Comments and Addenda

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Consistency of Low-Energy Parameters and Soft-Pion Scattering Theory*

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The parameters of low-energy $\pi\pi$, πN, and $\pi N\rightarrow\pi\pi N$ scattering and their consistency with current-algebra predictions are discussed. It is found that the isospin-zero $\pi\pi$ scattering length is $(0.24\pm0.03)m_\pi^{-1}$ quite independent of the exact value of f_π. An effective value of f_π near 85 MeV seems to be preferred. Experiments are suggested which will clarify the situation further.

THE application of soft-pion techniques to scattering processes has always been subject to the uncertainty in the pion decay constant f_π. When evaluated directly from the pion decay lifetime[1] $f_\pi=94$ MeV is obtained, but when f_π is related to πN scattering through the Goldberger-Treiman (GT) relation we get $f_\pi=(M/G)g_A/g_V=82$ MeV. The purpose of this paper is to reexamine this problem and some related ones in the light of recently published work.

The unsubtracted, antisymmetric π-N dispersion relation evaluated[2] using the most recent experimental data gives the following relation between the isospin scattering and the Born term:

$$a_{1/2}-a_{3/2}=5.25f^2-(0.122\pm0.002)\,. \quad (1)$$

In addition to the usually accepted value of the $\pi\bar{N}N$ coupling constant[3] $f^2=0.081\pm0.003$, independent determinations of $f^2=0.079\pm0.002$ and $f^2=0.076\pm0.004$ are obtained from one-pion exchange (OPE) fits to N-N data.[4] Using the sum rule (1), a value for the antisymmetric scattering length $a_{1/2}-a_{3/2}=0.29\pm0.02$ is obtained. Direct experimental evidence from low-energy $\pi^\pm p$ elastic scattering gives[5] $a_{1/2}-a_{3/2}=0.30\pm0.02$. The soft-pion calculation for πN scattering gives

$$a_{1/2}-a_{3/2}=(45/f_\pi)^2=0.23\,,\quad \text{for}\quad f_\pi=94\text{ MeV}$$
$$=0.30\,,\quad \text{for}\quad f_\pi=82\text{ MeV}.$$

A value f_π near 85 MeV is indicated.

For $\pi\pi$ scattering a sum for the p-wave scattering length has been proposed[6] which can be quite accurately evaluated. This rule has a number of attractive features, including the small contribution of threshold s waves and asymptotic part relative to the intermediate region (primarily the ρ meson). Using the latest values of the ρ resonance parameters[7] ($m_\rho=770\pm4$ MeV, $\Gamma_\rho=111\pm6$ MeV) from e^+e^- colliding beams, we find that (in units of m_π^{-3})

$$a_1=0.035\pm0.005\,.$$

The soft-pion calculation of a_1 is free of ambiguity and is given by[8,9]

$$a_1=(16/f_\pi)^2\,, \quad (2)$$

resulting in the value $f_\pi=86\pm6$ MeV, again closer to the GT value of 82 MeV than the π-decay value 94 MeV.

The soft-pion calculation of the mass-shell π-π amplitude is not unique. A new parameter[9] ξ measuring the deviation from the σ model must be given before the ratio

$$a_0/a_2=(\tfrac{5}{2}\xi-7)/(\xi+2) \quad (3)$$

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[1] See, for example, Stephen L. Adler and Roger F. Dashen, *Current Algebras and Applications to Particle Physics* (W. A. Benjamin, Inc., New York, 1968).

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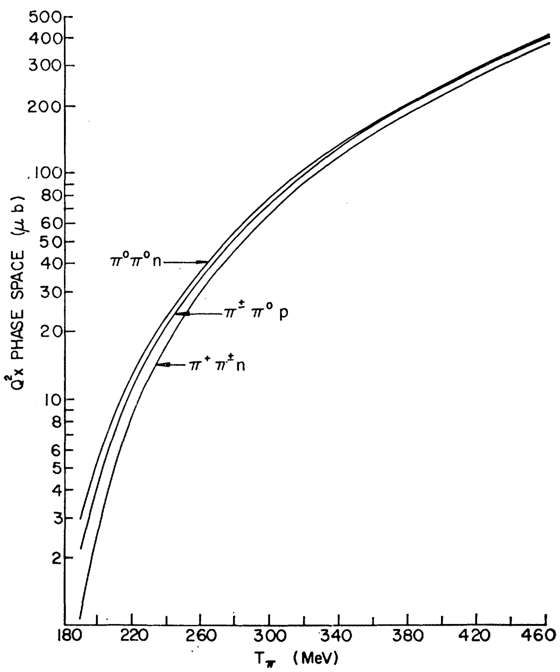

