Sound Propagation in Liquid ^{4}He†

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Measurements of the attenuation of sound in liquid ^{4}He down to 0.1 K have been performed at 12, 30, 36, 60, 84, 90, 108, 132, 150, and 208 MHz. Measurements of the temperature dependence of the velocity of sound were made at 12, 36, 60, and 84 MHz. These data are compared with recent theoretical work, particularly that of Khalatnikov and Chernikova. The attenuation data agree well with theory in the vicinity of the peak in the attenuation near 1 K but do not agree elsewhere, the observed attenuation being greater than that predicted by theory. The temperature dependence of the velocity at low temperatures is found to be less than that predicted by theory, while the frequency dependence of the velocity (at finite temperature) is opposite to that predicted by theory.

I. INTRODUCTION

The pioneering work at ultrasonic frequencies on the propagation of sound in liquid ^{4}He was done by Pellam and Squire.[1] They found that the attenuation is very high at the lambda point, has a minimum at about 1.9 K, and increases as the temperature is lowered. Khalatnikov[2,3] calculated the attenuation of sound for this temperature range. This theory, which has two parameters to be fixed by the experiment, was successful in explaining the later more precise data of Atkins and Chase[4] and Chase.[5] Further measurements at lower temperatures[6–14] showed that the attenuation reached a maximum around 0.9 K and then fell off as the temperature further decreased. The data below 0.9 K were however in disagreement with the theory of Khalatnikov. For the usual ultrasonic frequencies, the situation above and below 0.9 K are physically different. Above 0.9 K collisions between the quasiparticles, i.e., phonon-phonon, phonon-roton, and roton-roton, are frequent and therefore local equilibrium is established within a time $1/\omega$ where ω is the impressed sound frequency. This regime is called the hydrodynamic regime, and it is well known that the attenuation is given by

$$\alpha = (\omega^2/2\rho c^3)(\tfrac{4}{3}\eta + \xi) \ ,$$

where η and ξ are the first and second viscosity coefficients.[15] Khalatnikov has calculated these first and second viscosity coefficients. On the other hand, below about 0.9 K the collisions are infrequent, and we are in the collisionless regime where the attenuation does not follow the hydrodynamic result and a different approach is required. It is clear that the full temperature and frequency dependence of sound propagation in liquid ^{4}He are quite a difficult problem which requires the use of the kinetic equation. One has to take into account elastic (particle conserving) as well as inelastic (those not conserving the number of particles) collisions in the collision integral of the kinetic equation. Even so, Khalatnikov and Chernikova[16–18] recently derived an expression for the attenuation of sound without any adjustable parameters. The theory is very successful at low frequencies (1 to 6 MHz) and at higher frequencies it is especially successful around 0.9 K. As is well known, below 0.6 K the only excitations in the liquid are the phonons, and the problem would appear to be simpler. There is also great interest in this case because at the usual ultrasonic frequencies when we are in the collisionless regime we can attack the problem fundamentally as a phonon-phonon scattering problem. This problem is also of great interest for the attenuation of sound in a dielectric solid. The liquid, however, is a much simpler system theoretically and experimentally. It is isotropic, only longitudinal phonons exist, there are no dislocations, and for all practical purposes there are no impurities. Thus the only mechanism for the sound attenuation is the phonon-phonon interaction. This problem was attacked recently by many theoreticians using different methods which will be discussed in the next section. Because certain of the theoretical results were not in agreement with each other or with the existing experiments, the present work was initiated. The emphasis has been on the very low-temperature region which is dominated by phonon-phonon scattering, but some high-temperature data could also be taken at the lower frequencies. In these experiments we measured the attenuation of sound at 10 different frequencies[12,13] and the velocity of sound at four frequencies.[13] The experimental method is presented in Sec. III and the results are discussed in Sec. IV.

II. THEORY

Liquid ^{4}He, being a quantum liquid, has the property that for temperatures less than about 2 K, it is possible to describe approximately the state of the system in terms of a set of weakly interacting normal modes or elementary excitations possessing definite energies and momenta. The relation between the energy and momentum of an elementary excitation was first deduced by Landau from general considerations, and experimental heat capacity and second-sound propagation data.[19,20] It was later detemined in much greater detail from the inelastic neutron diffraction experiments of Henshaw and Woods[21] which are shown in Fig. 1. The solid line in Fig. 1 is a fit to the experimental data of the form

$$\epsilon^2 = A_1 p^2 + A_2 p^4 + A_3 p^6 + A_4 p^8 \ , \qquad (1a)$$

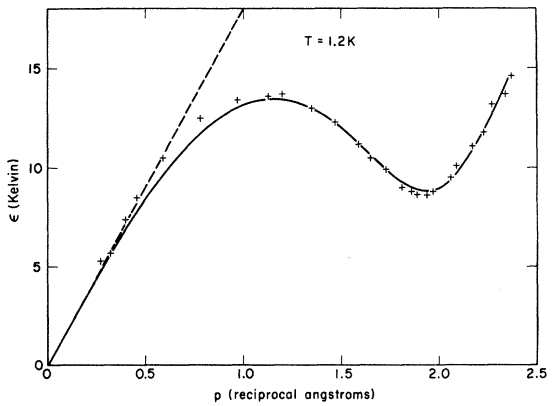

FIG. 1. The elementary excitation spectrum of liquid ^{4}He at 1.12 K as determined by the neutron diffraction measurements of Henshaw and Woods. The solid line is a four-term fit to the spectrum of the form suggested by Landau and Khalatnikov. The dashed line is the limiting slope determined from the velocity of sound.

where ϵ is measured in degrees Kelvin and momentum in reciprocal angstroms. This form was suggested by Landau and Khalatnikov.[22] The values used are $A_1 = (10^8 \hbar c/\kappa)^2 = 328$, where c is the sound velocity and κ is Boltzmann's constant, $A_2 = -189$, $A_3 = 36.6$, and $A_4 = -2.17$. In the long-wavelength limit this may be written as

$$\epsilon = cp(1 - \gamma p^2) \ , \qquad (1b)$$

where $\gamma = 2.59 \times 10^{37}$ in cgs units. While the overall fit is rather good, it is seen that the observed dispersion at small momenta is less than that predicted by the fit and thus this value of γ represents an upper limit. If we use the four-term power-series fit but include data only up to 1.2 Å^{-1}, then γ (which is related to A_2) is negative, and of order 10^{36}, while the magnitude of A_3 becomes much greater (the sign remaining the same). Sound-velocity measurements would, in principle, determine the value of γ but would require an accuracy of parts in 10^8 at 100 MHz. Though it is somewhat misleading, it has become customary to consider two regions of this curve separately; namely, the region near $p = 0$ where the spectrum is approximately linear (for which the name phonons is appropriate) and the region near the minimum (~2 Å^{-1}), (for which the name rotons has been adopted). Only the phonon and roton region of the spectrum are appreciably occupied at higher temperatures, while below 0.6 K only the phonon region plays a role.

At finite temperatures, elementary excitations may be annihilated, created, or scattered. This leads to a lifetime broadening or energy uncertainty in the excitations. Thus as the temperature is raised and the number of excitations and collisions increases, our description of the system in terms of elementary excitations becomes more and more imprecise. For example, it is known from the neutron data that the energy uncertainty for a roton is of the same order as its mean energy for temperatures above about 2 K. However, as the temperature is lowered, a description of the system in terms of elementary excitations becomes an increasingly accurate model to use for calculating various thermodynamic and transport properties. A measurement of sound propagation at very low temperature yields very useful data with which to compare the results of calculations based on the interacting elementary excitation model.

At absolute zero the elementary excitation spectrum is identical to the energy eigenvalue spectrum of a single excitation of the liquid (where these eigenvalues are numbered according to their momenta). For this to be the case, the lifetime of this single excitation must be infinite. That this is the case follows from the special form of the spectrum; once an excitation is created it cannot decay into two (or more) subsequent excitations since the laws of energy and momentum conservation forbid it. We will demonstrate this stability for long wavelengths. Let us consider the decay of a phonon of momentum p_i into two subsequent phonons of momentum p_f and p_s. The energy- and momentum-conservation theorems yield (where we have assumed that all the phonons are collinear since this is the most favorable case for positive γ):

$$p_i = p_f + p_s \qquad \text{(momentum conservation)},$$

$$p_i(1 - \gamma p_i^2) = p_f(1 - \gamma p_f^2) + p_s(1 - \gamma p_s^2) \qquad \text{(energy conservation)}.$$

Subtracting these two equations and taking the cube of the first equation yields the following set of equations

$$p_i^3 = p_f^3 + p_s^3, \quad p_i^3 = (p_f + p_s)^3 \ .$$

Since these equations have only the trivial solution $p_s = 0$, it is clear that energy and momentum conservation cannot be satisfied.

Let us now consider the case where there are two excitations in the liquid. In the same way that energy and momentum conservation will not allow one phonon to decay into two, two phonons cannot combine (inelastically scatter) to form one. Thus three phonon events (one going into two and vice verse) appear at first sight to be forbidden. For this reason Landau and Khalatnikov[22] considered only the four-phonon process, the elastic scattering of two phonons.

The matrix element for phonon-phonon scattering can be derived from the hydrodynamical model of Landau,[19] and we refer the reader to Khalatnikov's[23] or London's[24] works for a more detailed account. The equations of hydrodynamics are nonlinear, and monochromatic sound waves are not strictly normal modes but are weakly coupled by the nonlinear terms in these equations of motion. In quantum hydrodynamics these nonlinear terms result in phonon-phonon scattering. Since historically the four-phonon process was the first to be treated theoretically,[22] we shall begin our discussion of phonon-phonon scattering here. The calculation proceeds as follows. From quantum hydrodynamics we get third-order terms in the Hamiltonian coupling three phonons, and fourth-order terms connecting four phonons. Four-particle matrix elements follow from (a) third-order terms in the Hamiltonian treated in the second order of perturbation theory and (b) fourth-order terms in the Hamiltonian treated in first-order perturbation theory. There are six different diagrams contributing to process (a) and one to process (b). The diagram contributing to process (b) and five of the diagrams contributing to process (a) are argued[22] to be small or zero and are neglected. The remaining diagram may be substituted in the "golden rule" to yield a transition probability. In the final step the transition probability is substituted in a collision integral from which a phonon lifetime t_{pp} or attenuation coefficient α results. This calculation, as outlined here, was originally carried out by Landau and Khalatnikov.[22] If we treat sound attenuation from the point of view that it is simply a scattering of sound phonons by thermal phonons[25] (valid for $\omega\tau \gg 1$, where τ is some relaxation time), then we may use the phonon lifetime calculated by Landau and Khalatnikov to determine the attenuation. The result[25] (valid for $\hbar\omega \ll \kappa T$) is

$$\alpha = \frac{1}{ct_{pp}} = \frac{5}{8}\frac{(u+1)^4\kappa^6}{c^9\rho^2\gamma\hbar^6\pi^3}\,\omega T^6 \quad \text{(four phonon)} \ , \tag{2}$$

where $u = (\rho/c)\partial c/\partial\rho$, κ is Boltzmann's constant, and ρ is the density. On the other hand, if we had neglected dispersion in the energy spectrum [$\gamma = 0$ in Eq. (1b)], this would have allowed the three-phonon process to "go" since it would not violate energy- and momentum-conservation theorems (incidentally expression (2) would be divergent in this limit). If we take the third-order (three phonon) terms in the Hamiltonian, substitute this matrix element in the golden rule and again evaluate the collision integral, the following relationship results:

$$\alpha = \frac{\pi^3}{60}\frac{(u+1)^2}{\rho}\frac{\kappa^4}{\hbar^3c^6}\,\omega T^4 \quad \text{(three phonon)} \ . \tag{3}$$

So to sum up the theoretical point of view on phonon-phonon scattering, we expect that if, somehow, the three-phonon process is allowed we will get an ωT^4 behavior[26–32] for the attenuation, while if the three-phonon process is forbidden then the next higher-order process should lead to an ωT^6 behavior. The early low-temperature data ($T < 0.6$ K) of Chase and Herlin[6] seemed to support the three-phonon result rather than the four phonon, which was puzzling since it seemed at first sight to violate the energy- and momentum-conservation theorems described above. The solution to this dilemma was given independently by Kawasaki[28] and Simons[33] and the argument goes as follows. Since the thermal phonons are continually colliding with each other, we expect an energy uncertainty $\delta\epsilon \simeq \hbar/\tau$, where τ is some mean thermal phonon lifetime. If this energy uncertainty $\delta\epsilon$ is greater than the energy discrepancy $\Delta\epsilon$ (between the initial and final states in the three-phonon process) then the three-phonon process can take place. The energy discrepancy $\Delta\epsilon$ is easily calculated to be $\Delta\epsilon \simeq 3\gamma p^2\hbar\omega$, where p is the thermal phonon momentum. Thus for the three-phonon process to take place we must have $3\gamma p^2\omega\tau \ll 1$. One can phenomenologically incorporate this idea into the theory by using the correct energy dispersion law, but instead of the delta function in the golden rule we use a Lorentzian spectral function. The result of such a calculation is

$$\alpha = \frac{\pi^2}{30}\frac{(u+1)^2}{\rho}\frac{\kappa}{\hbar^3c^6}\,\omega T^4 \times[\tan^{-1}(2\omega\tau) - \tan^{-1}(3\gamma\bar{p}^2\omega\tau)] \ , \tag{4}$$

where $\bar{p}$ is the average thermal momentum $3\kappa T/c$. This result was first derived (using thermal Green's functions) by Kwok, Martin, and Miller[34] and independently by Pethick and ter Haar.[35] It was later rederived along the lines outlined above by Eckstein.[32]

In Refs. 34 and 35 the thermal Green's-function approach was used. The exact spectral function

was, of course, not evaluated in this work; a Lorentzian was substituted as an approximation. We note the result is identical to that obtained using the "golden rule," and considerably more difficult to arrive at.

