343(94)	1.381(95)	1.342(94)	1.340
	3.164(∼0)	2.991(0.1)	3.105(∼0)	2.889(∼0)	
$5/2^+$	2.233(96)	2.213(94)	2.250(95)	2.215(92)	2.210
	3.306(0.4)	3.134(0.7)	3.211(0.5)	3.004(1)	2.370
	3.694(0.1)	3.542(0.2)	3.592(0.2)	3.405(0.4)	(2.690)
$7/2^+$	1.907(96)	1.882(95)	1.912(96)	1.875(94)	1.900
	3.717(0.2)	3.553(0.3)	3.608(0.2)	3.414(0.4)	
$11/2^-$	1.508(97)	1.488(96)	1.511(96)	1.475(94)	1.500
	3.627(0.7)	3.499(1.4)	3.605(1.8)	3.446(3)	

with the corresponding data. This may be because the effective force is too weakly attractive, but it may also mean that our qp description of the odd isotopes is somewhat inappropriate (at least too crude). As for the weakness of the force, we note that the lowest levels are almost pure one-qp states while the second and higher levels are almost pure three-qp states (very little one qp-three qp mixing). Clearly, the lower the level, the more reliable the QTD13; for too high-lying states five-qp modes may become not negligible. The role of the unprojected higher-order spurions due to the nucleon-number nonconservation is not clear, particularly in the higher states. The only spurion projected out[3–5] has three-qp components only. Higher-order spurions contain five and more qp components, but also small one- and three-qp components (lying within our Hilbert space).

The s.p. (valence neutron) HF energies corresponding to the IIGE results of Tables I–IV are given in Table V. These energies refer to the $2d_{5/2}$ level. They all include the "kinetic" energy and the HF self-energy parts. Typically, the standard s.p. level sequence is reproduced (as $\frac{5}{2}^+$, $\frac{7}{2}^+$, $\frac{1}{2}^+$, $\frac{3}{2}^+$, and $\frac{11}{2}^-$), but at $A=119$ a sudden change occurs in the relative position of the

TABLE III. QTD13 energy eigenvalues of the first few levels of Sn^{119}. For details see caption of Table I.

$A=119$	Tabakin		Yale-Shakin		
J^π	$V_0=1$	$V_0=1.136$	$V_0=1$	$V_0=1.128$	Expt.
$1/2^+$	1.309(98)	1.298(97)	1.317(97)	1.298(96)	1.300
	3.461(0.4)	3.293(1)	3.382(1)	3.182(2)	
$3/2^+$	1.332(97)	1.317(96)	1.338(97)	1.314(95)	1.324
	2.968(∼0)	2.791(∼0)	2.920(∼0)	2.722(∼0)	
$5/2^+$	2.237(98)	2.228(97)	2.244(97)	2.227(95)	2.230
	3.101(0.2)	2.936(0.4)	3.024(0.2)	2.835(0.3)	2.400
	3.638(0.2)	3.509(0.4)	3.562(0.4)	3.407(1)	2.520
$7/2^+$	2.099(95)	2.079(94)	2.107(95)	2.076(93)	2.090
	3.607(0.4)	3.469(0.4)	3.531(0.4)	3.364(0.5)	
$11/2^-$	1.387(97)	1.370(96)	1.393(96)	1.364(94)	1.380
	3.405(1.4)	3.282(2)	3.383(2)	3.242(3)	

TABLE IV. QTD13 energy eigenvalues of the first few levels of Sn^{121}. For details see caption of Table I.

$A=121$	Tabakin		
J^π	$V_0=1$	$V_0=1.168$	Expt.
$1/2^+$	1.360(97)	1.343(96)	1.350
	3.562(0.3)	3.421(1)	
$3/2^+$	1.308(97)	1.292(96)	1.300
	3.036(∼0)	2.835(∼0)	
$5/2^+$	2.427(98)	2.416(96)	2.420
	3.129(∼0)	2.941(∼0)	2.700
	3.675(∼0)	3.547(0.1)	3.010
$7/2^+$	2.239(95)	2.218(94)	2.230
	3.569(0.3)	3.432(0.3)	
$11/2^-$	1.357(97)	1.341(96)	1.350
	3.426(1)	3.285(2)	

$2d_{3/2}$ level (a sharp crossing with the $3s_{1/2}$ level). The results obtained with $V_0=1$ and $V_0\neq 1$ are very similar (see Tables I–V).

The spectra of Tables I–IV can be compared with those obtained by two of us[19] with the Tabakin potential, with the same particle-hole excitation energies of the core nucleons but with the s.p. energies of the valence neutrons as assumed by Bando.[18] These latter energies are (except for $A=119$ and 121) not very different from those of Table V. While, by definition of IIGE, the first J^π levels of the present work are much better, the positions of $\frac{5}{2}_2^+$ are not (e.g., for $A=119$).

