Pomeranchuk Exchange and Low-Energy Theorems in Compton Scattering*

FARZAM ARBAB
Brookhaven National Laboratory, Upton, New York 11973

AND

RICHARD C. BROWER
Lawrence Radiation Laboratory, University of California, Berkeley, California 94720

(Received 26 November 1968)

The method, introduced in a previous work, of deriving the kinematical properties of zero-mass bosons from the limit of a theory with massive particles is here applied to the problem of Compton scattering. It is shown that the same mechanism that eliminates the factor of $\alpha_\pi(t)$ from the nonsense amplitude in pion photoproduction processes and allows the pion pole to occur in such amplitudes also removes the nonsense factor of α_P-1 from the residue of the Pomeranchuk trajectory in Compton scattering. This allows the Pomeranchuk exchange to contribute to the photoabsorption total cross section. The connection of this result with the vector-dominance model is discussed. A short derivation of the low-energy theorems for Compton scattering using this limiting procedure is also given.

I. INTRODUCTION

IN a recent paper,[1] the authors presented a discussion of zero-mass bosons in S-matrix theory, based on the assumption that the properties of the amplitudes for reactions containing one zero-mass boson could be derived by taking a smooth limit of a theory with massive bosons. Since in this approach we worked entirely with helicity amplitudes, the usual discussion of gauge invariance was avoided, even though the final amplitudes had all the properties usually derived from the principle of gauge invariance. In particular, using this method, we proved that charge, which is defined as the soft coupling of a zero-mass vector particle, is conserved. In the present work, we extend this treatment of the photon to the study of Compton scattering.

The main new result presented here is connected with the description of the Pomeranchuk exchange in Compton-scattering amplitudes, but for the sake of completeness, we have also given a derivation of low-energy theorems for such processes. Our primary purpose is to demonstrate that both the low-energy theorems and the Pomeranchuk exchange can be smoothly related to the "kinematical properties" of hadronic amplitudes. Moreover, it is hoped that the approximate extension of these results to the ρ^0 considered as a small-mass boson may give some insight into the ρ dominance model for Compton scattering. We shall first briefly review the problem of Pomeranchuk exchange, since it presents the greatest difficulty in any model using the ρ-photon analogy.

More than five years ago, Mur[2] pointed out that the usual treatment of Regge exchange for Compton scattering yields a Pomeranchuk trajectory that does not contribute to the forward direction ($t=0$) of the elastic (non-spin-flip in the s channel) amplitude. This is due to the special properties of the crossing matrix for zero-mass particles whereby a helicity-flip t channel amplitude is crossed into the s-channel elastic amplitude. In the usual treatment of nonsense vertices, such an amplitude would be proportional to a factor of $\alpha_P(t)-1$ (we consider the Compton scattering of the pions for simplicity), and therefore it would vanish at $t=0$ if $\alpha_P(0)=1$. Such a zero in the Pomeranchuk exchange for forward elastic Compton scattering is in conflict with a vector-dominance model, since the corresponding amplitudes for transverse ρ^0 in elastic scattering off any target T ($\rho_{\rm tr}{}^0+T\rightarrow\rho_{\rm tr}{}^0+T$), and for photoproduction of transverse ρ^0 ($\gamma+T\rightarrow\rho_{\rm tr}{}^0+T$) are not required to have this zero. More important, the zero in the Compton amplitude implies, by the optical theorem, that the total photoabsorption cross section goes to zero with the power of the next lower trajectory (P'), and this is in contradiction with the contribution of the photoproduction of ρ^0. In order to get a constant high-energy photoabsorption cross section, models for Compton scattering have been devised with singular residues arising directly from the left-hand cut[3] or indirectly from fixed poles at wrong-signature nonsense points.[3,4] As pointed out by Sakurai,[5] this suggests, by the ρ-photon analogy, that singular residues or fixed poles exist for the hadronic amplitudes, $\rho^0+T\rightarrow\rho^0+T$. While such a situation cannot be ruled out, we show in this paper that it is not necessary. Indeed, the amplitudes involving the ρ^0 without fixed poles or singular residues can be "smoothly" related to amplitudes involving the photon, with a total photoabsorption cross section which approaches a constant at high energies. Hence, our description of the photon may place the application of vector dominance to Compton scattering

* Work supported, in part, by the U. S. Atomic Energy Commission.

