Study of the $Sc^{45}(\alpha, t)Ti^{46}$ Reaction at 41 MeV

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Thirteen angular distributions have been measured from 8° to 70° for the $Sc^{45}(\alpha, t)Ti^{46}$ reaction at 41 MeV. Very selective population of the states of Ti^{46} was observed, and the populated states correlate well with states of the $Sc^{45}(He^3, d)Ti^{46}$ reaction which have large $l_p=3$ spectroscopic strengths. The angular distributions have little structure and exhibit the same basic shape. The distorted-wave Born-approximation (DWBA) noncutoff calculations for the stripping of an $l_p=3$ proton provides an adequate description of the angular distributions. The relative spectroscopic strengths extracted from the DWBA analysis agree favorably with those deduced from a similar study of the $Sc^{45}(He^3, d)Ti^{46}$ reaction. Theoretical spectroscopic strengths calculated from the McCullen-Bayman-Zamick single-particle wave functions for the ground state of Sc^{45} and five low-lying states of Ti^{46} are in reasonable agreement with experiment.

INTRODUCTION

THE study of reactions where a single nucleon is transferred between a projectile and a target can yield information which is rich in details of the nuclear structure. The success in extracting this information depends, however, on the extent to which the mechanism of the reaction is understood. Of the proton transfer reactions (d, n), (He^3, d), and (α, t), the former two have generally been characterized as stripping-type processes and have been compared with direct-reaction theories such as the distorted-wave Born approximation (DWBA). Analysis of a few (α, t) reactions using current stripping theories have been less successful,[1-4] even though at sufficiently high incident energy the angular distributions have many of the attributes of the other reactions. The interpretation has been that the (α, t) process is direct but not entirely of the stripping type. The principal difference between the (α, t) reaction and the (d, n) and (He^3, d) proton transfer reactions is that high orbital angular momentum transfers are favored in the (α, t) process. This condidition is attributed to the momentum mismatch, and at medium incident energies results in the preferential population of residual states requiring three to five units of transferred angular momentum.[5,6] The choice of a suitable target to study the (α, t) reaction as a spectroscopic tool should be governed by the following considerations. First, the nucleus should have structure which is well understood from independent model calculations, and secondly, there should be an abundance of states for which the orbital angular momentum of the captured proton is 3 or 4. Nuclei with protons in the $1f_{7/2}$ shell satisfy this requirement. The $Sc^{45}(\alpha, t)Ti^{46}$ reaction is particularly suited because reasonable model wave functions are available,[7] and two studies of the $Sc^{45}(He^3, d)Ti^{46}$ reaction are available for comparison.[8,9] The results of an investigation of the (α, t) reaction on Sc^{45}, using 41-MeV α particles, are reported in this paper.

EXPERIMENTAL

The Lewis Research Center fixed-energy Cyclotron provided the source of 41.0 ± 0.2-MeV α particles. The Sc^{45} target was a self-supporting, evaporated film having an areal density of 0.926 ± 0.050 mg/cm^2. The tritons were detected in a 4500-μ-thick lithium-drifted silicon detector. The defining aperture of this counter was rectangular and subtended a solid angle of 1.24×10^{-4} sr. Tritons were discriminated from other $Z=1$ reaction products by a ΔE-E counter telescope and a particle identifier similar to the design of Goulding *et al.*[10] The effectiveness of this system can be assessed from the mass spectrum shown in Fig. 1. A triton spectrum for a laboratory angle of 20° is shown by the open circles in Fig. 2. An evaluation of this spectrum yields an over-all energy resolution of 150 keV full width at half-maximum (FWHM).

The first three triton groups in Fig. 2 are completely resolved and correspond to production of the ground and first two excited states of Ti^{46}.[11] There are also a number of prominent, but only partially resolved groups in the spectrum. These groups more than likely do not correspond to excitation of a discrete state but do represent preferential production of a state. They were analyzed by the following procedure. For each angle the three resolved groups were fitted with a skewed Gaussian function using a least-squares computer program. This

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procedure yielded the width parameter, peak position, and areas under the curve of the Gaussian function. An average value of the width parameter for the three fits was extracted and this value was held fixed for the fitting of the remaining prominent triton groups. Typical results using this method are illustrated by the solid curve in Fig. 2. The standard deviation between computer calculations and the total number of counts obtained by summing 66 resolved spectral peaks was calculated to be 5.5%. Differential cross sections for production of the states labeled t_0 through t_2 were calculated using triton yields obtained by summing the counts in the groups. Those for the states labeled t_3 through t_{12} were calculated using the yields obtained from the computer fits. The over-all uncertainty in the cross sections for these two sets is assessed at 15 and 25%, respectively. The uncertainty in the relative cross sections is estimated at 10 and 20%, since a fixed monitor counter was used to normalize the data. The c.m. differential cross sections plotted against the c.m. reaction angle are presented in Fig. 3. The errors shown reflect only the statistical uncertainty in the cross sections.

DISCUSSION

Several studies of the states of Ti^{46} have been reported.[8,9,12–17] The most useful of these for the present study are those of Mo *et al.*,[17] who studied the $Ti^{46}(p, p'\gamma)Ti^{46}$ reaction, and Broman and Pullen[8] and Barnard and Jones,[9] who studied the $Sc^{45}(He^3, d)Ti^{46}$ reaction. Their results are summarized in columns (a) through (g) of Table I. The mean Ti^{46} excitation energies for the present study are listed in column (h). These were obtained from the analytic form of an energy calibration curve based on a linear least-squares fit between the Q values and peak positions obtained from the computer fits to the spectra for the t_0, t_1, t_2, and t_8 groups. Those states designated in column (h) correlate well with those states in the $Sc^{45}(He^3, d)Ti^{46}$ reaction having large $l_p=3$ spectroscopic strengths. This is expected from the momentum-mismatch argument, which can be formulated as follows. The momentum transfer vector for a stripping reaction is defined as

