To convert Ω from the ring frame to inertial space, multiply by:

$$F \text{ wave}, \quad c/(c-R\omega_r);$$

$$B \text{ wave}, \quad c/(c+R\omega_r).$$

When F radiation with frequency Ω_F is backscattered, it appears in the B mode with frequency $\Omega_F(c-R\omega_r)/(c+R\omega_r)\simeq\Omega_F(1-2R\omega_r/c)$. Similarly, B radiation appears in the F mode with frequency $\simeq\Omega_B(1+2R\omega_r/c)$. In the ring laser, all frequencies present are very near the cavity resonant frequency, so all frequency shifts are approximately of magnitude $|\delta\Omega/\Omega_0|\simeq 2R\omega_r/c$. Note that if the ring were a regular polygon, this formula could have been computed as the first-order approximation to the Doppler shift experienced by a wave backscattered from a mirror moving with the ring instantaneously but not accelerating with it. However, the Doppler formula would not be correct to higher orders in the mirror velocity, and would not even be applicable for a general irregular polygon, where different mirrors lie at different radii from the rotation axis.

APPENDIX C: LASER GAIN

At steady state, the laser gain $G_S(\Omega_0,\Omega)$ for each of the four frequencies Ω_{S1}, Ω_{S2} present equals half the corresponding energy loss rate, which is ϵ_0^{-1} times a fictional conductivity $\sigma_S(\Omega_0,\Omega)$, which in turn is given by a self-consistency equation derived in the preceding paper. Neglecting pulsations in the excitation density, it is found that

$$\epsilon_0^{-1}\sigma_S(\Omega_0,\Omega)=G_S(\Omega_0,\Omega)=\frac{\Omega|\mu_{ab}|^2}{2\epsilon_0\hbar\gamma}\int_v N(v)e^{\mp\mu}\mathfrak{L}_S,$$

where the various symbols are defined in the preceding paper.

PHYSICAL REVIEW VOLUME 181, NUMBER 2 10 MAY 1969

Channeling of MeV He$^+$ Ions in Tungsten and Other Crystals: An Intercomparison of Rutherford Scattering and of Characteristic L and M X-Ray Yields

J. A. Davies, L. Eriksson,* N. G. E. Johansson,* and I. V. Mitchell
Chalk River Nuclear Laboratories, Chalk River, Ontario, Canada
(Received 13 May 1968; revised manuscript received 14 January 1969)

Measurements of wide-angle (150°) scattering and of L and M x-ray yields in tungsten single crystals are reported as a function of crystallographic orientation with respect to an incident beam of 1.4-MeV helium ions. Comparison of these yields establishes both lower and upper limits for the minimum impact parameter ($r_{\min}$) between a channeled ion and the tungsten-lattice atoms; these limits are consistent with Lindhard's estimate that $r_{\min}\sim\mathbf{a}$, the Thomas-Fermi screening distance (i.e., ~0.11 Å for He in W). A similar comparison between wide-angle scattering and x-ray yield curves is reported for several other lattices—Al, Si, GaP, GaSb, and UO_2; again the results are consistent with the predicted relationship: $r_{\min}\sim\mathbf{a}$. Anomalies in published orientation studies of K, L, and M x-ray yields are shown to be due to depth effects.

I. INTRODUCTION

PREVIOUS theoretical[1] and experimental[2,3] work has established that close-encounter processes such as wide-angle Rutherford scattering exhibit extremely strong attenuations whenever the incident beam is aligned within a predicted critical angle of a major axis or plane. For example, in tungsten along a major axis such as the ⟨111⟩, attenuation factors of up to 100 have been observed, indicating that as much as 99% of the incident beam is being channeled on entering the crystal. These earlier Rutherford-scattering measurements can be used to establish upper and lower limits to $r_{\min}$. On the one hand, for the channeled fraction to be as large as 99%, the "forbidden" area $\pi(r_{\min})^2$ around each atomic row must be less than 1% of the available area. This sets an upper limit of 0.13 Å for $r_{\min}$ in the case of ⟨111⟩ tungsten. On the other hand, the existence of such a strong orientation dependence also requires that $r_{\min}$ cannot be less than $\bar{p}$ where $\bar{p}$ is the impact parameter of the particular close impact process. For wide-angle Rutherford scattering of MeV projectiles, $\bar{p}$ is typically 10^{-3}–10^{-4} Å; hence, these upper and lower limits for $r_{\min}$ are about two orders of magnitude apart.

