Effect of np Repulsive Core in Λd Scattering*

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(Received 28 February 1969)

The Λd spin-$\frac{3}{2}$ elastic and total cross sections are calculated for incident Λ laboratory momentum $p_\Lambda \leq 400$ MeV/c using a Faddeev three-body formalism. Only ${}^3S_1$ ΛN and ${}^3S_1$ np interactions are used. The former is represented by a sum of two separable potentials, one attractive, the other a repulsive core. The np interaction is represented by two such potentials adjusted to both the low-energy data and the phase shift at 150 MeV in the c.m. system, or by a single separable potential adjusted to only the low-energy data. The presence of the np core potential changes the S-wave Λd cross sections by at most 15% (this at $p_\Lambda = 400$ MeV/c), while with all significant partial waves included the maximum change in the Λd cross sections is less than 2%.

I. INTRODUCTION

In a previous paper[1] a simple model was used to show that low-energy lambda-deuteron (Λd) elastic and total cross sections were strongly dependent on the presence (or absence) of a repulsive core in the S-wave lambda-nucleon (ΛN) interaction. In this paper for the same model it is shown that these cross sections are insensitive to the short-range behavior of the neutron-proton (np) potential.

The motivation for investigating the Λnp system is that it is the simplest multinucleon system containing a Λ. What is desired from a study of such a multiparticle system is that it say something about the two-body scattering amplitudes that cannot be obtained from two-body calculations and experiments directly. Both low-energy (say, incident Λ laboratory momentum $p_\Lambda \gtrsim 200$ MeV/c) Λp[2,3] and Λd[4] scattering experiments are now feasible. With the aid of a theoretical model low-energy ΛN scattering parameters (that is, the low-energy ΛN "on-shell" amplitude) may be extracted from the former.[2,3] A theoretical study of the latter could then be used to obtain information on the ΛN off-shell amplitude, or the ΛN on-shell amplitude at higher energies, or the ΛNN amplitude. At somewhat higher energies, near or above the threshold for $\Lambda N \rightarrow \Sigma N$, the sensitivity of the Λd elastic, breakup, and Σ production cross sections to various symmetry models for the ΛN interaction could be investigated.[5]

Certainly a major condition for the successful implementation of such an undertaking is that "noise" from the uncertainties as to the correct form for the np amplitudes does not wash out the information on the features of the ΛN and ΛNN amplitudes being studied. In fact, it is necessary that rather rough treatments of the np amplitude be adequate. Along with the complexities of the ΛN amplitude, which is a multichannel, spin-dependent, and possibly non-charge-symmetric amplitude, the necessity for using a "realistic" np potential (e.g., a full-blown one-boson-exchange potential) would make the calculation unfeasible.

The bound state of the Λnp system—the $J=\frac{1}{2}$, $T=0$ hypertriton ${}_\Lambda H^3$—would seem to be a prime candidate for study in this regard. Because the binding energy of the Λ is so small ($B_\Lambda = 0.06 \pm 0.06$ MeV)[6] the effect of the np ${}^3S_1$ amplitude in ${}_\Lambda H^3$ is completely dominated by the deuteron pole.[7] However, the present uncertainty in the experimental value of B_Λ is a serious drawback.[8] Not much can be gained from calculations of B_Λ using different sets of low-energy ΛN scattering parameters other than to confirm that the values obtained from analyses of Λp scattering experiments are the right size.[9]

For the Λd scattering problem considered here, the hope was that calculations could be carried out at energies that were, on the one hand, low enough that the deuteron pole still dominated the contribution of the ${}^3S_1$ np amplitude and, on the other hand, were high enough that the Λd scattering cross sections were sensitive to the details of the ΛN amplitude. With the use of a simplified model it was shown in Ref. 1 that, unlike the bound-state case, the presence of a ΛN repulsive core shows up very strongly in the Λd cross sections with $p_\Lambda = 100$–300 MeV/c. With the use of the same sort of simplified model it is shown below that the above hope is fulfilled for at least this same range of p_Λ.

The simplified problem considered was that of Λd

* A preliminary report of this work was given at the 1968 San Diego meeting of the American Physical Society, Bull. Am. Phys. Soc. 13, 1642 (1968).

