Inelastic Scattering Calculations with Projected Hartree-Fock Wave Functions

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The 0^+-2^+ cross sections for inelastic proton scattering have been calculated for several target nuclei in the $2s$-$1d$ shell. Inelastic cross sections are calculated in the distorted-wave Born approximation. Hartree-Fock wave functions projected onto states of good angular momentum represent the initial and final states of the nucleus. The cross sections are generally correct in shape, but too small in magnitude; it is suggested that the latter feature results from the assumption of an inert core. To test this hypothesis, the Ne^{20} cross section is recalculated using a 20-particle wave function. The result compares more favorably with experiment.

1. INTRODUCTION

A MAJOR objective of any theory of nuclear reactions is to relate the observed fluctuations in cross sections to the detailed structure of the many-body system composed of the target nucleus plus the projectile. Heretofore, most reaction studies have made use of only relatively simple nuclear wave functions, namely, those based on the spherical shell model or the macroscopic collective model. The use of these models in reaction studies has been discussed exhaustively in the literature.[1] Although the results so obtained have been fairly successful, it is well known that such primitive nuclear wave functions are inadequate for nuclear-structure calculations. This raises considerable doubt as to their reliability for the scattering calculations. The point has been emphasized by Glendenning and Veneroni, who, in a study of inelastic proton scattering from the even nickel isotopes, found that inelastic scattering is rather sensitive to the details of a microscopic description of the target.[2] Satchler[3] has given a discussion of inelastic scattering based on the shell-model description of nuclei situated near closed shells; reaction studies which have been done since, namely, $A(p, p')A^*$ and $A(p, n)B$, indicate that a more detailed description of the nuclear form factor is required.

The macroscopic model has served a useful purpose in the study of inelastic scattering from deformed nuclei. It enables one to determine the multipolarity of the transition under consideration and provides a small amount of information about nuclear structure—namely, the deformation parameter. While such a model is convenient from a practical point of view, one would nevertheless prefer a description of such processes in terms of detailed microscopic wave functions such as would be used in structure calculations. The use of such wave functions enables one to study effects which are difficult or impossible to analyze if the macroscopic model is used. For example, the use of many-particle wave functions makes it possible to study exchange effects between the projectile and target nucleons, and it becomes relatively easy to study scattering to unnatural parity states, which could be excited via spin flip if nucleons are used as projectiles.

A problem which does not arise when the macroscopic model is used in distorted-wave Born-approximation (DWBA) reaction calculations is that of choosing an effective interaction. Instead, it is assumed that the inelastic processes result from nonspherical components in the optical potential. A Taylor expansion of the optical potential in terms of the deformation parameter is made and the first-order term in the expansion is retained as the interaction which gives rise to inelastic scattering.[4] In the microscopic description, the choice of effective interaction is not so simple. A sum of effective two-body forces between the projectile and the target nucleon will be assumed in this paper. The justification of such a choice will only be apparent after the results of detailed calculations are compared with experiment. As Satchler has suggested, the use of such an effective interaction assumes that multiple scattering is not important; however, if one uses phenomenological optical potentials to describe the distorted waves then multiple scattering is neglected only in the off-diagonal parts of the scattering matrix. There is little that can be said about the analytic form of the effective interaction since it cannot, as yet, be calculated. The nucleon-nucleon interaction may be chosen to be either the two-nucleon t matrix, or a spin-dependent phenomenological interaction. The nucleon-α or nucleon-deuteron t matrix may be chosen in a similar fashion. It is suggested, however, that for the case of nucleon-nucleus scattering the phenomenological projectile-target nucleon interaction (for intermediate energies, 10–50 MeV) should not be radically different from the residual two-body force in the structure problem since the interaction occurs in the field of nuclear matter.

In recent years, there has been considerable progress in our understanding of the underlying microscopic structure of deformed nuclei. The Hartree-Fock (HF) method of deformed orbitals[4a] has played a major role in

[1] N. K. Glendenning, Phys. Rev. **114**, 1297 (1959); R. H. Bassel *et al.*, *ibid.* **128**, 2693 (1962); K. A. Amos, V. A. Madsen, and I. E. McCarthy, Nucl. Phys. **A94**, 103 (1967).
[2] N. K. Glendenning and M. Veneroni, Phys. Rev. **144**, 839 (1966).
[3] G. R. Satchler, Nucl. Phys. **77**, 481 (1966).
[4] E. Rost and N. Austern, Phys. Rev. **120**, 1375 (1960).
[4a] F. Villars, in *Proceedings of the International School of Physics "Enrico Fermi", Course XXIII, Nuclear Physics* edited by V. F. Weisskopf, (Academic Press Inc., New York, 1963); G. Ripka, in *Advances in Nuclear Physics* edited by M. Baranger and E. Vogt (Plenum Publishing Corp., New York, 1968), Vol. 1.

clarifying many of the ideas which produced a detailed description of these nuclei. The HF method has been applied to several nuclei in the $(1p, 2s\text{-}1d)$ shells since many of these nuclei are believed to have deformed ground-state equilibrium shapes. An advantage of working in this region of the Periodic Table is that the number of particles is sufficiently small that it is possible to carry out calculations in which the usual closed-shell or inert-core assumption is dropped.

The major objective of this paper is to formulate a microscopic theory of inelastic scattering from deformed nuclei using projected HF wave functions and then to apply the theory to inelastic scattering of protons from the even-even, $N=Z$, nuclei in the $2s\text{-}1d$ shell. The nuclear wave functions for the target and residual nucleus are obtained by projecting states of good J and M from the "intrinsic" HF state. These wave functions are then used to calculate nuclear form factors for reaction studies.

In Sec. 2, the transition amplitude is discussed and nuclear form factors are defined; detailed expressions are given for the nuclear form factors for the case in which the nuclear states are described by projected HF wave functions. The results of DWBA calculations of inelastic cross sections are presented in Sec. 3, and some of the details of the calculations are discussed.

2. TRANSITION AMPLITUDE AND FORM FACTORS

A. General Formulation

Under the assumption that the projectile-target interaction can be written as a sum of two-body interactions, the DWBA transition amplitude for inelastic scattering from a target containing A nucleons is

$$T_{if}=A\langle\psi_f^{(-)}(0)\Psi_f(1\cdots A)\mid t(0,1)\mid\Psi_i(1\cdots A)\psi_i^{(+)}(0)\rangle. \tag{1}$$

The initial and final nuclear states are represented by Ψ_i and Ψ_f, and the initial and final projectile states by the distorted waves $\psi_i^{(+)}$ and $\psi_f^{(-)}$. We assume that integration over the internal coordinates of the projectile has already been carried out, so that 0 stands for the spin, isospin, and c.m. coordinates of the projectile. The projectile-nucleon interaction is represented by $t(0,1)$; it may be the t matrix, for instance, or a phenomenological pseudopotential. For nucleon-nucleus scattering, the major exchange term may be included simply by writing

$$t(0,1)\,(1-P_{01}{}^H)$$

in place of $t(0,1)$, where $P_{01}{}^H$ is the (Heisenberg) operator which exchanges all the coordinates of particles 0 and 1.

Because of the assumption that the projectile-target interaction is two-body in character, it is possible to express the transition amplitude in terms of single-particle transition amplitudes. We begin by expanding the initial and final nuclear states in a complete set $\{\varphi_{jm}\}$ of single-particle wave functions, where j stands for all the quantum numbers $(nlj\tau)$ except for the z component of angular momentum, which is denoted by m. (Often, however, we shall write a for the set j_am_a, the set j_a, or even the set $n_al_a\tau_a$ when no confusion will result.) Thus

$$\Psi_i(1\cdots A)=A^{-1/2}\sum_a\varphi_a(1)\Psi_{ia}(2\cdots A), \tag{2a}$$

$$\Psi_f(1\cdots A)=A^{-1/2}\sum_b\varphi_b(1)\Psi_{fb}(2\cdots A). \tag{2b}$$

The scattering amplitude then becomes

$$T_{if}=\sum_{ab}\langle\Psi_{fb}\mid\Psi_{ia}\rangle T_{ab}, \tag{3}$$

where the single-particle transition amplitude T_{ab} is given by

$$T_{ab}=\langle\psi_f^{(-)}(0)\varphi_b(1)\mid t(0,1)\mid\varphi_a(1)\psi_i^{(+)}(0)\rangle. \tag{4}$$

