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Nernst Effect and Flux Flow in Superconductors. II. Lead Films*

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We have studied thermally induced and current-induced flux motion through the Nernst effect and the flux-flow resistivity, respectively, in superconducting lead films. The film thickness ranged between 1 and 7 μ. In addition to the transport entropy of a fluxoid and the flux-flow resistivity, the critical temperature gradient and the critical current, at which thermally induced or current-induced flux motion sets in, were determined. At 375 Oe and 4.2°K, the transport entropy per unit flux S_φ/φ was found to increase with increasing film thickness up to a thickness of about 3 μ. For thicker films, S_φ/φ decreased with increasing film thickness. For a film thickness near 3 μ, S_φ/φ was close to the value expected for a large flux bundle from the difference in entropy density of normal and superconducting material. The critical current density and thereby the critical Lorentz force per unit length of fluxoid were found to increase strongly with decreasing film thickness, whereas the critical thermal force varied little with film thickness. The critical apparent Lorentz force was larger than the critical thermal force by 2–3 orders of magnitude, the discrepancy increasing with decreasing film thickness. These results suggest that in a thin film the critical current flows predominantly along the surface in such a pattern that there is very little interaction with the fluxoids in the film. Apparently, at and below the critical current, the electrical current flow in a thin film is such that it contributes only very little to the Lorentz force on the fluxoids.

I. INTRODUCTION

FLUX flow under the influence of an electrical current and the longitudinal voltage associated with it has been studied in the mixed or intermediate state of a superconductor in a number of experiments.[1–6] These current-voltage measurements are usually carried out with a flat, long superconducting specimen in a transverse magnetic field. For all materials investigated, the voltage has been found to increase linearly with current for currents somewhat larger than J^*, the minimum current necessary to cause a nonvanishing voltage. At currents just above J^*, the voltage-current curves show some curvature before they enter the linear region. Jones *et al.*[7] suggested recently that this nonlinearity in the voltage-current characteristics at intermediate currents may be because of inhomogeneities in the flux-pinning property of the specimens.

Recent experiments have shown[8] that the surface of a type-II superconductor can support a transport supercurrent. Swartz and Hart[8] have suggested that, in a rather defect-free specimen, the critical currents are surface currents and that these surface currents increase as the magnetic field becomes oriented parallel to the surface. Swartz and Hart have proposed a surface flux-pinning model. According to this model, the fluxoids are pinned at surface pinning sites such that there will be no flux motion below a critical surface current. This view has been challenged by Joiner and Kuhl,[5] who argued that it is the *bulk* critical current which is influenced by surface pinning.

The question whether the critical current flows through the bulk of the specimen with homogeneous density or whether it is highly inhomogeneous, being large at some preferred current paths, say, along the surface, and small in the remainder of the sample, cannot be answered by flux-flow resistivity measurements alone. However, this question may be answered to some extent by studying flux motion under the influence of a temperature gradient in combination with current-induced flux motion.[9]

We have studied the flux flow under the influence of a temperature gradient in lead films through the Nernst effect. Also the longitudinal voltage caused by an electrical current was measured with the same specimens. The film thickness ranged between 1 and 7 μ. From an extrapolation of the linear part in the plots of voltage versus temperature gradient and voltage versus current to zero voltage, the critical temperature gradient and the critical current were obtained, respectively. The critical current density and thereby

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[7] R. G. Jones, E. H. Rhoderick, and A. C. Rose-Innes, Phys. Letters **24A**, 318 (1967).

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[9] R. P. Huebener and A. Seher, preceding paper, Phys. Rev. **181**, 702 (1969), hereafter referred to as I.

TABLE I. Electrical resistance ratio, critical current density, critical temperature gradient, the ratios S_1/S_2 and $|j_c/(\partial T/\partial x)_c|$, and the critical thermal force at 4.2°K and 375 G for Pb films of different film thickness.