$$\mathbf{q} \equiv \mathbf{k}_i - (M_T/M_R)\,\mathbf{k}_f, \qquad (1)$$

where $\mathbf{k}_i$ and $\mathbf{k}_f$ are the c.m. wave vectors of the incident and exit particles and M_T and M_R are the masses of the target and residual nuclei. $|\mathbf{q}|$ depends not only on $|\mathbf{k}_i|$ and $|\mathbf{k}_f|$, but on the reaction angle. $|\mathbf{q}|$ is a minimum for a reaction angle of 0°, is zero if $|\mathbf{k}_i| = (M_T/M_R)|\mathbf{k}_f|$ (that is if the momenta are matched), and increases monotonically with angle. If $|\mathbf{k}_i|$ and $(M_T/M_R)|\mathbf{k}_f|$ are unequal the minimum $|\mathbf{q}|$ can be substantially different from zero. In a simple stripping reaction we expect the maximum in the differential cross section to occur at an angle corresponding to $|\mathbf{q}|\,R=l$, where R is the radius of the nuclear surface, and $l\hbar$ is the orbital angular momentum of the transferred nucleon.[18] This is indeed the case for the $Sc^{45}(He^3, d)Ti^{46}$ data.[8] For the $Sc^{45}(\alpha, t)Ti^{46}$ reaction $|\mathbf{k}_\alpha|$ differs enough from $|\mathbf{k}_t|$ that the minimum value of $|\mathbf{q}|\,R>1$. Hence the condition $|\mathbf{q}|\,R=l$ can never be satisfied for the $l_p=1$ proton transfers and these transitions should be suppressed. Other evidence for the suppression of the $l_p=1$ components of the mixed $l_p=1$ and $l_p=3$ transitions can be seen from the shapes of the (α, t) angular distributions. There are no distinguishing characteristics in, for example, the angular distributions for production of the first three states in Ti^{46}, as shown in Fig. 4. This is in contrast to that observed in the (He^3, d) reaction, where the $l_p=1$ component can easily be observed at the forward angles.

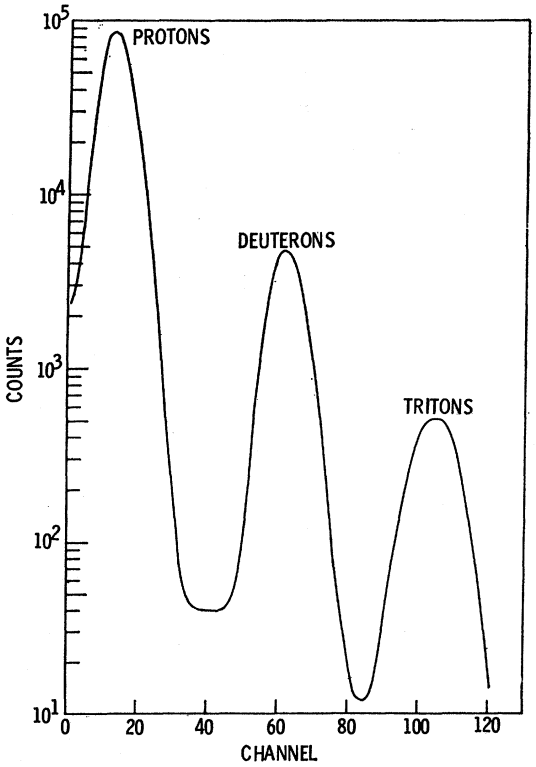

FIG. 1. Mass-identifier-output spectrum for the $Z=1$ reaction products.

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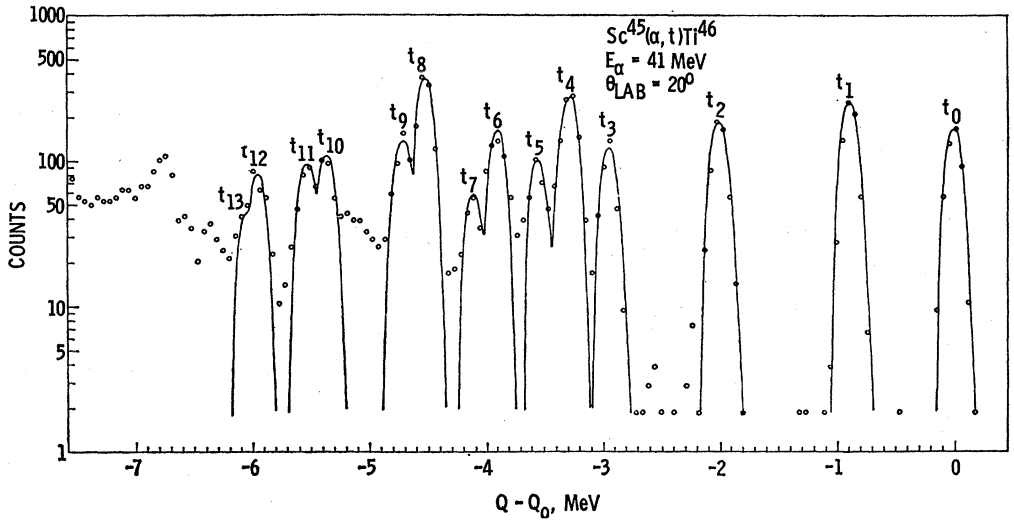

FIG. 2. Triton spectrum obtained at 20° for the $Sc^{45}(\alpha, t)Ti^{46}$ reaction at 41 MeV. The solid line is an analytical fit to the spectrum using skewed Gaussian functions to describe the peak shapes. The calibration curve used to calculate the excitation energies shown is based on a linear least-squares fit between the Q values and peak positions obtained from the computer fits to the spectra for the t_0, t_1, t_2, and t_8 groups. The Q values for these four excitations were taken from Ref. 8.

Even though the energy resolution in the present experiment was not adequate for resolving discrete states above 3 MeV in Ti^{46}, an analysis of the angular distributions presented in Fig. 3 within the framework of current DWBA stripping reaction formalisms seems reasonable because of the suppression of the $l_p=1$ transitions and the selectivity of the (α, t) reaction for certain residual states. The data are all consistent and all angular distributions display the same basic shape.