It is convenient to express both T_{if} and T_{ab} as sums of amplitudes for transfer of total angular momentum (J,M),

$$T_{if}=\sum_{JM}\langle J_iM_i, JM\mid J_fM_f\rangle T_{if}{}^{JM}, \tag{5a}$$

$$T_{ab}=\sum_{JM}\langle j_am_a, JM\mid j_bm_b\rangle T_{ab}{}^{JM}, \tag{5b}$$

and also to express Φ_{ia} and Φ_{fb} in terms of angular momentum eigenstates

$$\Psi_{ia}=\sum_{J_AM_A}\langle J_AM_A, j_am_a\mid J_iM_i\rangle\Psi_{ia}{}^{J_AM_A}, \tag{6a}$$

$$\Psi_{fb}=\sum_{J_BM_B}\langle J_BM_B, j_bm_b\mid J_fM_f\rangle\Psi_{fb}{}^{J_BM_B}. \tag{6b}$$

Then the relationship

$$T_{if}{}^{JM}=\sum_{ab}S_J(if\mid ab)\,T_{ab}{}^{JM} \tag{7}$$

holds, where the spectroscopic amplitude S_J is given by

$$S_J(if\mid ab)=\sum_{J_C}U(j_aJ_CJJ_f\mid J_ij_b)\,\langle\Psi_{fb}{}^{J_C}\mid\mid\Psi_{ia}{}^{J_C}\rangle. \tag{8}$$

It is possible to decompose T^{JM} still further into amplitudes for transfer of definite L and S

$$T_{if}{}^{JM}=\sum_{LM_L;SM_S}\langle s_f\mu_f, SM_S\mid s_i\mu_i\rangle\times\langle LM_L, SM_S\mid JM\rangle T_{if}{}^{LSJM_L}, \tag{9}$$

$$T_{if}{}^{LSJM_L}=\sum_{ab}S_J(if\mid ab)\,T_{ab}{}^{LSJM_L}. \tag{10}$$

Here $s_i=s_f$ is the spin of the projectile, and μ_i and μ_f are its initial and final projections upon the z axis. The sum over initial and final states of the entire system

may easily be carried out to yield

$$\left(\frac{d\sigma}{d\Omega}\right)_{if}=\left(\frac{m}{2\pi\hbar^2}\right)^2\frac{k_f}{k_i}\frac{2J_f+1}{2J_i+1}\sum_{LSJ}\frac{\sigma_{LSJ}}{(2L+1)(2S+1)}, \quad (11)$$

$$\sigma_{LSJ}=\sum_{M_L}|T_{if}{}^{LSJM_L}|^2. \quad (12)$$

With Eq. (7), the first step of our program is complete, namely, to separate details of the reaction mechanism (given by T_{ab}) from details of the nuclear structure (given by S_J). The single-partical transition amplitude is of secondary concern to us; it may be evaluated in a number of ways,[5] depending on the specific forms of $t(0,1)$ and the distorted waves. For example, if $t(0,1)$ is central and spin-dependent, it may be written

$$t(0,1)=\sum_{LSJM}(-1)^{L+S+J+M}t_{LSJ}(r_0,r_1)\times\mathfrak{Y}_{LSJ}{}^{-M}(0)\mathfrak{Y}_{LSJ}{}^{M}(1), \quad (13)$$

where the spin-angle tensor operator $\mathfrak{Y}_{LSJ}{}^{M}$ is defined by

$$\mathfrak{Y}_{LSJ}{}^{M}=\sum_{M_LM_S}\langle LM_L,SM_S\,|\,JM\rangle Y_L{}^{M_L}(\hat{r})\sigma_S{}^{M_S}.$$

The operator $\sigma_S{}^{M_S}$ is a rank-s tensor in the spin space of the particle; for spin-$\frac{1}{2}$ particles we take $\sigma_0=1$ and $\sigma_1=2\mathbf{S}$, where $\mathbf{S}$ is the spin operator. The distorted waves for the projectile are of the form

$$\psi_{\mathbf{k}}{}^{(+)}(\mathbf{r})=\sum_{lj}\psi_{lj}{}^{(+)}(k,r)\times\sum_{m_lm}\langle lm_l,s\mu\,|\,jm\rangle Y_l{}^{m_l}(\hat{k})^*\mathcal{Y}_{lsj}{}^{m}; \quad (14)$$

the spin-angle function $\mathcal{Y}_{lsj}{}^{m}$ may be expressed in terms of the spin states $\chi_s{}^{\mu}$ of the projectile as

$$\mathcal{Y}_{lsj}{}^{m}=\sum_{m_l\mu}\langle lm_l,s\mu\,|\,jm\rangle Y_l{}^{m_l}(\hat{r})\chi_s{}^{\mu}.$$

The various angular integrations and spin summations may be carried out using the techniques of Racah algebra, with the result

$$T_{ab}{}^{LSJM_L}=\sum_{l_im_i}\sum_{l_fm_f}\langle l_fm_f,LM_L\,|\,l_im_i\rangle\times Y_{l_i}{}^{m_i}(\hat{k}_i)^*Y_{l_f}{}^{m_f}(\hat{k}_f)\sum_{j_ij_f}\sum_{L'S'}Q_{LSJ}{}^{if}\langle j_i\,||\,\mathfrak{Y}_{L'S'J}\,||\,j_f\rangle\times\int_0^\infty\psi_{l_fj_f}{}^{(-)}(k_f,r)^*f_{L'S'J}{}^{ab}(r)\psi_{l_ij_i}{}^{(+)}(k_i,r)r^2dr, \quad (15)$$

where[6]

$$Q_{LSJ}{}^{if}\equiv(\hat{j}_i{}^2\hat{j}_f{}^2\hat{L}^2\hat{S}^2/\hat{l}_i\hat{s}_i\hat{j}_f\hat{J})\begin{Bmatrix}l_i & s_i & j_i\\ l_f & s_f & j_f\\ L & S & J\end{Bmatrix},$$

$$\langle j_i\,||\,\mathfrak{Y}_{LSJ}\,||\,j_f\rangle=(\hat{J}^2/\hat{L}^2\hat{S}^2)(l_i{}^2\hat{s}_i{}^2/\hat{j}_i{}^2)\langle s_i\,||\,\sigma_S\,||\,s_f\rangle\times\langle l_i\,||\,Y_L\,||\,l_f\rangle Q_{LSJ}{}^{if},$$

and the single-particle form factor $f_{LSJ}{}^{ab}$ is given by

$$f_{LSJ}{}^{ab}(r_0)=\langle j_b\,||\,\mathfrak{Y}_{LSJ}\,||\,j_a\rangle\times\int_0^\infty\varphi_b{}^*(r_1)t_{LSJ}(r_0,r_1)\varphi_a(r_1)r_1{}^2dr_1. \quad (16)$$

It is convenient to define a nuclear form factor $f_{LSJ}{}^{if}(r_0)$ by

$$f_{LSJ}{}^{if}(r_0)\equiv\sum_{ab}S_J(if\,|\,ab)f_{LSJ}{}^{ab}(r_0); \quad (17)$$

then one has an equation for $T_{if}{}^{LSJM_L}$ identical to Eq. (15) except that $f_{LSJ}{}^{ab}(r_0)$ is replaced by $f_{LSJ}{}^{if}(r_0)$.

The expressions given above are not commonly used in scattering theory, although Satchler[7] does refer briefly to them. We find the calculations easier to program in this representation, and a convenient check on the case with no spin-orbit coupling is provided by the fact that

$$\sum_{j_ij_f}Q_{LSJ}{}^{if}\langle j_i\,||\,\mathfrak{Y}_{L'S'J}\,||\,j_f\rangle=\langle s_i\,||\,\sigma_S\,||\,s_f\rangle\times\langle l_i\,||\,Y_L\,||\,l_f\rangle\delta_{LL'}\delta_{SS'}.$$

The fact that in general L and S may be different from L' and S' (which are the angular momenta transferred to the struck nucleus) simply reflects the fact that spin flip may also occur before and after the collision as a result of the spin-orbit interaction.

If the two-body interaction $t(0,1)$ is isospin-dependent, $t_{LSJ}(r_0,r_1)$ will be a function of the isospin coordinates of the projectile and bound nucleon. One could introduce a general formalism for handling isospin at this point. However, when HF wave functions are used, there occurs a natural separation of the spectroscopic amplitude S_J into a neutron part and a proton part. This will be discussed in Sec. 2 B.