Sample No.	52	48	49	65g	66g	64g	63g
Film thickness (μ)	0.89	1.8	2.9	4.3	4.3	6.6	6.6
$R(295°\text{K})/R(4.2°\text{K})$[a]	250	280	385	580	600	980	840
$j_c(A/\text{cm}^2)$	1200	125	50.4	31	74	25	28
$(\partial T/\partial x)_c(°\text{K/cm})$	1.86	0.71	0.29	0.23	0.34	0.59	0.8
$(S_1/S_2)\,(\text{Oe}/°\text{K})$	0.50	2.3	6.85	3.8	4.5	1.6	1.4
$\left\lvert\frac{j_c}{(\partial T/\partial x)_c}\right\rvert\left(\frac{\text{Oe}}{°\text{K}}\right)$	811	220	221	170	272	52	45
$\frac{S_1}{S_2}\left(\frac{\partial T}{\partial x}\right)_c\left(\frac{A}{\text{cm}^2}\right)$	0.74	1.30	1.58	0.70	1.22	0.7	0.89
$\left\lvert\frac{j_c/(\partial T/\partial x)_c}{S_1/S_2}\right\rvert$	1620	96	32	45	61	32	33

[a] $R(4.2°\text{K})$ is measured in a transverse magnetic field of 600 Oe.

the critical Lorentz force per unit length of fluxoid were found to increase strongly with decreasing film thickness, whereas the critical thermal force varied little with film thickness. The critical apparent Lorentz force was larger than the critical thermal force by 2–3 orders of magnitude, the discrepancy increasing with decreasing film thickness. These results suggest that in a thin film the critical current flows predominantly along the surface in such a pattern that there is very little interaction with the fluxoids in the film and that the critical current contributes only very little to the Lorentz force on a fluxoid.

From the data the transport entropy per flux, associated with the flux bundles in the films, was determined as a function of film thickness. Preliminary results of the present investigation have been reported earlier.[10–12] The theoretical scheme used in the data analysis and an explanation of the terminology used below can be found in I.

II. EXPERIMENTAL

The lead films were deposited on a glass slide under a vacuum of about 1×10^{-6} mm Hg. The specimen material was 99.999% pure lead. The depositions were made using Joule-heated tantalum boats. The distance between the boat and the glass slide was about 15 cm. During the deposition the glass slide was at or slightly above room temperature. The shape of the lead films is shown in Fig. 1. Before the deposition, various potential leads and current leads were attached to the glass slide through small dots of indium. The lead wires were Teflon-insulated niobium wire. The leads marked 1 and 2 in Fig. 1 were used to measure the transverse Nernst voltage caused by a longitudinal temperature gradient. The longitudinal voltage associated with a longitudinal current was measured with leads 1 and 3. Wires 4 and 5 served as current leads. The broad end on the right side of the film was clamped against a copper block acting as a heat sink in the cryostat. A heater was clamped from both sides to the film such that the Nernst probes 1 and 2 were located in the middle between heater and heat sink. The location of heater and heat sink is indicated in Fig. 1 by the dashed areas. The inner distance between heater and heat sink was 0.6–0.7 cm. The width of the lead films was 1.2–1.3 cm. The film thickness ranged between 1 and 7 μ. It was measured gravimetrically from a glass slide of 2.2-cm diam and 0.025-cm thickness, placed near the specimen slide during the deposition. A check of the gravimetrically obtained film thickness with room-temperature resistivity measurements of the films indicated satisfactory agreement. The film thickness and the electrical resistance ratio $R(295°\text{K})/R(4.2°\text{K})$ for the different specimens are listed in Table I. The value $R(4.2°\text{K})$ is measured in a transverse magnetic field of 600 Oe.