For the transfer of a nucleon with l units of orbital angular momentum it is customary to write the differential cross sections as[19]

$$\frac{d\sigma}{d\Omega} \propto \left(\frac{2J_f+1}{2J_i+1}\right) S_l \sigma_{\mathrm{DWBA}}, \qquad (2)$$

where J_i and J_f are the angular momentum quantum numbers of the target and residual nuclei, respectively, S_l is the spectroscopic factor, and σ_{DWBA} is the differential cross section computed from the DWBA formalism. If the isospin formalism is used, then S_l should be replaced by C^2S_l, where C is the Clebsch-Gordan coefficient for the isospin.[20] Since the theoretical values of the S_l used in this analysis did not include the isospin factor, the form of the differential cross section in Eq. (2) will be used to determine experimental values of S_l. S_l contains the structure information for the nuclei involved and is defined for single-particle wave functions as[7]

$$S_l = n[\langle \psi_{M^{J_f}} | (\psi^{J_i} \phi^j)_{M^{J_f}} \rangle]^2, \qquad (3)$$

where n is the number of equivalent nucleons of the type transferred in the residual nucleus, ψ^{J_f} and ψ^{J_i} are the wave functions of the residual and target nuclei, and ϕ^j is the single-particle wave function of the captured nucleon. Equation (2) is a proportionality rather than an equality because the formalism involves an overlap integral for the dissociation of an α particle into a triton and proton and this quantity is not normally calculated for the (α, t) reaction.

The DWBA formalism of Tobocman[21] was used for the description of σ_{DWBA} in Eq. (2). The numerical calculations employed the zero-range approximation for the nuclear interaction, noncutoff radius calculation for the radial integrals, and were made with the direct reaction FORTRAN code written by Gibbs *et al.*[22] The wave functions for the incident and exit channels were generated using a Woods-Saxon potential of the form

$$V_c - V/(1+e^x) - iW/(1+e^{x'}), \qquad (4)$$

where

$$x=(r-r_0A^{1/3})/a, \qquad x'=(r-r_0'A^{1/3})/a',$$

and V_c is the Coulomb potential for a uniformly charged sphere of radius $r_{0c}A^{1/3}$. The potential parameters for the incident $\alpha+Sc^{45}$ system were obtained from calculations of the differential cross sections for the elastic scattering of 41-MeV α particles from Sc^{45}. The experimental data used for comparison with the theoretical calculations were taken with the same target used in the (α, t) experiment. The theoretical calculations determined

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TABLE I. Summary of pertinent spectroscopic information for Ti^{46}.

a Excitation (MeV)	b Excitation (MeV)	c L transition	d Spectroscopic strength	e Excitation (MeV)	f L transition	g Spectroscopic strength	h Excitation (MeV)	i Triton group	j Spectroscopic strength
0.00	0.000	3	0.27	0.000	3	0.53	0.000	T-0	0.27
0.89	0.887	1, 3	0.06+0.28	0.891	1, 3	0.25+0.96	0.890	T-1	0.40
2.01	2.006	1, 3	0.04+0.22	2.014	1, 3	0.20+0.72	2.012	T-2	0.30
2.61									
2.96	2.96	3	0.22	2.974	3	0.50	2.963	T-3	0.20
3.06				3.074	2	0.09			
3.17	3.17	3	0.58						
3.23				3.247	1, 3	0.01+0.24			
3.29				3.310	3	0.90	3.294	T-4	0.55
3.44	3.44	3	0.16	3.455	2	0.27			
3.58				3.598	1, 3	0.02+0.36	3.575	T-5	0.20
3.73				3.737	1	0.06			
				3.861	1, 3	0.06+0.31			
3.90	3.90	1, 3	0.09+0.36				3.920	T-6	0.33
				3.955	1, 3	0.09+0.57			
4.05				4.049	1	0.08			
				4.158	1, 3	0.02+0.14	4.133	T-7	0.14
4.21				4.206	3	0.23			
4.35				4.394	1	0.004			
4.45				4.533	3	2.11	4.533	T-8	0.76
4.64				4.620	1	0.06			
4.72				4.723	1, 3	0.03+0.59	4.733	T-9	0.27
4.83				4.846	1, 3	0.03+0.08			
				4.999	1, 3	0.01+0.05			
				5.045	0	0.06			
				5.098	1	0.13			
				5.187	1, 3	0.02+0.17			
				5.326	1	0.02			
				5.383	1, 3	0.07+0.40	5.370	T-10	0.26
				5.557	1, 3	0.14+0.41	5.549	T-11	0.25
				5.618	1	0.06			
				5.816	0	0.06			
				5.899	1, 3	0.02+0.04			
				5.982	1, 3	0.02+0.31	5.967	T-12	0.22
				6.029	1	0.05			

[a] Energy levels of Ti^{46} from J. N. Mo *et al.* (Ref. 17). The uncertainty is quoted as ±10 keV.

[b–d] States of Ti^{46} studied and spectroscopic strengths for the $Sc^{45}(He^3, d)Ti^{46}$ reaction, from R. W. Barnard and G. D. Jones (Ref. 9).

[e] Energy levels of Ti^{46} from Lars Broman and D. J. Pullen (Ref. 8). The uncertainty is quoted as ±12 keV for $E \leq 3.955$ MeV, 15 keV for $4.049 \leq E \leq 5.982$ MeV, and 20 keV for $6.029 \leq E \leq 7.565$ MeV.

[f,g] Spectroscopic information for the reaction $Sc^{45}(He^3, d)Ti^{46}$ at 15 MeV. Extracted from Table I of Ref. 8.

[h] Excitation energies of Ti^{46} corresponding to the triton groups designated in Fig. 2 (present paper). The uncertainty is estimated as ±30 keV.

[i] Triton group designated in Fig. 2 (present paper).

[j] Experimental spectroscopic strengths as deduced from the present study of the reaction $Sc^{45}(\alpha, t)^{46}$.