In order to establish narrower experimental limits for $r_{\min}$, we need to investigate the orientation dependence of processes for which $\bar{p}$ is much larger than 10^{-3} Å. For this purpose, the characteristic inner-shell x-ray yields are particularly suitable, since they cover the desired range of impact parameters. Unfortunately, a quantitative relationship between the mean radius of

* Permanent address: Research Institute for Physics, Stockholm 50, Sweden.

[1] J. Lindhard, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **34**, 14 (1965).

[2] E. Bøgh and E. Uggerhøj, Nucl. Instr. Methods **38**, 216 (1965).

[3] J. A. Davies, J. Denhartog, and J. L. Whitton, Phys. Rev. **165**, 345 (1968).

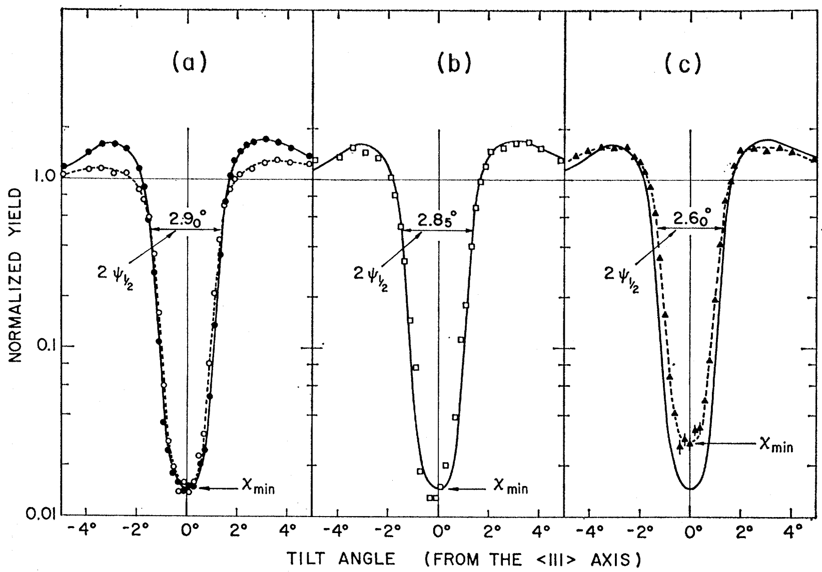

FIG. 1. Comparison of Rutherford scattering and of L and M x-ray yields in tungsten as a function of the angle between the incident beam (1.4-MeV helium) and the ⟨111⟩ axis: (a) Rutherford-scattering yields—● at 1200 Å depth, ○ at 5000 Å depth; (b) L x-ray yield (□) compared to the 1200 Å Rutherford-scattering curve (——); (c) M x-ray yield (▲) compared to the 1200 Å Rutherford scattering curve (——). All yield curves approach the value of 1.0 (i.e., the normal "random" yield) at larger tilt angles.

a particular electron shell and the mean impact parameter ($\bar{p}$) for x-ray production is not yet available. We assume, therefore, as a first approximation, that these two distances are comparable in magnitude; and hence an estimate of $\bar{p}$ for each of the x-ray processes can be obtained from the mean radius $\bar{r}$ of that particular electron shell. As seen from Table I,[4] the K, L, and M x-rays in W cover a region of impact parameters that extends almost to **a**.

The extensive experiments of Khan and collaborators,[5–7] using a proton beam, have already established that x-ray yields depend strongly on the orientation of a crystal relative to the incident beam direction. Their experiments, however, cannot readily be used in the present intercomparison, as they did not adequately monitor the depth from which the x rays originated. In the present work, we have used a helium beam instead of protons, in order to restrict x-ray production to the first few thousand Å. Our measured yield energy curves indicated that, for 1.4-MeV He, the beam loses energy sufficiently rapidly that 95% of the L and M x-rays originate within 6000 Å of the surface. In the case of the M x-rays, their strong self-absorption in tungsten further limits the effective measured depth to about 2000 Å from the surface. Previous studies[3] of the rate at which channeled particles become scattered (i.e., "dechanneled") have shown that in tungsten, along the main *axes*, negligible dechanneling (<1%) occurs within the first 6000 Å, and hence depth effects can be ignored in the present intercomparison. On the other hand, for planar channeling, the rate of dechanneling is roughly an order of magnitude greater[3] than in the axial case, and hence even at 6000 Å depth effects are not completely negligible, as will be discussed below (Fig. 3).