Let us now examine various limits of this expression. For $\omega\tau \gg 1 \gg 3\gamma\bar{p}^2\omega\tau$ we get the usual three-phonon result of Eq. (3). For $3\gamma\bar{p}^2\omega\tau \gg 1$ this process goes to zero, as it should, since the three-phonon process is forbidden in this limit. There is, however, a problem. It is believed that the attenuation follows directly from the lifetime calculated in the thermal Green's-function approach. Khalatnikov maintains that the attenuation must go over into the four-phonon result $\alpha \propto \omega T^6$ asymptotically at low temperatures. The limit we get however is $\alpha \propto T^2/\gamma\tau$. It is difficult to reconcile these conflicting results, and we can only conclude that the present theory is incomplete. It is not known whether the failure of the low-temperature limit of the thermal Green's-function approach is due to the use of an incorrect spectral function or the exclusion of vertex corrections. The possibility that the ωT^6 form of Khalatnikov may be in error should not be excluded. Furthermore, since we have introduced a phenomenological parameter τ the theory is incomplete in this respect also.

So far we have discussed sound attenuation strictly from the point of view of phonon-phonon scattering, and we must now examine the validity of such a point of view. When $\omega\tau \gg 1$, the thermal phonon relaxation time is much longer than the period of a sound wave; however, the thermal phonon distribution is assumed to relax to an equilibrium distribution function. Since we were using equilibrium distribution functions in the derivations outlined before we may assume that these results are valid (if anywhere) in the $\omega\tau \gg 1$ region, or since $\tau \propto T^{-n}$, in the very low-temperature high-frequency limit. As the temperature is raised two things happen. The thermal phonon lifetime τ becomes shorter and, above 0.6 K, the rotons begin to contribute to the attenuation process. We neglect the rotons for the moment, deferring this discussion until later.

When $\omega\tau \ll 1$, the thermal phonon distribution relaxes in a time short compared with a sound period. Since the sound wave sets up periodic variations in the local properties of the medium, the thermal phonon distribution function relaxes to some value characteristic of local equilibrium but not to the thermal equilibrium value; consequently, the derivations we have described break down. Thus in this region the sound wave periodically modulates the thermal phonon distribution function, and one needs a formalism that allows for spatial variations in the distribution function. Such a formalism is provided by the Boltzmann or kinetic equation. Actually, the results of a Boltzmann-equation treatment are valid for both regions ($\omega\tau \geq 1$ and $\omega \leq 1$). Indeed, the low-temperature limits of the results based on the kinetic equation are identical to the scattering and Green's-function methods we have described previously. The sound attenuation will now be treated as the attenuation of a classical density wave propagating through the liquid, rather than as a scattering problem.

Two additional equations and some assumptions about the nature of the collision integral in the kinetic equation are required to solve the problem. The additional equations are the equation of continuity and the equation of motion of the condensate or superfluid. The problem of how to handle the collision integral has recently been approached from two different points of view. Disatnik[36] employed a "collision-time" approximation, with energy and momentum conservation directly built in. Such a model has the advantage of simplicity but has the disadvantage of introducing a phenomenological phonon-phonon relaxation time τ whose temperature dependence is unknown. Khalatnikov and Chernikova,[16–18] in their treatment, used a previous calculation by Landau and Khalatnikov,[37] of the viscous or wide angle scattering time τ_{pp}. In this calculation it is assumed that the collinear four-phonon scattering time t_{pp} is extremely short. This results in a quasiequilibrium among the thermal phonons moving in a specific direction which can be characterized by some temperature $T_p(\theta)$ which, as indicated, is angular-dependent. It is furthermore assumed that an inelastic (collinear) five-phonon scattering time τ_{3-2} (where three phonons combine to form two) is so small compared with other important scattering times that it results in a vanishingly small chemical potential for the thermal phonons. This is because the chemical potential for excitations whose number are not conserved vanishes in thermal equilibrium and, since the five-phonon processes change the number of particles, equilibrium at the local temperature can be established. Thus the nonequilibrium distribution function can be written in the form of a Planck distribution with an anisotropic temperature. The resultant relaxation time τ_{pp} is proportional to T^{-9}. The results for the attenuation of sound calculated by Khalatnikov and Chernikova and by Disatnik are identical to those given previously in the limit $\omega\tau \gg 1$ provided the τ entering the phenomenological treatments is equated to τ_{pp}. Khalatnikov and Chernikova state, however, that their results are valid only when $t_{pp} \ll \tau_{pp}$. The kinetic-equation approach of Refs. 16–18 and 36 and also the Green's-function approach of Ref. 35 allow, in addition, a determination of the temperature and frequency dependence of the sound velocity. The result in the limit $\omega\tau_{pp} \gg 1$ is

$$c(T)-c(0)=\frac{\pi^2}{60}\frac{(u+1)^2}{\rho\hbar^3}\left(\frac{\kappa T}{c}\right)^4 \times\ln\frac{1+(2\omega\tau_{pp})^2}{1+(3\gamma\bar{p}^2\omega\tau_{pp})^2}. \quad (5)$$

Andreev and Khalatnikov[38] obtain a nearly identical result.

For the ultrasonic frequencies employed to date, rotons contribute strongly at temperatures where $\omega\tau_{pp}\leq 1$ and thus the expressions based only on phonon-phonon scattering are not valid in this limit. The expressions do, however, go over into the classical hydrodynamic expression for first viscosity attenuation ($\alpha\propto\omega^2\tau_{pp}$). The theory could be verified experimentally only at very low frequency and at temperatures less than 0.6 K.

The appearance (above 0.6 K) of rotons in the liquid greatly complicates the calculation of transport processes. The rotons may scatter elastically and inelastically both among themselves and also with the phonons. The theoretical treatment must now include two coupled kinetic equations, one for phonons and one for rotons. Nevertheless, Khalatnikov and Chernikova proceeded to investigate this problem.[16–18]

The matrix element for phonon-roton scattering may be calculated more or less exactly from a knowledge of the density dependence of the parameters Δ and p_0, which describe the roton, and further from the requirements imposed by Galilean invariance.[22,23] Roton-roton scattering can be calculated only on the basis of some model, and Landau and Khalatnikov[22,23] used a delta-function potential. With this potential (the strength of which is fixed by experiment) the elastic roton-roton scattering time t_{rr} was shown to be very short and thus a Planck distribution with a local temperature is appropriate to describe the rotons. Inelastic phonon-roton and roton-roton scattering times are shown to be long compared with the other important scattering times; thus the roton distribution function must contain a local nonvanishing chemical potential since particle equilibrium (zero chemical potential) is achieved only through the inelastic collisions. The remaining scattering time τ_{pr} for elastic phonon-roton collisions turns out to be the most important scattering time in the region of temperature 0.9–1.2 K. For $\omega\tau_{pr}>1$ the expressions go over into the hydrodynamic form where now, however, a second viscosity coefficient ($\propto\tau_{pr}$) appears. Near 1 K where $\omega\tau_{pr}=1$ a large attenuation and dispersion result. As the temperature is raised above 1.2 K the rapidly decreasing phonon-roton scattering time τ_{pr} becomes equal or less than the five-phonon scattering time $\tau_{3\rightarrow 2}$. When this happens, local number equilibrium can no longer be established in the phonon gas and a nonvanishing chemical potential in the phonon distribution function must be included. The treatment in this region proceeds by ignoring the effect of wide angle scattering and includes only τ_{pr} and $\tau_{3\rightarrow 2}$.

Khalatnikov and Chernikova have derived two different sets of expressions for the temperature and frequency dependence of the attenuation and velocity of sound in liquid ^{4}He. The expressions are rather lengthy and have been collected in the Appendix. One set describes the range less than 1.2 K while the second is for temperatures above 1.2 K. The treatment above 1.2 K would not be valid, of course, near the lambda point since the whole character of the problem is different in this region.[39] It should also be pointed out that the equations for $T>1.2$ K do not join with the corresponding ones for $T<1.2$ K. Furthermore, Khalatnikov and Chernikova made it clear that their treatment for $\omega\tau_{pp}>1$ is not valid when $\omega t_{pp}\geq 1$, i.e., when the reciprocal of the angular frequency becomes comparable with the collinear four-phonon scattering time. Since t_{pp} is proportional to γ/T^7, and the value of γ is somewhat uncertain, it is not known at what temperature their treatment becomes invalid. Disatnick suggests that his treatment with a phenomenological scattering time may still be valid in this limit. We note again, however, that when $3\gamma\bar{p}^2\omega\tau>1$ (very low temperature and very high frequency) none of the phenomenological treatments go over into the four-phonon result and thus, we believe, must be incorrect. Nor is it clear that the τ in the Green's-function and scattering approaches should be identified with the τ_{pp} of Khalatnikov and Chernikova's treatment. Indeed we would expect t_{pp} to be reasonable for the energy uncertainty $\delta\epsilon$. Since a large portion of the data to be described was taken at low temperatures where possibly $\omega t_{pp}\geq 1$, it should be understood that the data may well lie outside the range of validity of existing theoretical treatments.

III. EXPERIMENT

At first sight it would appear that the simplest way to study the attenuation of ultrasound in ^{4}He would be to observe the exponential decay of the amplitude of a sound pulse which has undergone repeated reflections in a cavity containing the liquid. Such a technique is not feasible in ^{4}He because of several spurious attenuation mechanisms which can be so severe as to completely mask the true attenuation.[40] This is particularly true at low temperatures and high frequencies. The dominant source of spurious attenuation comes from nonparallelism of the reflecting surfaces in the sonic cavity, these surfaces usually being the quartz piezoelectric transducers. The nonparallelism causes the different regions of

the wave front to arrive at different times or, equivalently, different phases; consequently, the relative phase across the wave front changes on each reflection. Thus the transducer, which really measures an average over the wave front, detects a different amplitude on each reflection even if there were no attenuation in the liquid. In addition some energy must be extracted on each arrival of the wave front for the purpose of detection. Obviously some other technique is required. The problem has been attacked by three different approaches.

Pellam and Squire[1] constructed a sonic cavity where the receiving transducer could be moved relative to the transmitting crystal. The attenuation may then be studied by observing the change in amplitude of the signal when the transmitter and receiver are moved relative to each other by some known distance d. The technique works as long as the relative nonparallelism of the receiving and transmitting transducers can be maintained. Chase[5] has improved the technique by allowing an independent adjustment of the relative inclination and the distance between the transmitting and receiving transducers.

A radically different approach was employed by Woolf, Platzman, and Cohen.[41] What they did was to use the density variations set up by a transmitting transducer as a moving diffraction grating to scatter laser light. This is possible since the index of refraction is slightly density-dependent. Since the wave is attenuated in the liquid the angle at which diffraction maxima are observed will have a finite width. The attenuation follows immediately from the magnitude of the width. There are difficulties in practice but the method looks promising, particularly at high temperature and frequency.

The third method, the one used in these experiments, measures only the change in attenuation as a function of temperature. In order to determine the absolute value of attenuation one must resort to theory. One can in general write that

$$\alpha(T) = \alpha(T) - \alpha(T_{\min}) + \alpha(T_{\min})$$

$$\text{or} \quad \alpha(T) = (\Delta\alpha)_{\min} + \alpha(T_{\min}) \ ,$$

where $(\Delta\alpha)_{\min}$ is the change in attenuation in going from the lowest temperature reached $(T_{\min})$ to the temperature in question, and $\alpha(T_{\min})$ is the true attenuation at the minimum temperature. If we can demonstrate that $\alpha(T_{\min})$ is less than the resolution of the instrument than one can neglect $\alpha(T_{\min})$ and set $\alpha(T) = (\Delta\alpha)_{\min}$. The true attenuation according to theory is proportional to T^n at very low temperatures, where $n = 4$ or 6 for the three- and four-phonon processes, respectively i.e., the true attenuation rapidly approaches zero at low temperatures. Assuming then that $\alpha = aT^n$, it can be shown that

$$\alpha(T_{\min}) = \frac{\alpha(T) - \alpha(T_{\min})}{(T/T_{\min})^n - 1} = \frac{(\Delta\alpha)_{\min}}{(T/T_{\min})^n - 1} \ . \quad (6)$$

All of the parameters on the right, except for n, can be measured experimentally. Within the resolution of our instrument (0.02 – 0.1 dB), we usually observed no change in the attenuation from the minimum temperature up to some higher temperature (T_c). In this situation, assuming the most unfavorable case of $n = 4$, Eq. (6) can be written as

$$\alpha(T_{\min}) = \frac{(\Delta\alpha)_{\text{resolution}}}{(T_c/T_{\min})^4 - 1} \ .$$

We observed in all cases that $(T_c/T_{\min})^4 \geq 2$; consequently $\alpha(T_{\min})$ was always less than the resolution of our instrument and hence can be set equal to zero.

If $T_{\min}$ is greater than 0.1 K, one may try to obtain the absolute value of the attenuation by a least-squares fitting of the attenuation change to some assumed temperature dependence. In particular one may again assume the form

$$\Delta\alpha = \alpha(T) - \alpha(T_{\min}) = aT^n - \alpha(T_{\min}) \ ,$$

where a, n, and $\alpha(T_{\min})$ are determined from the fit. Such a procedure is reliable only if it is known for certain that the true attenuation may be accurately represented by such a fit. As we shall see later the value of n is slightly temperature-dependent and thus such a normalization procedure is risky.