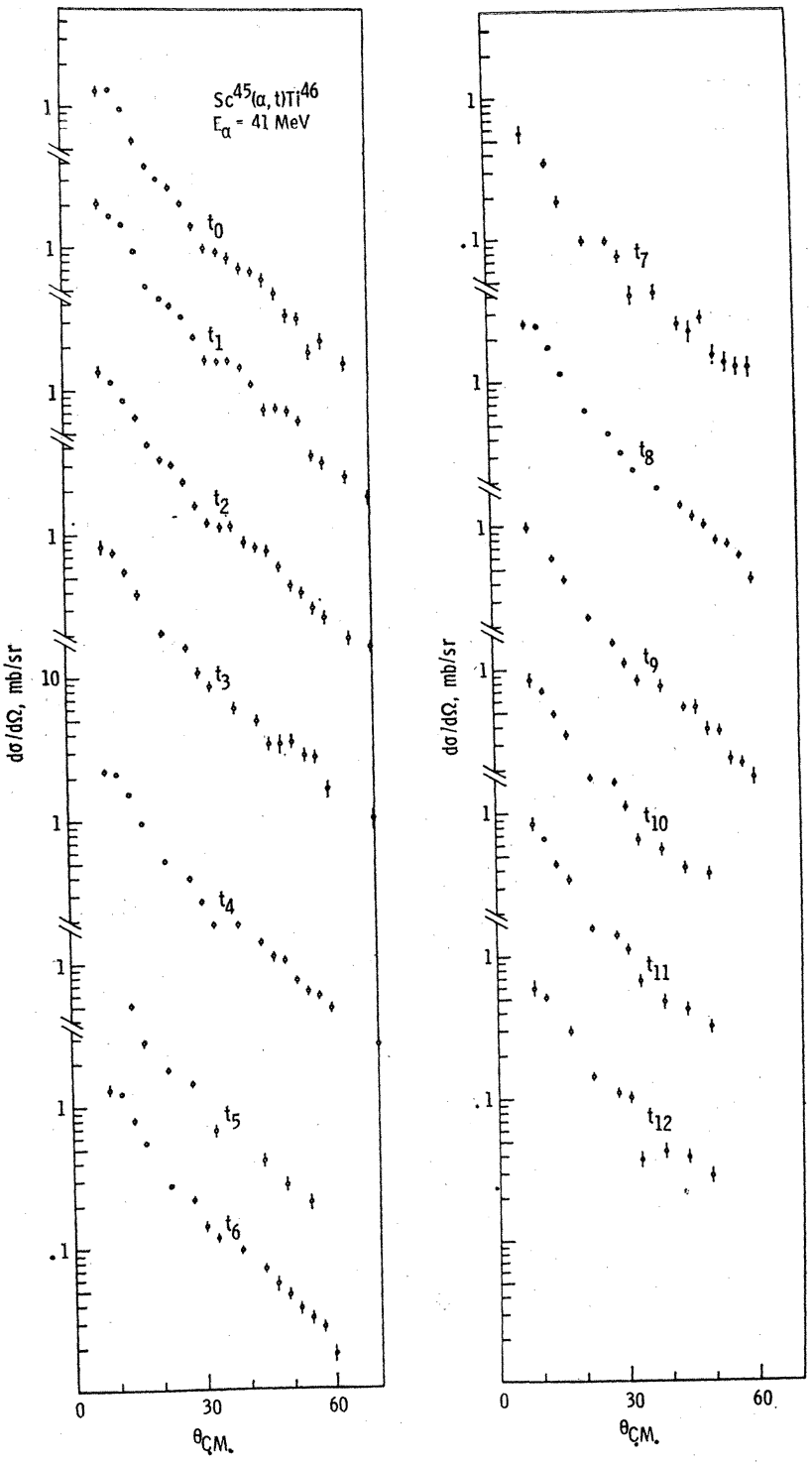

FIG. 3. Summary of the measured triton angular distributions for the $Sc^{45}(\alpha, t)Ti^{46}$ reaction at 41 MeV. The errors shown are due only to statistical uncertainties. The t_i notation refers to the triton group designated in Fig. 2. The mean excitation energies in Ti^{46} corresponding to these groups can be inferred from Table I.

those potential parameters which minimized the χ^2 function defined by

$$\chi^2 \equiv \sum_{i=1}^{N} \left(\frac{\sigma_i^{\text{expt}} - \sigma_i^{\text{theoret}}}{\Delta\sigma_i^{\text{expt}}} \right)^2. \quad (5)$$

Four sets of equivalent parameters are given in Table II. The calculation using the "deep-well" parameters (set 4) is displayed along with the experimental results in Fig. 5. Since similar data were not available for the exit $t+Ti^{46}$ system, the triton optical-model parameters

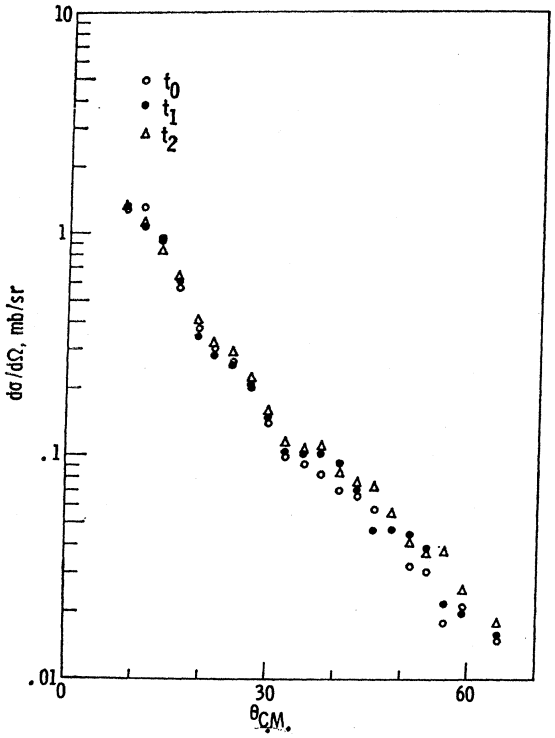

FIG. 4. Normalized angular distributions for the t_0, t_1, and t_2 triton groups in the $Sc^{45}(\alpha, t)Ti^{46}$ reaction.

were estimated from the 20-MeV triton elastic scattering results of Hafele *et al.*[23] The potential well depths were calculated from the expressions $V=0.057A+148$ and $W=-0.097A+29.4$. This dependence of V and W on A for $52\leq A\leq 116$ was deduced by Hafele *et al.* using fixed geometrical parameters. The complete set of parameters is listed as set 5 in Table II.

