Does the Electromagnetic Current Have a Unitary Singlet Component?*

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A possibility that the electromagnetic current may have a unitary singlet component has been investigated in connection with $V_9 \to l\bar{l}$ decays. The Maki-Hara model of elementary particles seems to be experimentally ruled out. As for the three-triplet model, it has been pointed out that we must have additional 1^- vector mesons ρ', ω', and ϕ' with nonzero charm quantum numbers in order to saturate a new sum rule.

RECENTLY, there has been a great deal of theoretical and experimental interest in the leptonic decays of the vector mesons. Especially the validity of the sum rule[1,2]

$$\tfrac{1}{3}m_\rho\Gamma(\rho\to l\bar{l})=m_\omega\Gamma(\omega\to l\bar{l})+m_\phi\Gamma(\phi\to l\bar{l}) \qquad (1)$$

is of great interest. Although the experimental analysis of the DESY group[3] shows that this relation is fairly well satisfied, the Orsay data[4] seem to suggest a possible violation. Hence, it would be worthwhile to reexamine its validity from a theoretical standpoint. The sum rule Eq. (1) has been obtained under the following three conditions: (i) the validity of the first Weinberg sum rule[5] with respect to the $SU(3)$ group, (ii) the vector-dominance approximation, and (iii) the octet nature of the electromagnetic current without any unitary singlet component. Although any violation of these conditions can modify[6] the sum rule Eq. (1), we shall restrict ourselves in this paper for a possible modification of condition (iii) together with some remarks on (ii). Specifically, we shall ask the following questions:

(a) Does the electromagnetic current of hadrons contain a piece that transforms as a singlet under $SU(3)$?

(b) To what extent can one differentiate between various types of quark models?

A study of the leptonic decays of vector mesons may settle[7,8] questions (a) and (b) by means of the sum rule Eq. (1). A great deal of interest is also attached to question (b), particularly in view of the fact that the three-triplet quark model of Han and Nambu[9] seems to be preferred over the usual quark model from considerations of finiteness[10] of the lowest-order radiative corrections to β decays, from an analysis of the decay[11] $\pi^0\to 2\gamma$, and from the possibility of achieving a symmetric S-wave configuration for the nucleon as a bound state of three quarks.

We first discuss questions (a) and (b), using only the first Weinberg sum rule. Towards the end, we shall use the second sum rule as modified in Ref. 1, and list some detailed predictions for the leptonic vector boson decays.

Weinberg's first sum rule[1] may be written

$$\int_0^\infty \frac{1}{m^2}\rho_{\alpha\beta}{}^{(1)}(m^2)dm^2=a\delta_{\alpha\beta}+b\delta_{\alpha0}\delta_{\beta0} \qquad (\alpha,\beta=0,1,\cdots,8), \qquad (2)$$

which, under the usual assumption of pole dominance leads,[12] among other results, to the relations

$$G_\rho{}^2/m_\rho{}^2=G_\omega{}^2/m_\omega{}^2+G_\phi{}^2/m_\phi{}^2, \qquad (3)$$

$$\sigma_\omega G_\omega/m_\omega{}^2+\sigma_\phi G_\phi/m_\phi{}^2=0, \qquad (4)$$

where the various coupling constants are defined by the matrix elements

$$\langle 0|V_\mu{}^{(3)}(0)|\rho^0(k)\rangle=G_\rho\varepsilon_\mu{}^{(\rho)}(k)(2k_0V)^{-1/2}, \qquad (5a)$$

$$\langle 0|V_\mu{}^{(8)}(0)|\omega,\phi(k)\rangle=G_{\omega,\phi}\varepsilon_\mu{}^{(\omega,\phi)}(k)(2k_0V)^{-1/2}, \qquad (5b)$$

$$\langle 0|V_\mu{}^{(0)}(0)|\omega,\phi(k)\rangle=\sigma_{\omega,\phi}\varepsilon_\mu{}^{(\omega,\phi)}(k)(2k_0V)^{-1/2}. \qquad (5c)$$

In terms of the ordinary quark field $q(x)$, the vector currents are defined by

$$V_\mu{}^{(\alpha)}(x)=\tfrac{1}{2}i\bar{q}(x)\gamma_\mu\lambda_\alpha q(x), \qquad (6)$$

where $\alpha=0,1,\cdots,8$, with $\lambda_0=\sqrt{\tfrac{2}{3}}$ as usual, although we need not assume the explicit form Eq. (6) in what follows.

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[1] T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **19**, 470 (1967).