3. SPECTRA AND ELECTROMAGNETIC PROPERTIES OF THE EVEN ISOTOPES OF TIN

For calculating the spectra and wave functions of the even isotopes we have used the same reduced matrix elements of the effective forces (Tabakin and Yale-Shakin). The BCS input parameters were, for each value of A, calculated with the HF s.p. energies of the valence neutrons (including the valence self-energy parts) taken to be the arithmetical means of the energies $\tilde{E}_n^0$ of the neighboring odd isotopes $A-1$ and $A+1$ as taken from Table V (for $A=120$ and the Yale-Shakin force we take the values of $A-1=119$ of Table V); the nuclear effective force is not modified ($V_0=1$). The prescription of the arithmetical means for the s.p. HF energies is somewhat arbitrary. We have examined its consequences by repeating all our calculations, taking for the isotope A the HF energies of $A-1$; the corresponding results were only slightly different from those we present below. The qp parameters E, u, and v are calculated for the even isotopes from the BCS equations by using these HF energies (corresponding to $V_0=1$). Two theories are applied: (a) the pure two-qp theory (QTD) and (b) the QSTD theory (or MTDA of Ref. 8) in which nuclear states are superpositions of zero-, two-, and four-qp excitations. All the spurions due to the nucleon-number nonconservation which are entirely contained within a given Hilbert space are projected out from the secular matrices. All the equations and descriptions of the method are given in Refs. 7, 10, and 11. The FORTRAN codes used for the computations of the present work are those of Refs. 7, 10, and 11.

In Tables VI and VII we present some of our QTD and QSTD eigenvalues for $A=116$, 118, and 120 for the even parity (0^+, 2^+, 4^+) and odd parity (3^-, 5^-, 6^-,

TABLE V. The HF s.p. energies (including the valence self-energy parts) $\tilde{E}_a^0$, of odd Sn isotopes (117–121) obtained by the IIGE method (second iteration) using the core renormalized (i) Tabakin and (ii) Yale-Shakin reaction matrix for $V_0=1$ and $V_0\neq 1$.

	115		117		119		121
$V_0=1$	Tabakin	Yale-Shakin	Tabakin	Yale-Shakin	Tabakin	Yale-Shakin	Tabakin
$\tilde{E}_{7/2^0}-\tilde{E}_{5/2^0}$	0.5562	0.5604	0.5010	0.4788	0.1743	0.1166	0.2775
$\tilde{E}_{1/2^0}-\tilde{E}_{5/2^0}$	1.7630	1.7566	1.8937	1.8839	1.8410	1.8417	2.0265
$\tilde{E}_{3/2^0}-\tilde{E}_{5/2^0}$	2.8958	2.8233	2.5391	2.4057	1.2916	1.3437	1.7675
$\tilde{E}_{11/2^0}-\tilde{E}_{5/2^0}$	3.2452	3.2958	2.9934	3.0567	2.8180	2.9120	2.8810
	115		117		119		121
$V_0\neq 1$	Tabakin	Yale-Shakin	Tabakin	Yale-Shakin	Tabakin	Yale-Shakin	Tabakin
$\tilde{E}_{7/2^0}-\tilde{E}_{5/2^0}$	0.5612	0.5563	0.5123	0.4832	0.1635	0.1105	0.2670
$\tilde{E}_{1/2^0}-\tilde{E}_{5/2^0}$	1.7619	1.7575	1.8914	1.8864	1.8438	1.8444	2.0288
$\tilde{E}_{3/2^0}-\tilde{E}_{5/2^0}$	2.8930	2.8270	2.5228	2.4053	1.2885	1.3436	1.7800
$\tilde{E}_{11/2^0}-\tilde{E}_{5/2^0}$	3.2432	3.2971	2.9840	3.0608	2.8327	2.9241	2.8924

TABLE VI. QTD and QSTD eigenvalues of the first few 0^+ levels of the even Sn isotopes with $A=116$, 118, and 120. Percentages of the four-qp components of the corresponding QSTD eigenvectors are indicated (in parentheses). Experimental energies are given for comparison.

116							
Tabakin			Yale-Shakin				
QTD	QSTD(I)	QSTD(II)	QTD	QSTD(I)	QSTD(II)	Expt.	
0.000	(−0.346)(5.1)	(−0.206)(36.2)	0.000	(−0.333)(5.0)	(−0.204)(36.1)	0.000	
1.713	1.505(5.7)	1.520(6.0)	1.827	1.583(6.9)	1.600(7.2)	1.76	
2.797	2.364(16.0)	2.366(16.2)	2.826	2.281(21.1)	2.282(21.3)	2.02	
118						120	
Tabakin	Yale-Shakin				Tabakin	Yale-Shakin	
QTD	QTD	QSTD(I)	QSTD(II)	Expt.	QTD	QTD	QSTD(I)	QSTD(II)	Expt.
0.000	0.000	(−0.230)(3.6)	(−0.119)(35.7)	0.000	0.000	0.000	(−0.247)(4.4)	(−0.140)(35.3)	0.000
1.828	1.883	1.636(11.1)	1.653(11.2)	1.75	1.756	1.728	1.409(13.1)	1.451(14.3)	1.872
2.480	2.529	2.003(16.3)	2.015(17.5)	2.043	2.459	2.531	2.126(10.9)	2.155(12.8)	2.170

and 7^-) for lowest-lying levels. The percentage of the four-qp components of the QSTD eigenvectors is indicated in parentheses.