The bound-state wave function for the captured proton was an eigenfunction of a Woods-Saxon Hamiltonian with eigenenergy equal to the binding energy of the proton in the residual nucleus. The potential did not contain a spin-orbit term. The radius and diffuseness parameters of the potential functions were 4.5 and 0.65 F. The depth of the potential was chosen to reproduce the binding energy.

The numerical calculations employing the "deep-well" optical-model potential parameters (currently favored in the literature) for the incident $\alpha+Sc^{45}$ system are displayed in Fig. 6. Essentially equivelent fits can be obtained with any of the sets of optical-model potential parameters listed in Table II if a cutoff radius calculation is used. We find, however, that for the "deep-well" parameters the calculations are much less sensitive to choice of the cutoff radius and that the difference in magnitude and shape of the theoretical angular distributions for zero and nonzero cutoff calculations are less pronounced. The theoretical calculations were adjusted in magnitude by determining the normalization parameter α, which minimized the χ^2 function defined by

$$\chi^2=\sum_{\theta=8^\circ}^{35^\circ}\left(\frac{\alpha\sigma_\theta^{\text{theoret}}-\sigma_\theta^{\text{expt}}}{\Delta\sigma_\theta^{\text{expt}}}\right)^2. \quad (6)$$

The theoretical fits are seen to be quite satisfactory in the angular region 8°–35° and are reasonable at the larger angles. Relative spectroscopic strengths were extracted by assuming Barnard and Jones's value of 0.27 for the spectroscopic strength of the ground-state transition in the $Sc^{45}(He^3, d_0)Ti^{46}$ reaction. These spectroscopic strengths are listed in Table I and are displayed graphically in Fig. 7. Some appropriate $l_p=3$ spectroscopic strengths from the works of Broman and Pullen and of Barnard and Jones[9] are also shown. The strengths from the paper of Broman and Pullen have been arbitrarily normalized to the ground-state strength from Barnard and Jones. In view of the uncertainties in the experimental data and the theoretical fits the agreement between the spectroscopic strengths obtained from the (α, t) and (He^3, d) reactions is good. All spectroscopic strengths, with the possible exception of that for the t_8 transition, agree within the associated errors. The good agreement is probably due to the large proton orbital angular momentum transfer ($l_p=3$)

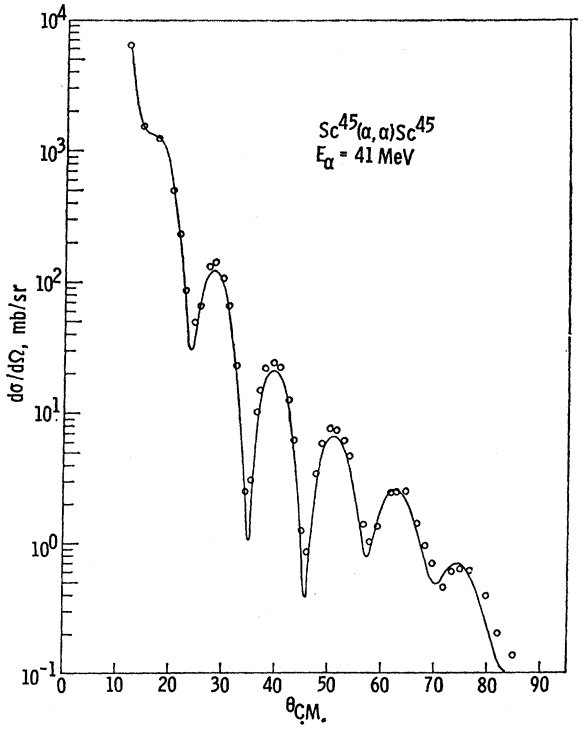

FIG. 5. Optical-model fit to the $Sc^{45}(\alpha, \alpha)Sc^{45}$ angular distribution for an α-particle energy of 41 MeV. The parameters used were from set 4 shown in Table II. The fits obtained with other sets of parameters shown in Table II were practically indistinguishable.

[23] J. C. Hafele, E. R. Flynn, and A. G. Blair, Phys. Rev. **155**, 1238 (1967).