[1] L. H. Schick, Nuovo Cimento Letters 1, 313 (1969).

[2] B. Sechi-Zorn, B. Kehoe, J. Twitty, and R. A. Burnstein, Phys. Rev. 175, 1735 (1968).

[3] G. Alexander, U. Karshon, A. Shapira, G. Yekutieli, R. Engelmann, H. Filthuth, and W. Lughofer, Phys. Rev. 173, 1452 (1968).

[4] G. Alexander (private communication).

[5] Results of this type of study could, of course, be analyzed further in variation calculations of light hypernuclear binding energies; see, e.g., Ref. 26 and work cited therein.

[6] B. Bohm, J. Klabuhn, U. Krecker, F. Wysotski, G. Coremans, W. Gajewski, C. Mayeur, J. Sacton, P. Vilain, G. Wilquet, D. O'Sullivan, D. Stanley, D. H. Davis, E. R. Fletcher, S. P. Lovell, N. C. Roy, J. H. Wickens, A. Filipkowski, G. Garbowska-Pniewska, T. Pniewski, E. Skrzypczak, T. Sobczak, J. E. Allen, V. A. Bull, A. P. Conway, A. Fishwick, and P. V. March, Nucl. Phys. B4, 511 (1968).

[7] See, e.g., B. Ghaffary Kashef and L. H. Schick, Nuovo Cimento 50B, 395 (1967).

[8] The three-nucleon system does not suffer from these defects. See, e.g., R. Stagat, Ph.D. thesis, Rensselaer Polytechnic Institute, 1968 (unpublished); or A. N. Mitra, Rutherford Laboratory Report No. RPP/A 50, 1968 (unpublished).

[9] A. J. Toepfer and L. H. Schick, Phys. Rev. 175, 1253 (1968).

scattering in the spin-$\frac{3}{2}$ state with only S-wave two-body potentials present. The neutron and proton were taken to be identical spin-$\frac{1}{2}$ particles so that with the Λ having spin $\frac{1}{2}$, only two potentials ($^3S_1$ *np* and $^3S_1$ ΛN) were needed. Each of these was represented by a nonlocal separable (NLS) potential or a sum of two NLS potentials. These potentials are discussed in Sec. II. A Faddeev[10] type of multiple-scattering formalism that incorporated these potentials was used to calculate elastic and total Λd cross sections for c.m. energies $E \lesssim 43$ MeV (i.e., $p_\Lambda \leq 400$ MeV/c).[11] The results of these calculations are given and discussed in Sec. III.

The "model" nature of these calculations must be emphasized. Even at the low energies considered here, tensor forces as well as non S-wave forces in both the *np* and the ΛN interactions would not be negligible. Further, even though the threshold for Σ production is $\approx$40 MeV above the highest energy considered, the results of Ref. 9 on the effect of Λ-Σ conversion in $_\Lambda H^3$ indicate the effect of the closed Σ channel is not negligible. The values used for various ΛN potential parameters as well as the numerical results obtained for the Λd cross sections may bear little resemblance to the "correct" values of a more realistic calculation. However, what is being discussed in this work—the effect of the *np* $^3S_1$ repulsive core potential on the Λd cross sections—is in all practicality independent of these other considerations.

II. ΛN AND *NP* POTENTIALS

Each $^3S_1$ two-body potential was taken to have the configuration space matrix elements

$$V(r,r')=\lambda_1 v_1(r)v_2(r')+\lambda_2 v_2(r)v_2(r'), \qquad (1)$$

where

$$v_j(r)=(4\pi r)^{-1}\exp(-\beta_j r) \qquad (j=1,2) \qquad (2)$$

and $r(r')$ is the distance between the two particles. This potential has been previously discussed in some detail.[12] Conventionally the subscript 1 was used for the attractive potential and the subscript 2 for the repulsive potential. To represent a purely attractive potential $\lambda_2=0$ was used. Otherwise, to insure a strong repulsion at short distances,

$$\lambda_2 > -\lambda_1 > 0 \quad \text{and} \quad \beta_1 < \beta_2$$

were used. Potential 2 was therefore designated as the "core" and $V(r,r')$ with $\lambda_2=0$ designated a "no-core" potential.