TABLE I. Comparison between the electron-shell radii ($\bar{r}$) and the Thomas-Fermi screening distance (**a**).

Target atom	$\bar{r}$ (in Å)[a]			**a**
	K shell	L shell	M shell	
Al	0.06	···		0.18
Si	0.05_5	···		0.17
P	0.05	···		0.17
Ga	0.02	**0.10**		0.14
Sb	0.01	0.06		0.12
W	0.007	0.04	**0.10**	0.11
U		0.03	**0.08**	0.10

[a] Calculated from Herman and Skillman's (Ref. 4) electron-density-distribution tables. Bold-faced values are the only radii that approach **a** in magnitude; these are also the three x rays whose yield curves do *not* agree with the Rutherford-scattering behavior.

II. RESULTS AND DISCUSSION

The experimental technique is identical to that used in the earlier Rutherford-scattering experiments,[3] except that a gas-filled proportional counter has been added in order to measure the x-ray yields. This counter together with the solid-state detector system enabled us to observe simultaneously the orientation dependence of several processes: for example, in tungsten, L and M x-ray production and wide-angle (150°) Rutherford scattering.

A. Axial Channeling in Tungsten

The results of such a comparison are shown in Fig. 1. In the case of Rutherford scattering [Fig. 1 (a)],

[4] T. Hermann and S. Skillman, *Atomic Structure Calculations* (Prentice-Hall, Inc., Englewood Cliffs, N. J., 1963).
[5] W. Brandt, J. M. Khan, D. L. Potter, R. D. Worley, and H. P. Smith, Phys. Rev. Letters **14**, 42 (1965).
[6] J. M. Khan, D. L. Potter, R. D. Worley, and H. P. Smith, Phys. Rev. **148**, 413 (1966).
[7] J. M. Khan, D. L. Potter, R. D. Worley, and H. P. Smith, Phys. Rev. **163**, 81 (1967).

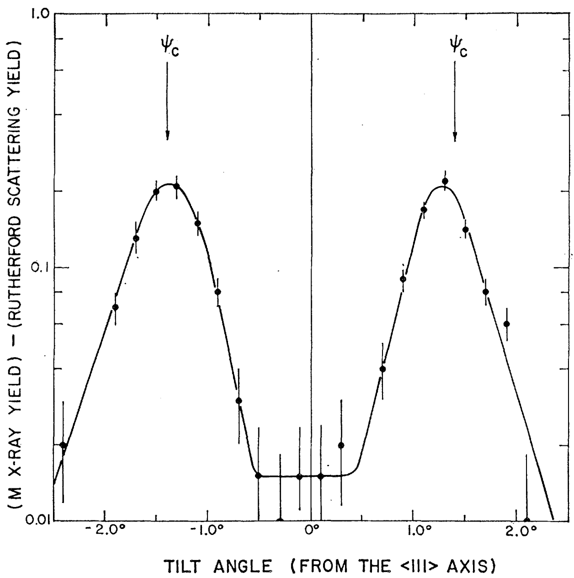

FIG. 2. Orientation dependence of the "excess M yield" [defined as the difference between the normalized values of the M yield and the 1200 Å Rutherford-scattering yield curves in Fig. 1 (c)].

energy analysis of the scattered beam[3] permits us to measure the yield as a function of depth, and hence to establish the rate of dechanneling. As can be seen, the 1200 and 5000 Å curves are indistinguishable (except in the "shoulder" region, where depth effects are always much more pronounced,[2,3] thereby confirming that negligible dechanneling has occurred within the depth from which the x rays are being produced.