B. Evaluation of Spectroscopic Factor Using Projected HF Wave Functions

We turn now to our primary concern, evaluation of S_J using HF wave functions. As is well known, the HF method consists of approximating the nuclear wave function by a single Slater determinant,

$$\Phi=(A!)^{-1/2}\det\{\varphi_\lambda\}, \quad (18)$$

where $\{\varphi_\lambda\}$ is a set of single-particle wave functions, or orbitals. These functions are determined by the requirement that Φ minimize the expectation value of the nuclear Hamiltonian H, which leads to the coupled eigenvalue equations

$$h\varphi_\lambda=\epsilon_\lambda\varphi_\lambda, \quad (19)$$

where

$$h(i)=\frac{p_i{}^2}{2m}+\sum_\mu\int d\tau_j\,\varphi_\mu{}^+(j)v(i,j)(1-P_{ij}{}^H)\varphi_\mu(j), \quad (20)$$

[5] G. R. Satchler, Nucl. Phys. **A95**, 1 (1967).

[6] Symbol $\hat{x}$ stands for $(2x+1)^{1/2}$, and $\{\ \}$ is a standard 9-j symbol; see Ref. 19.

[7] G. H. Satchler, Nucl. Phys. **55**, 1 (1964).

with the μ sum restricted to those orbitals used to construct Φ.

Generally speaking, the determinant Φ is not an eigenstate of J^2 or J_Z, but this deficiency is easily remedied. Since the rotation operator $R(\Omega)$ commutes with the nuclear Hamiltonian, the function $R(\Omega)\Phi$ yields the same expectation value of H as Φ does. This "degeneracy" suggests that we use a linear combination of the functions $R(\Omega)\Phi$, weighted so as to produce an eigenstate of J^2 and J_Z. One finds that the appropriate combination is

$$\Psi_{JM}=\frac{2J+1}{8\pi^2}\sum_K\int d\Omega\, D_{MK}{}^J(\Omega)^*R(\Omega)\Phi, \quad (21)$$

which we write symbolically as

$$\Psi_{JM}=P_M{}^J\Phi=\sum_K P_{MK}{}^J\Phi. \quad (22)$$

In general, the state Ψ_{JM} yields a higher expectation value of H than Φ does, because of the cross terms, but one hopes the discrepancy is not large. Calculations in the *s-d* shell confirm this hope, at least for low-lying nuclear levels.[8] A better procedure would be to determine the set $\{\varphi_\lambda\}$ by minimizing $\langle\Psi\mid H\mid\Psi\rangle$ directly, but this is a formidable task and until recently[9] has not been attempted.

The nucleus is often represented as N nucleons outside a closed-shell core, and the core orbitals are not varied in the HF procedure. In that case Eq. (20) becomes

$$h(i)=h_c(i)+\sum_\mu{}'\int d\tau_j\,\varphi_\mu{}^+(j)v(i,j)(1-P_{ij}{}^H)\varphi_\mu(j), \quad (23)$$

where $h_c(i)$ is the Hamiltonian [Eq. (20)] for the core orbitals and the sum over μ is limited to the extra-core orbitals in Φ. Usually h_c is not dealt with directly; instead, its eigenfunctions are assumed to be shell-model orbitals (harmonic-oscillator functions, for instance), and its eigenvalues are taken from experiment. As before, eigenstates of J^2 and J_z are obtained using Eq. (21), but because of the closed-shell core a simplification occurs. The determinant Φ may be written as the antisymmetrized product of two determinants, one of which represents the core nucleons. This latter determinant has $J^2=0$ and is thus invariant under rotations. Hence, the net effect of the rotation operator in Eq. (21) is to rotate only the extracore orbitals in Φ.

Whatever method is used to determine $\{\varphi_\lambda\}$, we assume that the nuclear wave functions are single Slater determinants projected onto states of good J and M by means of Eq. (22). In most of the calculations, we assume the existence of a closed-shell core, identical in initial and final states; however, in our final results the core can always be taken to contain no particles. Finally, we assume that the set $\{\varphi_\lambda\}$ has been made orthonormal, since any nonorthogonal components would make no contribution to the Slater determinant.

Our first task is to show that the summation over single-particles in Eq. (3), and consequently in Eq. (7), is restricted to states outside the core. This is easiest to show in the notation of second quantization, using fermion creation and annihilation operators, for then the overlap integral in Eq. (3) may be written

$$\langle\Psi_{fb}\mid\Psi_{ia}\rangle=\langle\Psi_f\mid\eta_b{}^+\eta_a\mid\Psi_i\rangle$$
$$=\delta_{ab}\langle\Psi_f\mid\Psi_i\rangle-\langle\Psi_f\mid\eta_a\eta_b{}^+\mid\Psi_i\rangle. \quad (24)$$

The first term on the right side of Eq. (24) vanishes for inelastic transitions, since the initial and final nuclear states are orthogonal. The second term will also vanish, if either a or b refers to a core state, because the core has been unaffected by the projection procedures of Eq. (22) (except for antisymmetrization) and is consequently still filled.

We next turn to evaluation of the spectroscopic amplitude S_J. If the nucleus is not spherical, the orbits φ_λ will, in general, be of the form

$$\varphi_\lambda=\sum_{jm}C_{jm}{}^\lambda\varphi_{jm}, \quad (25)$$

where $\{\varphi_{jm}\}$ is the complete set used earlier. Consequently, an expansion of the determinant Φ leads to

$$\Phi=A^{-1/2}\sum_{\lambda jm}C_{jm}{}^\lambda\varphi_{jm}(1)\Phi_\lambda(-1), \quad (26)$$

where $\Phi_\lambda(-1)$ is the cofactor of $\varphi_\lambda(1)$ in Φ and is itself a normalized Slater determinant for $A-1$ nucleons.

The projection operator $P_{MK}{}^J$ must now be applied to Φ, and for this purpose we need a theorem concerning it. From the definition of $P_{MK}{}^J$, it may be shown that $P_{MK}{}^J$ picks out states whose J^2 and J_z eigenvalues are $J(J+1)$ and K, respectively, and then changes the J_z eigenvalue from K to M. Consequently, $P_{MK}{}^J$ transforms under rotation like an irreducible tensor A_{JM}. The product of another irreducible tensor A_{jm} with $P_{M'K'}{}^{J'}$ may therefore be written

$$A_{jm}P_{M'K'}{}^{J'}=\sum_{J_1M_1}\langle J'M',jm\mid J_1M_1\rangle B_{J_1M_1}(j;J'K') \quad (27)$$

and application of $P_{MK}{}^J$ then yields

$$P_{MK}{}^JA_{jm}P_{M'K'}{}^{J'}=\langle J'M',jm\mid JK\rangle B_{JM}(j;J'K')$$
$$=\langle J'M',jm\mid JK\rangle\sum_{m'\mu}\langle J'\mu,jm'\mid JM\rangle$$
$$\times A_{jm'}P_{\mu K'}{}^{J'} \quad (28)$$

(note that B_{JK} has been singled out and then trans-

[8] W. H. Bassichis, B. Giraud, and G. Ripka, Phys. Rev. Letters **16**, 980 (1965); M. R. Gunye and C. S. Warke, Phys. Rev. **156**, 1087 (1967).

[9] M. Bouten, P. Van Leuven, and H. Depuydt, Nucl. Phys. **A94**, 687 (1967); L. Satpathy and S. C. K. Nair, Phys. Letters **26B**, 716 (1968).

formed to B_{JM}). Setting $K'=M'$ and summing both sides of Eq. (28) over all J' and M' results in

$$P_{MK}{}^{J}A_{jm}=\sum_{m'}A_{jm'}\sum_{J'M'\mu}\langle J'M',jm\,|\,JK\rangle \times\langle J'\mu,jm'\,|\,JM\rangle P_{\mu M'}{}^{J'}, \quad (29)$$

which is the desired theorem.