It is straightforward to show that for two particles of reduced mass μ at a c.m. energy $\mathcal{E}$ the potential given above yields (in units where $\hbar=1$) the following form for the off-shell momentum-space matrix elements of the t matrix $t(p,q;k=(2\mu\mathcal{E})^{1/2})$:

$$t(p,q;k) = \sum_{i=1}^{2}\sum_{j=1}^{2} v_i(p)\tau_{ij}(k)v_j(q), \qquad (3)$$

where

$$v_i(p)=1/(p^2+\beta_i^2),$$

$$\tau_{ii}=\lambda_i[1-\lambda_j g_{jj}(k)]/D(k) \qquad (i\neq j),$$

$$\tau_{ij}=\lambda_i\lambda_j g_{ij}(k)/D(k) \qquad (i\neq j),$$

and

$$g_{ij}(k)=(-\mu/2\pi)/[(\beta_i+\beta_j)(\beta_i-ik)(\beta_j-ik)].$$

The Fredholm denominator $D(k)$ has the form

$$D(k)=[1-\lambda_1 g_{11}(k)][1-\lambda_2 g_{22}(k)]-\lambda_1\lambda_2[g_{12}(k)]^2. \qquad (4)$$

The on-shell scattering amplitude $f(k)$ is given by

$$f(k)=-(\mu/2\pi)t(k,k;k), \qquad (5)$$

but $t(p,q;k)$ itself is referred to below as the two-body (off-shell) amplitude.

Ideally in the true spirit of this work the above potential parameters should be determined by direct appeal to the experimental phase shifts. However, for the ΛN potential such phase shifts are not well enough determined to serve the purpose of fixing the potential parameters. For the *np* potential it makes more sense to use the phase shifts predicted by local potentials that give good results for the physical parameters of three- and four-nucleon systems than to fit experimental phase shifts at high energies that do not directly enter the three-body problem at hand.

For the ΛN potential the $^3S_1$ scattering length a_Λ and effective range $r_{0\Lambda}$ determined from the charge-symmetric (CS) part of the Herndon-Tang (HT) potential H[13]

$$a_\Lambda=-1.95\ \text{F}, \qquad r_{0\Lambda}=3.50\ \text{F} \qquad (6)$$

were the low-energy ΛN scattering parameters fit in all cases. The $^3S_1$ part of this HTCS potential is a local potential consisting of a hard core inside an attractive exponential well:

$$V(r)=\infty \qquad r<r_c$$
$$=-V_0\exp[-\kappa(r-r_c)] \qquad r>r_c \qquad (7)$$

[10] L. D. Faddeev, *Mathematical Aspects of the Three-Body Problem in Quantum Scattering Theory* (Daniel Davey and Co., Inc., New York, 1965).
[11] Nonrelativistic kinematics were used throughout this work.
[12] L. H. Schick and J. H. Hetherington, Phys. Rev. 156, 1602 (1967).
[13] R. C. Herndon and Y. C. Tang, Phys. Rev. 159, 853 (1967).

TABLE I. ΛN $^3S_1$ potentials with $a_\Lambda=-1.95$ F and $r_{0\Lambda}=3.50$ F.

Potential	$1/\beta_1$ (F)	$1/\beta_2$ (F)	$-\lambda_1$ (MeV)$^2/(20\pi)^3$	λ_2 (MeV)$^2/(20\pi)^3$	$\delta_{0\Lambda}$ (deg) at c.m. energy (MeV) 20	40	60	80
N	0.766	···	0.952	0.0	25.20	20.20	16.30	14.00
B	0.335	0.300	2326	5898	21.95	11.88	2.854	−4.933
Local					21.96	11.61	2.215	−6.074

with $V_0=676.9$ MeV, $\kappa=3.935$ F^{-1}, and $r_c=0.60$ F.[13] When a no-core NLS ΛN potential was used, a_Λ and $r_{0\Lambda}$ as given in Eq. (6) were sufficient to determine all the ΛN potential parameters, namely, λ_1 and β_1. In addition, when a core was included, the core range $1/\beta_2$ was fixed at some arbitrary value and the value of the $^3S_1$ phase shift $\delta_{0\Lambda}$ at an appropriate energy was matched to that calculated from the potential of Eq. (7). It was shown in Ref. 1 that the Λd cross sections were independent of the precise value of β_2 used in this procedure if $1/\beta_2$ lay in the range 0.2–0.35 F.[14] Since in most of the Λd calculations performed here the ΛN c.m. energy was $\lesssim 20$ MeV, this value was the energy at which $\delta_{0\Lambda}$ was fit.