The film specimens were mounted in a cryostat shown schematically in Fig. 2. A thin-walled stainless-

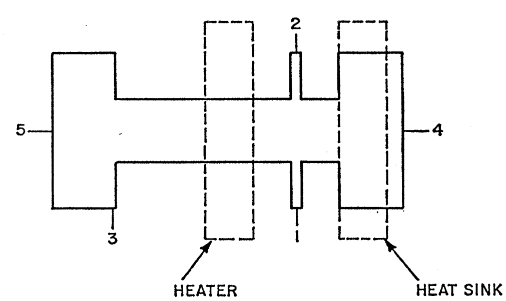

FIG. 1. Geometry of the films.

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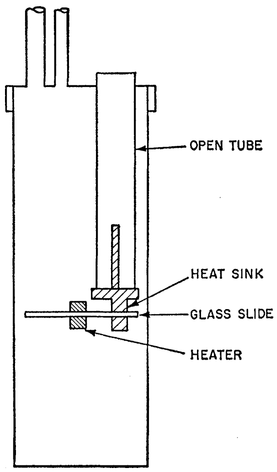

FIG. 2. Scheme of the cryostat.

steel tube which is open at the top is soldered into the top of the vacuum can of the cryostat. The stainless-steel tube is closed at the bottom with a copper plug which carries a rectangular extension acting as heat sink. A copper pin attached to the copper plug inside the tube serves to improve the thermal contact between the heat sink and the liquid helium within the tube. The glass slide carrying the film and the heater is clamped horizontally against the heat sink. Thermal contact at heater and heat sink is aided using vacuum grease. Electrical insulation of the specimen from the cryostat is provided through cigarette paper varnished on the area of the heat sink in contact with the film. All incoming wires are thermally grounded at the stainless-steel tube.

During the measurements of the Nernst voltages, the cryostat was placed in liquid helium and was evacuated. To establish a temperature gradient along the film, one side of the film was cooled below 4.2°K by pumping on the helium bath, whereas the other side was heated. Since the heat conductivity of the glass slide was found to be nearly temperature-independent in the range investigated, the temperature in the middle between heater and heat sink, where the Nernst voltages were measured, was close to the average temperature between the hot and the cold side. Furthermore, in the middle between heater and heat sink the temperature gradient is very insensitive to deviations from the temperature independence of the heat conductivity of the material. By properly heating and cooling both sides of the film, the temperature gradient was varied such that the temperature at the Nernst probes stayed constant at 4.2°K. The temperature gradients were typically between 0.3 and 3.5 °K/cm. In this way the Nernst voltage was measured at 4.2°K as a function of magnetic field, temperature gradient, and film thickness.

Flux-flow resistivity measurements were carried out isothermally at 4.2°K. During these measurements the cryostat was filled with helium-exchange gas. Further details on the experimental techniques can be found in I.

III. RESULTS

In Fig. 3 we show the transverse voltage U_{12} measured between leads 1 and 2 for a film of 2.9-μ thickness as function of magnetic field for different temperature gradients. The temperature at the Nernst probes is kept close to 4.2°K. The temperatures at the heater and the heat sink are listed in the figure. Curves similar to those shown in Fig. 3 were obtained for the other films with different thickness. As in niobium,[9] the voltages U_{12} appear only above a certain value B^* of the magnetic field, pass through a maximum, and vanish at about the critical field. The sign of the Nernst voltage corresponds to flux motion from the hot end

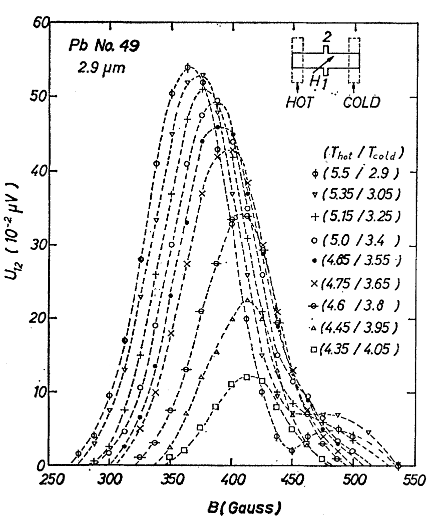

FIG. 3. Nernst voltage U_{12} as function of magnetic field at 4.2°K for different temperature gradients. Film thickness = 2.9 μ.