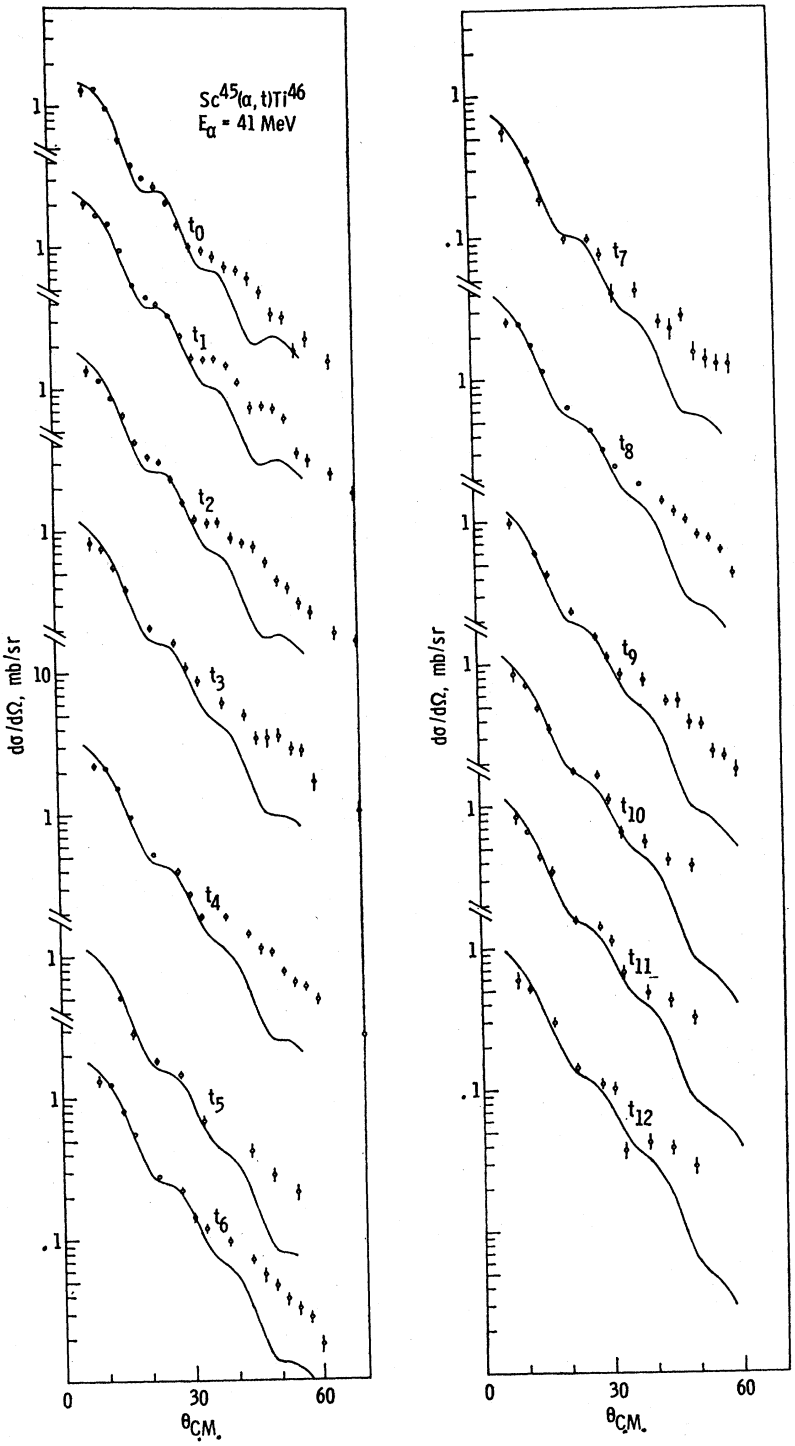

FIG. 6. Theoretical fits to the triton angular distributions for the $Sc^{45}(\alpha, t)Ti^{46}$ reaction using the DWBA calculations described in the text.

which, for the (α, t) reaction, favors the direct reaction mechanism.[3]

The wave functions of McCullen, Bayman, and Zamick[7] (hereafter referred to as MBZ) for Sc^{45} and Ti^{46} were used in the theoretical calculation of the spectroscopic factors. They assume that these nuclei are described by wave functions composed of a Ca^{40} core plus $Z-20$ protons and $N-20$ neutrons in the $1f_{7/2}$ shell. Experimental evidence from the $Sc^{45}(He^3, d)Ti^{46}$ reaction and comparison of theoretical and experimental energy spectra (Fig. 8) indicates that this is only an approximation since the wave function for many

TABLE II. Optical-model parameters obtained from fitting $Sc^{45}(\alpha, \alpha)Sc^{45}$ data at 41 MeV. The set with $V=200.2$ MeV was used in the DWBA calculations.[a]

Set particle	V (MeV)	r_0 (F)	a (F)	W (MeV)	r_0' (F)	a' (F)	r_{0c} (F)	χ^2	χ^2/N
1 α	27.0	1.7	0.591	11.7	1.7	0.591	1.7	125	2.72
2 α	64.6	1.6	0.538	14.3	1.6	0.538	1.6	86	1.87
3 α	103.6	1.515	0.544	17.5	1.515	0.544	1.515	104	2.22
4 α	200.2	1.395	0.565	26.4	1.395	0.565	1.395	144	3.16
5 Triton	150.6	1.24	0.678	25.0	1.45	0.841	1.24		

[a] The triton potential parameters were estimated from the 20-MeV triton elastic scattering results of Hafele *et al.* (Ref. 23).

of the excited states of Ti^{46} clearly contain admixtures of $2p_{3/2}$ configurations. They write the Ti^{46} and Sc^{45} wave functions in states α and β as

$$\Psi_{1234,p_1p_2}(Ti^{46}\,\alpha J_f M)=\sum_{L_n,\nu_n,L_p} D^{\alpha J_f}(L_p, L_n, \nu_n) \times[\Psi_{12}(j^2L_p)\Psi_{1234}(j^4\nu_nL_n)]_M{}^{J_f}, \quad (7)$$

$$\Psi_{1234,p_1}(Sc^{45}\,\beta J_i M)=\sum_{\nu'_n,L'_n} C^{\beta J_i}(\nu'_nL'_n) \times[\Psi_{1234}(j^4\nu'_nL'_n)\Psi_1(j)]_M{}^{J_i}, \quad (8)$$

where ν is the seniority quantum number, L is the angular momentum quantum number of the coupled $Z-20$ protons or $N-20$ neutrons, and $D^{\alpha J_f}$ and $C^{\beta J_i}$ are the components of the normalized eigenvectors. Following MBZ the theoretical spectroscopic factor for the $Sc^{45}(\alpha, t)Ti^{46}$ reaction becomes

$$S=2\,|\sum_{L_p,\nu_nL_n}(-1)^{L_n}D^{\alpha J_f}(L_p, \nu_nL_n)C^{1J_i}(\nu_nL_n) \times U(jjJ_fL_n; L_pJ_i)|^2, \quad (9)$$

where U is the normalized Racah coefficient in Jahn's

TABLE III. Single-proton-transfer theoretical spectroscopic strengths for the stripping reaction as deduced from the MBZ single-particle wave functions for Sc^{45} and Ti^{46}.

Excitation energy (MeV)	J_f	S	$\left(\frac{2J_f+1}{2J_i+1}\right)S$	Excitation energy (MeV)	J_f	S	$\left(\frac{2J_f+1}{2J_i+1}\right)S$
0.0000	0	2.0000	0.2500	5.6391	4	0.0012	0.0013
1.1009	2	1.3200	0.8250	5.6761	3	0.0696	0.0609
2.1726	4	0.3885	0.4370	5.6966	7	0.0534	0.1001
2.7710	2	0.3328	0.2080	5.9327	8	0.0000	0.0000
3.2186	4	0.00079	0.00089	5.9423	7	0.0720	0.1350
3.2639	6	0.3523	0.5724	5.7980	2	0.0000	0.0000
3.6944	2	0.1955	0.1221	6.0965	4	0.0533	0.0599
3.7482	3	0.0616	0.0539	6.1339	5	0.0205	0.0281
3.9031	4	0.8996	1.0120	6.2678	6	0.0004	0.0007
3.9966	1	0.0641	0.0240	6.4425	6	0.0752	0.1222
4.0224	5	0.0005	0.00069	6.5901	7	0.0117	0.0219
4.1870	6	0.8517	1.3840	6.6614	5	0.0046	0.0064
4.8897	6	0.0011	0.0017	6.8694	1	0.0032	0.0012
4.9821	8	0.0000	0.0000	6.9170	6	0.0059	0.0097
5.1973	0	0.0000	0.0000	7.0542	3	0.0000	0.0000
5.2026	5	0.0587	0.0807	7.1207	0	0.0003	0.0000
5.2110	4	0.0013	0.0014	7.1236	2	0.00041	0.0003
5.4751	3	0.0435	0.0380	7.1894	4	0.0061	0.0069
5.4989	2	0.0003	0.00019	7.2550	6	0.0237	0.0386