The parameters for the ΛN potentials used in this work are shown in Table I. Potential N is the no-core potential. Most of the results of the Λd calculations with this potential were taken from Ref. 1. They were used here for comparison of the effect of a ΛN core relative to the effect of an np core in the three-body problem.

The bulk of the Λd calculations were performed using potential B of Table I. The phase shift $\delta_{0\Lambda}$ for this potential, as well as that for potential N and the local potential of Eq. (7), at ΛN c.m. energies from 20 to 80 MeV are also given in this table. Note that $\delta_{0\Lambda}$ for B gives a good fit to the local potential $\delta_{0\Lambda}$ throughout most of this range. Note also how rapidly $\delta_{0\Lambda}$ for B differs from that of N as the energy increases. This is due to the relatively long core range, $1/\beta_2$ being almost as large as $1/\beta_1$. This in turn may be related to the lack of a long-range one-pion-exchange potential as exists in the np interaction.

For the $^3S_1$ np potential a procedure almost identical to that used for the ΛN potential was carried out. The low-energy "scattering" parameters used here were the deuteron binding energy ϵ and the $^3S_1$ scattering length

$$\epsilon=2.225 \text{ MeV} \quad \text{and} \quad a=5.3858 \text{ F}. \tag{8}$$

The local potential used to fix the phase shift at higher energies was taken from Herdon and Tang.[15] It has the same form as the potential of Eq. (7) but with $V_0=549.26$ MeV, $\kappa=2.735$ F^{-1}, and $r_c=0.45$ F. The phase shift δ_0 from the sum of NLS potentials was chosen here to vanish at the same value of the np c.m. energy (156.35 MeV) as did that from this local potential. This energy for $\delta_0=0$ is somewhat higher than the value 120 MeV used by other authors,[16] but this local potential does give good results in the two-, three-, and four-nucleon systems.[15]

Table II contains the potential parameters, effective range, and phase shifts for a number of different separable potentials as well as the phase shifts obtained from the HT local np potential. Note how much shorter the range of the core is here (as compared to the range of the attractive potential) than in the ΛN potential B and the consequent slow deviation of the phase shifts for potentials with cores from the no-core phase shift as the energy increases. Note also the large variation in λ_2 (two orders of magnitude) between potentials 1 and 3.

For completeness and to aid in the later discussion the poles in the k plane, k_1 through k_4, of the off-shell two-body np amplitude $t(p,q;k)$ are given in Table III for the NLS np potentials listed in Table II. These singularities were found by setting $D(k)$ of Eq. (5) equal to zero for each set of potential parameters. Note that the introduction of the core leads to two new poles, k_3 and k_4. These are very far away from the region of interest (see Sec. III) in either the low-energy two- or three-body problems. The fact that they are not on the imaginary k axis, as they should be,[17] is of little consequence. The k_1 np singularity is of course the deuteron pole.[18]

The on-shell amplitudes have further poles at $k=i\beta_1$ and (when a core is present) $k=i\beta_2$, where the $v_j(k)$ become infinite. These are denoted k_5 and k_6 in Table III. They are "range singularities," being the analogs of the well-known infinite set of poles on the positive imaginary k axis that occurs when the local potential of Eq. (7) is used to calculate the on-shell amplitude. The two-body amplitudes that appear in the three-body problem are, of course, off-shell amplitudes.

Finally, the deuteron radial wave function $u(r)=$

[14] This is a reasonable range to consider because it may be shown that for $\lambda_2\to\infty$ $2/\beta_2$ plays the role of the core radius when the intrinsic ranges of the potentials in Eqs. (1) and (7) are compared.

[15] R. C. Herndon and Y. C. Tang, Phys. Rev. **153**, 1091 (1967).

[16] See, e.g., F. Tabakin, Phys. Rev. **174**, 1208 (1968).