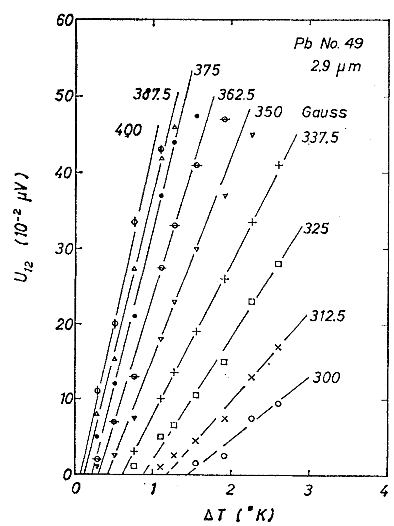

FIG. 4. Nernst voltage as function of the temperature difference between hot and cold side for different magnetic field. Average temperature is 4.2°K; film thickness is 2.9 μ.

to the cold end of the film. The value of B^* decreases with increasing temperature gradient. Apparently, below B^* flux motion does not occur because here the pinning force is larger than the thermal force. For the higher values of the temperature gradient, the voltage curves show a tail and sometimes even a second maximum on the high-field side. A similar tail on the high-field side has been observed by Otter and Solomon[13,14] in measurements of the Ettingshausen effect in a type-I superconductor. These authors have found that the transverse temperature gradient caused by a longitudinal current drops suddenly at about $\frac{1}{2}$–$\frac{2}{3}$ of the critical field with a tail extending to the critical field.

Figure 4 shows the data of Fig. 3 in a plot of U_{12} versus the temperature difference ΔT between the hot and the cold ends of the film. U_{12} is seen to increase linearly with ΔT above a critical temperature gradient. Since the heat conductivity of the glass slides is nearly temperature-independent and since the Nernst probes were located in the middle between heater and heat sink, the temperature gradient at the Nernst probes can be obtained simply by dividing ΔT by the distance between heater and heat sink. By extrapolating the curves linearly to zero voltage, the critical temperature gradient was found.

In Fig. 5 the longitudinal voltage caused by a longitudinal current is plotted versus current for different magnetic fields for the same lead film of 2.9-μ thickness and for the temperature 4.2°K. The voltage-current curves show the usual behavior: a curved section at small currents and a linear region for higher currents. By extrapolating the linear parts to zero voltage, the critical currents were obtained. Curves similar to those shown in Fig. 5 were found for the other lead films investigated.

After determining the slopes S_2 and S_1 of the linear parts in the curves of voltage versus current and voltage versus temperature gradient, respectively, the transport entropy per flux, $S_\varphi/\varphi=S_1/S_2$, was calculated. Figure 6 shows S_φ/φ calculated from the slopes as a function of magnetic field for the film with 2.9-μ thickness and 4.2°K. The transport entropy per flux passes through a maximum at intermediate fields and vanishes at about the critical field. Curves similar to that of Fig. 6, with a maximum at about 375 G, were obtained for all specimens. Since below 300 G flux pinning made thermally induced or current-induced flux motion rather difficult, no transport entropy values were obtained in this region.

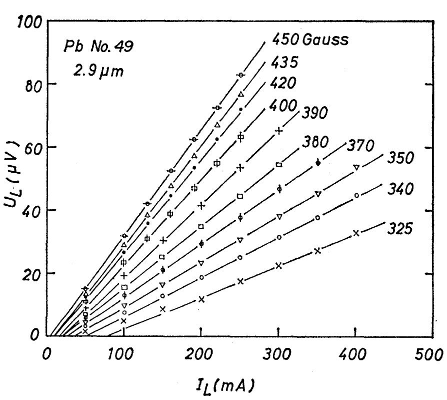

FIG. 5. Current-induced longitudinal voltage as function of current for different magnetic fields at 4.2°K; film thickness is 2.9 μ.