Using Eqs. (26) and (29) to construct the nuclear wave function Ψ_{JM} yields

$$\Psi_{JM}=\sum_{K}P_{MK}{}^{J}\Phi =A^{-1/2}\sum_{jm}\varphi_{jm}(1)\sum_{J'\mu}\langle J'\mu,jm\,|\,JM\rangle\sum_{\lambda Km'M'}C_{jm'}{}^{\lambda} \times\langle J'M',jm'\,|\,JK\rangle P_{\mu M'}{}^{J'}\Phi_{\lambda}(-1). \quad (30)$$

From the definitions of $\Psi_{ia}{}^{J_AM_A}$ and $\Psi_{fb}{}^{J_BM_B}$, given by Eqs. (2) and (6), it follows by comparison with Eq. (30) that

$$\Psi_{ia}{}^{J_AM_A}=\sum_{\lambda_1M'_A;m_aK_i}C_a{}^{\lambda_1}\langle J_AM_A',j_am_a\,|\,J_iK_i\rangle \times P_{M_AM_A'}{}^{J_A}\Phi_{i\lambda_1}(-1), \quad (31a)$$

$$\Psi_{fb}{}^{J_BM_B}=\sum_{\lambda_2M'_B;m_aK_f}C_b{}^{\lambda_2}\langle J_BM_B',j_bm_b\,|\,J_fK_f\rangle \times P_{M_BM_B'}{}^{J_B}\Phi_{f\lambda_2}(-1). \quad (31b)$$

The overlap integral needed for evaluation of S_J is therefore given by

$$\langle\Psi_{fb}{}^{J_C}\,||\,\Phi_{ia}{}^{J_C}\rangle=\sum_{\lambda_1\lambda_2}\sum_{m_am_b}(C_b{}^{\lambda_2})^{*}C_a{}^{\lambda_1} \times\sum_{M_AM_B,K_iK_f}\langle J_CM_A,j_am_a\,|\,J_iK_i\rangle\langle J_CM_B,j_bm_b\,|\,J_fK_f\rangle \times\langle\Phi_{f\lambda_2}\,|\,P_{M_BM_A}{}^{J_C}\,|\,\Phi_{i\lambda_1}\rangle. \quad (32)$$

For simplicity, the normalization of the initial and final nuclear states has been ignored in the derivation of Eq. (32). It is clear, however, that

$$\sum_{K}P_{MK}{}^{J}\Phi$$

will not be normalized to unity, even if Φ is, and Eq. (32) should therefore be multiplied by the normalization factor

$$N=(N_iN_f)^{-1/2}, \quad (33)$$

where

$$N_i=\langle\Psi_i\,|\,\Psi_i\rangle=\sum_{KK'}\langle\Phi_i\,|\,P_{KK'}{}^{J_i}\,|\,\Phi_i\rangle, \quad (34a)$$

$$N_f=\langle\Psi_f\,|\,\Psi_f\rangle=\sum_{KK'}\langle\Phi_f\,|\,P_{KK'}{}^{J_f}\,|\,\Phi_f\rangle. \quad (34b)$$

The integrals appearing in Eqs. (32) and (34) are evaluated by means of the integral representation [Eq. (22)] for $P_{MK}{}^{J}$. Consider the normalization integral, which contains terms of the form

$$\int d\Omega\, D_{MK}{}^{J}(\Omega)^{*}\langle\Phi\,|\,\tilde{\Phi}\rangle, \quad (35)$$

where $\tilde{\Phi}=R(\Omega)\Phi$. The effect of $R(\Omega)$ on Φ is to rotate each of the N extra core orbitals in Φ. The core orbitals in Φ and $\tilde{\Phi}$ are identical, and so we find that

$$\langle\Phi\,|\,\tilde{\Phi}\rangle=\det(B), \quad (36)$$

where B is an $N\times N$ matrix, with elements

$$B_{\lambda\mu}=\langle\varphi_\lambda\,|\,\tilde{\varphi}_\mu\rangle=\langle\varphi_\lambda\,|\,R(\Omega)\,|\,\varphi_\mu\rangle. \quad (37)$$

The integral in Eq. (32) is treated in a similar fashion, except that now terms of the form

$$\int d\Omega\, D_{MK}{}^{J}(\Omega)^{*}\langle\Phi_{f\lambda}\,|\,\tilde{\Phi}_{i\mu}\rangle \quad (38)$$

are encountered. Here $\Phi_{f\lambda}$ and $\tilde{\Phi}_{i\mu}$ are normalized Slater determinants for $A-1$ particles, again with identical core orbitals. It can be shown in this case that $\langle\Phi_{f\lambda}\,|\,\tilde{\Phi}_{i\mu}\rangle$ is equal to the cofactor of $B_{\lambda\mu}$ in the determinant $\det(B)$. An elementary theorem concerning cofactors of determinants then permits us to write

$$\langle\Phi_{f\lambda}\,|\,\tilde{\Phi}_{i\mu}\rangle=b_{\lambda\mu}\det(B), \quad (39)$$

where $b_{\lambda\mu}$ is an element of the matrix $b=B^{-1}$.

Most HF calculations are made without isospin mixing, which means that each orbital is either a pure neutron state or proton state. The summation in Eq. (32) is therefore restricted by the condition $\tau_{\lambda_1}=\tau_{\lambda_2}$, and the result vanishes unless $\tau_a=\tau_b$. One may thus speak of the proton part $S_J{}^{p}$ or the neutron part $S_J{}^{n}$ of the spectroscopic factor defined in Eq. (8).

3. PROTON SCATTERING FROM Ne^{20}, Mg^{24}, Si^{28}, and S^{32}

A. Structure of $2s$-$1d$ Shell Nuclei

The projected HF method has been used with considerable success in nuclear-structure studies of $2s-1d$ shell nuclei.[10] In most structure calculations it has been assumed that the nuclei in this region are axially symmetric, although Bar-Touv and Kelson have made a study of intrinsic HF spectra which indicates that there are regions of asymmetry in the $2s-1d$ shell.[11] An additional assumption which is generally made is that the O^{16} closed-shell core is inert; in this case Eq. (22) becomes

$$\Psi_{JM}=P_{MK}{}^{J}\Phi_K \quad (40)$$

and the orbitals φ_λ will be of the form

$$\varphi_\lambda=\sum_{j}C_j{}^{\lambda}\varphi_{jm_\lambda} \quad (41)$$

and the subspace of the basis states is limited to the $2s$-$1d$ shell (i.e., $1d_{5/2}$, $2s_{1/2}$, and $1d_{3/2}$). The radial dependence of the basis functions is taken to be that of the harmonic oscillator, and the spherical single-particle energies appearing in Eq. (23) are taken from experiment. The residual two-body interaction is chosen

[10] See Refs. 4a, 8, and 9.

[11] J. Bar-Touv and I. Kelson, Phys. Rev. **138**, B1035 (1965).

TABLE I. Energies and wave functions for axially symmetric, even-even, $2s$-$1d$ shell nuclei. E_{HF} is the total HF energy, e_λ the energy of each HF orbit, Δ the gap between occupied and unoccupied orbits, and m_λ the z component of angular momentum of each HF orbit. The last six columns give the projection of each HF orbit onto the spherical basis vectors.

Nucleus	E_{HF} (MeV)	e_λ, (MeV)	Δ, (MeV)	nlj / m_λ	$1d_{5/2,1/2}$	$1d_{5/2,3/2}$	$1d_{5/2,5/2}$	$2s_{1/2,1/2}$	$1d_{3/2,1/2}$	$1d_{3/2,3/2}$
Ne^{20}	−34.82	−14.27	8.2	$\frac{1}{2}$	0.7273			−0.5729	−0.3780	
Mg^{24}	−68.55	−16.12	1.6	$\frac{1}{2}$	0.7188			−0.6320	−0.2897	
		−10.96		$\frac{3}{2}$		0.9674				−0.2533
Si^{28}	−116.41	−17.94	7.0	$\frac{1}{2}$	0.5290			0.7967	−0.2922	
		−17.55		$\frac{5}{2}$			1.000			
		−14.07		$\frac{3}{2}$		0.6682				0.7440
S^{32}	−159.06	−18.76	5.0	$\frac{1}{2}$	0.5757			0.7598	−0.3020	
		−17.76		$\frac{5}{2}$			1.000			
		−16.11		$\frac{1}{2}$	0.6928			−0.6495	−0.3133	
		−14.48		$\frac{3}{2}$		0.7333				0.6799
Ar^{36}	−211.89	−20.68	7.1	$\frac{5}{2}$			1.000			
		−20.40		$\frac{1}{2}$	0.4868			0.8716	−0.0576	
		−19.81		$\frac{3}{2}$		0.9960				0.0895
		−16.87		$\frac{1}{2}$	0.6828			−0.3386	0.6475	
		−15.42		$\frac{3}{2}$		−0.0895				0.9960

to be a Rosenfeld mixture with Gaussian radial dependence similar to that used by Ripka[12]:

$$v(ij)=V_0\exp(-\mu r^2)\tfrac{1}{3}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j(0.3+0.7\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j), \quad (42)$$

where $V_0=-50$ MeV and $\mu=0.29$ F^{-2}. There has been no correction for c.m. motion. The determinantal wave function Φ_K is constructed in such a way that, along with each single-particle state [Eq. (41)], one also includes the time-reversed state

$$\varphi_{-\lambda}=\sum_j C_j^\lambda(-1)^{j-m_\lambda}\varphi_{j-m_\lambda}; \quad (43)$$

that is, a proton and neutron are put into each of the states [Eqs. (41) and (43)].