[17] T. R. Mongan, Phys. Rev. **175**, 1260 (1968).

[18] The Λd amplitude has k_1 on the negative imaginary k axis but the rest of its singularity structure is similar to that of the np amplitude.

TABLE II. *np* $^3S_1$ potentials with $a=5.3858$ and $\epsilon=2.225$.

Potential	$1/\beta_1$ (F)	$1/\beta_2$ (F)	$-\lambda_1$ (MeV)^2/$(20\pi)^3$	λ_2 (MeV)^2/$(20\pi)^3$	r_0 (F)	δ_0 (deg) at c.m. energy (MeV) 30	90	150
0	0.6952	···	3.33	0.0	1.7266	59.35	32.93	22.61
1	0.4981	0.155	29.9	49 950	1.7357	55.72	19.97	1.529
2	0.4973	0.165	29.4	4 288	1.7358	55.71	19.96	1.528
3	0.4858	0.225	32.0	400.1	1.7359	55.65	19.84	1.510
4	0.4550	0.300	54.3	180.7	1.7362	55.54	19.62	1.474
Local					1.7287	56.49	21.11	1.686

$r\psi(r)$ is given by

$$u(r)=N[\lambda_1\Phi_1(r)+\lambda_2'\Phi_2(r)], \qquad (9)$$

where

$$\Phi_j(r)=[\exp(-\beta_j r)-\exp(-\alpha r)]/(\beta_j^2-\alpha^2),$$

$$\lambda_2'=\lambda_2\lambda_1 g_{12}(i\alpha)/[1-\lambda_2 g_{22}(i\alpha)],$$

$$\alpha=(2\mu\epsilon)^{1/2},$$

and N is a normalization factor. This function is plotted in Fig. 1 for three of the NLS potentials given in Table II. The wave function for the potentials with λ_2 nonzero only differ significantly from the no-core wave function for $r\lesssim 0.7$ F. Unlike the wave function for a local hard-core potential [such as given in Eq. (7)] the wave functions for the sums of the NLS potentials used here do not vanish completely for small r, but they do become very small and they change sign. Potentials which give a ground-state wave function with a node[16] have been previously unfavorably criticized by other authors.[19] The Λd bound-state and scattering-state results discussed in Sec. III, however, indicate that no great violence has been done to the three-body calculations by the use of the *np* potentials described above.

III. Λd SCATTERING

The details of the Faddeev equations as applied to the present problem have been covered elsewhere.[20] The modifications of previous work to accommodate the sums of NLS potentials for the two-body interactions used here are straightforward and will not be discussed further except for the following two points.

First, since the number of coupled one-dimensional integral equations to which the three-body formalism can be reduced is of interest, it should be noted that for spin-$\frac{3}{2}$ Λd scattering with the central S-wave ΛN and *np* potentials used here the number of equations is the same as for three spinless particles. With the neutron and proton treated as identical particles there are two, three, or four such equations depending on the use of a core potential in none, one, or both two-body interactions. In fact, for the elastic-scattering problem the multiple-scattering amplitude which begins (or ends) with an *np* scattering may be entirely eliminated from the formalism.[21] This reduces the number of equations to one or two depending only on the absence or presence of the ΛN core *independent* of the number of separable terms used to represent the *np* interaction. Although it was not needed here, this latter procedure will no doubt prove useful when a separable series expansion for the t matrix of a local potential[22,23] is used for the *np* interaction.

Second, most of the scattering calculations were performed for energies above the threshold for deuteron breakup. For these calculations the on-shell relative momentum variable for any pair of particles, k of Eq. (4), was given by $k=(2\mu W)^{1/2}$, with

$$W=E-\tfrac{1}{2}q^2(M^{-1}+m^{-1}) \qquad (10)$$

and Im$k>0$. Here μ is the reduced mass and M is the total mass of the scattering pair, m is the mass of the third particle, and E is the c.m. energy at which the Λd scattering occurs. E is positive (negative) above (below) the deuteron breakup threshold. The integration variable in the Faddeev formalism is q, the momentum of the third particle relative to the c.m. of the scattering pair. The original contour of integration lies along the positive real q axis with the usual $i0^+$ being added to E to define the integrals. Here the contour rotation method[24] was used to smooth the integrands of

[19] J. E. Levinger, A. H. Lu, and R. Stagat, Rensselaer Polytechnic Institute Report (unpublished); and Bull. Am. Phys. Soc. **13**, 1401 (1968).