[1] F. Arbab and R. C. Brower, Phys. Rev. **178**, 2470 (1969).

[2] V. D. Mur, Zh. Eksperim. i Teor. Fiz. **17**, 1458 (1963); **18**, 727 (1964) [English transls.: Soviet Phys.—JETP **44**, 2173 (1963); **45**, 1051 (1964)]; H. D. I. Abarbanel and S. Nussinov, Phys. Rev. **158**, 1462 (1967); H. K. Shepard, *ibid.* **159**, 1331 (1968); **165**, 1934(E) (1968).

[3] H. D. I. Abarbanel, F. E. Low, I. J. Muzinich, S. Nussinov, and J. H. Schwarz, Phys. Rev. **160**, 1329 (1967); A. H. Mueller and T. L. Trueman, *ibid.* **160**, 1306 (1967).

[4] S. Mandelstam and L.-L. Wang, Phys. Rev. **160**, 1490 (1967).

[5] J. J. Sakurai, Stanford Report No. SLAC-TN-68-11, 1968 (unpublished).

on the same basis as in processes involving only a single photon.

Before delving into the problem of Pomeranchuk exchange in detail, we first give a short Reggeized version of our treatment of amplitudes containing one photon in Sec. II. This will serve to remind the reader of the fact that certain nonsense factors, such as the factor of $\alpha_\pi(t)$ in the contribution of the pion trajectory to the helicity-1 amplitude in the photoproduction of pions, are absent when we consider reactions containing zero-mass particles. The absence of the $\alpha_P(t)-1$ factor in the Compton-scattering amplitude is closely related to this phenomenon as will be seen in detail in Sec. III B. Section III A is devoted to the derivation of low-energy theorems.

As was discussed in Ref. 1, our basic assumption, apart from the usual Lorentz invariance, analyticity, and crossing properties, is the assumption of smoothness in the mass of the external vector particle. In the case of photoproduction this property takes the form of the requirement that $m_\gamma M_0$ should vanish as m_γ goes to zero, where M_0 is the amplitude for a zero-helicity massive vector particle. This condition is necessary and sufficient to obtain, in the limit $m_\gamma=0$, the correct transformation properties of the photon amplitude under the Lorentz group. Thus, the assumption of smoothness in mass seems to be intimately connected with the fact that the photon is an external particle transforming according to a one-dimensional representation of the Lorentz group. If one accepts the condition on $m_\gamma M_0$ as a physical requirement for Lorentz-invariant photon amplitudes, this limiting process presents a rigorous approach to the peculiar kinematical properties of zero-mass particles. Note that the unitarity condition has not been used in our discussion of the kinematics of zero-mass bosons. This condition will no doubt give further information about M_0. For example, if we assume a convergent perturbation expansion in e^2 and require that the unitarity condition for the zero-mass case be obtained from the unitarity equation for the massive case in *each order*, then the condition that $M_0\to$ const (or $M_0\to 0$) as $m_\gamma\to 0$ corresponds to a unitary theory *with* (or *without*) a final zero-spin zero-mass particle. It is important to note that these stronger conditions do not invalidate any of the results presented in Ref. 1 or in this paper. (See Appendix and Ref. 6.)

II. NONSENSE FACTOR IN PHOTOPRODUCTION AMPLITUDES

Let us consider the photoproduction of charged pions from spinless targets (the nucleon target has been treated elsewhere[7]). The Reggeized t-channel amplitudes $(t:\gamma\pi^+\to ab)$, when the photon has a finite mass m_γ, are written as

$$M_1{}^t=\frac{\phi^{1/2}}{\mathcal{T}}\sum_k \frac{\alpha_k}{\sin\pi\alpha_k}\gamma_1{}^k(t)(1\pm e^{-i\pi\alpha_k})(s/s_0)^{\alpha_k-1}, \quad (1)$$

$$M_0{}^t=\frac{1}{\mathcal{T}}\sum_k \frac{1}{\sin\pi\alpha_k}\gamma_0{}^k(t)(1\pm e^{-i\pi\alpha_k})(s/s_0)^{\alpha_k}, \quad (2)$$

where k labels the different trajectories with reduced kinematic-singularity-free residues $\gamma_0{}^k$ and $\gamma_1{}^k$, ϕ is the Kibble boundary function, and $\mathcal{T}$ is given by

$$\begin{aligned}\mathcal{T}&=[(t-t^+)(t-t^-)]^{1/2},\\ t^\pm&=(m_\pi\pm m_\gamma)^2.\end{aligned} \quad (3)$$