In Table VI we present our $J^\pi=0^+$ results. Two variants of the QSTD theory are considered (labeled I and II). A detailed description of these variants is given in Ref. 11. The distinction between I and II concerns a different treatment of the spuriousness due to the nucleon-number nonconservation. In the case I the qp vacuum (the BCS ground state) is assumed to be totally free of spuriousness and thus the qp vacuum ($|\tilde{0}\rangle$) components of the spurions to be projected out are omitted ($\langle\tilde{0}\,|\,\psi_{sp}'\rangle=0$). In the case II we project out exactly all the spurions $|\psi_{sp}\rangle$ entirely within the QSTD Hilbert space [together with their constant (or $|\tilde{0}\rangle$) components, i.e., $\langle 0\,|\,\psi_{sp}\rangle\neq 0$]. In this latter case, the most important BCS fluctuations of the number-squared operator are eliminated [the spurion $\mathfrak{N}(\hat{N}^2-N_0^2)\,|\,\tilde{0}\rangle$, where N_0 is the actual neutron number]. The only practical difference between I and II is the rather high (~36%) percentage of the four-qp components of the ground state (0_1^+) in case II. This is due to the depletion of the $|\tilde{0}\rangle$-component of $|0_1^+\rangle$ through the projection of the spurion of $\hat{N}^2-N_0^2$. Because of the four-qp correlations, the energy of the 0_1^+ level of QSTD is depressed in relation to the BCS ground (a negative shift). In principle, one should compensate all the remaining QSTD eigenvalues for this small shift. We prefer to avoid this reinterpretation for several reasons. For example, if one turns on the two-qp–six-qp coupling by the same part of the qp-transformed Hamiltonian H_{40}, which is responsible for the 0_1^+ energy depression, we may in the other states get similar small shifts which could compensate for that of 0_1^+.

From Table VI we see that the agreement of the position of the calculated 0_2^+ level with experiment is rather good. In fact, it is better in the present work than in that of Refs. 10 and 11. The four-qp percentages of $|0_{2,3}^+\rangle$ are not very high.

From Table VII we see that the over-all agreement with experiment of the calculated even-parity levels $2_{1,2,3}^+$ and $4_{1,2,3}^+$ is rather impressive; again, it is better than that of Refs. 10 and 11. Unfortunately, no satisfactory agreement with experiment is obtained for the odd-parity states, particularly for the 3_1^- and 7_1^- levels. This failure to reproduce the odd-parity levels is probably due mainly to the incorrect position of the HF energy of the $1h_{11/2}$ s.p. level in relation to the remaining HF energies (all these s.p. levels of the valence neutrons are of positive parity). It probably means that the interpretation of the $\frac{1}{2}^-_1$ levels in the odd isotopes as QTD13 eigenvalues for the application of the IIGE method is much too crude or may be that the experimental (IGE input) information on these $\frac{1}{2}^-_1$ levels is questionable.

The over-all fit to the data is usually measured in terms of the quantity

$$\chi^2=\sum[(E_{calc}-E_{expt})^2/E_{expt}], \qquad (1)$$

where E_{expt} is the observed level energy and E_{calc} is the corresponding calculated energy; the summation runs over all the observed levels with assigned spin and parity of a given nucleus. We have calculated χ^2 values for the levels of Tables VI and VII. For example, for the isotope 116 and the QTD theory with the Yale-Shakin force we find $\chi^2=1.268$ MeV for the 13 known levels (including 0_1^+) and $\chi^2=0.542$ MeV if the worst fit levels, 3_1^- and 7_1^-, are excluded from the analysis. To compare this with our previous calculations we may

TABLE VII. QTD and QSTD eigenvalues of the first few 2^+, 4^+, 3^-, 5^-, 6^-, and 7^- levels of the even Sn isotopes with $A=116$, 118, and 120. Percentages of the four-qp components of the corresponding QSTD eigenvectors are indicated (in parentheses). Experimental energies are given for comparison.

	116					118				120			
	Tabakin		Yale-Shakin			Tabakin	Yale-Shakin			Tabakin	Yale-Shakin		
	QTD	QSTD	QTD	QSTD	Expt.	QTD	QTD	QSTD	Expt.	QTD	QTD	QSTD	Expt.
2^+	1.654	1.505(4.4)	1.533	1.353(5.1)	1.291	1.420	1.335	1.151(5.7)	1.229	1.413	1.329	1.139(6.0)	1.166
	2.359	2.124(7.5)	2.397	2.092(10.3)	2.108	2.184	2.243	1.897(11.9)	(2.040)	2.083	2.029	1.719(10.3)	(2.089)
	2.574	2.346(7.3)	2.572	2.216(11.3)	2.224	2.299	2.291	2.008(7.6)	...	2.528	2.718	2.482(7.5)	...
4^+	2.523	2.324(3.6)	2.512	2.397(3.4)	2.391	2.302	2.267	2.076(7.0)	2.278	1.885	1.810	1.618(6.1)	2.183
	2.782	2.569(5.9)	2.844	2.652(6.0)	2.531	2.790	2.785	2.563(10.4)	(2.480)	3.050	3.031	2.757(39.3)	...
	3.020	2.800(6.7)	3.079	2.871(6.2)	2.803	3.242	3.362	3.127(9.1)	...	3.582	3.672	3.043(68.7)	...
3^-	3.445	3.331(4.8)	3.460	3.332(5.0)	2.267	3.366	3.354	...	2.321	3.277	3.203	3.108(6.2)	2.391
	3.613	3.456(5.0)	3.791	3.601(6.1)	...	3.715	3.888	...	...	3.757	3.926	3.517(66.4)	...
5^-	2.315	2.231(2.6)	2.394	2.301(2.7)	2.364	2.199	2.243	2.133(3.5)	2.319	2.026	2.025	1.919(3.7)	2.285
	2.587	2.499(2.7)	2.663	2.565(3.0)	...	2.379	2.432	2.315(3.5)	...	2.302	2.422	2.270(5.1)	...
6^-	2.544	2.436(3.6)	2.640	2.490(4.7)	2.774	2.313	2.372	...	...	2.222	2.245	...	...
	2.587	2.453(4.2)	2.677	2.544(4.5)	...	2.478	2.543	...	...	2.307	2.424	...	...
7^-	2.319	2.198(3.9)	2.374	2.229(4.7)	2.909	2.122	2.201	2.043(4.7)	2.572	2.148	2.298	2.148(4.7)	2.483
	3.262	3.107(4.5)	3.373	3.189(5.1)	...	3.361	3.466	3.251(6.8)	...	3.394	3.462	3.246(10.2)	...

mention that for the corresponding QTD levels of Table V of Ref. 11 we had $\chi^2=1.369$ and 1.023 MeV if 3_1^- and 7_1^- are excluded; for the corresponding QTD levels of Table VI of Ref. 11, we had $\chi^2=3.221$ and 2.442 MeV if 3_1^- and 7_1^- are excluded.