[20] J. H. Hetherington and L. H. Schick, Phys. Rev. **139**, B1164 (1965).

[21] G. Doolen, Ph.D. thesis, Purdue University, 1968 (unpublished).

[22] M. G. Fuda, Phys. Rev. **174**, 1134 (1968).

[23] J. S. Ball and D. Y. Wong, Phys. Rev. **169**, 1362 (1968).

[24] J. H. Hetherington and L. H. Schick, Phys. Rev. **137**, B935 (1965).

TABLE III. k-plane poles of the np amplitude.

Potential	k_1 (MeV/c)	k_2 (MeV/c)	$k_{3,4}$ (MeV/c)	k_5 (MeV/c)	k_6 (MeV/c)
0	$i45.706$	$-i330$	$\cdots$	$i284$	$\cdots$
1	$i45.706$	$-i835$	$\pm 19\ 064 - i1274$	$i396$	$i1273$
2	$i45.706$	$-i815$	$\pm 5\ 722 - i1207$	$i397$	$i1196$
3	$i45.706$	$-i725$	$\pm 1\ 924 - i943$	$i406$	$i877$
4	$i45.706$	$-i655$	$\pm 1\ 264 - i787$	$i434$	$i658$

the integral equations; i.e., the replacement

$$q \to x \exp(-i\rho) \qquad (0<\rho<\tfrac{1}{4}\pi)$$
$$0 \leq x \leq +\infty \qquad (11)$$

was used.

In Fig. 2 the original contour and the rotated contour in the k plane are both shown. The segment $0\leq k\leq (2\mu E)^{1/2}$ is just the physical region where the on-shell two-body t matrix has been adjusted to predetermined values. From Table III none of the singularities of the np amplitude (or of the ΛN amplitude either[18]) has been crossed by the contour during the process of distortion.

For all values of E considered the Λd elastic angular distribution was calculated directly from a partial-wave expansion of the Λd elastic scattering amplitude, the elastic cross section was obtained by integration of this angular distribution, and the total cross section was obtained, by use of the optical theorem, from the forward-scattering amplitude. The results for the cross sections are given in Tables IV and V.[25]

Table IV contains the S-wave Λd S-matrix element (S_0) as well as the elastic (σ_{el}) and total (σ_{tot}) Λd cross sections. Each of these is shown for five values of the incident Λ laboratory momentum p_Λ—or, equivalently, the Λd c.m. energy E—and, as labeled in the right-hand columns, different combinations of the ΛN and np potentials of Tables I and II, respectively. For each value of p_Λ, row 1 was obtained with purely attractive ΛN and np potentials, row 2 was obtained with a core only in the ΛN interaction, and row 3 was obtained with a core in both interactions. Due to limitations of the numerics used in these calculations the values for σ_{el} and σ_{tot} are estimated to be accurate only to within 1%. For example, it is reasonable that the differences that exist between the use of the np potentials 0 and 3 in σ_{el} and σ_{tot} for the ΛN potential B and a fixed p_Λ be as shown, but these differences cannot be guaranteed to be real.

The first thing to note in this table is that for a fixed set of potentials, $S_0=\exp(2i\delta_0)$, considered as an energy-dependent vector in the complex plane with its

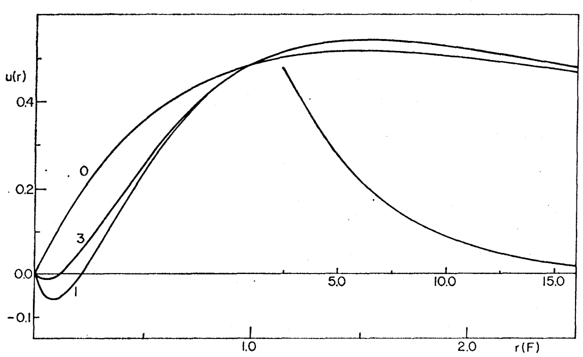

FIG. 1. The deuteron radial wave function for three of the potentials of Table II. The distance scale on the $u=0$ axis applies only to the tail of the wave function.