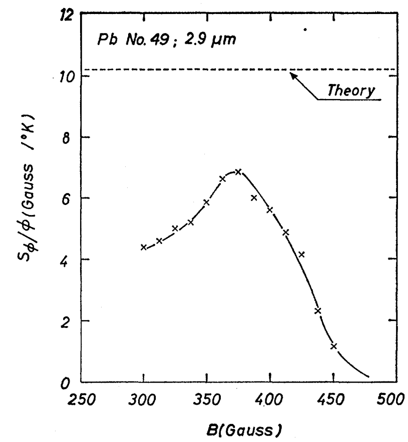

FIG. 6. Transport entropy per unit length and per flux calculated from the slopes S_1 and S_2 as function of magnetic field at 4.2°K. Film thickness is 2.9 μ. The dashed curve represents Eq. (6).

A theoretical estimate of the transport entropy of a flux bundle in a type-I superconductor can readily be obtained from the entropy densities of normal and superconducting material. From the difference of the entropy density in the normal and superconducting state,

$$S_n-S_s=-\frac{H_c(T)}{4\pi}\frac{\partial H_c(T)}{\partial T}, \tag{1}$$

with

$$H_c(T)\approx H_c(0)(1-T^2/T_c^2), \tag{2}$$

we find

$$S_n-S_s=\frac{H_c(0)H_c(T)}{2\pi}\frac{T}{T_c^2}. \tag{3}$$

The excess entropy per unit length of a flux bundle is

$$S_\varphi=(S_n-S_s)F, \tag{4}$$

where F is the cross-sectional area of the bundle. The flux φ in the bundle is given by

$$\varphi=H_c(T)F. \tag{5}$$

From (3)–(5), we find

$$\frac{S_\varphi}{\varphi}=\frac{H_c(0)}{2\pi}\frac{T}{T_c^2}. \tag{6}$$

[13] F. A. Otter, Jr., and P. R. Solomon, Phys. Rev. Letters **16**, 681 (1966); P. R. Solomon and F. A. Otter, Jr., Phys. Rev. **164**, 608 (1967).
[14] P. R. Solomon (unpublished).

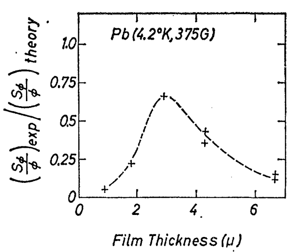

FIG. 7. S_φ/φ calculated from the slopes S_1 and S_2 and normalized to the value of Eq. (6) at 4.2°K and 375 Oe as function of film thickness.

The theoretical dashed line in Fig. 6 indicates the value calculated from Eq. (6).

Figure 7 shows the quantity S_φ/φ calculated from S_1 and S_2 and normalized to the value calculated from (6) at 4.2°K as a function of film thickness. The data are given for a magnetic field of 375 G at which S_φ/φ versus B shows a maximum. S_φ/φ increases first with increasing film thickness, passes through a maximum, and in the thicker films decreases with increasing film thickness. The decrease of S_φ/φ with increasing film thickness in the thicker films is in agreement with earlier results on lead.[15] Curves very similar to that of Fig. 7 were also found recently [11,16] in thin films of tin and indium. We note from Fig. 7 that at an intermediate thickness of about 3 μ, S_φ/φ is only slightly smaller than the theoretical value of Eq. (6).