We turn now to the electromagnetic properties of some of the calculated excited states. Let us concentrate on one isotope, $A=116$. One of the most important observables is the collective $E2$ transition $2_1^+\to 0_1^+$. Since the respective eigenvectors do not contain protons explicitly, we must use the concept of a constant effective charge e_2 or construct the effective $E2$ operator from a microscopic theory.[23,24] Following closely the theory of Ref. 24 [Eq. (11)] and, for consistency, assuming the s.p. energies of Bando,[18] we computed the appropriate effective $E2$ operator (and the corresponding theoretical effective charge matrix); we include in the computation all the s.p. proton levels contained between the magic numbers 8 and 126. Such a large number of subshells involved is, in contrast to the core polarization of the force, necessary here because of the importance of the upper major shell. The elements of this $E2$-operator (and thus of the effective charge matrix) are very close to those of Ref. 23. For example, the elements of the calculated effective charge matrix are only very slightly smaller than those of Table I of Ref. 23. They are clustered in two groups: one with values slightly over 0.6 and one with values between 1.0 and 1.2.

In Table VIII we give the calculated QSTD values of the reduced transition rate $B(E2,\ 2_1^+\to 0_1^+)$ and of the quadrupole moment of the 2_1^+ state, $Q(2_1^+)$. The most recent published experimental values of these two observables appear to be $420\ e^2\ \mathrm{F}^4$,[25] and $+0.40\pm 0.30$ b,[26] respectively. The predictions of Table VIII are too small by a factor of 2–4 for both the observables [$Q(2_1^+)$ is of the correct sign]. This is in contrast to the results of Refs. 23 and 24, where values much closer to observation were reported. This is connected with the fact that the partial contributions of the two-qp–two-qp, two-qp–four-qp, four-qp–two-qp terms (the terms four-qp–four-qp are small) do not all add up coherently with the vectors $|\ 2_1^+\rangle$ and $|\ 0_1^+\rangle$ of the present work. Incidentally, the corresponding QTD values of $B(E2,\ 2_1^+\to 0_1^+)$ are significantly larger than those of Table VIII [$B(E2,\ 2_1^+\to 0_1^+)=138.7\ e^2\ \mathrm{F}^4$ with $e_2(n,n')$], while the QTD values of $Q(2_1^+)$ are smaller. The calculated $B(E2,\ 2_1^+\to 0_1^+)$ for the isotope $A=120$ are still considerably smaller than those of Table VIII, thus in a more dramatic disagreement with experiment. The specific difficulty with $A=120$ may be a direct consequence of the incorrectness of the HF energies of $A=119$ and $A=121$ as predicted by the IGE method (as in Table V).

TABLE VIII. Values of the reduced transition rate $B(E2,\ 2_1^+\to 0_1^+)$ and of $Q(2_1^+)$ of Sn^{116} calculated with $e_2=1$ and with the theoretical effective $E2$ operator. The QSTD (I and II explained in the text) eigenvectors are those of Tables VI and VII; the force is Yale-Shakin.

	$e_2=1$		$e_2(n,n')$	
$A=116$	QSTD (I)	QSTD (II)	QSTD (I)	QSTD (II)
$B(E2,\ 2_1^+\to 0_1^+)$ (in $e^2\mathrm{F}^4$)	196.5	145.0	115.9	85.8
$Q(2_1^+)$ (in b)	+0.05		+0.04	

An interesting observable is the static magnetic moment $\mu(5_1^-)$ of the 5_1^- state. Bodenstaedt *et al.*[27] give the following values of the corresponding g factor:

$$g_{5_1^-}(A=116)=-0.065\pm 0.005,$$

$$g_{5_1^-}(A=120)=-0.058\pm 0.007.$$

If one uses the "bare" $M1$ operator (involving the valence neutrons only) and applies it to our calculated QSTD eigenvectors $|\ 5_1^-\rangle$, one obtains too large a value: $g_{5_1^-}{}^{(\mathrm{bare})}(A=116)=0.121$ and $g_{5_1^-}{}^{(\mathrm{bare})}(A=120)=-0.062$. The inclusion of all the contributions of the diagrams of virtual excitations and deexcitations of the core neutrons and protons leads to very important reductions of $g_{5_1^-}$ in the direction of a better agreement with experiment. Again using the same s.p. basis as in the preceding calculation of the effective $E2$ operator and following closely the method (even the FORTRAN code is the same) of Ref. 24, we compute the total effective $M1$ operator for the valence neutrons. With this the g factors are then recalculated and we find (QSTD):

$$g_{5_1^-}{}^{(\mathrm{theor})}(A=116)=-0.081,$$

$$g_{5_1^-}{}^{(\mathrm{theor})}(A=120)=-0.027.$$

The agreement with experiment is much worse here than in Ref. 24. The absolute value of g for $A=116$ is too large because the theoretical "effective $M1$ reduction matrix" elements of the excitations of the core nucleons are too large (give too little reduction), larger by about 50% than the corresponding elements of Ref. 24. This too small reduction is due to the way (variant $S1$, see Ref. 10) in which we have chosen here to take into account the Pauli principle in the propagators. Due to the particular selection rules of the $M1$ operator this is the only case in which the choice of the propagators is of importance. As for $A=120$, the too small absolute g follows from the properties of the