[2] R. J. Oakes and J. J. Sakurai, Phys. Rev. Letters **19**, 1266 (1967).

[3] U. Becker *et al.*, Phys. Rev. Letters **21**, 1504 (1968).

[4] J. E. Augustin *et al.*, Phys. Letters **28B**, 503 (1969); also the CERN-Bologna data indicate a possible violation in the same direction. See A. Zichichi in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), pp. 70, 71.

[5] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967); T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **18**, 761 (1967).

[6] H. Sugawara, Phys. Rev. Letters **21**, 772 (1968) has proposed to modify the first Weinberg sum rule on the basis of his model and derived a different sum rule. However, an extra assumption with regard to $SU(3)$ invariance is used.

[7] The possibility of detecting the unitary singlet component in electromagnetic current in $V_q\to l\bar{l}$ decay was originally suggested by A. Salam, in *Proceedings of the Twelfth Annual Conference on High-Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965).

[8] S. Okubo, Progr. Theoret. Phys. (Kyoto) Suppl. **37**, 114 (1966).

[9] M. Y. Han and Y. Nambu, Phys. Rev. **139**, B1006 (1965).

[10] K. Johnson, F. E. Low, and H. Suura, Phys. Rev. Letters **18**, 1224 (1967); N. Cabibbo, L. Maiani, and G. Preparata, Phys. Letters **25B**, 31; **25B**, 132 (1967).

[11] S. L. Adler, Phys. Rev. **177**, 2426 (1969); S. Okubo, *ibid.* (to be published).

[12] Here we remark that if the so-called ρ' meson exists, Eq. (3) will not be valid in general. However, if the mass of ρ' is much larger than that of ρ, its contribution to Eq. (3) will be negligible. Similar remarks apply also to possible contributions from analogous ω' and ϕ'.

Now, if the electromagnetic current has a unitary singlet term

$$j_\mu^{\mathrm{em}}(x)=V_\mu^{(3)}(x)+\tfrac{1}{3}\sqrt{3}V_\mu^{(8)}(x)+\epsilon V_\mu^{(0)}(x), \quad (7)$$

then the decay rate for $V\to l\bar{l}$ is given by

$$\Gamma(V\to l\bar{l})=\frac{4\pi}{3}\left(\frac{e^2}{4\pi}\right)^2\frac{f_V^2}{m_V^3}\left[1+O\left(\frac{m_l}{m_V}\right)^4\right], \quad (8)$$

where f_V's are expressed as[13]

$$f_\rho=G_\rho,\quad f_\omega=\tfrac{1}{3}\sqrt{3}G_\omega+\epsilon\sigma_\omega,\quad f_\phi=\tfrac{1}{3}\sqrt{3}G_\phi+\epsilon\sigma_\phi. \quad (9)$$

For the usual Gell-Mann-Zweig quark model, we have $\epsilon=0$, while the Maki-Hara model[14] corresponding to the charge assignment of (1,0,0,0) for the four quarks gives $\epsilon=\sqrt{\frac{2}{3}}$. As for the three-triplet model, the situation is a bit more complicated, and we shall discuss it separately.

Using Eqs. (3) and (4), we obtain from Eqs. (8) and (9) the following modification to Eq. (1):

$$\tfrac{1}{3}(1+Z)m_\rho\Gamma(\rho\to l\bar{l})=m_\omega\Gamma(\omega\to l\bar{l})+m_\phi\Gamma(\phi\to l\bar{l}), \quad (10)$$

where Z is given by

$$Z=3\epsilon^2\left(\frac{\sigma_\omega^2}{m_\omega^2}+\frac{\sigma_\phi^2}{m_\phi^2}\right)\frac{m_\rho^2}{G_\rho^2}=3\epsilon^2\frac{m_\phi^2}{m_\omega^2}\frac{\sigma_\omega^2}{G_\phi^2}\geqslant 0. \quad (11)$$

Note that the linear term proportional to σ_ω and σ_ϕ has disappeared because of Eq. (4). Thus, if the electromagnetic current has a unitary singlet term, one expects to have a departure from Eq. (1) given by

$$\Sigma=\tfrac{1}{3}m_\rho\Gamma(\rho\to l\bar{l})-m_\omega\Gamma(\omega\to l\bar{l})-m_\phi\Gamma(\phi\to l\bar{l}), \quad (12)$$

so that

$$\Sigma=-\tfrac{1}{3}Zm_\rho\Gamma(\rho\to l\bar{l})<0. \quad (13)$$

This negative sign for Σ agrees with the analysis of the Orsay group.[4] However, since the experimental error is fairly large, it would be premature to conclude at present that a unitary singlet piece exists in the electromagnetic current. It will certainly be of great interest to have more accurate experimental data.