In Table I we list the critical current density j_c (obtained from the critical current divided by the cross-sectional area of the film), the critical temperature gradient $(\partial T/\partial x)_c$, the ratio of the slopes S_1/S_2, the critical thermal force $(S_1/S_2)(\partial T/\partial x)_c$, and the ratio $|j_c/(\partial T/\partial x)_c|$ at 4.2°K and 375 G for the lead films as a function of film thickness. The critical current density, which is identical with the critical Lorentz force per flux and per unit length of fluxoid, is seen to increase strongly with decreasing film thickness, whereas the critical thermal force varies only slightly with film thickness. Further, the quantity (S_1/S_2) $(\partial T/\partial x)_c$, which is identical with the critical thermal force per flux and per unit length of fluxoid, is much smaller than the corresponding quantity j_c, the discrepancy increasing with decreasing film thickness.

IV. DISCUSSION

A. Transport Entropy of the Fluxoids

From Fig. 6 we see that the transport entropy per unit flux, S_φ/φ, is not constant as a function of the magnetic field as indicated by Eq. (6), but rather decreases to zero as one approaches the critical field. The vanishing of S_φ/φ as H approaches H_c is due to the following reason. As we approach H_c, the normal regions in a type-I superconductor become larger and larger until, at H_c, the whole specimen consists of a single normal region. On the other hand, in order to feel a thermal force, the regions containing the excess entropy density must be rather localized. As we approach H_c, the normal regions become less and less localized, and therefore the thermal force and the transport entropy vanishes.

These arguments may also provide an interesting possibility to explain the second maximum in the Nernst voltage at higher magnetic fields observed at the higher temperature gradients[10] and shown in Fig. 3. Using a powder technique for decorating the magnetic structure in the intermediate state of a flat type-I superconductor in a transverse magnetic field, some authors[14,17] have shown recently that the structure of the intermediate state shows a transition from rather localized normal regions in a superconducting environment at low magnetic fields to rather localized superconducting regions in a normal environment near H_c. Near H_c the thermomagnetic effects are associated more with the localized superconducting regions in the normal environment than with the widely spread normal regions. Apparently, here the thermal force acts upon a small superconducting island and pushes it *from the cold to the hot end* of the sample. The direction of the thermal force is now reversed, since the entropy density in the region considered is *smaller* than in its environment. The motion of a region without flux in a magnetic field is equivalent to the motion of the same region with flux in the opposite direction. In this way we see that the thermal forces acting upon an isolated superconducting region in a normal environment and upon an isolated normal region in a superconducting environment result in a Nernst voltage of the same sign. Therefore, it seems possible that the transition in the magnetic structure of the intermediate state from isolated normal spots at low fields to isolated superconducting spots near H_c can cause a second maximum in the Nernst voltage at higher magnetic fields.

It appears that the shape and the size of the normal spots in the intermediate state play a role in the thermomagnetic effects of type-I superconductors. This influence has been investigated and discussed extensively by Otter and Solomon.[13,14] As mentioned in Sec. III, these authors have found from the Ettingshausen effect in a type-I superconductor (Sn+0.05 at.% In) that the transport entropy drops to a small value at an intermediate magnetic field with a tail extending to the critical field. From magnetic coupling experiments[14,18] and from studies of the intermediate-state structure using a powder technique, they concluded that at about half the critical field flux flow disappears in the resistance mechanism. Similar results have been indicated in the noise measurements of Van Gurp.[19]

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[17] F. Haenssler and L. Rinderer, Helv. Phys. Acta **40**, 659 (1967).

[18] P. R. Solomon, Phys. Rev. Letters **16**, 50 (1966).

[19] G. J. Van Gurp, Phys. Letters **24A**, 528 (1967); G. J. Van Gurp, Phys. Rev. **166**, 436 (1968).