In order to be able to accurately resolve small attenuation changes, it is necessary that the measurement technique be unaffected by changes in the input power level applied to the transmitting transducer, or changes in the gain of the amplifier-detector following the receiving transducer. The methods adopted for these measurements were variations of the "pulse-comparison" technique and the "pulse-interference" technique.[40] A simplified block diagram is shown in Fig. 2. Power from a continuously running oscillator was split into two channels. Appropriately timed pulses were generated in each channel by means of two coaxial rf switches. One pulse, the signal pulse, was applied to the sonic cavity; a second pulse, the reference pulse, was applied to a high-precision attenuator which was followed by an accurately calibrated (in nanoseconds) delay line. The output from the sonic cavity and from the 50-Ω attenuator following the delay line were combined, amplified, detected, and finally

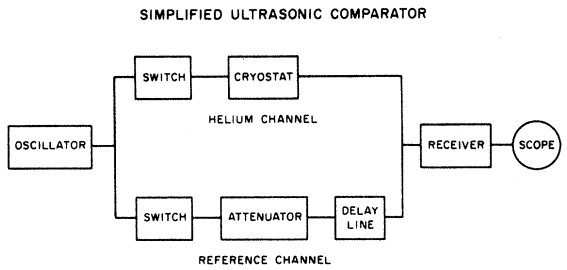

FIG. 2. Simplified block diagram of the ultrasonic comparator.

displayed on an oscilloscope. The attenuation-measurement technique consists of initially matching the amplitude of the two channels at some temperature (usually $T_{\min}$) and then heating the sonic cavity and observing the change in the attenuator setting (in the reference channel) necessary to re-establish the amplitude balance in the two channels. Such a measurement technique is obviously insensitive to changes in transmitter power or receiver gain. As the attenuation increases the power level of the transmitter may be raised in order to re-establish a favorable signal-to-noise ratio. Care must be taken, however, to keep the power level in the helium below some critical value (which increases with increasing temperature); otherwise the attenuation becomes amplitude-dependent. This was easily observed in practice by establishing a balance at some low power level and then increasing the power level to the point where the balance was upset.

In the pulse-comparison technique the switches in the reference and signal channels were activated on alternate cycles of a basic clock frequency. The amplitudes of the two channels were then equated by visual comparison with an oscilloscope. In the pulse interference method the two channels were activated at very nearly (but not quite) the same time (with each cycle of the clock), the time difference being equal to the transit time of the sound wave in the liquid. This means that both signals arrived at the receiver input at the same time and, providing they were coherent, would interfere. By proper adjustment of the delay line and the attenuator a null was achieved. Since the velocity is temperature-dependent, a change in temperature will cause a change in transit time or, equivalently, a change in phase of the signal traversing the helium. By observing the change in the delay line setting Δt required to restore the null we could study, in addition, the temperature dependence of the velocity of sound between two temperatures T and $T_{\min}$ through the relation

$$\Delta t/d = 1/c(T) - 1/c(T_{\min}),$$

where d is the length of the sound cavity.[40] It is understood of course that if the attenuation changes we must adjust the attenuator also to achieve the null; thus we obtain a simultaneous measurement of the temperature dependence of the attenuation. Since both the reference and signal pulses were derived from a continuous oscillator, coherence was assured. For the velocity measurements it is further required, however, that the long-term stability of the oscillator be uncommonly good, since a change in frequency δf is observed as an apparent change in transit time δt (or velocity δc) through the relation

$$\delta t = (d/c)(\delta f/f) \quad (\text{or } \delta c/c = \delta f/f).$$

The oscillator used had a stability of one part in 10^7 per hour which was satisfactory. The frequency was monitored with an electronic frequency meter.

It is quite important that stray rf coupling to the receiver be carefully avoided, because it would interfere with the received signal. Since the velocity (or phase) is temperature- dependent then the relative phase of the leakage and the received signal would vary in the course of the experiment and the measured attenuation would be spurious. The rf leakage was reduced to a negligible level by the following precautions. The receiver and signal generator were kept well separated and all interconnecting cables were double shielded. Furthermore the rf switches used had a minimum of 40-dB isolation.

Figure 3 shows the cell which contained the sonic cavity. The sonic cavity was formed by a $\frac{1}{2}$-in. o.d. $\times$ $\frac{1}{4}$-in. i.d. $\times$ 1.022-cm-long fused quartz annulus. The ends of the cavity were unloaded, coaxially plated, x-cut quartz transducers. Both 12- and 30-MHz fundamental frequencies were used in these experiments. The transducers were clamped to the quartz spacer by flat polished stainless-steel rings which also facilitated one of the electrical contacts to the transducer. A small coil spring about 0.050 in. in diameter was wound with 0.002-in.-diameter Pt wire, and it served to make the second electrical contact. The spring was such that it caused a minimal warping of the transducer (thus reducing spurious attenuation) while still providing the necessary electrical contact. A Teflon "sound spoiler" filled the remainder of the stainless-steel clamping ring and greatly attenuated the signals returning to the transducers from this region. The remainder of this section of the sonic cell was filled with coaxial washers which completed the electrical connections to the transducers. The end caps of the sonic cell (which are necessary for the assembly of the components of the sonic cavity) were sealed to

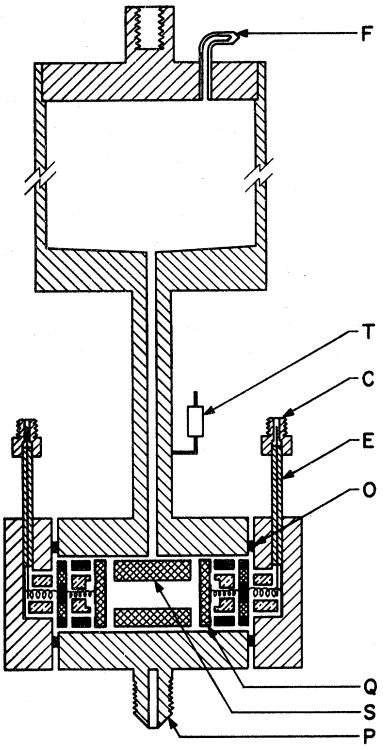

FIG. 3. Cross sectional view of the sonic cell. The following items are indicated: (F) copper tubing for pressurizing and sealing the cell. (T) resistance thermometer, (C) coaxial connector for rf input, (E) epoxy-sealed stainless-steel transmission line, (O) lead O ring, (Q) X-cut quartz transducer, (S) fused quartz spacer, and (P) threaded stud for attaching magnetic thermometer.

the sonic cell by lead O-rings[42] (made by bending wire in a circle and twisting the ends) which were compressed by four screws. Radio-frequency power was brought into and out of the sonic cell (through the end caps) by vacuum and pressure-tight coaxial transmission lines. The outer conductor of this transmission line was a 0.100-in. o.d. $\times$0.010-in. wall stainless-steel tube while the inner conductor was a 0.025-in. o.d. $\times$0.021-in. i.d. stainless-steel tube. The annulus was filled with Emmerson and Cummings Stycast-filled epoxy[43] (which has a much smaller thermal expansion coefficient than pure epoxy). A good bond to the stainless-steel tube was assured by abrading the inside of the tube. Most of the seals of this type withstood numerous cyclings between room and helium temperature, as did the lead o-ring seals. A copper coil spring was soldered to the end of the inner conductor which made contact to the previously mentioned coaxial washers. Miniature coaxial connectors were employed on the opposite end of the transmission line. A magnetic thermometer was thermally and mechanically attached to the bottom of the sonic cell.

For a variety of reasons it is advisable to construct the sonic cell as a bomb (the main difficulty being associated with heat conduction associated with film flow). By this we mean that the sonic cell contains a large enough storage region so that when it is pressurized to some nominal value at room temperature [500 psi (gauge) in our case] there will be enough liquid to fill the sonic cavity at helium temperature. The upper region of the sonic cell shown in Fig. 3 serves as this storage region. Pressurized gas from a helium cylinder was passed through a liquid-helium cold trap, and then through a soft copper tube which entered the cell at the top. When the pressure reached 500 psi (gauge) the copper tube was "crimped" off and the end soldered closed.

The use of unloaded transducers is quite important (by an unloaded transducer we mean one loaded only with helium and with no bonded solid backing), since an uncommonly good electrical impedance match can be obtained to a quartz transducer loaded only with liquid helium.[40] The advantage of a good electrical match is that the sonic power level in the helium may be much lower because of the increased detection efficiency that results from a good electrical match. Thus reliable data may be taken with a much higher signal-to-noise ratio before the previously discussed amplitude dependent effect contributes. Also since the efficiency of a transducer falls off rapidly with increasing frequency, we can carry out measurements to much higher frequencies (208 MHz in these experiments) if we take advantage of the better electrical match achieved with an unloaded transducer. The peak electrical power input to the sonic cell was never greater than $\frac{1}{50}$ W in these measurements and a signal-to-noise ratio as high as 90 dB was achieved while working at the fundamental of a 30-MHz transducer. Also the clock frequency was never high enough to cause the average power input to heat the sonic cell. The price of the good electrical match is a very narrow bandwidth. This results in a slow rise time for the signal pulse (4–10 μsec in our case). Since the transit was on the order of 50 μsec, however, there was ample time for the pulse to build up.

We now proceed to the cryogenic aspects of the experiment. The sonic cell, together with an iron-ammonium alum thermal guard salt, potassium-chromium alum refrigerator salt, and cerium-magnesium-nitrate magnetic thermometer were located inside a copper can (inner can) which was cooled by a ^{3}He evaporation refrigerator (shield). The inner can was isolated from the ^{4}He bath by means of a vacuum space between the inner can and an outer can. Both the magnetic thermometer and the refrigerator salt were thermally anchored to the sonic cell. Coaxial stainless-steel transmission lines were used to bring the rf power into and out of the ap-

paratus. The inner and outer conductors were insulated from each other with Teflon "spaghetti" and the annuli were filled with high-viscosity silicone oil to facilitate thermal grounding of the inner conductor. The outer conductor was thermally anchored at several points with copper wire, which was soldered to the outside of the tube. Resistance thermometers monitored the refrigerator salt, sonic cell, and guard pill, and were useful in determining thermal equilibrium following a heat input to the apparatus.

The magnetic thermometer was calibrated against the vapor pressure of ^{3}He between the temperatures of 600 to 1600 mK. Helium-3 exchange gas was admitted to the inner space; the salts and cell were then cooled by pumping on the ^{3}He shield. After the inner parts had cooled to about 500 mK the pump valve was closed and heat was supplied to bring the temperature to about 600 mK. One hour was allowed for equilibrium before making pressure, susceptibility, and resistance measurements and before heating for the next point. The temperature drift was 1 mK per minute or less, and all readings were taken as a function of time so that all parameters could be interpolated to the same time. The ^{3}He pressure was read on an oil manometer or on a mercury manometer with a cathetometer. Four to five points at equal intervals of $1/T$ were taken, and the calculated temperature was within $\pm 0.2\%$ of the measured temperature using the equation $T = A/(M - B)$, where M is the susceptibility bridge reading and A and B are constants determined from the calibration.

The low temperatures were reached by adiabatic demagnetization from 500 mK. The heat of magnetization was removed with a small auxiliary ^{3}He evaporation refrigerator from which all the ^{3}He was ultimately pumped out. With copper-nickel or brass sonic cells, temperatures as low as 30 mK were routinely achieved.

Following demagnetization, it was observed that the upward temperature drift of the apparatus was never more than 1 mK per hour. After waiting a period of time to allow for thermal equilibrium, a reference level of the attenuator (and when measuring velocities a delay line setting) was established. The cell was then heated and after temperature equilibrium had been reestablished the new attenuator and delay line settings were determined. This process was continued until the signal disappeared into the noise, or an amplitude dependence precluded further measurements. At the lower frequencies it was possible to follow the attenuation up to the lambda point. Measurements above the lambda point were not made in these experiments. At the higher temperatures, where the velocity change causes the delay time difference to be greater than the period $(1/f)$, we re-established the null setting by subtracting one period from the delay line setting.

Measurements of the temperature dependence of the attenuation were made at 12, 30, 36, 60, 84, 90, 108, 132, 150, and 208 MHz. Measurements of the temperature dependence of the velocity of sound were made at 12, 36, 60, and 84 MHz. A detailed discussion together with a comparison of these data with theory will be carried out in the next section. We will also compare our results with the experimental results of other authors.

IV. DATA AND INTERPRETATION

Figures 4 through 13 show the temperature dependence of the attenuation at 12, 30, 36, 60, 84, 90, 108, 132, 150, and 208 MHz. The data have been plotted in a log-log manner so that if the attenuation were proportional to some power of the temperature it would appear as a straight line with a corresponding slope. Note that plotting in this manner appears to amplify the experimental error associated with reading small attenuations; thus the data necessarily show more scatter at low temperatures. The 30-, 90-,

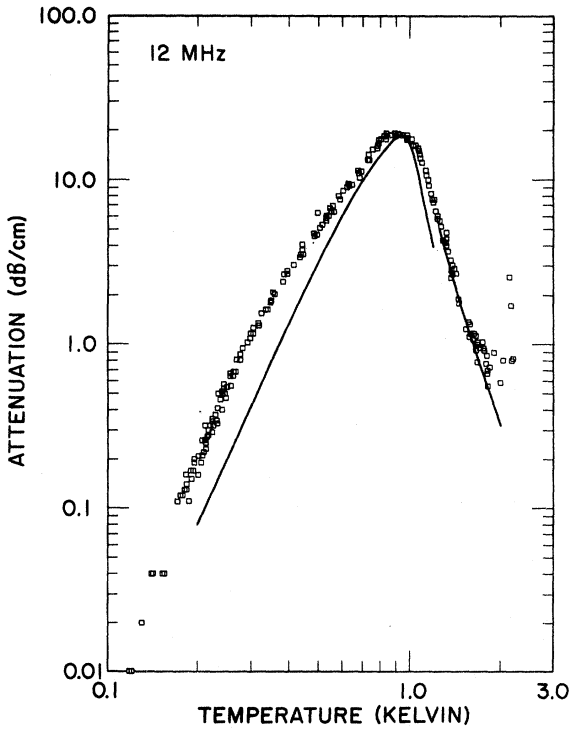

FIG. 4. The 12-MHz attenuation data. The solid line is the theory of Khalatnikov and Chernikova. The break in the theoretical curves at 1.2 K results from the different expressions which must be used above and below this temperature.