Comparison of the x-ray yields with the 1200 Å Rutherford-scattering curve shows that (i) the orientation dependence of the yield of L x-rays [Fig. 1 (b)] is indistinguishable from that of the Rutherford-scattering yield, indicating that the channeled beam is unable to interact with the L-shell electrons, and (ii) the yield curve of M x-rays [Fig. 1 (c)] is slightly narrower and shallower than that of the other processes, indicating that a channeled beam does penetrate somewhat into the M-electron region, and that $r_{\min}$ is therefore comparable in magnitude to the mean impact parameter for ejecting an M-shell electron.

These observations may seem at first to contradict the earlier x-ray results for *proton* channeling in W and Cu reported by Khan *et al.*,[7] since these workers found that the yield curves for K and L x-rays were much shallower than for the M x-rays. In their analysis, however, Khan *et al.* did not correct for the significant variation in mean depth from which (in the case of protons) the different x-rays originate. The observed M x-rays are still restricted to the first ~2000 Å because of their strong self-absorption; on the other hand, due to the much smaller rate of energy loss of a proton beam (compared to helium), the K x-rays originate from a mean depth of several microns. Hence, for protons, dechanneling effects can no longer be ignored. We therefore extended the present intercomparison between Rutherford scattering and x rays to include the case of proton channeling in tungsten. The apparent anomaly disappears completely, provided the yield curves are measured at comparable depths: i.e., we observe (i) good agreement between our proton data and those of Khan *et al.* for each of the three x-ray curves, (ii) good agreement between the K (or the L) x-ray curve and the corresponding Rutherford-scattering curve *measured at the same mean depth*, and (iii) a significantly narrower and shallower M x-ray curve than the corresponding one for Rutherford scattering.

In the perfectly aligned (i.e., 0°) case, the excess M yield [defined as the difference between the normalized values of the M yield and the 1200 Å Rutherford scattering yield curves in Fig. 1 (c)] is only about 2% (Fig. 2); hence, for a *well*-channeled beam, most of the ions are evidently being steered at distances greater than the mean impact parameter for M x-ray production. However, in the vicinity of the critical angle, where the channeled particles all have sufficient transverse energy to approach $r_{\min}$ before being steered away, the excess M yield rises to a peak of ~25% of its normal (or random) yield, indicating that an appreciable fraction of a *barely* channeled beam is able to penetrate into the M electron shell.

B. Axial Channeling in Other Crystals

Similar experiments have been carried out in Al, Si, GaP, GaSb, and UO_2, again using a helium ion beam (1–2 MeV) in order to restrict the depth from which the observed x rays originate, and so minimize dechanneling effects. The results are summarized in Table II. Within experimental error, the K x-ray yields all display the same orientation dependence as the back-scattered yield curve, as also do the L x-rays

TABLE II. Comparison of Rutherford-scattering and x-ray data[a] (using a helium beam).

Crystal		Beam energy (MeV)	x-ray shell	$\psi_{1/2}$ values From scattering yield (deg)	$\psi_{1/2}$ values From x-ray yield (deg)	$\chi_{\min}$ values From scattering yield	$\chi_{\min}$ values From x-ray yield
Al	⟨110⟩	1.4	K	0.57	0.55	0.26	0.29
Si	⟨110⟩	1.4	K	0.55	0.55	0.03	0.03
W	⟨111⟩	1.4	L	1.45	1.43	0.01_4	0.01_3
			M	1.45	1.30	0.01_3	0.02_4
GaP	⟨110⟩	1.0	L(Ga)	0.96	0.83		
	⟨111⟩	1.0	L(Ga)	0.70	0.62		
GaSb	⟨110⟩	1.0	L(Sb)	1.04	1.04		
	⟨111⟩	1.0	L(Sb)	0.85	0.78		
UO_2	⟨100⟩	1.4	M(U)	1.3	1.1	0.02_5	0.05
UO_2	⟨100⟩	1.9	L(U)	1.02	0.98	0.03	0.03_5
			M(U)	0.99	0.84	0.03	0.08

[a] In each case, the scattering yield was measured at the same mean depth as that of the observed x ray.