Recently, Bassichis *et al.*[13] have performed HF calculations in which the O^{16} core assumption was dropped and the basis space extended to include the $1s$, $1p$, $2p$, and $1f$ states as well as the $2s$-$1d$ states. The over-all results of their calculations are encouraging, and we will make use of their Ne^{20} wave functions, since the core-polarization effects are included explicitly.

TABLE II. $B(E2)$ rates in e^2F^4 for $2s-1d$ shell nuclei.

Nucleus	Basis $\nu=0.27$	Basis WS	Experiment
Ne^{20}	115.3	115.9	286.5
Mg^{24}	33.7	30.3	436.7
Si^{28}	225.4	221.2	327.0
S^{32}	20.0	18.1	264.9

The results of HF calculations using the residual two-body force given in Eq. (42), and a value of 0.27 F^{-2} for the harmonic-oscillator parameter $\nu=m\omega/\hbar$, are given in Table I. The particular choice of ν used here is based on a study of HF spectra and $B(E2)$ rates[14] using basis functions with Wood-Saxon radial dependence.[15] The $E2$ transition rates were calculated from the expression

$$B(E2)=(2J_i+1)^{-1}\sum_{\mu M_i M_f}|\langle\Psi_f|\sum_{k=1}^{A}e_k r_k^2 Y_2^\mu(k)|\Psi_i\rangle|^2, \quad (44)$$

with e_k equal to e for protons and 0 for neutrons. The results are compare with experiment in Table II. For completeness we also give the Wood-Saxon results since only the Ne^{20} result appears elsewhere. The Mg^{24} and S^{32} results are very low for both sets of basis functions and it is quite clear that the introduction of the usual effective charges of $1.5e$ for protons and $0.5e$ for neutrons will not be sufficient to yield reasonable agree-

[12] G. Ripka, in *Lectures in Theoretical Physics* (University of Colorado Press, Boulder, Colo., 1965), Vol. VIII c.

[13] W. H. Bassichis, A. K. Kerman, and J. P. Svenne, Phys. Rev. **160**, 746 (1967).

[14] W. F. Ford and R. C. Braley, Nucl. Phys. **A126**, 671 (1969).

[15] W. F. Ford and R. C. Braley, Bull. Am. Phys. Soc. **13**, 1654 (1968).

TABLE III. Parameters used in the optical-model potential $U(r)$ defined in Eq. (48). (Radii are obtained in the usual fashion, $R_x = r_x A^{1/3}$.)

(E_p) (MeV)	V_0 (MeV)	W_s (MeV)	$V_{1\cdot s}$ (MeV)	r_0 (F)	r_s (F)	r_{1s} (F)	a_0 (F)	a_s (F)	a_{1s} (F)
Ne^{20} (18.2)	47.45	7.5	5.1	1.185	0.942	1.042	0.721	0.568	0.488
Ne^{20} (18.2)	45.2	6.18	8.5	1.20	1.20	1.20	0.65	0.47	0.65
Mg^{24} (17.5)	49.14	8.064	5.29	1.174	1.19	1.06	0.736	0.562	0.546
Mg^{24} (17.5)	47.3	5.73	7.65	1.20	1.20	1.20	0.64	0.50	0.64
Si^{28} (17.5)	55.54	6.53	7.13	1.10	1.37	1.10	0.633	0.600	0.585
Si^{28} (17.5)	42.8	11.3	7.5	1.30	1.25	1.30	0.62	0.44	0.62
S^{32} (17.5)	52.14	6.12	5.56	1.15	1.32	0.85	0.654	0.547	0.41
S^{32} (17.5)	42.4	8.62	7.5	1.30	1.30	1.30	0.62	0.44	0.62

ment with experiment. Future studies of the asymmetric solutions for these nuclei should clarify some of the structure problems generally associated with these nuclei. The value of $B(E2)$ for Ne^{20} using the wave functions of Bassichis *et al.* was found to be $80e^2\,F^4$. This is quite low but is to be expected since they used a rather large value of $\nu(0.385\ F^{-2}$; see Ref. 13).

B. Direct Interaction

The calculation of microscopic form factors for inelastic scattering requires a knowledge of the nucleon-nucleus interaction. At the present stage of development, theories of the nucleus and reactions are not sufficient to provide such a force. Instead, either a phenomenological interaction or the two-nucleon t matrix must be used.

We have chosen a phenomenological spin-dependent interaction which was used by Glendenning and Veneroni in a recent study of inelastic scattering. These authors assumed the two-body interaction to be important only in even states and used an interaction of the form

$$V(r) = -52\exp[-(r/1.85)^2](P_{\mathrm{TE}}+0.6P_{\mathrm{SE}}), \tag{45}$$

where P_{TE} and P_{SE} are projection operators for the triplet-even and singlet-even states, respectively. This force is found to approximately reproduce the low-energy neutron-proton data.

In terms of single-particle operators

$$V(r) = -52[0.3(1+\tfrac{1}{2}\tau_0\cdot\tau_1) - 0.1(\tfrac{1}{2}+\tau_0\cdot\tau_1)\sigma_0\cdot\sigma_1] \times \exp[-(r/1.85)^2], \tag{46}$$

where τ_0 and τ_1 refer to the isospin of the projectile and bound nucleon, respectively. Comparing Eq. (46) with Eq. (13), we see that

$$t_{L0J}(r_0, r_0) = -15.6(1-\tfrac{1}{2}\tau_0\cdot\tau_1)\exp[-(r/1.85)^2] \tag{47a}$$

and

$$t_{L1J}(r_0, r_1) = 5.2(\tfrac{1}{2}+\tau_0\cdot\tau_1)\exp[-(r/1.85)^2]. \tag{47b}$$

Because the isospin state of the projectile does not change during the scattering event, the matrix element of $\tau_0\cdot\tau_1$ will be $+1$ when projectile and target nucleon are like nucleons and -1 when they are unlike nucleons. Hence, the form factor can be written as a sum of two terms, one for protons and one for neutrons. We point out that the strength of the free two-nucleon potentials like that above must usually be increased in order to account for the participation of core nucleons in the reaction. In Sec. D following, this matter is discussed in greater detail when the result for a 20-particle calculation on Ne^{20} is presented.

The distorted waves used to represent the projectile are solutions to the Schrödinger equation with an optical potential

$$U(r) = V_c(r) - V\rho V(r) - (4i/a_S)W_S\rho_S(r) - V_{LS}(\sigma\cdot\mathbf{l}/a_{LS}r)\rho_{LS}(r), \tag{48}$$

where $V_c(r)$ is the Coulomb term, V is the strength of the real potential, W_S is the strength of the surface

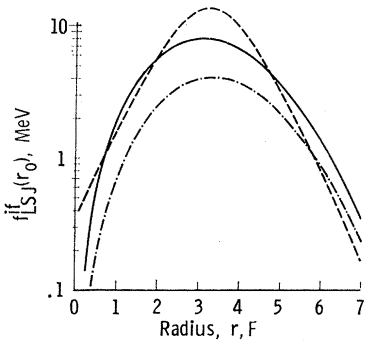

FIG. 1. Scalar form factors for 2^+ states in Ne^{20}. (Macroscopic model ---; Hartree-Fock ——; unrestricted Hartree-Fock ——).