We remark that if the ordinary limiting procedure to obtain the Schwinger term for the equal-time commutator $[V_4^{(\alpha)}(x),V_\mu^{(\beta)}(y)]_{x_0=y_0}$ is used, then we get[15] one more condition $b=0$ in Eq. (2) for the quark model as well as for the Maki-Hara model. This corresponds to the case of the exact nonet model, and leads to the following additional relation:

$$G_\rho^2/m_\rho^2=\sigma_\omega^2/m_\omega^2+\sigma_\phi^2/m_\phi^2, \quad (14)$$

which in turn gives $Z=3\epsilon^2$ from Eq. (11). Hence, the Maki-Hara model gives $Z=2$, so that Eq. (10) is now written

$$m_\rho\Gamma(\rho\to l\bar{l})=m_\omega\Gamma(\omega\to l\bar{l})+m_\phi\Gamma(\phi\to l\bar{l}), \quad (15)$$

which is ruled out by the experimental data already available.

Next, let us proceed to a discussion of the three-triplet model. Using the $SU(3)\otimes SU(3)$ notation,[16] one has in this case

$$j_\mu^{\mathrm{em}}(x)=V_\mu^{(3,0)}(x)+\tfrac{1}{3}\sqrt{3}V_\mu^{(8,0)}(x)+\tfrac{2}{3}\sqrt{3}V_\mu^{(0,8)}(x), \quad (16)$$

with

$$V_\mu^{(\alpha,\beta)}(x)=i(\tfrac{3}{8})^{1/2}\bar{q}(x)\gamma_\mu\lambda_\alpha\rho_\beta q(x), \quad (17)$$

where λ_α and ρ_β ($\alpha,\beta=0,1,\cdots,8$) refer[16] to two independent sets of 3×3 matrices corresponding to the ordinary $SU(3)$ and to the charm $SU(3)$ space, respectively. Since the commutation relations among $V_3^{(\alpha,\beta)}(x)$ now form an algebra corresponding to the $SU(9)$ group rather than $SU(3)\otimes SU(3)$, one can use the method of Glashow, Schnitzer, Weinberg[17] to obtain an extension of the first Weinberg sum rule Eq. (2):

$$\int_0^\infty \frac{1}{m^2}\rho_{\alpha\alpha',\beta\beta'}{}^{(1)}(m^2)dm^2=a\delta_{\alpha\beta}\delta_{\alpha'\beta'}+b\delta_{\alpha0}\delta_{\beta0}\delta_{\alpha'0}\delta_{\beta'0}, \quad (18)$$

where the unprimed and primed indices refer to the ordinary and charm $SU(3)$ spaces, respectively. Note that the asymptotic $SU(9)$ formula Eq. (18) can also be derived[15] from the standard limiting procedure of obtaining the Schwinger term.

Defining the coupling constants for the couplings of ρ, ω, and ϕ to the three pieces of electromagnetic current in Eq. (16) analogously to Eq. (5), we obtain the following results:

(i) If we have exact invariance with respect to the charm $SU(3)$, the charm current $V_\mu^{(0,8)}(x)$ could not couple to ω and ϕ (which are assumed as usual to belong to charm singlet states), so that from Eq. 11 we would have $Z=0$, and the original sum rule Eq. (1) would be valid. However, the sum rule Eq. (18) for $\alpha'=\beta'\neq 0$ in this case cannot be saturated unless one has a total set of 81 vector mesons.

(ii) If we do not assume invariance under the charm $SU(3)$, we may avoid the introduction of extra vector mesons, and the sum rule Eq. (18) can be saturated by the usual ρ, ω, and ϕ alone. However, in this case, one gets Eq. (14) again from Eq. (18). Since $\epsilon=2/\sqrt{3}$ in this case, we get $Z=4$ from Eq. (11). But the sum rule Eq. (10) for $Z=4$ is badly violated experimentally.

(iii) If we give up the literal identification Eq. (17), then Eq. (18) may not be valid. In that case, one can avoid all difficulties mentioned above. We may, for

[13] The notation used in Refs. 3 and 4 is related to ours as follows: $\gamma_V=\frac{1}{2}g_V=m_V^2/2f_V$.