An understanding of the dependence of the transport entropy per flux on film thickness, shown in Fig. 7, requires a discussion of the variation of the flux distribution in the film with film thickness. According to the model by Tinkham,[20] sufficiently thin films of a type-I superconductor, when placed in a transverse magnetic field, exist in the mixed state with one flux quantum contained in each vortex. Lasher[21] has shown theoretically, that, as the film thickness increases, a mixed state will be assumed in which the individual fluxoids contain more and more flux quanta, until in sufficiently thick films the domain structure of the intermediate state is reached. From magnetic coupling experiments with tin films, Sherrill[22] has demonstrated the decrease in fluxoid size with decreasing film thickness. In a lead foil of 50-μ thickness at 1.2°K Träuble and Essmann[23] have shown the existence of a more or less triangular structure of fluxoids in which each fluxoid contains about 50 flux quanta. These authors recently[24] observed the variation of the fluxoid size in lead films with film thickness. They used a decoration method in combination with an electron or an optical microscope. From studies of the transverse magnetization and the ac susceptibility of pure lead films and foils, Cody and Miller[25] have concluded that at 4.2°K lead films up to 1-μ thickness exist in the mixed state described by Tinkham.[20] According to these authors, lead films with a thickness above 1 μ behave more and more like material exhibiting an intermediate state and a positive surface energy.

A possible interpretation of the dependence of the transport entropy on film thickness, shown in Fig. 7, can be as follows. We assume that at constant magnetic field the fluxoid size decreases with decreasing film thickness as suggested from the work mentioned above. Then the distance between the individual fluxoids becomes smaller and smaller as the film thickness decreases. If the magnetic field is not low enough, the distance between the fluxoids may become so small that there is some interaction between neighboring fluxoids, resulting in a reduced localization of the normal fluxoid cores. As the film thickness increases, the individual fluxoids contain more flux quanta and become more localized, since they are more separated from each other. This would then explain the decrease of S_φ/φ with decreasing film thickness found for the films with a thickness below 3 μ. If this model is correct, this decrease of S_φ/φ should be less pronounced as the magnetic field is lowered. Unfortunately, measurements at smaller magnetic fields are very much perturbed by flux pinning. The variation of S_φ/φ with film thickness in the thicker films may be influenced by the fact that here the flux-flow properties are sensitive to the *shape* of the normal and superconducting regions. It is clear that any detailed discussion of these matters would require a direct study of the intermediate-state structure in the specimens using the recently improved direct observation methods.[23,24,26]

We note that for a film thickness around 3 μ the ratio S_1/S_2 is close to the theoretical value of Eq. (6). The derivation of Eq. (6) is valid only for a rather large fluxoid, containing many flux quanta, for which the contribution of the surface energy can be neglected. The surface-energy contribution, which becomes appreciable for *small* fluxoids in a type-I superconductor, tends to increase the expected value of S_φ/φ above the value from Eq. (6).

B. Critical Temperature Gradient and Critical Current Density

We see from Table I that the critical current density j_c, and thereby the critical Lorentz force per flux and per unit length of fluxoid, increases strongly with decreasing film thickness. A similar dependence of j_c on specimen thickness has been found by Joiner and Kuhl[5] using thin foils of type-II material in a transverse magnetic field. On the other hand, the critical thermal force per flux and per unit length of fluxoid, that is, the product $(S_1/S_2)(\partial T/\partial x)_c$, does not vary consistently with film thickness.

In I it was pointed out that the ratios $|j_c/(\partial T/\partial x)_c|$ and S_1/S_2 would be equal, if the critical current flows through the specimen with homogeneous density. We note from Table I that $|j_c/(\partial T/\partial x)_c|$ is larger than S_1/S_2 by 2–3 orders of magnitude, the discrepancy increasing with decreasing film thickness. In the same sense j_c, the critical Lorentz force per flux and per unit length of fluxoid, is much larger than $(S_1/S_2)(\partial T/\partial x)_c$, the critical thermal force per flux and per unit length of fluxoid, the discrepancy increasing with decreasing film thickness. It is unlikely that this large discrepancy is caused by the variation of the superconducting properties of the sample in the direction of the temperature gradient. From the results presented in Table II we can argue similarly as in I. The increase of j_c with decreasing film thickness suggests that the critical current is either a surface current (advocated by Swartz and Hart[8]) or a bulk current limited by surface pinning (advocated by Joiner and Kuhl[5]). The fact that the apparent critical Lorentz force is much larger than the critical thermal force indicates that at and below its critical value the current flows through special channels in such a way that there is very little interaction with the fluxoids and that these current channels contribute very little to the Lorentz force on a fluxoid. The fact that the discrepancy between the