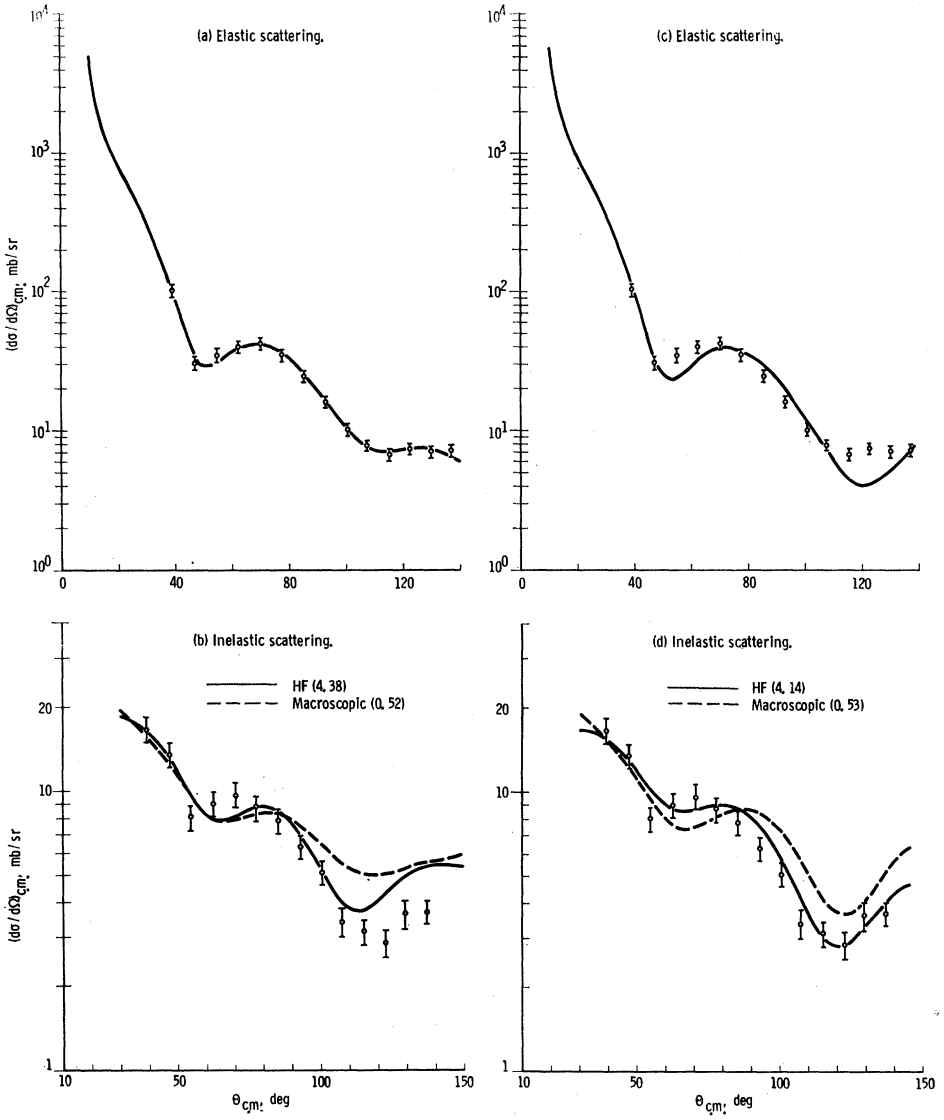

FIG. 2. Cross sections for elastic and inelastic ($Q=-1.63$ MeV) scattering of 18.2-MeV protons from Ne^{20}. In (a) and (b) the optical potentials are chosen to optimize the elastic scattering fit, while in (c) and (d) the potentials optimize the inelastic scattering fit.

absorption, and V_{LS} is the strength of the spin-orbit term. The term $\rho V(r)$ is the usual Wood-Saxon form factor, and $\rho_S(r)$ and $\rho_{LS}(r)$ are derivatives of functions like ρV, but with appropriate radius and diffuseness.

C. (0^+-2^+) Form Factors for Ne^{20}

The microscopic nuclear form factor $f_{LSJ}{}^{if}(r_0)$ for inelastic scattering was defined in Eq. (17). For the inelastic scattering of nucleons S will be 0 or 1, since $s_0=\frac{1}{2}$. The form factor $f_{LLJ}{}^{if}(r_0)$ is called the vector form factor and it is this term which gives rise to spin-flip transitions. The macroscopic form factor is proportional to the derivative of the optical potential which is used to fit the elastic scattering.[4] In keeping with convention, this form factor is assumed spin-independent, and has a derivative Wood-Saxon radial dependence:

$$f_{J0J}{}^{if}(r_0)=C_J\left[2+\exp\left(\frac{r_0-R_0}{a}\right)+\exp\left(\frac{R_0-r_0}{a}\right)\right]^{-1}, \tag{49}$$

where C_J depends on the multipolarity of the transition, the optical-model parameters, and the deformation (β_J) of the nucleus. The well parameters, R_0 and a, refer to the radius and diffuseness of the optical potential, and r_0 is the separation between the projectile and the nuclear surface. If U_0 is the strength of the optical potential,

$$C_J=\beta_J(U_0R_0/a\hat{J}).$$

In the discussion which follows, we shall refer to restricted and unrestricted form factors. By unrestricted

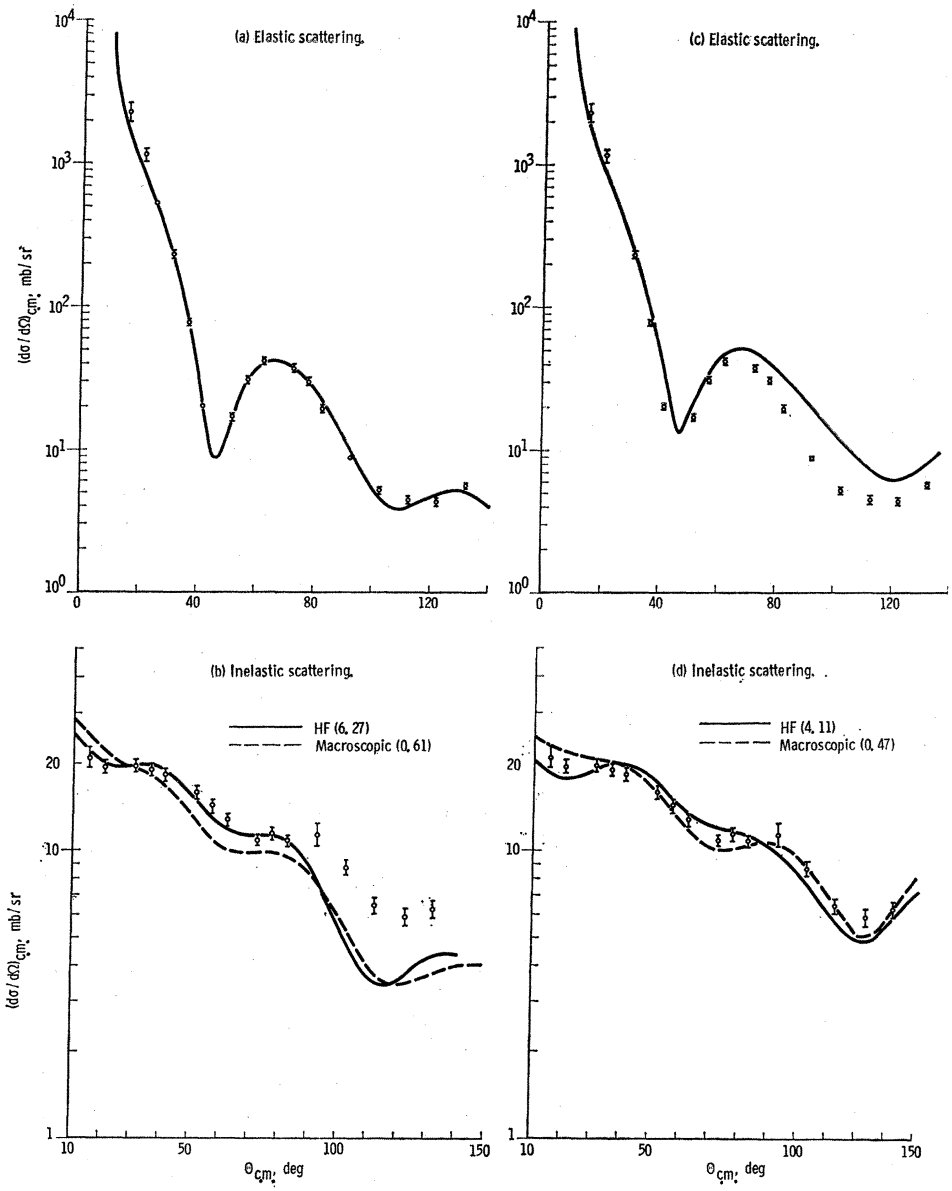

FIG. 3. Cross sections for elastic and inelastic ($Q=-1.37$ MeV) scattering of 17.5-MeV protons from Mg^{24}. In (a) and (b) the optical potentials are chosen to optimize the elastic scattering fit, while in (c) and (d) the potentials optimize the inelastic scattering fit.

we mean that the HF calculations are carried through without an O^{16} core and with the enlarged basis space ($1s$, $1p$, $2s$, $1d$, $2p$, $1f$) as was done by Bassichis *et al.* The results of our calculations which were presented in Table I are of the restricted type.