[14] Z. Maki, Progr. Theoret. Phys. (Kyoto) **31**, 331 (1964); Y. Hara, Phys. Rev. **134**, B701 (1964).

[15] S. Okubo, lecture notes at the University of Islamabad, 1967 (unpublished); also, in *Proceedings of the International Conference on Particles and Fields, Rochester*, 1967 (Wiley-Interscience, Inc., New York, 1967).

[16] Here we follow the notation given in J. Otokozawa and H. Suura, Phys. Rev. Letters **21**, 1295 (1968). Our $SU(3)\otimes SU(3)$ should not be confused with the chiral $SW(3)$ group. Also, we shall denote the first and second $SU(3)$'s as the ordinary and charm $SU(3)$ groups, respectively.

[17] S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 139 (1967).

instance, select a suitable subset of the currents in Eq. (17) and invoke asymptotic $SU(3)\otimes SU(3)$ rather than $SU(9)$ without any contradiction in the saturation of the resulting sum rule. But we have no way to calculate σ_ω, and hence Z will be a free parameter.

We shall now consider the second Weinberg sum rule for asymptotically broken $SU(3)$ and revert to the notation used before:

$$\int_0^\infty \rho_{\alpha\beta}{}^{(2)}(m^2)dm^2 = c\delta_{\alpha\beta}+d\delta_{\alpha 0}\delta_{\beta 0}+ed_{8\alpha\beta}\,. \tag{19}$$

In principle, we could add[18] a term proportional to $d_{8\alpha\gamma}d_{8\beta\gamma}$ to the right-hand side of Eq. (19). However, we adopt the atitude that such a second-order $SU(3)$-violating effect is small, if it exists at all, in the asymptotic sum rule, so that Eq. (19) would still be valid to a good approximation. Also, we assume in a similar spirit that there is no cross-term proportional to $\delta_{0\alpha}\delta_{8\beta}+\delta_{8\alpha}\delta_{0\beta}$ in Eq. (19).

Now the sum rules Eqs. (2) and (19) give us the following relations for G_ω and G_ϕ:

$$G_\omega{}^2 = \frac{m_\omega{}^2(m_\phi{}^2-m_8{}^2)}{m_\rho{}^2(m_\phi{}^2-m_\omega{}^2)}G_\rho{}^2+\frac{4}{3}\frac{m_\omega{}^2 m_{K^*}{}^2}{m_\phi{}^2-m_\omega{}^2}G_\kappa{}^2\,, \tag{20a}$$

$$G_\phi{}^2 = \frac{m_\phi{}^2(m_8{}^2-m_\omega{}^2)}{m_\rho{}^2(m_\phi{}^2-m_\omega{}^2)}G_\rho{}^2-\frac{4}{3}\frac{m_\phi{}^2 m_{K^*}{}^2}{m_\phi{}^2-m_\omega{}^2}G_\kappa{}^2\,, \tag{20b}$$

$$m_8{}^2 = \tfrac{1}{3}(4m_{K^*}{}^2-m_\rho{}^2)\,, \tag{20c}$$

where G_κ denotes the coupling of the scalar κ to the vector current defined by

$$\langle 0|V_\mu{}^{(4-i5)}(0)|\kappa^+(k)\rangle = \sqrt{2}G_\kappa(2k_0V)^{-1/2}k_\mu\,. \tag{21}$$

Equations (20) reduce to those obtained in Ref. 1 when we set $G_\kappa=0$. For simplicity in further analysis, we set $m_\omega=m_\rho$ which is experimentally correct with high accuracy. One then gets two solutions involving σ_ω and σ_ϕ:

$$\sigma_\omega = G_\phi = 0 \tag{22}$$

or

$$\sigma_\phi = -\tfrac{1}{2}\sqrt{2}G_\phi\,,\quad \sigma_\omega = \tfrac{1}{2}\sqrt{2}\frac{m_\omega{}^2}{m_\phi{}^2}\frac{G_\phi{}^2}{G_\omega}\,. \tag{23}$$