notation.[24] Results obtained for the evaluation of Eq. (9) for the first 38 states of Ti^{46} predicted by the MBZ model are shown in Table III. The numerical values of the C^{lJ_i} and $D^{\alpha J_f}$ were taken from Ref. 25. For excitations above 3 MeV in Ti^{46} the few spin and parity assignments are very tentative, and it is thus impossible to identify which states correspond to the MBZ predictions. The spin and parity of the first four states of Ti^{46} are predicted by the MBZ model and the level at 3.310 MeV which has been given a tentative assignment[26] of 6^+ appears to be the first 6^+ level predicted by the MBZ model. The MBZ spectroscopic factors are displayed along with the experimental values in Fig. 9. The agreement is quite satisfactory especially for those transitions shown to be pure $l_p=3$ angular momentum transfer in the (He^3, d) reaction. The good agreement between the MBZ and experimental spectroscopic factors for the 3.310-MeV state supports the contention that the spin and parity of this state is 6^+.

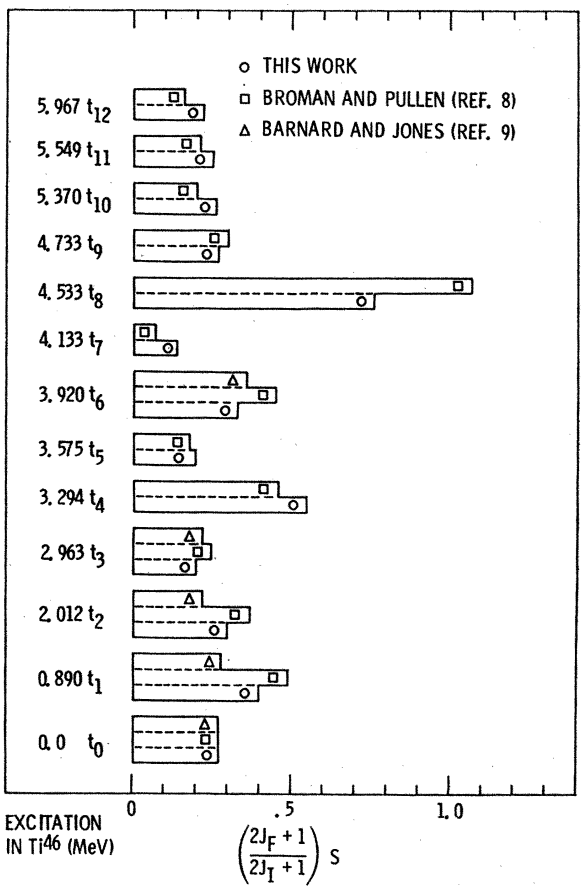

FIG. 7. Comparison of spectroscopic strengths for the (α, t) and (He^3, d) reactions on Sc^{45}. Only the $l_p=3$ component of the (He^3, d) spectroscopic strength is shown.

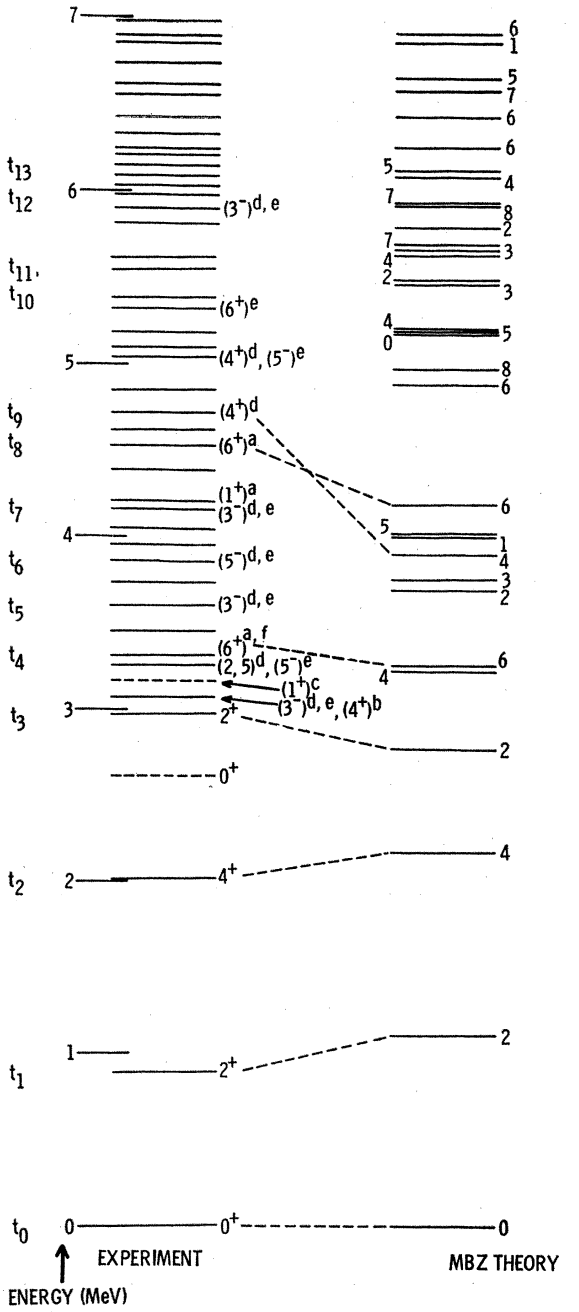

FIG. 8. Energy levels of Ti^{46} as reported by Broman and Pullen (Ref. 8). The t_i notation indicates those levels studied in the $Sc^{45}(\alpha, t)Ti^{46}$ reaction. The two levels shown as dashed lines were not reported by Broman and Pullen but have been by several other investigators. The interconnecting dashed lines indicate those levels which can possibly be identified with the predictions of McCullen *et al.* The legend for the spin assignments is a (Ref. 8), b (Ref. 17), c (Ref. 27), d (Ref. 15), e (Ref. 14), and f (Ref. 26).