[23] M. Gmitro, A. Rimini, J. Sawicki, and T. Weber, Phys. Rev. Letters **20**, 1185 (1968).
[24] M. Gmitro, A. Rimini, J. Sawicki, and T. Weber, Phys. Rev. **175**, 1243 (1968).
[25] P. Stelson *et al.*, Phys. Rev. **170**, 1172 (1968).
[26] J. De Boer, J. Phys. Soc. Japan Suppl. **24**, 203 (1968).
[27] E. Bodenstaedt *et al.*, Z. Physik **168**, 370 (1962); Cooperation of the Angular Correlation Groups of Bonn and Hamburg, Nucl. Phys. **89**, 305 (1966).

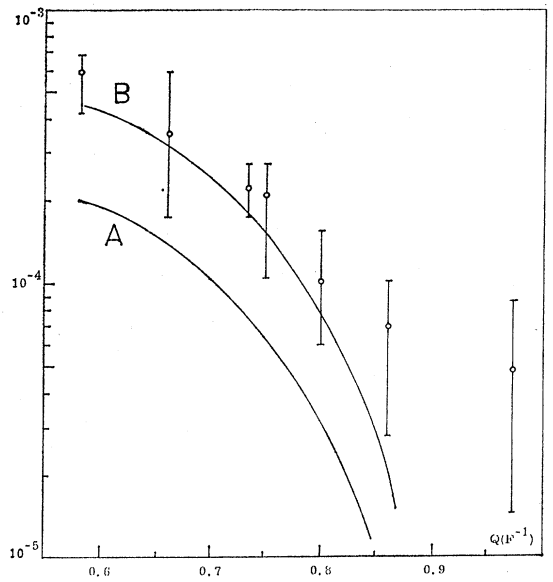

FIG. 1. Inelastic form factor squared $|F_{\text{in}}|^2$, in Born approximation corrected as in Ref. 24, for the reaction $Sn^{116}(e, e')$ $Sn^{116}(2_1^+)$ at 150 MeV. Curve A reproduces the results of the present work for theory QSTD(I). The theoretical effective operator is used. The force is that of Yale-Shakin. Curve B reproduces the corresponding results of Ref. 24. The experimental error bars are taken from Ref. 28.

$|5_1^-\rangle$ vector. Thus $g_{5_1^-}$ is yet another observable badly predicted for $A=120$ by the IIGE method.

Similarly, only poor results are obtained for the inelastic electron scattering from the even isotopes with the excitation of the 2_1^+ and 3_1^- states. The data are those of Barreau and Bellicard[28] and refer to the bombarding electron energy of 150 MeV, and the scattering angle varied between 45° and 80°. The $(e, e')E\lambda$ form

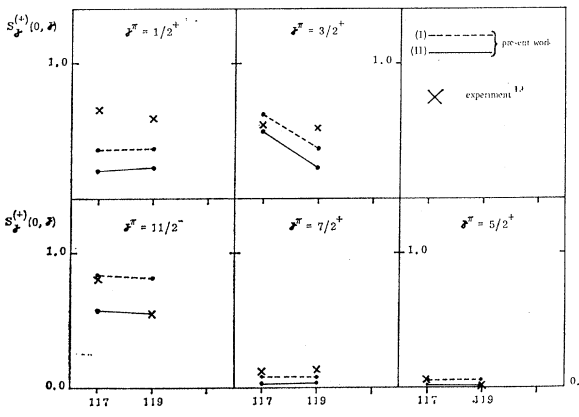

FIG. 2. Spectroscopic factors $S_J^{(+)}(0_1^+, J_1^\pi)$ for (d, p) reactions on the even tin targets with $A=116$ and 118, leading to the lowest state of spin J^π of the odd-mass isotopes. The experimental values (Ref. 20) are indicated as $\times$, the present results are connected by a dashed line (variant I of the QSTD theory) and a solid line (variant II). The nuclear force is that of Yale-Shakin.

[28] P. Barreau and J. B. Bellicard, Phys. Rev. Letters **19**, 1444 (1967).

factors squared are extracted from the differential cross sections as

$$|F_{\text{in}}|^2=\sigma(E_0, \theta)/Z^2\sigma(\text{Mott})\,(Z=1).$$

In Fig. 1 we give the angular distribution of $|F_{\text{in}}|^2$ for the excitation of the 2_1^+ state in Sn^{116} [the reaction $Sn^{116}(e, e')$ $Sn^{116}(2_1^+)$] at 150 MeV. The theoretical curve A was calculated as in Ref. 24 with the theoretical effective $E2$ operator here based on the HF s.p. energies of the present paper and with the QSTD (I) eigenvectors $|0_1^+\rangle$ and $|2_1^+\rangle$. It is compared with the theoretical curve B of Ref. 24 and with the error bars of the data of Ref. 28. The angular distribution of the theoretical curve A is reasonably consistent with the data, but the absolute values are too small by a factor of the order of 2–2.5. The theoretical prediction of Ref.