The first solution Eq. (22) gives $\frac{1}{3}\Gamma(\rho\to l\bar{l})=\Gamma(\omega\to l\bar{l})$ irrespective of the value of the parameter ϵ. This being in apparent contradiction with the experimental result, we discuss only the second solution, Eq. (23). Then one gets[19]

$$\frac{\Gamma(\phi\to l\bar{l})}{\Gamma(\rho\to l\bar{l})} = \frac{1}{3}\left(\frac{G_\phi}{G_\rho}\right)^2\left(\frac{m_\rho}{m_\phi}\right)^3[1-(\sqrt{\tfrac{3}{2}})\epsilon]^2\,, \tag{24a}$$

$$\frac{\Gamma(\omega\to l\bar{l})}{\Gamma(\rho\to l\bar{l})} = \frac{1}{3}\left(\frac{G_\omega}{G_\rho}\right)^2\left[1+(\sqrt{\tfrac{3}{2}})\epsilon\frac{m_\omega{}^2 G_\phi{}^2}{m_\phi{}^2 G_\omega{}^2}\right]^2. \tag{24b}$$

For the Maki-Hara model, we have $\epsilon=\sqrt{\tfrac{2}{3}}$ so that $\Gamma(\phi\to l\bar{l})=0$, in contradiction to experiment. This result [together with Eq. (15)] has been previously obtained[8] by assuming the validity of $SU(6)$, and it is also obtainable from the naive quark-model considerations where the ϕ meson is taken to be a bound state of the $q_3\bar{q}_3$ system. However, here we have obtained this result from a more general consideration. Together with Eq. (15), this would imply that the Maki-Hara model is experimentally ruled out in our approach.

Finally, we shall briefly comment about the determination of the parameter δ introduced by Schnitzer and Weinberg[20] from the formula[21]

$$\frac{1}{m_\rho{}^2}\Gamma(\rho\to 2\pi)\Gamma(\rho\to l\bar{l}) = \frac{1}{36}\left(\frac{e^2}{4\pi}\right)^2\left(\frac{3-\delta}{4}\right)^2\left[1-\left(\frac{2m_\pi}{m_\rho}\right)^2\right]^{3/2}. \tag{25}$$

Using the experimental value given by Ting,[22] one finds $\delta=-1.01\pm0.24$. Note that $\delta=-1$ implies that the electromagnetic form factor of the pion satisfies an unsubtracted dispersion relation.

Similarly, if we define the parameter x by

$$x = G_\rho/m_\rho f_\pi\,, \tag{26}$$

where f_π is the decay constant for $\pi\to\mu\nu$ ($f\pi\approx 130$ MeV), the value $x=1$ corresponds to the exact validity of the KSRF relation.[23] We may determine x experimentally from the relation

$$\Gamma(\rho\to l\bar{l}) = \frac{4\pi}{3}\left(\frac{e^2}{4\pi}\right)^2\frac{1}{m_\rho}f_\pi{}^2x^2. \tag{27}$$

Using the experimental value,[22] one finds $x=1.12\pm0.07$.

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[18] Several modifications of Eq. (19) have been proposed. See Refs. 6 and 15; also, I. Kimel, Phys. Rev. Letters **21**, 177 (1968); T. Akiba and K. Kang, Phys. Rev. **172**, 1551 (1968); K. Kang, *ibid.* **177**, 2439 (1969).

[19] Note that if we assume moreover the validity of Eq. (14) together with Eqs. (20) and (23), then one can compute $G_\kappa{}^2$ to be given by $m_{K^*}{}^2G_\kappa{}^2=(1/4m_\rho{}^2)(4m_{K^*}{}^2-m_\rho{}^2-m_\omega{}^2-2m_\phi{}^2)G_\rho{}^2\approx-0.03G_\rho{}^2$. Hence we have a contradiction unless we have $G_\kappa\approx 0$, so that $4m_{K^*}{}^2=m_\rho{}^2+m_\omega{}^2+2m_\phi{}^2$ which is the familiar formula (see Ref. 15) and is not so badly satisfied. If we do not assume Eq. (14), we can obtain from Eq. (20b) only an upper limit $m_{K^*}{}^2G_\kappa{}^2<(1/4m_\rho{}^2)(4m_{K^*}{}^2-m_\rho{}^2-3m_\omega{}^2)G_\rho{}^2\simeq 0.33G_\rho{}^2$ or $G_\kappa{}^2<0.24\, f_\pi{}^2$. The result of S. L. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968) in our notation is $G_\kappa{}^2=0.17\, f_\pi{}^2$ and is consistent with our upper limit.

[20] H. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967).

[21] δ is defined here by $G_\rho g_{\rho\pi\pi}=m_\rho{}^2\times\frac{1}{4}(3-\delta)$, so that our formula Eq. (25) does not depend upon the validity of the KSRF relation.

[22] S. C. C. Ting, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna*, 1968, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968).

[23] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); Riazuddin and Fayazuddin, Phys. Rev. **144**, 1071 (1966).