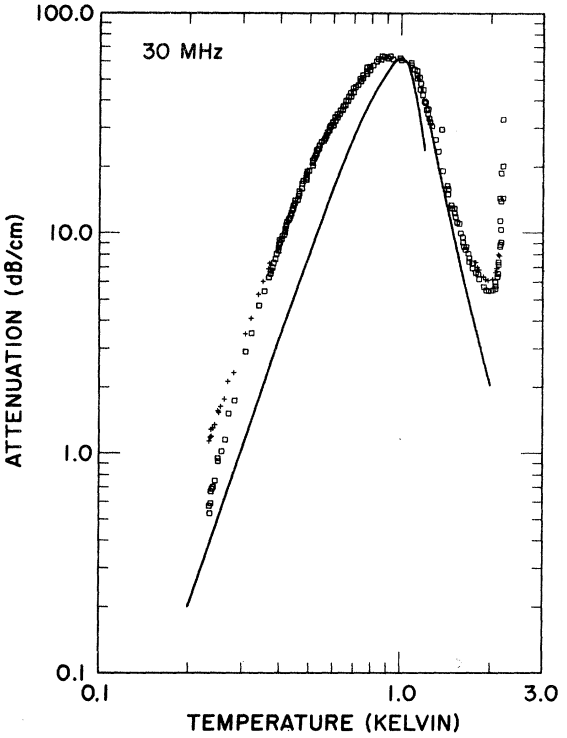

FIG. 5. The 30-MHz attenuation data. The solid line is the theory of Khalatnikov and Chernikova. The pluses show the data corrected by 0.6 dB to account for a possible normalization error. The break in the theoretical curves at 1.2 K results from the different expressions which must be used above and below this temperature.

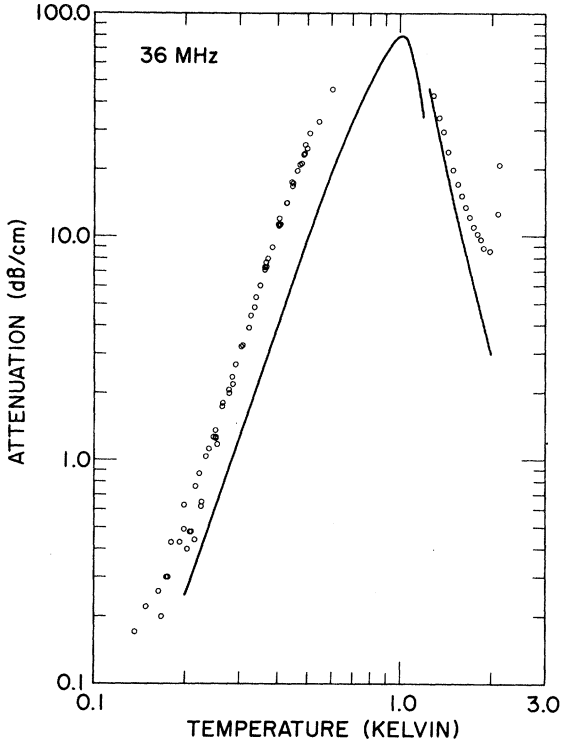

FIG. 6. The 36-MHz attenuation data. The solid line is the theory of Khalatnikov and Chernikova. The break in the theoretical curves at 1.2 K results from the different expressions which must be used above and below this temperature.

and 150-MHz data were taken with a 30-MHz fundamental frequency transducer, while the 12, 36, 60, 84, 132, and 208 were taken with a 12-MHz fundamental frequency transducer. In general a much better signal-to-noise ratio was achieved with the 30-MHz transducer. Signals at the odd harmonics between 132 and 208 MHz were observed but no runs were made. Signals were also observed at still higher frequencies but the amplitude-dependent effect prevented the taking of meaningful data. Were greater care taken to insure a more parallel alignment of the transducers, there is no reason why the techniques could not be extended to still higher frequency.

The 12- and 30-MHz data were observed continuously from the lowest temperature where an attenuation change was resolvable right up to the lambda point. We note the following general characteristics of these 12- and 30-MHz data. At low temperature the attenuation rises rapidly (with temperature) with a slope that corresponds to a nearly T^4 behavior. The attenuation reaches a maximum near 1 K and then falls rapidly finally reaching a minimum near 1.9 K. Near the lambda point (2.2 K) an extremely rapid rise is

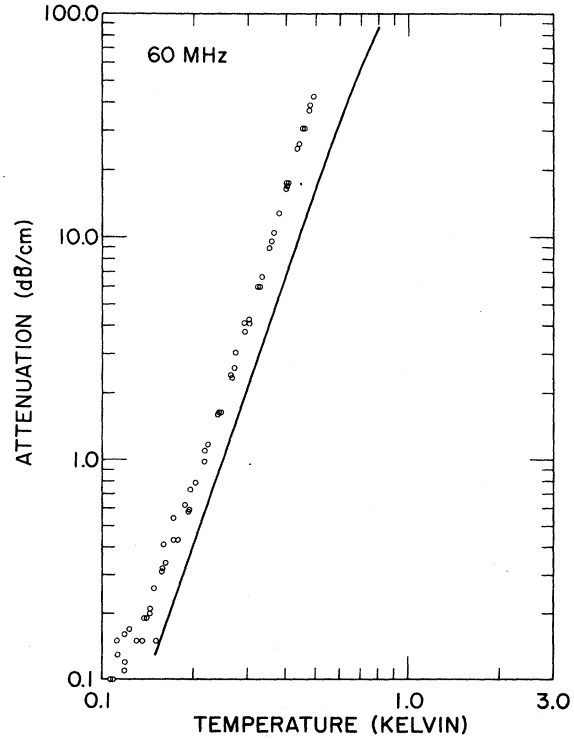

FIG. 7. The 60-MHz attenuation data. The solid line is the theory of Khalatnikov and Chernikova.

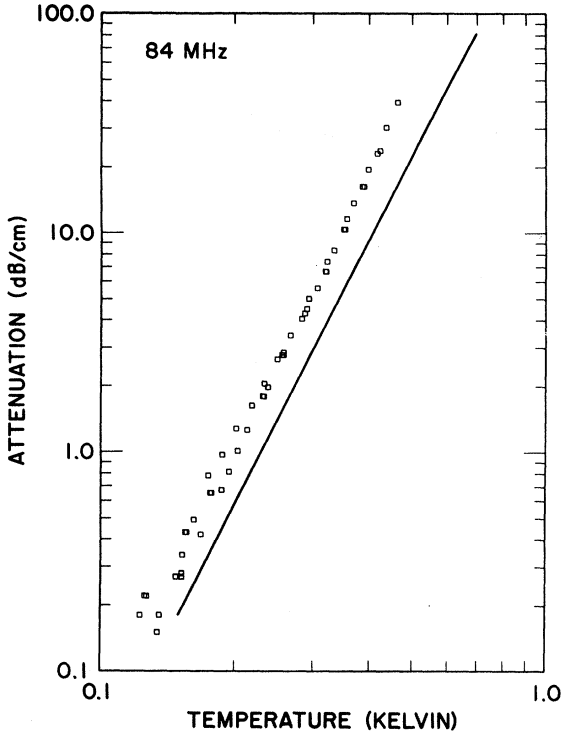

FIG. 8. The 84-MHz attenuation data. The solid line is the theory of Khalatnikov and Chernikova.

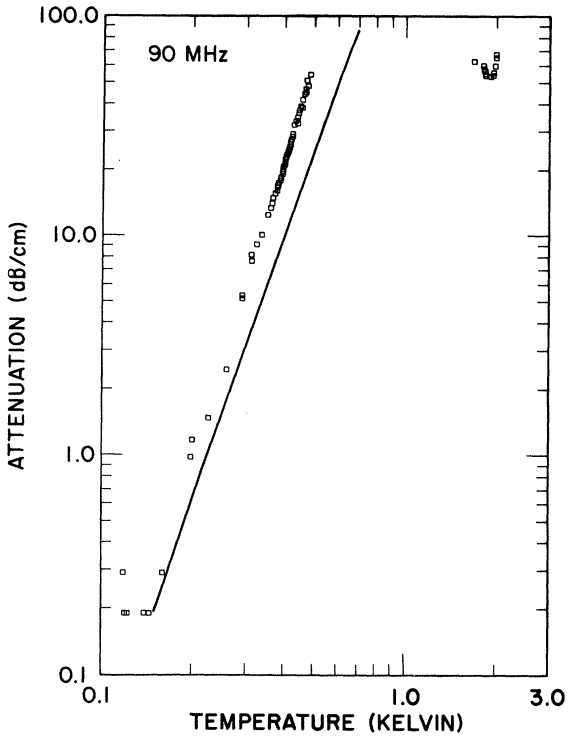

FIG. 9. The 90-MHz attenuation data. Because of the excellent signal-to-noise ratio of the 30-MHz transducers some data were observed near 1.9 K. The solid line is the theory of Khalatnikov and Chernikova.

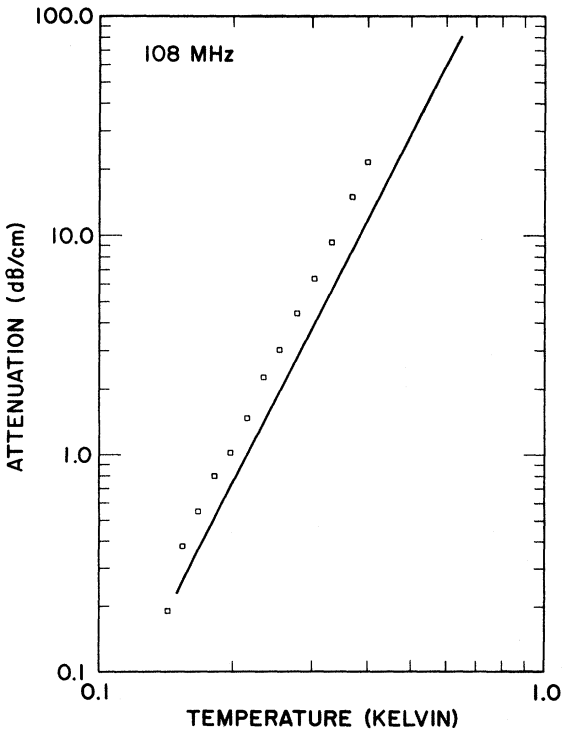

FIG. 10. The 108-MHz attenuation data. The solid line shows the theory of Khalatnikov and Chernikova.

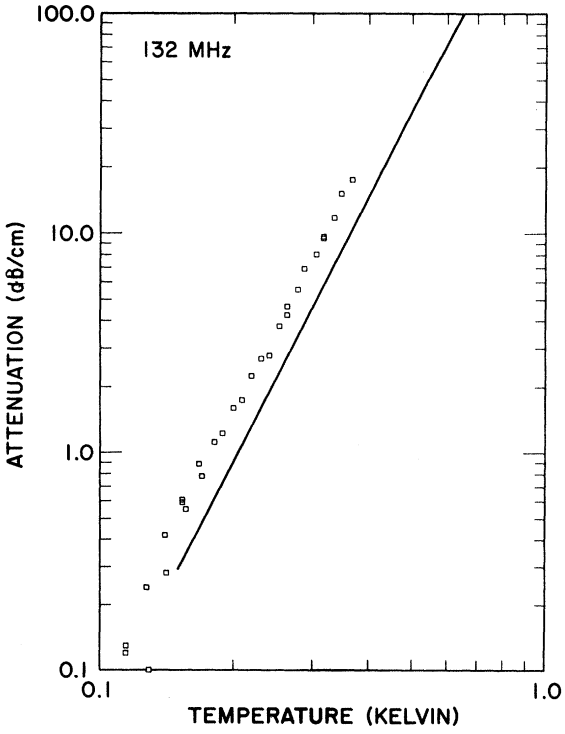

FIG. 11. The 132-MHz attenuation data. The solid line shows the theory of Khalatnikov and Chernikova.

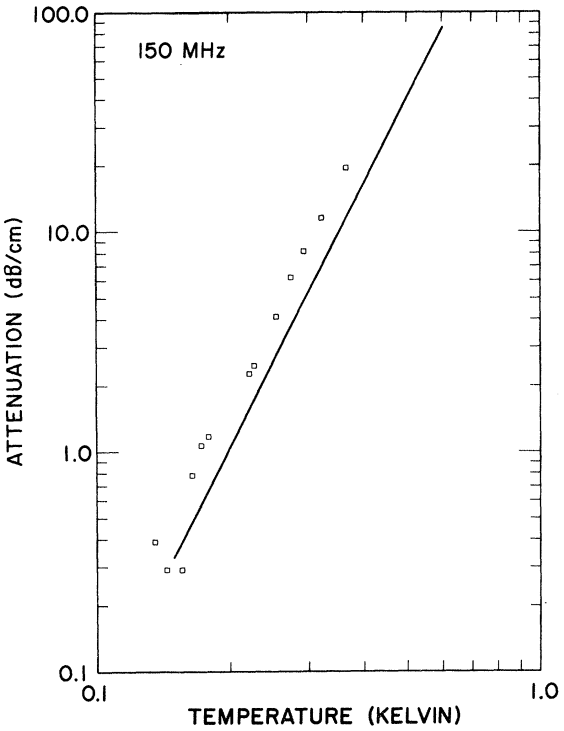

FIG. 12. The 150-MHz data. The solid line shows the theory of Khalatnikov and Chernikova.