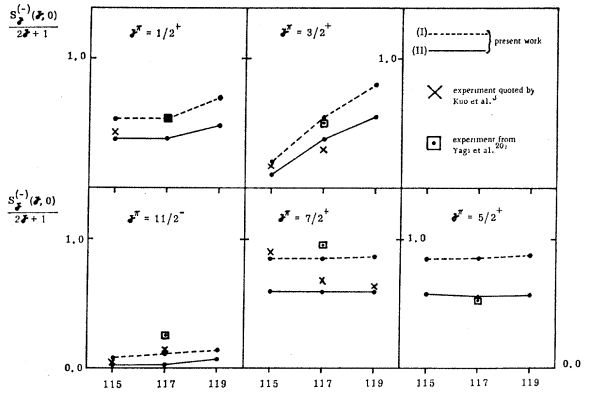

FIG. 3. Spectroscopic factors $S_J^{(-)}(J_1^\pi, 0_1^+)/2J+1$ for pickup reactions on the even tin targets with $A=116$, 118, and 120 leading to the lowest state of spin J^π of the odd-mass isotopes with $A=115$, 117, and 119, respectively. The experimental data of Ref. 21 for $Sn^{118}(p, d)Sn^{117}$ are marked ⊡; the experimental data for (d, t) reactions quoted in Ref. 3 are marked $\times$. Present results are connected by a dashed line (variant I) and by a solid line (variant II). The nuclear force is that of Yale-Shakin.

24 (curve B) is in much better agreement with experiment than the results of the present work. This fact is consistent with the too small values of the $B(E2)$ of Table VIII. Had the constant effective charge $e_2=1$ been assumed, curve A would have been shifted upwards by 35%, still below the experimental error bars.

4. SPECTROSCOPIC FACTORS FOR ONE-NEUTRON TRANSFER REACTIONS

In computing the spectroscopic factors[29] for stripping (d, p) and pickup (p, d), (d, t) reactions both on the even- and the odd-mass tin isotopes, we have used the formulas and the FORTRAN codes of Refs. 5 and 19. In Figs. 2–4 we compare our results with the spectroscopic factors extracted in the experimental works of Refs. 20 and 21. The nuclear force is that of Yale-Shakin but almost equal results were obtained for the

[29] E. Baranger and T. T. S. Kuo, Nucl. Phys. **A97**, 289 (1967).

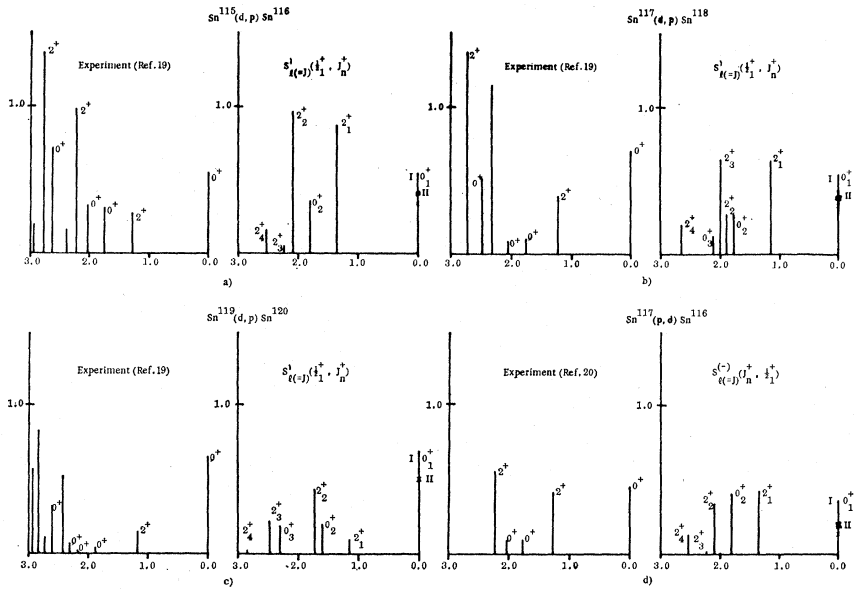

FIG. 4. (a)–(c) Spectroscopic factors $S'_{l(=J)}(\frac{1}{2}_1^+, J_n^+) = \frac{1}{2}(2J+1)\, S_{l(=J)}{}^{(+)}(\frac{1}{2}_1^+, J_n^+)$ for (d, p) reactions on the odd-mass targets with $A=115$, 117, and 119, respectively, with $J^\pi = 0^+$ and 2^+ up to 3-MeV excitation energy. Present results are plotted on the right. Among the experimental data (Ref. 20), plotted on the left, the states not labeled explicitly have an ambiguous (1^+, 2^+, 3^+) spin assignment. The nuclear force is that of Yale-Shakin. (d) Spectroscopic factors $S_{l(=J)}{}^{(-)}(J_n^+, \frac{1}{2}_1^+)$ for the (p, d) reactions on Sn^{117}, leading to the states of Sn^{116} with $J^\pi = 0^+$ and 2^+ up to 3-MeV excitation. Present results are plotted to the right and the experimental values of Ref. 21 to the left. The nuclear force is that of Yale-Shakin.

Tabakin potential. The two variants of the QSTD theory for the eigenvectors of the even isotopes, discussed in Sec. 3 and labeled I and II, are considered. The agreement between the predicted and the "experimental" data are about as good as that found by two of us[19] with the renormalized Tabakin potential of Ref. 10 and s.p. energies of Bando.[18] The results of the variant II are always lower than those of the variant I and in the case of the reactions on even targets the experimental data generally lie between them. Figure 4 shows that the experimental peaks for the lowest 0_1^+ and 2_1^+ final states of the reactions on the odd-mass targets are well reproduced. In particular, for the 0_1^+ states the variant I of the QSTD theory gives better results, the peaks of the variant II always lying too low. This depression of the results with the variant II can be explained by the depletion of the $|\tilde{0}\rangle$ component of the ground-state eigenvectors $|0_1^+\rangle$ as already pointed out in Sec. 3.