In Fig. 1, we present the scalar form factors for Ne^{20}. The other nuclei (Mg^{24}, Si^{28}, S^{32}) have form factors with the same general shapes but the magnitudes vary widely when the restricted wave functions are used. The vector form factors for the microscopic calculation are not shown as it was found that their magnitudes were smaller by a factor of several hundred than the scalar form factors, indicating that the spin-flip mechanism is relatively unimportant in the excitation of this type of state. A similar result was found by Glendenning and Veneroni for the excitation of the 2_1^+ states in the even Ni isotopes. The radial shapes of the scalar and vector form factors differ very little.

The projected HF form factors in Fig. 1 differ in magnitude by a factor of 2—the unrestricted form factor being the larger. We estimate that about $\frac{1}{3}$ of the enhancement is due to the use of different values of ν. Clearly then, the core contribution is very important. The positions of the maxima of the form factors differ by about 0.4 F with the unrestricted one having its peak at 3.1 F. The macroscopic form factor shown for comparison was calculated using the parameters (see Table III):

$\beta_2=0.53$, $U_0=45.2$ MeV, $R_0=3.26$ F, $a=0.65$ F.

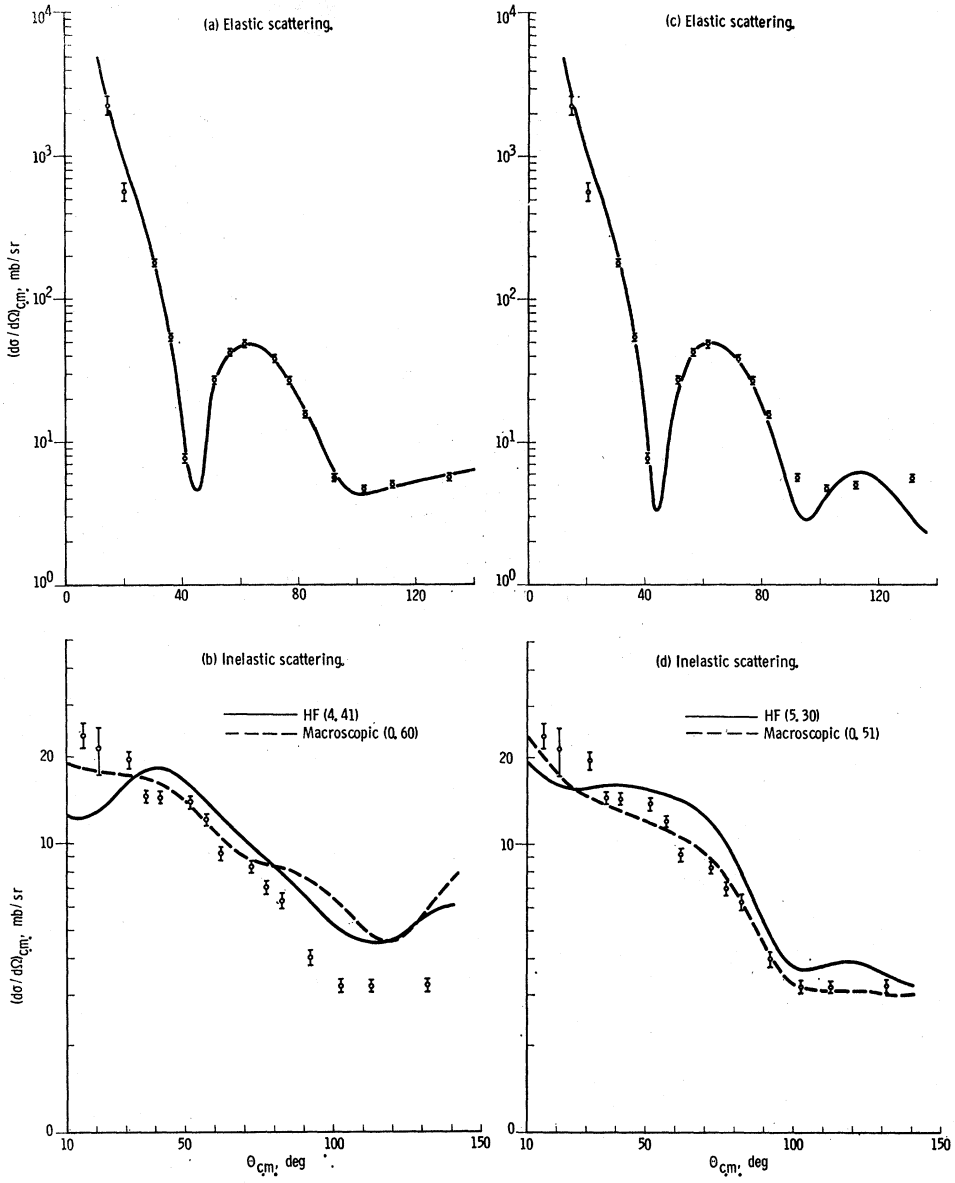

FIG. 4. Cross sections for elastic and inelastic ($Q=-1.78$ MeV) scattering of 17.5-MeV protons from Si^{28}. In (a) and (b) the optical potentials are chosen to optimize the elastic scattering fit, while in (c) and (d) the potentials optimize the inelastic scattering fit.

D. Inelastic Cross Sections (0^+–2^+)

The differential cross sections are calculated in the DWBA, and exchange effects between the projectile and target nucleons are neglected. While one would expect DWBA to be reliable for the excitation of collective states, the question of the importance of exchange effects cannot be answered hastily without a careful study in which these effects are accounted for explicitly.[16]

The differential cross sections for elastic and inelastic scattering have been calculated and the results are compared with experiment[17] in Figs. 2–5. The nuclear wave functions used in each of these inelastic calculations are of the restricted type. Each set of cross sections has been calculated with two sets of optical potentials. Table III gives the optical-model parameters [see Eq. (48)] which were used. (For each nucleus, the first set gives the best fit to the elastic scattering.) Parts (a) and (b) of each figure correspond to results obtained using optical potentials which yield a best fit to the elastic scattering, while in (c) and (d) the optical potentials yield the best results for inelastic

[16] K. A. Amos, V. A. Madsen, and I. E. McCarthy, Nucl. Phys. **A94**, 103 (1967).

[17] G. M. Crawley and G. T. Garvey, Phys. Rev. **160**, 981 (1967); G. M. Crawley, Ph.D. thesis, Princeton University, 1967 (unpublished).