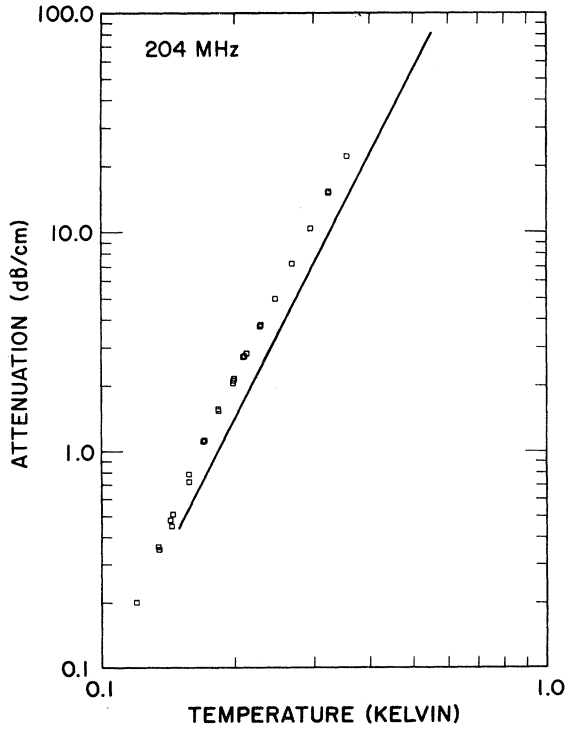

FIG. 13. The 208-MHz data. The solid line shows the theory of Khalatnikov and Chernikova.

observed. The 12-MHz data show a clear change in slope near 0.35 K. This frequency was repeated five times and all features were completely reproducible. If one had attempted to determine the absolute value of the attenuation by fitting only data taken above 0.35 K (by using the least-squares fit procedure described in the last section), one would have obtained an erroneous result. Thus carrying the measurements to low temperature is essential if a reliable determination of the absolute value of the attenuation is to be obtained. On one 12-MHz run, data were taken on the fifth rather than the first echo (the points shown in Fig. 4 below 0.1 dB/cm are from the fifth echo measurements.) Since the attenuation and velocity changes are multiplied by a factor $(2n+1)$ where n is the echo number, it was possible to resolve smaller attenuation and velocity changes and thus extend the measurements to lower temperatures.

The first data taken in these experiments were at 30 MHz. In this run the comparison pulse was derived from a second oscillator. The power output of these oscillators had some tendency to drift relative to each other. Equating of the power outputs was overlooked in establishing the low-temperature reference point, and by the time the system had been heated to where the first data point was taken some drift may have occurred. Care was taken throughout the remainder of the run to equate the power outputs and thus the data as given are accurate to within 0.1 dB as differences, but the absolute value may be off by more than this amount. As can be seen, the data do not lie on a straight line at low temperatures. By adding 0.6 dB to all points a straight line results (at low temperature) corresponding to a temperature dependence of the attenuation proportional to $T^{3.99}$ (pluses in Fig. 5). Circumstances did not permit retaking the low-temperature data (the high-temperature region was repeated twice and normalized to the first run at the minimum near 1.9°K). We have included this data so that we may later compare it at high temperature with the theory of Khalatnikov and Chernikova. A 0.6-dB correction contributes negligibly at high temperature. In all future runs both pulses were derived from the same oscillator.

The lower-signal-to-noise ratio achieved with the 12-MHz transducers (and thus the earlier onset of the amplitude dependence) prevented extending the 36-MHz data to the peak at 1 K. The rest of the curve has been resolved in detail, however. The 36-MHz data show the same general features as the 12 and adjusted 30-MHz data, namely an essentially T^4 behavior at low temperature, rising to a peak near 1 K and then falling off to a minimum near 1.9 K.

From 60 to 208 MHz only the low-temperature

portion could be studied in detail although some data near the minimum at 1.8 K were resolvable. In all cases these low-temperature high-frequency data fall on a fairly good straight line with the slope near (but rarely equal to) four.

In order to communicate the full accuracy of the data we have collected the results of all experimental attenuation runs in Tables I through X. The letters (A, B, etc.) indicate different experimental runs. The temperatures are as calculated from the calibration fit and the last decimal place is not significant. The last decimal place of the attenuation (and velocity) measurements is only partially significant. If an accurate frequency is not quoted then the frequency is the nominal value and is accurate to approximately one percent.

A very interesting feature of the low-temperature data is displayed if we plot the frequency dependence of the attenuation for a few temperatures. Since measurements were taken at ten different frequencies we obtain a fairly complete picture in the range 12 to 208 MHz. Figure 14 shows the results of such a plot at 0.2, 0.25, 0.3, 0.35, and 0.4 K. The solid lines are smooth curves drawn through the points. At low temperature the errors are due to inaccuracies in measuring the attenuation. At higher temperatures the errors are due to inaccuracies in measuring the temperature and calibrating the thermometer. Clearly the attenuation is proportional to the frequency between 12 and 36 MHz but "flattens out" at higher frequency (above 0.3 K). This is qualitatively what we would expect on the basis of Eq. (4) and we are encouraged to attempt a closer comparison. We must bear in mind, however, that since this formula does not go over asymptotically to the four-phonon result a close comparison may not be justified. It is, however, interesting to pursue the analysis. We will adopt

TABLE I. Temperature dependence of attenuation and velocity of sound in liquid ^{4}He at 12 MHz.

Temperature (mK)	Attenuation (dB/cm)	$\frac{\Delta c}{c} \times 10^6$	Temperature (mK)	Attenuation (dB/cm)	$\frac{\Delta c}{c} \times 10^6$
		Series A 11.92 MHz $T_{min} = 80.0$ mK			
210.7	0.32	0.4	681.4	10.38	650.1
212.4	0.32	0.4	787.6	15.66	531.6
242.0	0.50	4.2	780.5	15.66	531.6
245.4	0.50	4.2	838.3	17.84	306.7
276.7	0.87	10.7	831.6	17.84	306.7
278.2	0.87	10.7	899.1	18.84	−155.1
301.3	1.16	19.1	892.9	18.84	−155.1
305.5	1.16	19.1	980.0	17.85	−912.2
335.2	1.63	39.9	980.0	17.85	−912.2
340.5	1.63	39.9	1100.1	12.81	−2282.7
386.5	2.68	81.9	1100.5	12.81	−2282.7
394.1	2.68	81.9	1230.6	5.86	−4138.1
440.8	3.56	135.2	1371.3	2.56	−6751.4
446.1	3.56	135.2	1520.2	•••	−9969.7
490.7	4.66	191.0	1700.4	•••	−19596.7
493.0	4.66	191.0	1849.0	•••	−32292.3
562.8	6.46	308.5	2004.1	•••	−57000.4
564.0	6.46	308.5	2154.2	•••	−84898.1
681.4	10.38	650.1			
		Series B 12.02 MHz Echo No. 5 $T_{min} = 108.4$ mK			
109.1	0.00	0.0	187.8	0.11	1.5
118.8	0.01	0.4	211.5	0.23	3.6
120.3	0.01	0.4	213.3	0.23	3.6
130.1	0.02	0.7	227.7	0.32	4.8
130.7	0.02	0.7	229.2	0.32	4.8
141.4	0.04	0.9	257.6	0.64	10.2
142.6	0.04	0.9	260.2	0.64	10.2
153.9	0.04	1.0	297.3	1.08	24.0
155.3	0.04	1.0	299.0	1.09	24.0
187.2	0.11	1.5			

TABLE I (*Continued*)

Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)
		Series C Normalized to series F			
171.7	0.11	529.0	6.03	1560.8	1.86
176.8	0.12	558.7	6.98	1565.5	1.84
180.2	0.13	606.3	8.63	1569.5	1.84
184.0	0.14	621.4	9.15	1627.6	1.95
189.7	0.17	671.7	11.31	1631.0	1.94
193.0	0.17	733.5	14.45	1633.3	1.88
201.0	0.21	791.8	17.72	1733.9	1.44
206.4	0.26	838.5	19.22	1740.3	1.31
214.2	0.27	911.3	19.46	1802.0	0.99
216.5	0.30	804.6	17.89	1822.5	0.88
221.1	0.32	1034.3	16.41	1827.5	0.92
224.2	0.35	1085.0	13.61	1910.1	0.72
228.6	0.37	1124.3	10.04	1914.0	0.78
233.2	0.41	1146.9	8.78	1924.2	0.98
238.4	0.46	1170.6	7.80	1932.1	0.89
244.3	0.51	1195.1	6.87	1947.1	1.09
250.1	0.55	1214.7	6.23	1969.7	1.23
263.7	0.68	1238.1	5.53	1997.5	1.14
277.8	0.83	1262.4	4.87	2002.9	0.80
292.5	1.03	1314.3	3.74	2152.7	2.37
317.9	1.31	1353.7	3.09	2158.0	3.21
358.6	2.03	1384.6	2.70	2161.4	4.33
412.8	3.07	1438.5	2.48	2163.7	6.53
442.6	3.75	1476.6	2.45		
482.3	4.70	1527.6	2.18		
		Series D Normalized to series F			
176.5	0.12	212.6	0.25	258.7	0.63
184.2	0.13	217.8	0.28	257.2	0.58
205.3	0.19	227.1	0.34		
207.2	0.21	237.4	0.50		
		Series E Normalized to series F			
178.0	0.12	212.0	0.26	244.9	0.57
182.3	0.13	214.0	0.25	255.9	0.67
183.3	0.16	223.5	0.33	269.5	0.81
191.0	0.15	235.3	0.50	282.1	0.95
200.9	0.16	244.8	0.54	305.6	1.26
326.3	1.56	1097.8	12.87	1619.7	1.17
325.2	1.54	1132.0	11.50	1657.3	1.14
355.0	2.07	1151.6	10.12	1674.4	1.05
395.0	2.81	1158.2	9.34	1691.8	1.01
443.1	4.07	1178.7	8.36	1704.0	0.96
531.5	6.03	1189.2	7.65	1706.2	1.01
546.0	6.74	1205.9	6.93	1716.9	1.03
586.0	8.03	1215.7	6.59	1717.9	1.04
632.4	9.40	1232.3	6.06	1741.4	0.93
677.4	11.06	1245.9	5.69	1741.9	0.95
726.7	13.30	1263.1	5.27	1764.4	0.77
755.5	15.37	1286.9	4.83	1788.5	0.56
789.8	17.38	1306.0	4.41	1810.3	0.73
819.0	18.46	1329.1	4.02	1860.6	0.90

TABLE I (*Continued*)

Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)
844.9	19.14	1350.1	3.70	1925.9	0.59
874.5	18.95	1370.4	3.28	1992.5	0.81
894.0	19.14	1389.3	3.03	2072.0	2.59
925.1	19.14	1407.2	2.91	2108.4	1.73
979.5	18.95	1429.9	2.71	2136.4	0.80
1019.4	17.87	1550.6	1.25	2170.2	0.82
1046.1	16.40	1576.2	1.12		
1080.0	14.54	1598.0	1.17		
		Series F	$T_{min} = 146.6$ mK		
182.6	0.13	257.6	0.57	434.9	3.43
181.9	0.13	258.0	0.57	438.9	3.48
181.9	0.13	266.8	0.67	444.6	3.62
195.1	0.20	267.1	0.68	484.7	4.57
195.4	0.19	276.7	0.80	484.9	4.60
195.4	0.20	277.6	0.81	496.7	6.33
209.8	0.22	298.0	1.08	504.9	5.13
209.8	0.22	297.6	1.09	514.9	5.37
223.9	0.29	297.6	1.09	530.7	5.70
225.2	0.29	317.2	1.34	530.7	5.86
225.2	0.29	317.2	1.34	532.8	5.94
232.8	0.33	349.1	1.81	538.9	6.05
233.1	0.34	349.9	1.84	550.9	6.47
240.9	0.40	382.5	2.41	552.1	6.50
241.4	0.40	382.2	2.40	597.7	7.81
248.8	0.47	416.1	3.11	590.9	7.65
248.8	0.47	414.8	3.07	590.9	7.65
257.6	0.56	414.5	3.03	645.9	9.46
645.9	9.46	1066.7	15.68	1563.1	1.37
690.4	11.25	1066.7	15.81	1572.6	1.38
686.8	11.15	1139.1	10.82	1679.0	0.79
737.4	13.42	1134.1	11.14	1663.0	0.93
733.2	13.30	1126.7	11.53	1616.5	1.08
789.9	16.79	1205.0	7.30	1807.3	0.67
780.5	16.21	1196.7	7.45	1807.3	0.70
780.5	16.11	1207.8	7.59	1806.6	0.86
845.9	18.92	1221.9	6.55	1852.3	1.01
832.9	18.70	1219.1	6.50	1852.3	1.06
832.9	18.68	1216.2	6.47	1852.3	1.17
911.7	19.24	1317.8	4.17	1920.6	0.46
888.3	19.37	1307.9	4.35	1920.6	0.40
888.3	19.37	1304.6	4.35	1920.6	0.36
924.6	19.13	1391.8	2.70	1994.2	0.93
914.9	19.21	1380.7	2.85	1994.2	0.93
903.8	19.27	1380.7	2.85	2073.6	3.11
953.4	18.86	1462.1	1.79	2114.8	2.61
964.0	18.75	1454.0	1.83	2159.6	9.57
978.5	18.66	1449.9	1.89	2159.6	19.18
1071.1	15.19	1586.9	1.34		

TABLE II. Temperature dependence of the attenuation of sound in liquid ^{4}He at 30 MHz.

Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)
		Series A $T_{min} = 175.1$ mK			
235.3	0.53	416.5	10.58	1185.6	36.51
235.9	0.58	441.4	12.69	1185.6	36.44
237.1	0.67	460.5	14.84	1231.3	30.97
237.7	0.68	460.5	15.04	1231.3	30.71
237.7	0.59	488.7	17.43	1280.7	26.59
240.7	0.70	512.2	20.36	1298.1	23.52
245.0	0.75	512.2	20.53	1298.1	23.28
250.7	0.92	720.3	48.44	1315.9	19.11
250.7	0.95	720.3	49.82	1353.1	16.39
256.8	1.02	742.6	51.32	1455.9	12.97
263.9	1.15	754.3	52.26	1478.3	11.04
270.6	1.51	760.3	53.09	1550.1	9.62
283.4	1.73	766.4	53.97	1575.6	8.69
309.0	2.89	772.5	54.93	1629.2	7.34
322.5	3.50	778.8	56.04	1716.8	6.30
342.1	4.66	785.2	57.15	1780.6	5.77
355.2	5.43	791.7	58.03	1885.8	5.52
372.7	6.67	833.0	60.34	1963.1	5.51
395.8	8.28	799.6	62.94	1963.1	5.75
414.7	9.84	931.9	61.28	2091.7	6.53
414.7	9.98	1103.6	54.93	2138.3	14.41
414.7	10.11	1129.7	51.06	2187.2	9.16
415.6	10.30	1157.0	44.67	2187.2	20.23
416.5	10.51	1185.6	36.71	2208.6	32.69
		Series B Normalized to series A			
367.0	6.26	411.0	9.79	513.5	20.86
368.4	6.30	417.7	10.38	517.5	21.35
371.3	6.36	421.4	10.86	520.2	21.84
369.8	6.44	428.0	11.45	524.5	22.33
372.7	6.54	434.3	12.04	528.3	22.82
374.2	6.61	442.4	12.92	533.6	23.31
374.2	6.76	453.4	14.10	536.5	23.80
377.1	6.86	465.2	15.35	540.9	24.29
378.6	6.93	471.1	15.88	544.5	24.78
381.6	7.29	477.4	16.69	548.6	25.27
386.2	7.57	487.0	17.71	552.8	25.76
389.6	8.05	493.4	18.44	556.4	26.26
396.8	8.60	499.8	19.17	561.1	26.75
404.6	9.29	475.9	20.29	562.8	27.24
567.9	27.73	754.6	54.05	1446.0	15.90
571.2	28.22	770.9	56.16	1485.4	13.40
576.2	28.71	796.6	59.10	1516.8	12.39
579.2	29.20	824.2	61.06	1546.1	11.12
581.5	29.69	840.3	62.04	1595.1	10.02
589.3	30.67	869.6	63.64	1641.3	8.71
592.4	31.16	920.4	63.64	1679.9	8.14
596.3	31.65	945.0	62.49	1694.5	7.75
610.1	33.52	1082.9	59.30	1740.1	7.18
617.9	34.59	1106.9	55.60	1781.1	6.65
624.0	35.57	1136.8	52.49	1824.0	6.34

TABLE II (*Continued*)

Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)
629.7	36.06	1161.5	47.67	1902.7	5.72
640.5	37.53	1184.3	42.14	1968.2	5.49
646.3	38.51	1206.0	38.99	2014.2	5.51
654.2	39.49	1227.5	35.23	2071.5	6.02
670.3	41.45	1246.3	32.96	2128.4	7.37
679.7	43.41	1249.7	31.92	2151.3	8.71
696.1	45.37	1253.1	31.51	2154.6	11.33
709.9	47.33	1253.1	31.78	2161.2	13.95
731.5	50.27	1246.3	32.64	2167.9	18.71
		Series C Normalized to series A			
368.4	6.34	456.1	14.60	681.3	41.80
372.7	6.68	462.8	15.27	695.1	43.76
377.1	6.96	471.8	15.90	709.9	45.82
389.4	8.06	482.8	17.18	720.5	46.80
394.2	8.32	498.4	19.01	733.8	49.25
397.5	8.76	513.1	20.77	746.4	50.62
400.8	9.01	523.6	22.13	765.9	53.27
405.9	9.40	535.3	23.58	786.1	55.23
411.1	9.83	546.1	24.89	809.7	57.78
414.7	10.17	556.6	26.31	831.9	60.47
418.3	10.40	566.1	27.64	860.6	61.94
420.1	10.75	575.5	28.64	878.2	63.17
425.7	11.30	584.1	29.11	898.8	62.09
431.5	11.62	596.8	30.72	1004.3	61.31
435.4	11.95	608.0	32.28	1083.3	58.17
437.4	12.28	616.8	33.46	1136.8	50.52
439.4	12.73	625.6	34.89	1162.5	44.74
443.5	13.22	639.7	36.21	1182.3	42.38
447.6	13.63	652.1	37.97	1195.6	39.54
451.8	14.01	667.5	39.83	1216.6	36.40
1227.5	34.83	1679.9	8.03	1908.0	5.68
1244.1	33.17	1700.9	7.44	1940.4	5.56
1248.6	31.93	1744.5	6.89	1968.2	5.50
1255.4	31.68	1744.5	6.98	2029.0	5.57
1374.8	29.38	1744.5	7.29	2059.2	5.62
1447.6	15.05	1790.4	6.70	2080.8	5.98
1495.2	13.05	1814.3	6.49	2112.3	6.31
1528.8	11.91	1814.3	6.39	2134.9	7.22
1600.7	9.92	1814.3	6.38	2157.9	8.88
1625.6	9.10	1828.9	6.35	2167.9	10.42
1651.3	8.46	1838.7	6.21	2167.9	14.44

for the moment the point of view that the frequency dependence is accurately represented by Eq. (4) but that the coupling constant $(u+1)^2$, and the product of the dispersion constant γ, and relaxation time τ are to be determined from a least-squares fit. We obtained a value of u and $\gamma\tau$ at a specific temperature by fitting to the ten experimental frequencies. The same fit was then carried out at a second temperature and a new value of u and $\gamma\tau$ was determined. By repeating this process for several temperatures in the range 0.2 to 0.4 K the temperature dependence of u and $\gamma\tau$ are determined. A strong temperature dependence of u is not expected but some variation was observed. A "best value" of $u=5.30$ was selected which is to be compared with the accepted value of 2.65. Furthermore, we find $\gamma\tau \cong 6\times10^{29}/T^3$ which differs radically from the T^{-9} behavior expected on the basis of Landau and Khalatnikov's theory. Whitworth's[44] measurements of the thermal conductivity in narrow channels lead to a phonon lifetime $\tau \cong 1.6 \times 10^{-3}/T^{4.3}$ which lends some support to the T^{-3} behavior deduced from our fit. Actually most of the estimates of τ predict that below about 0.3 K the lifetime will be "size-effect limited" by scat-

TABLE III. Temperature dependence of attenuation and velocity of sound in liquid ^{4}He at 36 MHz.

Temperature (mK)	Attenuation (dB/cm)	$\frac{\Delta c}{c} \times 10^6$	Temperature (mK)	Attenuation (dB/cm)	$\frac{\Delta c}{c} \times 10^6$
Series A 35.89 MHz Normalized to series C, $\Delta c/c$ Normalized to series B					
202.5	0.40	3.3	367.4	7.66	22.0
203.3	0.40	3.3	384.2	8.97	28.3
225.7	0.62	3.1	404.5	11.40	39.5
226.7	0.65	3.1	407.0	11.40	40.0
253.8	1.18	3.8	427.1	14.18	55.8
255.1	1.18	3.8	429.5	14.18	55.8
285.9	2.18	8.0	450.7	17.38	79.0
323.1	3.92	9.8	472.4	21.01	109.3
336.5	4.82	11.9	478.9	21.21	112.1
351.5	6.03	16.4	486.5	23.46	136.9
Series B 35.89 MHz Normalized to series C, $\Delta c/c$ Normalized to $T_{min} = 174.0$ mK					
174.0	0.30	0.0	264.8	1.80	4.4
175.6	0.30	0.2	284.3	2.36	6.3
192.7	0.48	0.9	304.3	3.23	7.2
207.3	0.48	1.4	365.1	7.30	19.3
209.0	0.48	1.6	367.9	7.30	19.3
222.4	0.87	2.1	465.2	19.69	96.9
233.3	1.04	3.0	489.7	23.75	136.1
249.0	1.26	3.2	509.9	29.14	169.1
251.4	1.26	3.2			

Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)
Series C $T_{min} = 93.9$ mK					
94.9	0.00	363.8	7.09	1552.9	17.16
136.8	0.17	403.6	11.19	1600.6	15.20
148.9	0.22	448.4	16.90	1649.8	13.53
164.1	0.26	498.2	24.84	1699.9	12.22
180.4	0.43	546.5	32.68	1748.4	11.02
198.6	0.63	603.9	45.82	1797.9	10.26
216.4	0.76	1294.9	42.98	1845.7	9.70
239.2	1.12	1350.6	34.05	1887.2	8.91
263.8	1.74	1399.6	29.49	1976.7	8.62
292.3	2.69	1448.7	23.94	2101.2	12.59
327.2	4.43	1500.5	19.91	2124.3	20.88
Series D Normalized to series C					
167.3	0.20	277.5	2.06	406.4	12.07
198.6	0.49	278.2	2.00	446.1	17.59
215.0	0.44	307.8	3.26	492.5	25.77
251.4	1.25	341.1	5.35		
252.0	1.36	372.4	7.95		

tering off the boundaries of the chamber.[45] In this case an appropriate form of τ would be

$$1/\tau_{\text{total}} = 1/\tau_{\text{boundary}} + 1/\tau \ ,$$

where $\tau_{\text{boundary}} \cong d/c$.

By fitting a τ_{total} of this form to our data it is possible to obtain an approximate value for γ which we find is in the vicinity of 10^{36}. The temperature dependence of τ still goes approximately as T^{-3}. It is interesting to note in passing that the inclusion of boundary scattering changes the asymptotic form of Eq. (4) (as $T \to 0$) from $\alpha \propto T^2/\gamma\tau$ to $\alpha \propto \omega T^4$.

The 12-, 30-, and 36-MHz data can be compared with the theory of Khalatnikov and Chernikova for a wide range in temperature since

TABLE IV. Temperature dependence of attenuation and velocity of sound in liquid ^{4}He at 60 MHz.

Temperature (mK)	Attenuation (dB/cm)	$\frac{\Delta c}{c} \times 10^6$	Temperature (mK)	Attenuation (dB/cm)	$\frac{\Delta c}{c} \times 10^6$
		Series A 59.84 MHz $T_{min} = 89.2$ mK			
91.4	0.00	0.0	275.3	3.02	8.4
138.4	0.19	0.9	293.9	4.11	9.1
140.6	0.19	0.9	304.3	4.11	9.1
151.3	0.15	0.9	325.2	5.99	9.8
160.1	0.41	1.1	329.3	5.99	9.8
172.9	0.54	2.1	353.9	8.94	11.9
187.8	0.62	2.8	380.8	12.79	13.3
204.1	0.78	3.2	404.5	17.45	20.5
223.3	1.17	4.2	408.9	17.45	20.5
243.7	1.63	5.8	436.4	24.80	30.6
246.6	1.63	5.8	455.5	30.49	44.1
266.8	2.34	7.0	459.1	30.49	44.1
268.2	2.34	7.0	477.2	36.81	55.1
Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)
		Series B Normalized to series A			
112.6	0.15	172.3	0.43	303.9	4.26
119.3	0.16	173.4	0.43	304.3	4.26
119.3	0.11	193.4	0.58	305.0	4.28
119.6	0.12	194.2	0.59	335.2	6.61
130.4	0.15	217.7	0.98	335.2	6.63
130.7	0.15	217.7	0.98	367.6	10.43
144.1	0.20	243.1	1.61	367.6	10.43
144.5	0.21	244.3	1.61	405.8	16.84
158.2	0.31	272.4	2.58	446.9	27.41
158.6	0.32	272.8	2.59	493.0	42.31
		Series C Normalized to series A			
107.5	0.10	178.8	0.43	359.4	9.59
109.5	0.10	196.5	0.73	403.2	16.35
113.1	0.13	218.4	1.10	442.7	26.04
123.6	0.17	240.9	1.60	480.2	38.85
136.4	0.15	265.5	2.39	478.9	36.86
149.1	0.26	294.4	3.74		
162.7	0.34	325.7	5.96		

these data are fairly complete at the higher temperatures. The solid line in Figs. 4, 5, and 6 shows the theoretical results (see the Appendix for the theoretical expressions). Rather good agreement is obtained near the peak at 1 K (where $\omega\tau_{pr} = 1$), but the theoretical value is too small both above and below this temperature. The parameters used in this theory were the same as those used by Khalatnikov and Chernikova. The curves above and below 1.2 K result from the two different theoretical treatments used by these authors. As we stated before, they do not join smoothly with one another. If computational difficulties are surmountable, it would be useful to extend the theory to include simultaneously the effects of elastic phonon-roton (τ_{pr}), inelastic five phonon (τ_{3-5}) and wide angle phonon-phonon (τ_{pp}) scattering. The agreement is particularly poor at low temperatures. In these calculations the accepted value of $u = 2.65$ has been used. Since the Khalatnikov and Chernikova theory is asymptotically the same as Eq. (3), we know from our previous fit that $u = 5.30$ gives better agreement (at low temperature). Such a value of u would destroy the agreement at high temperatures, however.

Above about 1.2 K the attenuation should behave in a classical manner, that is

TABLE V. Temperature dependence of attenuation and velocity of sound in liquid ^{4}He at 84 MHz.