The factor α_k in M_1 is due to the fact that $M_1{}^t$ is a sense-nonsense amplitude and, for example, in the case of even signature it has no poles as $\alpha_k\to 0$. (Note that we are considering massive photons at this point.) As $m_\gamma\to 0$, the factor $\mathcal{T}^{-1}\to(t-m_\pi{}^2)^{-1}$, which as we will now show removes the factor of α_k only when k denotes the pion trajectory and normalizes the residue of the pion pole in $M_1{}^t$ in the limit $m_\gamma=0$. The two residues $\gamma_1{}^k$ and $\gamma_0{}^k$ are related through threshold and pseudothreshold relations at t^+ and t^-, respectively:

$$\pm\alpha_k(t^\pm)\gamma_1{}^k(t^\pm)=-\gamma_0{}^k(t^\pm)/(\sqrt{2}m_\pi s_0). \quad (4)$$

The quantity $\gamma_0{}^\pi(m_\pi{}^2)$ is known in terms of the charge of the pion e_π, and the coupling at the πab vertex g:

$$\gamma_0{}^\pi(m_\pi{}^2)=\tfrac{1}{2}\pi\alpha'(m_\pi{}^2)e_\pi g m_\gamma(4m_\pi{}^2-m_\gamma{}^2). \quad (5)$$

Equation (5) is the definition of the charge of the pion and is equivalent to Eq. (1.6) of Ref. 1.

We now expand the residues around the point $t=m_\pi{}^2$:

$$\begin{aligned}\gamma_0{}^k(t)&=a_0{}^k+b_0{}^k(t-m_\pi{}^2)+\cdots,\\ \alpha_k(t)\gamma_1{}^k(t)&=a_1{}^k+b_1{}^k(t-m_\pi{}^2)+\cdots.\end{aligned} \quad (6)$$

Note that, for the pion, $a_0{}^\pi$ is given by Eq. (5) and $a_1{}^\pi$ is equal to zero. Substituting this in Eq. (4), with the assumption that the parameters $a_i{}^k$, $b_i{}^k$, etc., remain bounded as $m_\gamma\to 0$, we obtain[8]

$$a_1{}^k=-\sqrt{2}m_\gamma b_0{}^k/s_0+O(m_\gamma{}^2), \quad (7)$$

$$a_0{}^k=-2\sqrt{2}m_\pi{}^2 m_\gamma s_0 b_1{}^k+O(m_\gamma{}^2). \quad (8)$$

---

[6] Y. Hara, Phys. Letters **23**, 696 (1966). Although we do not use the stronger condition [e.g., $M_{0,0}{}^s\sim O(m_\gamma{}^2)$ as $m_\gamma\to 0$], discussed in the Introduction, its implementation generally has no effect on our arguments. However, here it implies that Hara's theorem [i.e., $a_{1-1}{}^p(m_\gamma{}^2)\equiv 0$] cannot hold as $m_\gamma\to 0$, if the Pomeranchuk is to contribute in leading order to $M_{1,1}{}^s(t=0)$ for $m_\gamma=0$. Indeed to lowest order in $m_\gamma{}^2$ and t, one can easily demonstrate from $M_{0,0}{}^s\sim O(m_\gamma{}^2)$ and $M_{00}{}^t(t\neq 0)\sim O(m_\gamma{}^2)$ that $\gamma_{1-1}{}^k(t_1 m_\gamma{}^2)\simeq(t-2m_\gamma{}^2)b_{1-1}{}^k$ for any trajectory k. Work presently underway, indicates that for arbitrary masses m_{γ_1} and m_{γ_2}, the zero is located at $t=m_{\gamma_1}{}^2+m_{\gamma_2}{}^2+O(m_{\gamma_1}{}^4,m_{\gamma_1}{}^2m_{\gamma_2}{}^2,m_{\gamma_2}{}^4)$. Note that the t-channel amplitudes are nonuniform for $t=t^\pm$, hence $M_{00}{}^t\sim O(m_{\gamma_1}m_{\gamma_2})$ does not hold for $t=0$, $4m_{\gamma_1}{}^2$, or $4m_{\gamma_2}{}^2$. The nonuniformity of $M_{1-1}{}^t$ can be understood as due to the factor $(t-m_{\gamma_1}{}^2-m_{\gamma_2}{}^2)/\mathcal{T}^2$.