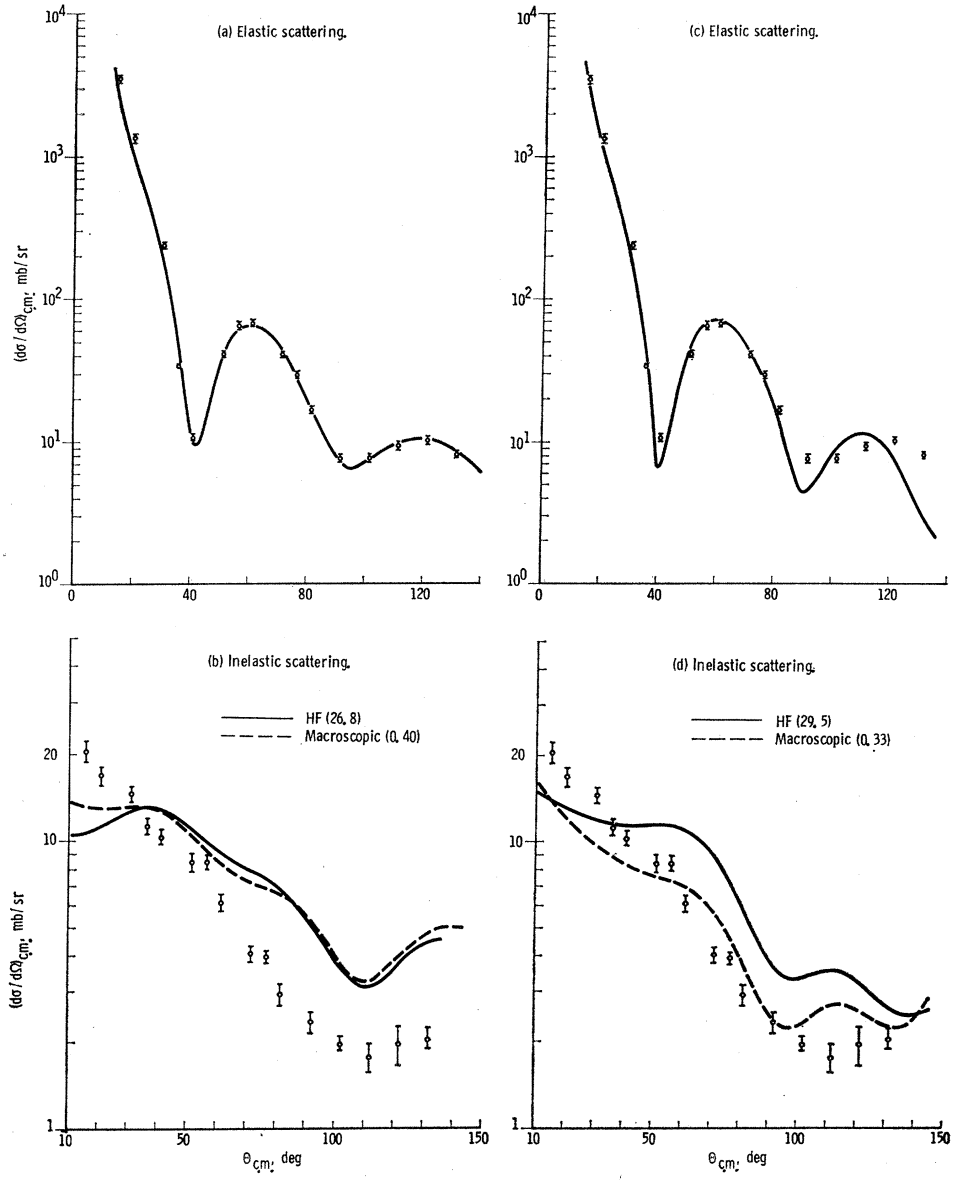

FIG. 5. Cross sections for elastic and inelastic ($Q=-2.24$ MeV) scattering of 17.5-MeV protons from S^{32}. In (a) and (b) the optical potentials are chosen to optimize the elastic scattering fit, while in (c) and (d) the potentials optimize the inelastic scattering fit.

scattering. The numbers in parentheses at the right of the legends for inelastic scattering are normalizations in the microscopic cases, and deformations in the macroscopic cases. The incident proton energy (lab) was 17.5 MeV for all cases except Ne^{20}, for which the incident proton energy was 18.2 MeV.

The results for Ne^{20} and Mg^{24} are presented in Figs. 2 and 3, respectively. For the case of Ne^{20}, the fit to the elastic scattering 2(a) is excellent, but the inelastic results 2(b) are not at all satisfactory for either the microscopic or macroscopic model. On the other hand, if the optical-model parameters are varied in such a way as to obtain the best inelastic results 2(d), then the quality of the elastic scattering 2(c) is worse. It is rather clear in this case that the microscopic model yields the better shape for the inelastic cross section, although the magnitude of the theoretical result is under estimated by a factor of 4.14. The deformation parameter obtained from the macroscopic model ($\beta_2=0.53$) is in poor agreement with experiment (0.87).[18]

In the case of Mg^{24} the situation is quite similar, with the inelastic fits being of generally low quality—especially for angles beyond 90° if the optical potential used is that which yields the best elastic scattering. If the inelastic optical potential is used then the magnitude for the inelastic cross section is still under estimated by a factor of 4.11 in the microscopic case, but it is difficult to say whether the microscopic model or

[18] *Nuclear Data*, Compiled by K. Way *et al.* (Academic Press Inc., New York, 1965).

the macroscopic model yields the better shape; both are quite good.

We turn now to the results for the cases of Si^{28} and S^{32}, which appear in Figs. 4 and 5. The elastic scattering "best-fit" potentials clearly give unsatisfactory results. Variation of the optical potential improves matters considerably for Si^{28} [4(d)], except in the forward direction. The S^{32} inelastic results do not have a good shape and the magnitude is very poorly predicted compared with results for the other nuclei.

We return now to the Ne^{20} calculation which was discussed in Sec. C above, i.e., the unrestricted (20-particle) calculation. The inelastic cross sections obtained using the above-mentioned form factors are presented in Fig. 6. As in the restricted case, the results obtained using the elastic optical potentials do not have the proper shape. The shape which results from use of the inelastic optical parameters (Table III) is quite satisfactory. Furthermore, we note in this case that the differential cross section is under estimated by a factor of only 2.9, indicating the importance of including the core nucleons in such calculations. Based on this result one finds that the vacuum interaction has to be enhanced by a factor of 1.7.

4. SUMMARY AND CONCLUSIONS

Detailed microscopic wave functions have been used to study the inelastic scattering of protons from several $2s$-$1d$ shell nuclei. The interaction between the projectile and target nucleus was assumed to be a sum of two-body forces. Transition amplitudes for the inelastic excitations were calculated using DWBA.

The nuclear wave functions have been obtained by projecting states of good angular momentum from deformed intrinsic states, where these states result from minimizing the HF energy for the system under consideration. Most of the calculations are based on the assumption that O^{16} constitutes an inert core but we include a reaction calculation for Ne^{20} in which this assumption is dropped. Relaxation of this assumption enables one to determine the enhancement of the vacuum interaction due to the participataion of core nucleons. For the case of Ne^{20}, it was found that the inclusion of the core reduces the required enhancement factor from 2.04 to 1.70.

Concerning the over-all comparisons with experiment, we have seen that the general shapes of the angular distributions are fairly good if one does not make use of the optical parameters which best fit the elastic scattering. However, we did find that the results for Si^{28} and S^{32} were not as satisfying as for Ne^{20} and Mg^{24}. Also, as one would expect, the magnitudes were quite low when the restricted type form factors were used. The poor quality of the inelastic results for Si^{28} and S^{32} could be due to the fact that the simple HF picture does not provide an adequate description of these nuclei. There are indications that both of these nuclei may require some admixture of spherical components in their wave functions in order to obtain a good description of their structure.[19]

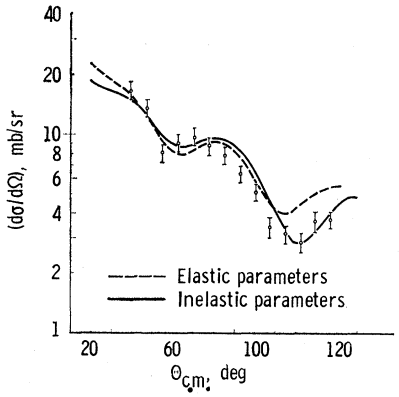

FIG. 6. Cross section for inelastic ($Q=-1.63$ MeV) scattering of 18.2-MeV protons from Ne^{20}, with form factors calculated using unrestricted HF wave functions. The solid and dashed lines show the results using optical parameters which optimize the inelastic and elastic fits, respectively.

The fact that the distorted waves which are generated by "elastic" optical potentials do not give good fits to the inelastic scattering is not surprising. In the search for such potentials, it is the asymptotic part of the scattered wave which is important; in the reaction calculation, the shape of the scattering wave function in the nuclear interior and near the nuclear surface is important. There have recently been investigations by Schenter[20] in which "true" optical potentials were calculated in terms of the nucleon-nucleon interaction and the ground-state wave function for the target. He found the wave function for the projectile to be greatly reduced in the nuclear interior when compared to wave functions generated by a phenomenological potential having the same phase shifts. The use of scattering wave functions based on this approach will, hopefully, be applied to inelastic scattering problems in the near future.

Some of the structure problems have already been mentioned. However, the incorrect asymptotic behavior of the harmonic-oscillator functions must not be overlooked. It would be more realistic to use radial functions such as those generated by a Wood-Saxon well. The use of such functions may have a significant affect on the form factors and hence the inelastic cross sections. A program for such a study is being undertaken by the present authors.

ACKNOWLEDGMENTS

Dr. R. W. Bercaw and Dr. E. T. Boschitz of the NASA Lewis Research Center provided the Ne^{20} data prior to publication.

[19] J. Bar-Touv and A. Goswami, Phys. Letters **28B**, 391 (1969).
[20] R. E. Schenter, Phys. Rev. Letters **18**, 465 (1967).