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[21] G. Lasher, Phys. Rev. **154**, 345 (1967).
[22] M. D. Sherrill, Phys. Letters **24A**, 312 (1967).
[23] H. Träuble and U. Essmann, Phys. Status Solidi **25**, 395 (1968).
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[25] G. D. Cody and R. E. Miller, Phys. Rev. Letters **16**, 697 (1966); Phys. Rev. **173**, 481 (1968).
[26] H. Kirchner, Phys. Letters **26A**, 651 (1968).

apparent critical Lorentz force and the critical thermal force increases with decreasing film thickness suggests that these current channels are associated with the surface of the specimen. Similar conclusions have been reached from studies of the Nernst effect and the flux flow in thin films of tin and indium.[16] It is interesting to note that the discrepancy between the critical thermal force and the critical Lorentz force disappears in a thick specimen where surface effects can be neglected.[27]

The present results appear to resolve the controversy between Joiner and Kuhl[5] and Swartz and Hart[8] in favor of the surface-current model. In their criticism of the surface flux-pinning model of Swartz and Hart, Joiner and Kuhl argue that apparently the critical current is not a surface current because otherwise the flux-flow resistivity would be reduced and would decrease much more strongly than observed as the magnetic field becomes more aligned with the sample surface. However, since the surface currents affect only the critical-current value and not the flux-flow resistivity, this argument of Joiner and Kuhl seems to be invalid.

[27] J. Lowell (private communication).

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Intrinsic Fluctuations in Superconducting Rings Containing One-Dimensional Weak Links

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The Langer-Ambegaokar statistical theory of dissipative fluctuations in narrow superconducting channels is extended to describe the quantum transitions of a closed superconducting ring containing a long one-dimensional weak-link section. The external magnetic flux Φ_e linking the loop is the independent thermodynamic variable. Based upon the Ginzburg-Landau free energy, the theory is expected to accurately describe single-quantum transitions for T near but slightly below T_c. Data reported by Lukens and Goodkind for thin-film Sn rings are reasonably consistent with the theory, but more definitive experimental tests are required.

I. INTRODUCTION

A STATISTICAL model based upon a free-energy function of the Ginzburg-Landau form has been proposed by Langer and Ambegaokar (LA) to describe the onset of dissipation in thin superconducting wires near T_c.[1] Their principal predictions for long, simply connected wires of uniform cross section have been confirmed by Webb and Warburton in experiments on whisker crystals of tin, structures with remarkable uniformity.[2,3] The original formulas were derived under the assumption that the gauge-invariant phase difference $\Delta\varphi$ across the ends of the wire was held fixed, but they also obtain for long wires under conditions of constant current.[4] In this paper we develop the corresponding theory of a closed ring of thin superconducting wire near T_c in a weak quasistatic external magnetic field. The total free energy includes that stored in the wire plus that stored in the field. The analysis is slightly more complicated than the cases considered previously in that the external magnetic field rather than the phase difference $\Delta\varphi$ or the current I is the independent thermodynamic variable.

We do not require that the ring have uniform cross section nor be everywhere thin compared to the coherence length $\xi(T)$, but we do assume that it contains a thin weak-link section of uniform cross-sectional area σ and of length $L \gg \xi(T)$. The weak-link section is "one-dimensional" in the sense that its maximum diameter is everywhere less than $\xi(T)$. Cases with nonuniform cross section or with $L \lesssim \xi(T)$ can be accommodated in the formalism; results for such systems will be described in a later paper. Briefly, the principal effect of variable σ and finite L is to broaden the temperature range of the resistive transition compared to that for a long uniform sample of the same average

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