FIG. 1. The pion lab kinetic energy (T_π) dependence of $Q^2\times$phase space for $\pi N\to\pi\pi N$.

of s-wave scattering lengths can be calculated. It can be further shown[9] that the same parameter ξ enters into the threshold amplitude for the process $\pi N\to\pi\pi N$. One such reaction $\pi^- p\to\pi^-\pi^+ n$ has been measured close to production threshold. The ratio of measured cross section to three-body phase space has the following dependence[10] on f_π and ξ

$$\left[\frac{\sigma(\pi^-\pi^+n)}{Q^2\times\text{phase space}}\right]^{1/2}=\left(\frac{82}{f_\pi}\right)^2(1.36-0.6\xi)=1.7\pm0.16. \quad (4)$$

The measurements used are at the two lowest energies where data are available.[11] The p-wave π-π scattering length is related to the s-wave lengths in the soft-pion scheme[8,9] by

$$2a_0-5a_2=18a_1=(68/f_\pi)^2. \quad (5)$$

Using (3)–(5), we find that

$$a_0=(0.20\pm0.02)+(16/f_\pi)^2, \quad (6)$$

so that a_0 is only slightly dependent on f_π. For the usual values of f_π, a_0 changes by only 0.01, giving

$$a_0=0.24\pm0.03. \quad (7)$$

The parameter ξ falls between zero and minus one for f_π ranging between 80 and 95 MeV. The isospin-two scattering length a_2 is, however, more sensitive to the value of f_π; $a_2=(0.08\pm0.01)-(29/f_\pi)^2$, giving an a_2 varying between -0.01 and -0.04. Thus the amplitude for $\pi^-p\to\pi^-\pi^+n$ is nearly proportional to a_0. Hence, a_0 is determined from the production cross section almost independently of f_π and ξ.

On the other hand, a measurement of $\pi^+p\to\pi^+\pi^+n$ near threshold would determine a_2 directly, since both of these quantities depend on ξ and f_π in similar ways. A third production reaction $\pi^-p\to\pi^0\pi^0n$ is useful, since it is sensitive to the relative sign of a_0 and a_2.

From Ref. 9 we can explicitly calculate the above cross sections:

$$\sigma(\pi^-p\to\pi^-\pi^+n)=|a(-+n)|^2Q^2\times\text{phase space},$$
$$\sigma(\pi^-p\to\pi^0\pi^0n)=|a(00n)|^2\tfrac{1}{2}Q^2\times\text{phase space},$$
$$\sigma(\pi^+p\to\pi^+\pi^+n)=|a(++n)|^2\tfrac{1}{2}Q^2\times\text{phase space},$$

where (with $f_\pi=82$ MeV)

$$a(-+n)=-1.36+0.6\xi,$$
$$a(00n)=2.11-0.3\xi,$$
$$a(++n)=1.51+0.6\xi.$$

The factor of $\frac{1}{2}$ appearing in the later two cross sections takes into account the identity of the final pions. The quantity $Q^2\times$phase space (where Q is the incident c.m. pion momentum) is shown in Fig. 1. Use of the above relations gives cross sections in μb. Other production cross sections near threshold are much smaller;

$$\sigma(\pi^+p\to\pi^+\pi^0p)=\sigma(\pi^-p\to\pi^-\pi^0p)=\tfrac{1}{4}\sigma(\pi^+p\to\pi^+\pi^+n).$$

The above predictions for production cross sections are expected to be accurate in an energy range where all the final particles are in relative s waves.

We have shown that by use of pion production data the s-wave $\pi\pi$ scattering lengths can be determined directly independent of the parameters ξ and f_π. A value of f_π consistent with all pion scattering processes is $f_\pi\simeq85$ MeV, which is close to the GT value. This f_π is an "effective" value containing all of the uncertainties of the zero-mass extrapolation, and seems to be the one appropriate for use in soft-pion calculations.

[10] Leaf Turner, Ph. D. thesis, University of Wisconsin, 1969 (unpublished). The notation used is described in detail later on in this article.

[11] Yu. A. Batusov, S. A. Bunyatov, V. M. Sidorov, and V. A. Yarba, Yadern. Fiz. 1, 526 (1965) [English transl.: Soviet J. Nucl. Phys. 1, 374 (1965)].