Temperature (mK)	Attenuation (dB/cm)	$\frac{\Delta c}{c} \times 10^6$	Temperature (mK)	Attenuation (dB/cm)	$\frac{\Delta c}{c} \times 10^6$
		Series A 83.79 MHz $T_{min} = 91.4$ mK			
92.6	0.00	0.0	257.6	2.76	7.0
125.7	0.22	1.4	286.7	4.29	10.0
127.2	0.22	1.4	289.8	4.50	9.3
155.3	0.43	2.1	318.6	6.66	10.5
156.3	0.43	2.1	320.6	6.66	10.5
176.4	0.65	2.5	349.9	10.41	12.6
177.6	0.65	2.5	352.5	10.41	12.6
203.3	1.01	4.2	385.3	16.38	13.0
203.3	1.01	4.2	388.2	16.38	13.0
231.3	1.80	6.5	416.1	23.20	16.5
232.3	1.80	6.5	416.8	23.20	18.9
256.3	2.76	7.0			
Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)
		Series B $T_{min} = 109.1$ mK			
119.1	0.04	194.9	0.81	333.5	8.33
127.8	0.04	194.9	0.84	333.5	8.35
127.8	0.02	213.4	1.26	368.9	13.68
134.8	0.15	237.6	1.98	422.9	23.78
152.6	0.27	257.6	2.86	464.4	32.47
152.6	0.28	257.6	2.89	495.3	48.49
168.4	0.42	282.8	4.06		
187.8	0.67	306.9	5.61		
		Series C $T_{min} = 85.2$ mK			
111.4	0.04	74.9	0.78	266.3	3.43
123.3	0.18	188.0	0.97	292.9	5.00
135.9	0.18	202.1	1.28	321.5	7.38
148.0	0.27	218.6	1.63	355.8	11.61
152.8	0.34	233.0	2.06	398.6	19.65
162.5	0.49	249.1	2.66	437.1	30.53

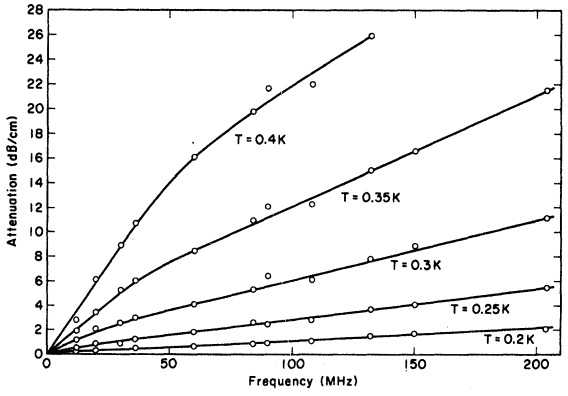

FIG. 14. The observed attenuation as a function of frequency for temperatures 0.2, 0.25, 0.3, 0.35, and 0.4 K. The solid lines are smooth curves drawn through the data. The 20-MHz points are the data of Waters, Watmough, and Wilks (Ref. 14).

$$\alpha = (\omega^2/2\rho c^3)(\tfrac{4}{3}\eta + \zeta) \ ,$$

as given in the introduction. Specifically the attenuation should be proportional to the square of the frequency. At the minimum in the attenuation near 1.9 K it was possible to observe an echo for frequencies up to 90 MHz. In Fig. 15 we have plotted the square root of the attenuation at 1.9 K as a function of frequency on a log-log plot. The data of Chase and of Woolf, Platzman, and Cohen have been included to extend the frequency range. We observe that the data follow a straight line with a slope of one.

After, the completion of the attenuation measurements equipment for the accurate measurement of the temperature dependence of the sound velocity was acquired. The measurement of sound velocity and attenuation are complementary. In the Green's-function theory, where one

TABLE VI. Temperature dependence of the attenuation of sound in liquid ^{4}He at 90 MHz.

Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)
		Series A T_{min} = 116.1 mK			
118.7	.29	324.7	9.11	1629.2	60.09
119.8	.19	337.3	10.09	1657.4	60.09
122.6	.19	352.2	12.45	1686.6	58.62
139.4	.19	367.0	14.80	1814.3	58.13
144.9	.19	383.2	17.84	1885.8	56.96
160.4	.29	400.8	21.17	1923.6	55.39
200.8	1.17	414.7	25.68	1963.1	54.21
198.7	.98	414.7	25.88	2091.7	53.43
226.4	1.47	439.4	32.54	2187.2	54.31
259.6	2.45	456.1	38.52	2187.2	56.17
291.1	5.29	471.9	51.37	2238.3	60.19
291.1	5.19	476.6	48.43	2469.0	65.98
312.0	8.13	486.2	54.70		
313.0	7.64	1602.0	63.03		
		Series B Normalized to series A			
361.5	13.33	399.1	21.39	425.7	28.67
361.5	13.36	400.8	22.10	425.7	28.88
361.5	13.35	402.5	22.37	425.7	28.88
362.8	13.51	404.2	22.86	425.7	28.90
364.2	14.06	405.9	23.35	425.7	28.98
367.0	15.08	409.4	23.84	425.7	28.89
372.7	15.52	411.1	24.33	425.7	29.18
377.1	16.02	412.9	24.82	429.5	32.16
378.6	16.49	414.7	25.31	435.4	33.33
381.6	16.88	416.5	26.00	439.4	34.53
383.2	17.26	416.5	26.44	443.5	36.18
386.2	17.94	418.3	26.88	445.5	37.20
389.4	18.45	418.3	27.22	451.8	38.94
392.6	19.08	420.1	27.45	458.3	41.79
394.2	19.65	422.0	28.24	462.8	44.14
395.8	20.12	423.8	28.30	467.3	45.22
395.8	20.61	425.7	28.44		
399.1	21.10	425.7	28.67		

TABLE VII. Temperature dependence of the attenuation of sound in liquid ^{4}He at 108 MHz, T_{min} = 109.7 mK.

Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)
120.3	0.03	181.6	0.80	277.5	4.44
131.5	0.00	197.1	1.02	303.3	6.39
143.0	0.19	214.3	1.47	331.8	9.37
154.2	0.38	233.6	2.26	368.9	15.02
167.4	0.55	253.8	3.01	399.5	21.61

calculates a self-energy, the velocity and attenuation are, respectively, the real and imaginary parts. When a kinetic-equation approach is used, one calculates a complex propagation constant $\kappa = \beta + i\alpha$, where $\alpha(\omega, T)$ is the attenuation and $\beta = 1/\lambda = \omega/c(\omega, T)$. The attenuation and velocity are, in fact, related by a dispersion relation.[46]

Measurements of the temperature dependence of the velocity were made at 12, 36, 60, and 84 MHz. Figure 16 shows the low-temperature

TABLE VIII. Temperature dependence of the attenuation of sound in liquid ^{4}He at 132 MHz.

Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)
		Series A T_{min} = 85.4 mK			
102.6	0.06	153.5	0.59	240.2	2.79
102.6	0.07	153.6	0.61	262.8	4.65
115.1	0.12	167.3	0.89	263.3	4.26
114.9	0.13	181.3	1.12	288.1	6.91
127.8	0.24	181.3	1.12	317.2	9.65
127.5	0.24	199.7	1.60	317.2	9.75
140.4	0.42	219.2	2.24	348.2	15.14
		Series B T_{min} = 117.9 mK			
129.4	0.10	189.1	1.23	278.5	5.55
141.4	0.28	209.2	1.74	306.0	8.04
156.5	0.55	230.2	2.68	335.6	11.80
170.2	0.78	252.9	3.78		

TABLE IX. Temperature dependence of the attenuation of sound in liquid ^{4}He at 150 MHz, T_{min} = 126.5 mK.

Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)
135.6	0.39	172.9	1.07	254.7	4.11
144.4	0.29	179.4	1.17	275.2	6.17
156.2	0.29	227.0	2.45	293.8	8.13
164.8	0.78	221.7	2.25	322.5	11.56

TABLE X. Temperature dependence of the attenuation of sound in liquid ^{4}He at 204 MHz, T_{min} = 98.0 mK.

Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)	Temperature (mK)	Attenuation (dB/cm)
120.2	0.20	171.2	1.12	212.8	2.81
134.5	0.36	183.9	1.53	228.2	3.72
135.0	0.35	184.2	1.55	228.7	3.78
143.3	0.48	197.8	2.06	246.6	4.97
144.1	0.45	198.4	2.09	268.9	7.20
144.9	0.51	199.4	2.14	295.6	10.40
157.7	0.78	209.8	2.68	324.3	15.11
157.7	0.72	210.2	2.71	324.5	15.30
170.0	1.11	212.4	2.80	356.3	22.25

portion of the data. Tables I, III, IV, and V contain the data for all four frequencies. As described earlier, the change in sound velocity between two temperatures T and T_{min} is calculated from the change in transit time through the relation

$$1/c(\omega, T) - 1/c(\omega, T_{min}) = \Delta t/d \ .$$

The temperature T_{min} at which these runs were started was about 35 mK. The theory tells us that the velocity change approaches zero as T^4 and if we couple this fact together with the fact that no velocity changes were observable below about 150 mK then we may approximate T_{min} by zero. Since the total observed velocity change was much smaller than the velocity itself we can make a first-order expansion and obtain

$$\Delta c/c(\omega, 0) \equiv [c(\omega, T) - c(\omega, 0)]/c(\omega, 0)$$
$$\cong - c(\omega, 0)\Delta t/d \ .$$

For $c(\omega, 0)$ we used the value of Whitney and Chase[47] 2.3827×10^4 cm/sec (for the usual ultra-

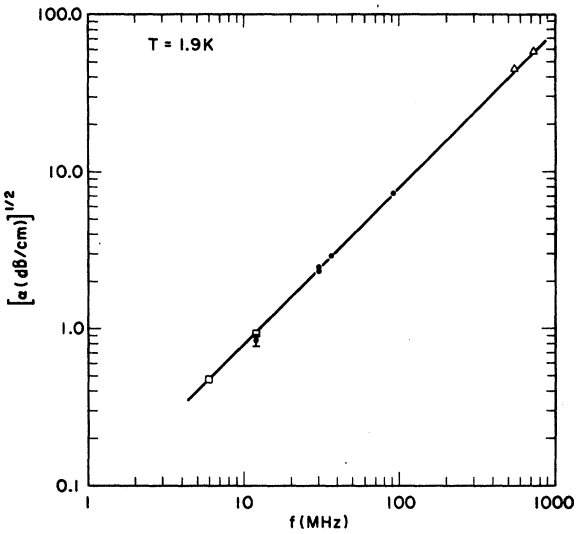

FIG. 15. A log-log plot of the square root of the attenuation as a function of the frequency. The circles are the 12-, 30-, 36-, and 90-MHz data from the present experiment. Shown also are the data of Chase at 6 and 12 MHz (squares) and also the data of Woolf, Platzman, and Cohen at 556 and 723 MHz (triangles). The straight line (with a slope of one) shows that the attenuation is proportional to the square of the frequency.

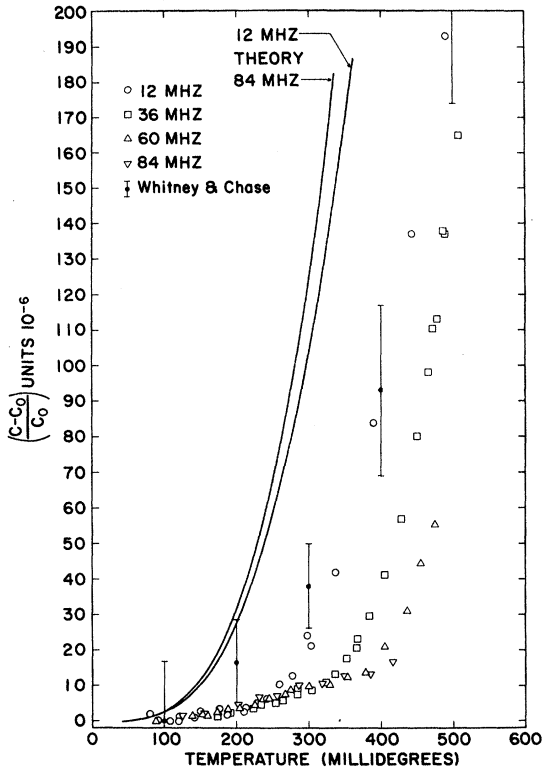

FIG. 16. The temperature dependence of the velocity of sound at 12, 36, 60, and 84 MHz. Only the low-temperature portion of the 12-MHz data is shown. Shown also is the 12- and 84-MHz theory of Khalatnikov and Chernikova. The points with vertical error bars are the data of Whitney and Chase.

sonic frequencies the dispersion at zero temperature is negligible). The solid lines in Fig. 16 are the theory of Khalatnikov and Chernikova for 12 and 84 MHz. The 11.9-MHz data of Whitney and Chase[48] have been included and agree with us within experimental error. We observe that the theory predicts a larger velocity change at higher frequency whereas the experimental data show quite the opposite behavior. Furthermore the observed velocity change is in all cases much smaller than the theoretical prediction. Instead of a T^4 dependence, an initial T^3 going over to T^6 behavior is observed. Had we used the "best value" of u from our previous fit, the discrepancy would be even larger. The behavior with frequency should be looked upon as the most serious disagreement. There is no way that we can obtain the observed frequency dependence from Eq. (5), and we can only conclude that the present low-temperature theory is inadequate.[49]

Figure 17 shows the full range of temperatures for the 12-MHz velocity data. Even though attenuation measurements were possible above 1 K for the 36 MHz, velocity measurements could not be made due to a lack of knowledge of the phase relative to low temperature. This occurred because the phase of the signal could not be followed through the attenuation maximum. Shown also in Fig. 17 is the theory of Khalatnikov and Chernikova. We see that the general behavior of the high-temperature side of the 12-MHz velocity data is adequately explained by the theory. We note the following general features. The velocity change rises to a peak and then passes through zero at a temperature very close to that at which the attenuation goes through a maximum. Thereafter the velocity continues to decrease with increasing temperature.