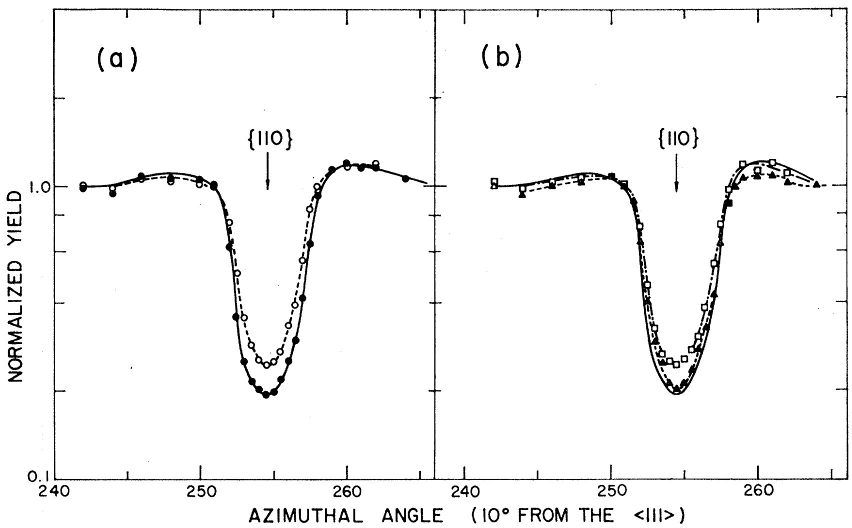

FIG. 3. Comparison of Rutherford scattering and of L and M x-ray yields in tungsten as a function of the angle between the incident beam (1.4-MeV helium) and the {110} plane: (a) Rutherford-scattering yields —● at 1200 Å depth, ○ at 5000 Å depth; (b) L x-ray yield (□) and M x-ray yield (▲) compared to the 1200 Å Rutherford scattering curve (——).

from Sb and from U. On the other hand, the L x-rays from Ga and the M x-rays from U exhibit a slightly narrower and shallower orientation effect than the back-scattered yield curve—i.e., they are analogous in behavior to the M x-ray in tungsten crystals [Fig. 1 (c)] —indicating that in these cases the mean impact parameter for x-ray production is comparable in magnitude to $r_{\min}$. The x-ray studies in GaP and GaSb form part of an extensive investigation of channeling in diamond-type crystals, which will be published in detail elsewhere.[8]

C. Planar Channeling in Tungsten

A brief investigation of planar channeling in tungsten has also been carried out, again using a 1.4-MeV helium beam as projectile. The results are summarized in Fig. 3. The most striking feature is the marked difference between the two Rutherford-scattering yield curves [Fig. 3 (a)]. Unlike the axial case [Fig. 1 (a)], there is now a considerable amount of dechanneling occurring even within the first 5000 Å, and this explains why in Fig. 3 (b) the observed L x-ray curve (originating from a mean depth of ~5000 Å) has a significantly weaker orientation dependence than the M curve. After an appropriate depth correction is applied, the Rutherford-scattering and L x-ray curves are again found to be identical, and the M x-ray curve is slightly narrower and shallower as in the axial case.

[8] S. T. Picraux, J. A. Davies, L. Eriksson, N. G. E. Johansson, and J. W. Mayer, Phys. Rev. **180**, 873 (1969).

III. SUMMARY

The present comparison between the orientation dependence of the yield curves for inner-shell x-ray production and that for Rutherford scattering clearly indicates that, in order to use x-ray measurements for a quantitative study of channeling behavior in single crystals, the depth effect must first be taken into account.

Inspection of the electron-shell radii ($\bar{r}$) in Table I shows that, in those three cases where the orientation dependence of the x-ray yield differs from that of the 150° scattering yield at the same depth (i.e., in the case of the L x-ray of Ga and the M x-rays of U and W), the values of $\bar{r}$ are only slightly smaller than **a**. In all the other cases, $\bar{r}$ is always at least a factor of 2 smaller than **a**, and the two processes show identical orientation dependences. This provides rather good support for Lindhard's prediction that $r_{\min}$ (the distance of closest approach of a channeled beam to an aligned row or plane) is comparable in magnitude to the Thomas-Fermi screening distance **a**.

ACKNOWLEDGMENTS

We wish to thank J. U. Andersen and K. B. Winterbon for many valuable discussions, and G. R. Bellavance and G. Sims for their technical assistance during these measurements. We are also particularly grateful to I. L. Fowler, H. McCready, and R. Toone of the Counter Development section, who designed and constructed the special x-ray counter required for these experiments.