[7] R. C. Brower and J. W. Dash, Phys. Rev. **175**, 2014 (1968).

[8] Here we are using the symbol $O(m_\gamma{}^2)$ to mean $m_\gamma\epsilon(m_\gamma)$, where $\epsilon(m_\gamma)$ is a quantity that goes to zero as $m_\gamma\to 0$.

For the pion, these two equations imply that b_0 goes to zero as $m_\gamma \to 0$ ($a_1{}^\pi=0$) and that, for $t\approx m_\pi{}^2$,

$$\alpha_\pi(t)\gamma_1{}^\pi(t)/(t-m_\pi{}^2)=-\pi\alpha'(m_\pi{}^2)ge_\pi/(\sqrt{2}s_0)\,. \quad (9)$$

For other trajectories, Eq. (8) implies that $a_1{}^k=O(m_\gamma)$, so that $\gamma_1{}^k(t)\propto(t-m_\pi{}^2)$ when $m_\gamma=0$. This factor of $(t-m_\pi{}^2)$ is cancelled with the $1/\mathcal{T}$, so that the factor of $\alpha_k(t)$ remains in the expression for $M_1{}^t$ for all k except the pion:

$$M_1{}^t=\phi^{1/2}\left[\frac{\bar{\gamma}^\pi(t)}{\sin\pi\alpha_\pi(t)}(1+e^{-i\pi\alpha_\pi})\left(\frac{s}{s_0}\right)^{\alpha_\pi-1}+\sum_k\frac{\alpha_k(t)\bar{\gamma}^k(t)}{\sin\pi\alpha_k(t)}(1\pm e^{-i\pi\alpha_k})\left(\frac{s}{s_0}\right)^{\alpha_k-1}\right], \quad (10)$$

where

$$\bar{\gamma}^\pi(m_\pi{}^2)=-\pi\alpha'(m_\pi{}^2)ge_\pi/(\sqrt{2}s_0)$$

and

$$\gamma_1{}^k(t)=(t-m_\pi{}^2)\bar{\gamma}^k(t)\,. \quad (11)$$

The general lesson that will carry over from the discussion of this section to the Pomeranchuk exchange in Compton scattering is that certain nonsense factors may not occur in amplitudes involving zero-mass particles.

We further note that at threshold the factor $(kp)^\alpha$ in the full residue requires that the Regge expansion in terms of $\nu=2kpz_t$ be given exactly by the leading term. For the pion, threshold coincides with the pole $[k=\frac{1}{2}(t-m_\pi{}^2)]$ so that the exact s dependence of the residue of the pole is determined by the Regge expansion. Using charge conservation $e_\pi=e_a-e_b$ and the kinematical fact that $\nu=(s-m_a{}^2)=-(u-m_b{}^2)$ when $t=m_\pi{}^2$, we see that our Regge expansion Eq. (10) is consistent with the decomposition of $M_1{}^t$ of Ref. 1:

$$M_1{}^t=\frac{\phi^{1/2}}{t-m_\pi{}^2}\left(\frac{\sqrt{2}ge_a}{s-m_a{}^2}+\frac{\sqrt{2}ge_b}{u-m_b{}^2}\right)+\phi^{1/2}\bar{B}_1{}^t. \quad (12)$$

We mention this because Eq. (12) contains the low-energy theorem (Kroll-Ruderman theorem) which is analogous to the low-energy theorem for Compton scattering derived in the next section. In Ref. 1, we proved Eq. (12) as well as charge conservation by the use of crossing, but these arguments need not be repeated here.