We now compare our results with those of other authors. Chase[5] has made detailed measurements of the attenuation at 2.0, 6.0, and 12.1 MHz for temperatures above 0.9 K. As described earlier a moving transducer technique was used in these measurements which allowed *in situ* alignment of the transducers and thus a determination of the absolute attenuation. Finite-amplitude effects were also observed by Chase; his reported data were taken at power levels than the threshold for this finite-amplitude effect. The 12.1-MHz data are essentially in agreement with our 12-MHz data. Near 1.9 K there is some small disagreement. Since the velocity of sound is temperature-dependent and amounts to a 4% shift at 1.9 K, there is some change in the amount of

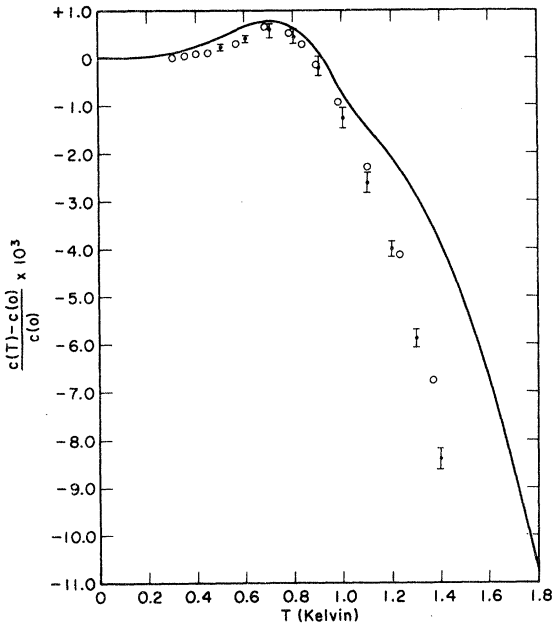

FIG. 17. The temperature dependence of the sound velocity at 12 MHz. The solid line is the theory of Khalatnikov and Chernikova. The points with vertical error bars are the data of Whitney and Chase.

energy coupled into the liquid at high temperature. The change in coupling is difficult to evaluate numerically. Our data are within Chase's experimental error, however. Later low-temperature measurements by Chase and Herlin[6] are (within their scatter) in agreement with our data on the low-temperature side ($T < 0.8$ K). The knee or change in slope observed in our experiments near 0.35 K is also clearly seen in the data of Chase and Herlin. Near 0.9 K a double peak was reported which was not observed in Chase's earlier work. In later measurements at 11.8 MHz Whitney[8] was able to show that this double peak was an instrumental effect resulting from using a single transducer. The magnitude of the peak measured by Whitney is in very good agreement with our value. Whitney also reports an indication of a change in slope (in a log-log plot) near 0.35 K.

Dransfeld, Newell, and Wilks[9,10] made measurements at 6.0 and 14.4 MHz. Since we have made no measurements at these frequencies we cannot compare their results directly with ours. Measurements at 6.0 MHz were made by Chase[5] as mentioned earlier and also more recently by Jeffers and Whitney.[11] The data of Jeffers and Whitney are in excellent agreement with the work of Chase[5] but are in rather poor agreement with the data of Dransfeld, Newell, and Wilks.[10] Because of the excellent agreement between Jeffers and Whitney and Chase, we conclude that there must be some error in the measurements of Dransfeld, Newell, and Wilks. Both the 6.0- and 14.4-MHz data of Ref. 9 appear quite flattened near the peak. Jeffers and Whitney attribute the discrepancy to the use of a single transducer for both transmitting and receiving the sound signals. The danger is that the received signal may interfere with either a prolonged ringing (of the transducer) following transmission, or a leakage of rf through the gating circuits used to generate the pulses. Indeed we observe that the 14.4-MHz data of Dransfeld, Newell, and Wilks appear as a double peak similar to that observed by Chase and Herlin, who also used only one transducer. Finite-amplitude effects are not discussed in Ref. 10 and may be another possible source of error.

Jeffers and Whitney have recently made an exhaustive study at low frequency. Measurements were made at 1.00, 2.02, 3.91, 6.08, 10.2, and 11.7 MHz. The 2.02- and 6.08-MHz data are in very good agreement with the high-temperature measurements of Chase. The data of Jeffers and Whitney were normalized by assuming the attenuation to be proportional to some power of the temperature and least-squares fitted in the manner described previously. As we have stated this is valid only as long as we know that the attenuation actually follows such a power law. Our 12-MHz data clearly show a break near 0.35 K and thus do not follow a power law. The normalized data of Jeffers and Whitney do not show a break near 0.35 K but instead show an over-all T^3 behavior at low temperature, whereas our data show a limiting slope near four at low temperature. We believe Jeffers and Whitney's normalization to be incorrect. Thus the over-all T^3 behavior observed (fitted) at other frequencies may be in error also. The normalization might also change the frequency dependence to make it closer to ω rather than $\omega^{3/2}$. We hasten to point out, however, that the small shift in $\alpha(T_{\min})$ necessary to change a T^3 behavior into a T^4 behavior at low temperature will have a negligible effect at the higher temperatures. It is also clear that the measured difference in attenuation between any two temperatures is also unaffected by the normalization.

One common criticism can be made of the low-temperature work of all the authors discussed. Warm-up times in their experiments ranged from 10 min to an hour whereas our warming rates were on the order of a few millidegrees per hour. This means that less time is available for an accurate determination of the amplitude and temperature. Some possibility also exists for the system to not be in thermal equilibrium.

Waters, Watmough, and Wilks[14] have recently made measurements at 20.7 and 61.1 MHz. The 61.1-MHz data are in excellent agreement with

our own 60-MHz data. We have not made any 20-MHz measurements, but their data fall nicely on the frequency dependence of fixed temperatures shown in Fig. 15. They have fit their data to an ωT^4 dependence of the attenuation at very low temperatures. Figure 14 shows that at low temperature the attenuation is approximately proportional to ω. Waters, Watmough, and Wilks state that the attenuation is proportional to T^4 at low temperatures but that the exponent becomes greater than 4 at higher temperatures. They attribute this to the ωT^6 term suggested by Khalatnikov.[25] According to formula 4 an ωT^4 behavior requires $3\gamma \bar{p}^2 \omega\tau \ll 1$. At higher temperatures (where they suggest the ωT^6 contribution) this inequality is even better satisfied. However, when this inequality is satisfied the perturbation theory used to calculate the four-phonon process (ωT^6) makes no sense. This is because the energy uncertainty will then be greater than the energy difference in the denominator of the second-order perturbation-theory expression used to calculate the four-phonon process. Indeed Khalatnikov[25] implies that the transition should be the other way around; namely, a transition from an ωT^4 high-temperature behavior to a ωT^6 low-temperature behavior. The data of Waters, Watmough, and Wilks appear to fit equally well a single power-law expression with n slightly greater than 4. On the whole we consider their data to be in good agreement with ours.

The temperature dependence of the sound velocity has recently been measured by Whitney and Chase.[48] Measurements were made at 1.00, 3.91, and 11.9 MHz from 0.2 to 1.1 K. In the frequency range 1.00 to 11.9 MHz Whitney and Chase find that (for a given temperature) the velocity increases with increasing frequency. This is the behavior expected from the Khalatnikov-Chernikova theory and is opposite to that observed by the present authors in the frequency range 12 to 84 MHz. Thus the frequency dependence of the velocity of sound at fixed temperature must pass through a maximum near 12 MHz in order to be consistent with both sets of data. This is a rather surprising and important result which should be tested again experimentally.

At temperatures below 0.7 K Whitney and Chase find that the measured temperature-dependent shift of the velocity is less than that predicted by Khalatnikov and Chernikova. We again have the situation where a single Grüneisen constant u cannot simultaneously fit the observed attenuation and velocity data. The 11.9-MHz data of Whitney and Chase are shown in Figs. 16 and 17 and are in good agreement with our 12.0-MHz results.

CONCLUSION

The theory of Khalatnikov and Chernikova agrees well with experiment only at the peak in the attenuation curve near 0.9 K. Agreement at higher and particularly at lower temperatures can be considered only qualitative. At higher temperatures there are a number of relaxation mechanisms contributing and also an elementary excitation model has less meaning. Thus disagreement at high temperature is easily understandable. At very low temperature one would think a more exact theoretical treatment would be possible since only phonons contribute. The disagreement at low temperature is most upsetting. As we have seen we can force fit Eq. (5) by fitting u and $\gamma\tau$. The temperature dependence of the τ we obtain by doing this is in complete disagreement with theory. While it is understood that there may be some experimental uncertainty in u, the value required by the fit will destroy the agreement of theory and experiment near 1 K. We again point out that Eq. (5) does not appear to have the right low-temperature limit. The situation with respect to the temperature and frequency dependence of the velocity is worse. The value of u which best fits the low-temperature attenuation is *greater* than the accepted value, while a *smaller* value of u is required to bring the theory in better agreement with the velocity. The temperature dependence of the velocity is also in disagreement with theory. Finally the frequency dependence of the observed velocity changes are in the opposite sense of that predicted by theory. We therefore conclude that the present theoretical formulation of sound propagation at very low temperatures is incomplete.

APPENDIX

We give here the expressions for the attenuation and velocity of sound as calculated by Khalatnikov and Chernikova.[16–18] For the low-temperature range (below 1.2 K) we present the formulas in a different notation than that used by Khalatnikov and Chernikova. The way they are given here is due to Whitney[50] and is convenient for machine computation.

ρ_n is the normal density of phonons

$$\frac{\rho_n}{\rho} = \frac{16\pi^5\kappa^4}{45h^3\rho c^5} T^4 = 1.218\times10^{-4}T^4 \ ,$$

where κ is Boltzmann's constant, h is Planck's constant, ρ is the density of the liquid, and c the velocity of sound.

τ_{pp} is the wide angle phonon-phonon relaxation time

$$\frac{1}{\tau_{pp}} = \frac{9\times13!}{2^{13}} \frac{\kappa^9}{h^7} \frac{(u+1)^4}{(\rho c^5)^2} T^9$$

$$= 3.134\times10^7 T^9 \ \text{sec}^{-1} \ ,$$

where $u=(\rho/c)\partial c/\partial\rho=2.64$.

τ_{pr} is the phonon-roton relaxation time

$$\frac{1}{\tau_{pr}}=(2\pi)^{\frac{17}{2}}\Gamma\frac{\kappa^{9/2}}{h^7}\left(\frac{p_0^4\mu^{1/2}}{\rho^2c^5}\right)T^{\frac{9}{2}}e^{-\Delta/\kappa T}$$
$$=1.28\times10^{12}T^{\frac{9}{2}}e^{-\Delta/\kappa T}\ \text{sec}^{-1}\ ,$$

where $\Gamma=\frac{2}{9}+\frac{1}{25}\left(\frac{p_0}{\mu c}\right)^2+\frac{2}{9}\left(\frac{p_0}{\mu c}\right)A+A^2=2.62$,

and $A=\left(\frac{\rho^2}{p_0c}\right)\frac{\partial^2\Delta}{\partial\rho^2}+\left(\frac{p_0}{\mu c}\right)\left(\frac{\rho}{p_0}\frac{\partial p_0}{\partial\rho}\right)^2=-0.1$,

(estimated by Khalatnikov and Chernikova). The values used are

$p_0=2.02\times10^{-19}$ g cm/sec ,

$\mu=1.06\times10^{-24}$ g, $\Delta/\kappa=8.65$ K ,

$c=2.383\times10^4$ cm/sec ,

$\rho=0.145$ g/cm^3 .

We define also $\theta_{pp}=\omega\tau_{pp}$, $\theta_{pr}=\omega\tau_{pr}$, $\theta=\omega\tau$ where

$$1/\tau=1/\tau_{pp}+1/\tau_{pr}\ .$$

The attenuation $\alpha(\omega,T)$ and the change in velocity of sound are given by

$$\alpha(\omega,T)=\tfrac{1}{2}(\omega/c)(\rho_n/\rho)\,\mathrm{Im}\Phi\ ,$$

$$c(\omega,T)-c(\omega,0)=-\tfrac{1}{2}c(\omega,0)(\rho_n/\rho)$$
$$\times[\mathrm{Re}\Phi+3\left(u^2-\frac{1}{4}\frac{\rho^2}{c}\frac{\partial^2c}{\partial\rho^2}\right)]\ ,$$

where

$$\mathrm{Re}\Phi=1-3(N_rD_r+N_iD_i)/(D_r^2+D_i^2)\ ,$$

$$\mathrm{Im}\Phi=1/\theta_{pr}-3(N_iD_r-N_rD_i)/(D_r^2+D_i^2)\ ,$$

$$N_r=\tfrac{1}{2}u^2\ln(1+4\theta^2)+B_rC_r-B_iC_i\ ,$$

$$N_i=-u^2\tan^{-1}2\theta+B_iC_r+B_rC_i\ ,$$

$$B_r=2u+1-(2\beta+1)/\theta_{pr}^2\ ,$$

$$B_i=[2(u+1)+\beta(1-\theta_{pr}^{-2})]/\theta_{pr}$$
$$-3u^2/\theta_{pp}\ ,$$

$$C_r=\tfrac{1}{2}\ln(1+4\theta^2)+\theta^{-1}\tan^{-1}(2\theta)-2\ ,$$

$$C_i=(2\theta)^{-1}\ln(1+4\theta^2)-\tan^{-1}(2\theta)\ ,$$

$$D_r=2-\left(\frac{1}{\theta}-\frac{\beta}{\theta_{pr}}\right)\tan^{-1}2\theta$$
$$+\frac{3}{\theta_{pp}}\left(C_i+\beta\frac{C_r}{\theta_{pr}}\right)\ ,$$

$$D_i=-\frac{1}{2}\left(\frac{1}{\theta}-\frac{\beta}{\theta_{pr}}\right)\ln(1+4\theta^2)$$
$$-\frac{3}{\theta_{pp}}\left(C_r-\beta\frac{C_i}{\theta_{pr}}\right)\ ,$$

$$\beta=(3\kappa/\mu c^2)T=0.688T\ .$$

For the higher temperatures (about 1.2 K) the only case of interest is $\omega\tau_{pr}\ll1$. For this case the five-phonon relaxation time is also included.

$$1/\tau_{3\to2}=\Lambda\kappa T^{12}/3N_p\ ,$$

where $N_p\approx2.4\times4\pi(\kappa T/h)^3$,

where Λ is taken from acoustic experiments and is 3.4×10^{43} and

$$\tau_{3\to2}=1.288\times10^{-8}T^{-9}\ \text{sec},$$

$$\alpha(\omega,T)=\frac{\omega^2\tilde{\tau}_{pr}}{c}\frac{\rho_n}{\rho}\left[\frac{2}{15}+\frac{1}{6\tilde{\beta}}(3u+1)^2\right]\ ,$$

$$\tilde{\tau}_{pr}=\tau_{pr}\left(1+\frac{0.376}{0.451(\tau_{pr}/\tau_{3\to2})+0.0036}\right),$$

$$\frac{1}{\tilde{\beta}}=\left(\frac{1.453}{T}+0.166\frac{\tau_{3\to2}}{\tau_{pr}}\right)\frac{\tau_{pr}}{\tilde{\tau}_{pr}}\ .$$

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