It is, of course, difficult to make definite spin and parity assignments to the states of Ti^{46} on the basis of a

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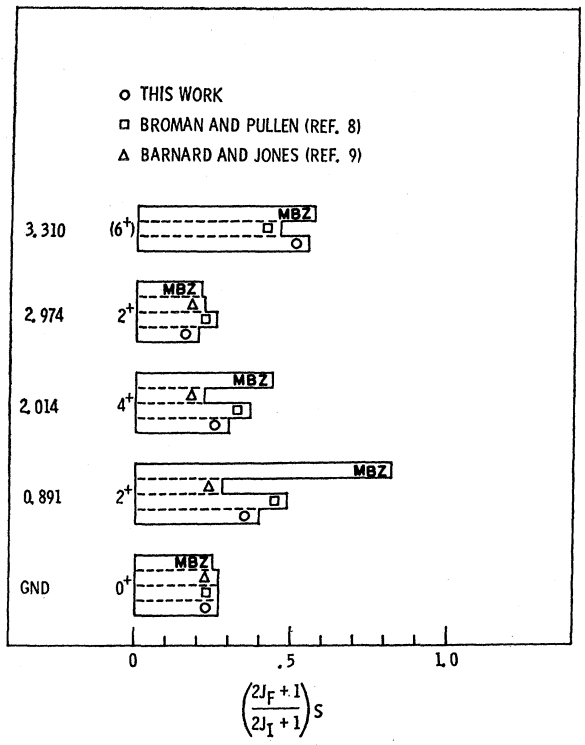

FIG. 9. Comparison of the theoretical spectroscopic strengths calculated from the Sc^{45} and Ti^{46} single-particle wave functions of MBZ with the experimental results of Refs. 8 and 9 and this work.

comparison of the experimental and theoretical spectroscopic strengths just presented. However, Table III shows particularly large spectroscopic strengths for a few transitions. It may be possible, then, to make some corroboration of a tentative assignment or some reasonable speculation for an assignment.

Broman and Pullen[8] have suggested that the states at 3.310, 4.206, and 4.533 MeV have spin and parity 6^+, 1^+, and 6^+, the reason being that these correspond to pure $l_p=3$ transitions and production of a 1^+ or 6^+ state by $l_p=1$ angular momentum transfer is forbidden by the selection rule for angular momentum. These states could correspond to the 6^+, 1^+, and 6^+ states at 3.2639, 3.9966, and 4.1870 MeV in the MBZ model. As discussed earlier the MBZ spectroscopic strength for the first 6^+ state is in good agreement with the experimental value for the 3.310-MeV state. The MBZ spectroscopic strength for the first 1^+ state is much lower than the experimental value for the 4.206-MeV state, so any verification of a tentative 1^+ assignment is dangerous. Since Lewis *et al.*[27] have identified a state at 3.17 MeV as having $J=1$ it would seem that this is a better candidate for the MBZ 1^+ state at 3.9966 MeV, since the next 1^+ state in the MBZ model occurs at 6.8694 MeV. The small MBZ spectroscopic strength for the state at 3.9966 MeV would explain why this state was not excited in either the high-resolution study of the $Sc^{45}(He^3, d)Ti^{46}$ reaction by Broman and Pullen[8] or the $Sc^{45}(\alpha, t)Ti^{46}$ reaction in the present study. The MBZ spectroscopic strength for the second 6^+ level is the largest of all the theoretical values and is in good agreement with the experimental value for the 4.533-MeV state. We can then substantiate Broman and Pullen's assignments to the 6^+ states but tend to disagree with the 1^+ assignment.

Figure 8 shows a number of odd-integral spin states. These have been identified as collective states[14,15] by means of inelastic scattering of α particles from Ti^{46}. Several of these states between 3.6- and 4.2-MeV excitation are excited both by the (α, t) and (He^3, d) reactions. Since odd-parity states are not predicted by the MBZ model we can make no statement about their assignments. However, the inelastic scattering studies[14,15] have yielded a tentative 4^+ assignment for the 4.723-MeV state.[15] This state is strongly excited in both the (He^3, d) and (α, t) experiments. The MBZ spectroscopic strength for the third 4^+ state at 3.9031 MeV is 1.01. This is, however, still a factor of 3–5 larger than the (α, t) and (He^3, d) spectroscopic strengths. Thus because of this large difference in the theoretical and experimental spectroscopic strengths and energies the corroborating evidence for this 4^+ assignment is weak.

Several states above 5-MeV excitation are strongly excited in both the (α, t) and (He^3, d) reactions. None of these states has been given spin assignments, so nothing quantitative can be said about them. The MBZ spectroscopic strengths (Table III) for this energy region are largest for the higher spins. Probably, then, these strongly excited states have spins in the range 3–7.

CONCLUSION

Transitions to the states of Ti^{46} via the $Sc^{45}(\alpha, t)Ti^{46}$ reaction have been shown to proceed primarily by $l_p=3$ proton angular momentum transfer, since the DWBA calculations for the stripping of a proton from the incident α particle yield reasonable fits to the triton angular distributions. Spectroscopic factors deduced from the DWBA analysis compare favorably with those deduced from a similar analysis of the $Sc^{45}(He^3, d)Ti^{46}$ reaction. Theoretical spectroscopic factors calculated in this work from the MBZ model for Ti^{46} are in reasonable agreement with experiment. The good over-all agreement between theory and experiment for both the (α, t) and (He^3, d) reactions on Sc^{45} indicates that the (α, t) reaction is a useful tool for nuclear spectroscopy where several units of orbital angular momentum are transferred.

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