III. COMPTON SCATTERING

In order to simplify the kinematics, we consider Compton scattering on a spinless target (pions; $\gamma\pi\to\gamma\pi$) with a massive photon. The t-channel helicity amplitudes $M_{\lambda_1\lambda_2}{}^t(\gamma\gamma\to\pi\pi)$ have the symmetry, $M_{\lambda_1\lambda_2}{}^t=(-1)^{\lambda_1+\lambda_2}M_{-\lambda_1-\lambda_2}{}^t$ due to parity, and $M_{\lambda_1\lambda_2}{}^t=M_{\lambda_2\lambda_1}{}^t$ due to statistics. There are four independent amplitudes, which we given in terms of the kinematic-singularity-free amplitudes $\bar{M}_{\lambda_1\lambda_2}$.

$$M_{11}{}^t=\frac{1}{t-4m_\gamma{}^2}\bar{M}_{11}{}^t,\qquad M_{10}{}^t=\frac{\phi^{1/2}}{t-4m_\gamma{}^2}\bar{M}_{10}{}^t,$$
$$M_{1-1}{}^t=\frac{\phi}{t(t-4m_\gamma{}^2)}\bar{M}_{1-1}{}^t,\quad M_{00}{}^t=\frac{1}{t-4m_\gamma{}^2}\bar{M}_{00}{}^t. \quad (13)$$

For the proof of the low-energy theorems, it is also necessary to write the s-channel amplitudes $M_{1;1}{}^s$ and $M_{1;-1}{}^s$ in terms of the kinematic-singularity-free parity-conserving amplitudes $F^\pm$ [$\pm$ refers to the j parity of the leading order poles, $P=\pm(-1)^J$]:

$$M_{1;1}{}^s=-\frac{\phi}{ts}\left(F^++\frac{1}{\mathcal{S}^2}F^-\right),$$
$$M_{1;-1}{}^s=t\left(F^+-\frac{1}{\mathcal{S}^2}F^-\right), \quad (14)$$

where $\mathcal{S}$ is defined similar to the factor $\mathcal{T}$ of the previous section. In general there is a conspiracy relation at $s=0$ of the form $F^+=-(1/\mathcal{S}^2)F^-$. Notice that $M_{1;\pm1}{}^s$ are nonsense-nonsense amplitudes at the pion pole, but the factor $1/\mathcal{S}^2$ will enable the pion pole to occur in $F^-/\mathcal{S}^2$ in the limit $m_\gamma=0$.

A. Low-Energy Theorems

Just as in the case of photoproduction, the low-energy theorems require the introduction of pole terms into the s- and u-channel amplitudes. As in Ref. 1, we define charge through the sense amplitudes by the limits

$$\lim_{s\to m_\pi{}^2}(s-m_\pi{}^2)M_{00}{}^s=-(4m_\pi{}^2-m_\gamma{}^2)e^2, \quad (15)$$

$$\lim_{u\to m_\pi{}^2}(u-m_\pi{}^2)M_{00}{}^u=-(4m_\pi{}^2-m_\gamma{}^2)e^2. \quad (16)$$

With the use of the helicity crossing matrix, the residues of these poles in $M_{1\pm1}{}^t$ and $M_{1;\pm1}{}^s$ are calculated. $M_{1\pm1}{}^t$ contain both the s and the u pole, while only the u pole contributes to $M_{1;\pm1}{}^s$:

$$M_{11}{}^t=\frac{1}{t-4m_\gamma{}^2}\left[\frac{2e^2[tm_\pi{}^2+O(m_\gamma{}^2)]}{s-m_\pi{}^2}+\frac{2e^2[tm_\pi{}^2+O(m_\gamma{}^2)]}{u-m_\pi{}^2}+B_{11}{}^t\right], \quad (17)$$

$$M_{1-1}{}^t=\frac{\phi}{t(t-4m_\gamma{}^2)}\left[\frac{2e^2}{s-m_\pi{}^2}+\frac{2e^2}{u-m_\pi{}^2}+B_{1-1}{}^t\right], \quad (18)$$