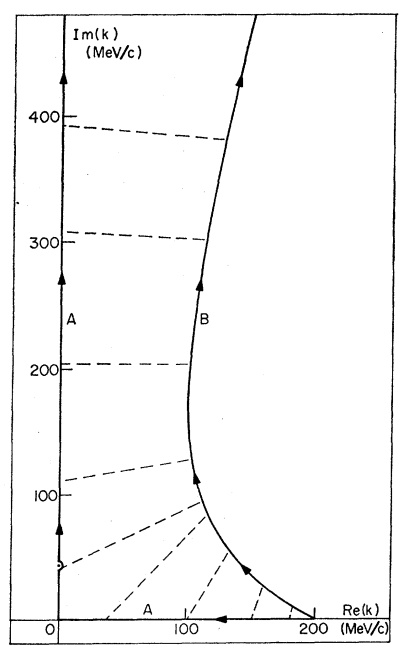

FIG. 2. Typical paths of integration in the np k plane for the original (A) and the rotated (B) q-plane contour. The dashed lines connect corresponding points on the two curves. The case shown is $E=42.8$ MeV and $\theta=15°$. The deuteron pole is at $k=i45.706$ MeV/c.

[25] All numerical work was performed on the Honeywell H-800 at the U.S.C. Computer Science Laboratory.

TABLE IV. Λd scattering.

P_Λ (MeV/c)	E (MeV)	S_0	σ_{el} (mb)	σ_{tot} (mb)	Potential np	ΛN
50	−1.52	$-0.891+i.455$	4700	4700	0	N
		$-0.962+i.262$	4870	4870	0	B
		$-0.955+i.293$	4870	4870	3	B
100	0.59	$-0.950+i.680$	1210	1220	0	N
		$-0.702+i.694$	1060	1070	0	B
		$-0.688+i.708$	1050	1060	3	B
200	9.02	$-0.277+i.741$	194	244	0	N
		$0.078+i.887$	153	190	0	B
		$0.091+i.891$	151	188	3	B
300	23.1	$0.278+i.727$	53.4	92.4	0	N
		$0.662+i.640$	34.0	60.8	0	B
		$0.676+i.631$	33.5	59.9	3	B
400	42.8	$0.609+i.570$	16.6	40.2	0	N
		$0.944+i.260$	6.43	17.5	0	B
		$0.950+i.246$	6.23	17.4	3	B

tail at the origin, is rotating counterclockwise as E increases from $-\epsilon$; i.e., as the Λd KE increases from zero. In all the cases shown there is a Λnp $J=\frac{3}{2}$ bound state; $\delta_0=\pi$ at zero KE. The binding energy B_Λ of the Λ was calculated for the potential combinations shown in the table. It was found that $B_\Lambda \lesssim 0.01$ MeV in all cases. B_Λ decreases when a ΛN core is used but, as in a previous work on the $J=\frac{1}{2}$ hypertriton bound state,[7] the np core has no effect on B_Λ. The existence of this quartet bound state is not peculiar to NLS potential calculations. Its presence has been noted before by Herndon and Tang.[26] Because of its small size and the uncertain-

TABLE V. Λd S-wave scattering.

P_Λ (MeV/c)	E (MeV)	S_0	σ_{el}^0 (mb)	σ_{tot}^0 (mb)	Potential np	ΛN
300	23.1	$0.267+i.729$	37.6	51.6	0	N
		$0.662+i.640$	18.1	23.4	0	B
		$0.683+i.628$	17.1	21.9	1	B
		$0.682+i.629$	17.1	22.0	2	B
		$0.676+i.631$	17.4	22.4	3	B
		$0.673+i.632$	17.5	22.6	4	B
400	42.8	$0.614+i.567$	9.15	15.0	0	N
		$0.944+i.260$	1.37	2.19	0	B
		$0.952+i.241$	1.17	1.86	1	B
		$0.952+i.242$	1.19	1.88	2	B
		$0.950+i.246$	1.22	1.96	3	B
		$0.948+i.248$	1.25	2.01	4	B

[26] R. C. Herndon and Y. C. Tang, Phys. Rev. **165**, 1093 (1968).

ties in the experimental values of the ΛN scattering parameters, it is certainly not to be worried over at this time.