5. SPECTROSCOPIC FACTORS FOR TWO-NEUTRON TRANSFER REACTIONS

The two-nucleon transfer reactions have recently received much attention both from the experimental and theoretical sides. The importance of these reactions lies in the natural, simple excitation of levels having two or more nucleons excited, particularly those of collective character. In 1962, Yoshida[30] pointed out the importance of the BCS pairing enhancement of the spectroscopic factors in ground-to-ground transitions in even-even vibrational nuclei where the superfluidity effect is present. Recently, Gyarmati and Sawicki[31] analyzed the problem of the spectroscopic (fractional parentage) factors of the two-nucleon transfer reactions in the framework of the qp theories QTD and QSTD of vibrational superfluid nuclei. These spectroscopic factors turn out to be quantities particularly sensitive to the details of the wave functions (QTD and QSTD eigenvectors). In particular, even small four-qp components of a QSTD eigenvector can lead to important two-qp–four-qp cross terms and significantly modify the two-nucleon transfer amplitudes.

Several experimental groups have studied the double-stripping and double-pickup [(t, p) and (p, t)] reactions in the even tin isotopes.[32] These nuclides do indeed well exemplify the vibrational region, and that is why a detailed analysis of these reactions is important. In the present work we do not attempt to analyze the angular distributions in terms of the DWBA or another theory, nor do we calculate the cross sections or their branching ratios. We merely confine ourselves to a study of the factors G_{NLSJT} defined by Glendenning[33] for states described in terms of harmonic-oscillator s.p. wave functions. The quantum numbers L, S, J, and T denote the orbital, spin, and total angular momenta and

[30] S. Yoshida, Nucl. Phys. **33**, 685 (1962).

[31] B. Gyarmati and J. Sawicki, Phys. Rev. **169**, 966 (1968); Nucl. Phys. **A111**, 609 (1968).

[32] G. Bassani *et al.*, Phys. Rev. **139**, B830 (1965); G. E. Holland *et al.*, Bull. Am. Phys. Soc. **12**, 19 (1967); J. H. Bjerregaard *et al.*, Nucl. Phys. **A110**, 1 (1968).

[33] N. K. Glendenning, Phys. Rev. **137**, B102 (1965).

TABLE IX. Spectroscopic factors G_{JN} for the reaction $Sn^{118}(p, t)Sn^{116}(J_n^\pi)$ calculated with the QTD and QSTD (I and II) eigenvectors of Tables VI and VII of the Yale-Shakin force.

	$0_1^+ \to 0_1^+$			$0_1^+ \to 0_2^+$		$0_1^+ \to 2_1^+$			$0_1^+ \to 4_1^+$	$0_1^+ \to 5_1^-$
N	QTD	QSTD (I)	QSTD (II)	QSTD (I)	QSTD (II)	QTD	QSTD (I)	QSTD (II)	QSTD (II)	QSTD (II)
0	0.0044	0.0041	0.0029	0.0017	0.0029	0.0062	0.0058	0.0042	0.0117	−0.00003
1	−0.0119	−0.0110	−0.0079	0.0007	−0.0079	−0.0261	−0.0250	−0.0195	−0.0873	−0.0361
2	0.0344	0.0305	0.0214	0.0164	0.0214	0.1017	0.0983	0.0768	0.5541	0.0925
3	−0.1599	−0.1464	−0.1053	0.0055	−0.1053	−0.6484	−0.6198	−0.4715	−0.0027	
4	0.6396	0.5972	0.4188	0.2354	0.4188	0.0031	0.0037	0.0016		
5	−0.0392	−0.0351	−0.0240	−0.0065	−0.0240					

the isospin of the transferred pair, respectively. The branching ratios and the absolute values of the cross sections are essentially determined by these G factors. They are specific superpositions of the actual fractional parentage factors of all the possible configurations of the two s.p. states of the two nucleons to be transferred.

In computing the G factors we have used the formulas and the FORTRAN codes of Ref. 31. We confine ourselves to the variant of the theory which takes into account the differences between the qp vacuum of the nucleus A and that of the nucleus $A\pm2$ (described as the case with $\Delta u \neq 0$ in Ref. 31). In fact, neglecting this effect leads to serious errors. In the case of (t, p) or (p, t) reactions we have $S=0$, $(L=J)$.

In Table IX we present some of our typical results for the factors G_{NJ}, $N=0, 1, 2, \cdots$, for the reaction $Sn^{118}(p, t)$ $Sn^{116}(J_n^\pi)$ calculated with the eigenvectors of Sec. 3 corresponding to the Yale-Shakin force. Since the characteristic phase factor with which G_{NJ} contributes to the forward scattering amplitude is $(-)^N$, it is those states for which all the G_{NJ} with even N are of sign opposite to all the G_{NJ} with odd N which are really strongly excited. In fact, it is the case with the $0_1^+ \to 0_1^+$ transition and with all the collective states. As stated above, the differences between the G_{NJ} of QTD and QSTD (II) are rather important (smaller G for QSTD), the G of QSTD (I) are intermediate between the two (more similar to those of QTD). Similar results were obtained for the Tabakin potential. For the latter force we found G_{NJ} only somewhat (by 5–30%) smaller on the average except for the $0_1^+ \to 0_2^+$ transition for which some of the G_N are reduced by about a factor of 2 in relation to those of Table IX.

By comparing the G_{NJ} factors of the present work with those of Ref. 31 we note that although they all have the same general qualitative and even semi-quantitative features, they differ considerably in detail. Unfortunately, we have as yet no DWBA or other results on the angular distributions or the cross sections in order to be able to examine critically the consequences of these differences. Some related DWBA calculations are now in preparation by Glendenning.[34] What we can say at the moment is that the general aspect, the coherence, and the order of magnitude of our G_{NJ} of Table IX are quite reasonable.

6. CONCLUSIONS AND FINAL REMARKS

The method of the IGE provides a unique determination of the s.p. HF energies of the valence nucleons. The fundamental assumption of the method is the interpretation of the appropriate energy levels of the odd-mass isotopes of the single closed-shell vibrational nuclei as simple pure one-qp excitations or, in the best case, such excitations corrected for some residual qp excitations. This assumption constitutes one of the weaknesses of the method. While a statistical-mechanical description such as the BCS model seems to be a reasonable approximation for the even isotopes where all the nucleons are paired off, it may be too crude for the neighboring odd isotopes of the same elements. In fact, the spectra of odd isotopes are generally rather delicate; i.e., sensitive to details of the nuclear force assumed. For example, if the excitations of the core nucleons are taken into account only via a core-polarization renormalization of the nuclear force, the IGE results may be poor owing to an incorrect treatment of the core parts (e.g., the s.p. energies of the core). The nucleon-number nonconservation and the related spuriousness may influence any IGE results significantly. One must at least minimize the fluctuation of the nucleon-number expectation value in the ground state. In the IIGE results of the present paper the number fluctuations δN are quite small ($<8\%$). A warning should be given here that all the conclusions are dependent on the uncertainties of the input data (e.g., the choice of the force, odd-even mass difference, observed levels with assigned spin and parity of odd-mass nuclei). This point has been demonstrated explicitly by one of

[34] D. G. Fleming, J. Cerny, and N. K. Glendenning, Phys. Rev. **165**, 1153 (1968).

us in Ref. 15 for the case of Ni isotopes. This feature is not revealed in the present paper; it constitutes one of its general weaknesses.

The results of the present work seem to be, at least in part, disappointing. They are limited to only two types of realistic nuclear forces. These forces are, however, very reasonable and, in a number of previous analyses of the structure of the Sn isotopes, have given satisfactory results.

For the odd isotopes the first levels of each J^π are brought to agreement with experiment by definition of the method. The second levels of the same J^π are essentially pure three-qp states (with very small one-qp components). The positions of the levels $(\frac{5}{2}^+)_{2,3}$ are in very poor agreement with the available experimental data. This may be due to the incorrectness of the QTD13 theory for describing any but the lowest-lying J^π levels; or it may be due to the incorrectness of the HF s.p. energies resulting from possibly inappropriate input data of the IGE method. The disagreement of the $(\frac{5}{2}^+)_{2,3}$ levels of QTD13 with the data is less marked for $A=119$ and 121.

The level positions of the first few low-lying even-parity states of the even isotopes are in remarkably good agreement with experiment except for the isotope 120. The QSTD eigenvectors have only small four-qp components and the pure two-qp approximation of QTD is reasonable except for calculating some "delicate" observables. The odd-parity states are, for all the even isotopes considered, in rather sharp disagreement with the available data. This is probably due mainly to the incorrectness of the HF energy of the $1h_{11/2}$ orbital following from the IGE method and the $(\frac{1}{2}^-)_1$ experimental levels.

The lack of coherence of all the two- and four-qp components of the QSTD eigenvectors involved is mainly responsible for the too small predicted transition rates $B(E2, 2_1^+ \rightarrow 0_1^+)$ and the 2_1^+ quadrupole moments, $Q(2_1^+)$. The disagreement with the observed $B(E2)$ is particularly bad for $A=120$. The specificity of $A=120$ is connected with a probably incorrect HF s.p. level sequence, in this case as predicted by IGE. The magnetic moment of the 5_1^- level is in tolerable agreement with experiment for $A=116$ and in poor agreement for $A=120$. The inelastic electron scattering form factors for the excitations of the 2_1^+ and 3_1^- states at 150-MeV bombarding electron energy are in poor agreement with the data. The spectroscopic factors for the one- and two-neutron transfer reactions are not at all inconsistent with the existing data. There are serious uncertainties in the extraction of the "experimental" spectroscopic factors. On the other hand, careful DWBA and other calculations of the cross sections and of the angular distributions are necessary before we can really critically evaluate the correctness of our predictions for the spectroscopic factors. The present study has been concerned with many aspects of several types of states of a number of odd and even isotopes of one element. We have seen how dangerous it is to draw general conclusions about the value of a theory based on only partial results. Indeed, while some properties of a given nuclear structure can be well reproduced, others may be in sharp disagreement with the data.

In summing up we may say that the present results are a warning against too much optimism about the indiscriminate applicability of the IGE method. It appears that, at least in some cases, the method demands too much and is a crude oversimplification of the physical reality due to its very rigid conditions, particularly suspect for the odd isotopes of single closed-shell vibrational nuclei. An explanation of the failures of the method could be sought in a stable deformation of the HF field (the present calculations are based on the assumption of a spherical nuclear s.p. basis). However, the above-mentioned criticisms, on the one hand, and the success of other non-IGE calculations based on a spherical basis (cf. Refs. 10, 11, 23, and 24) on the other, indicate that we are not yet obliged to abandon the spherical picture of the average nuclear field for tin and similar nuclei.

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