The primary conclusions of this work may also be drawn from Table IV. For each value of $p_\Lambda \leq 400$ MeV/c a comparison of the cross sections given in rows 2 and 3, on the one hand, and a comparison of the cross sections in rows 1 and 2, on the other, shows that inclusion of the *np* core in this calculation changes the cross sections by a negligibly small amount. The percentage change in either σ_{el} or σ_{tot} is in almost every case $<2\%$. Further, the absolute change in σ_{el} or σ_{tot} due to the *np* core is between one and two orders of magnitude smaller than the corresponding change due to the ΛN core.

Although S-wave Λd scattering completely dominates the elastic and total cross sections at the low-energy end of Table IV, for the higher energies shown higher partial waves become important. A comparison at these higher energies of the Λd P- and D-wave S-matrix elements when cores are used in both ΛN and *np* interactions with the case when only ΛN cores are used showed that the *np* core has, within the accuracy ($\approx 1\%$) of the calculations, absolutely no effect. Further work was therefore confined to just S-wave Λd scattering.

In Table V the Λd elastic and total S-wave cross sections, $\sigma_{tot}{}^0$ and $\sigma_{el}{}^0$, respectively, along with S_0, are given for the two highest energies used in Table IV. At each of these energies not only were results obtained for no cores (first rows) and ΛN cores only (second rows), but also for a ΛN core plus an *np* core with each of the four different *np* cores of Table II being employed (rows 3–6 at each energy).

At $p_\Lambda = 300$ MeV/c the presence of the *np* core decreases the Λd cross sections by 3–7% while at $p_\Lambda = 400$ MeV/c this decrease measures 9–18%, depending on which *np* core potential is used. A comparison of the size of the $p_\Lambda = 400$ MeV/c cross sections here with those in Table IV shows that the effect of the *np* core is cut down to $\approx 2\%$ by the fact that the S-wave cross section contributes such a small part to σ_{el} or σ_{tot} at this energy. Thus, results for σ_{el} or σ_{tot} calculated with any of the Table-II NLS *np* potentials not used in Table IV will not differ significantly from those in Table IV calculated with potential 3. The effect of the *np* core at the two higher values of p_Λ is certainly consistent with the above discussion of Fig. 2 and the *np* phase shifts at 30 MeV shown in Table II.

The reason for this insensitivity of the low-energy Λd cross sections to the *np* core is undoubtedly the presence of the deuteron pole in the *np* amplitude. From Table III and Fig. 2 it is seen that not only is the deuteron pole that singularity of the off-shell *np* amplitude that lies closest to the original integration contour in the k plane, it is the only one that lies "right in the middle" of this contour; i.e., it lies on the contour at a value of k for which the contribution of the *np* amplitude to the Λd scattering amplitude is not drastically reduced by the small values of the other functions (e.g., the propagator, or the deuteron wave function itself) that get folded into the calculation. Because of its location, the deuteron pole makes the effects of variation of the location (and the residues) of the other poles in the *np* amplitude unimportant.

It should not be thought, however, that the values of the *np* phase shift δ_0 play no role in the calculation. It is clear that the above argument breaks down if the core strength is made so large (i.e., if δ_0 goes to zero at a much faster rate than the δ_0 used here) that the *np* amplitude for a potential with a core differs significantly from the no-core amplitude even for relatively small values of k.

By way of contrast there is the strong dependence of the Λd cross sections as shown in Table IV on the presence of the ΛN core. The ΛN amplitude *does not* have a bound-state pole. The ΛN S-wave phase shift with the ΛN core present *does* go to zero relatively rapidly compared to the no-core phase shift.

Calculations extending the present work to other forms of *np* potential are in progress. It is of special interest to test whether the sensitivity of the Λd cross sections to the presence of the *np* core is significantly increased by the use of an *np* potential whose deuteron wave function at short distances more closely approximates that of a local potential with a hard core. This is one feature which might prevent any arbitrary *np* potential fit to the deuteron budding energy and the low-energy scattering data from yielding Λd scattering results similar to those found here.