$$M_{1;1}{}^s=-\frac{\phi}{t}\left[\frac{2e^2[s-m_\pi{}^2+O(m_\gamma{}^2)]}{\mathcal{S}^2(u-m_\pi{}^2)}+\frac{1}{s}\left(B_+{}^s+\frac{1}{\mathcal{S}^2}B_-{}^s\right)\right], \quad (19)$$

$$M_{1;-1}{}^{s}=-t\left[\frac{2e^2m_\pi{}^2[s-m_\pi{}^2+O(m_\gamma{}^2)]}{\mathcal{S}^2(u-m_\pi{}^2)}-B_+{}^s+\frac{1}{\mathcal{S}^2}B_-{}^s\right]. \quad (20)$$

As in Ref. 1, we assume that we can pass smoothly to the limit $m_\gamma\to 0$, and apply the crossing relations $M_{11}{}^t=M_{1-1}{}^s$, $M_{1-1}{}^t=M_{11}{}^s$ for $m_\gamma=0$. We find that the pole terms satisfy crossing by themselves (charge conservation has been assumed this time), and the background terms are constrained by

$$B_{11}{}^t=t^2\left(B_+{}^s-\frac{1}{(s-m_\pi{}^2)^2}B_-{}^s\right), \quad (21)$$

$$B_{1-1}{}^t=\frac{t}{s}\left(B_+{}^s+\frac{1}{(s-m_\pi{}^2)^2}B_-{}^s\right). \quad (22)$$

Assuming that the background terms actually do not become infinite as $m_\gamma\to 0$, from Eqs. (21) and (22) we can define the new kinematic-singularity-free background terms $\bar{B}$ by

$$B_{11}{}^t=t^2\bar{B}_{11}{}^t,\quad B_+{}^s=\bar{B}_+{}^s,$$
$$B_{1-1}=t\bar{B}_{1-1},\quad B_-{}^s=(s-m_\pi{}^2)^2\bar{B}_-{}^s, \quad (23)$$

with the conspiracy relation $\bar{B}_+{}^s\ (s=0)=-\bar{B}_-{}^s\ (s=0)$. The resultant expression for the Compton amplitudes have the full content of the low-energy theorems:

$$M_{1;-1}{}^s=M_{11}{}^t=\frac{-2e^2m_\pi{}^2t}{(s-m_\pi{}^2)(u-m_\pi{}^2)}+t\bar{B}_{11}{}^t,$$
$$M_{1;1}{}^s=M_{1-1}{}^t=-\frac{\phi}{t}\,\frac{2e^2}{(s-m_\pi{}^2)(u-m_\pi{}^2)}+\frac{\phi}{t}\bar{B}_{1-1}{}^t. \quad (24)$$

For example, it is usually stated that the Born term gives the zeroth- and first-order contributions in the momentum of the photon $k(s)$ for $M_{1;1}{}^s$ at fixed angle. We see this by the expression $\phi/t=-2k^2s(1+\cos\theta_s)$.

This derivation again emphasizes the point made by Abarbanel and Goldberger[9] that the low-energy theorems can be derived from kinematical considerations and that the asymptotic behavior of the amplitude is irrelevant to the theorems. The important condition is that, for example, $\bar{B}_{1-1}{}^t(k,\cos\theta_s)$ be bounded as $k\to 0$, which is of course true in lowest-order perturbation theory. However, the infinite set of cuts at $k=0$ may invalidate the theorem as an exact statement in zeroth and first order in k.[10]

[9] H. D. I. Abarbanel and M. C. Goldberger, Phys. Rev. **165**, 1594 (1968).

[10] S. M. Roy and Virendra Singh, Phys. Rev. Letters **21**, 861 (1968); T. P. Cheng, Rockefeller Report, 1969 (unpublished).

B. Pomeranchuk Exchange

We Reggeize our t-channel amplitudes according to the standard procedure:

$$\bar{M}_{\lambda_1\lambda_2}{}^t=\sum_k \gamma_{\lambda_1\lambda_2}{}^k\frac{1\pm e^{-i\pi\alpha_k}}{\sin\pi\alpha_k}s^{\alpha_k-|\lambda|}. \quad (25)$$

The residue of the Pomeranchuk trajectory in the sense-nonsense amplitude, $\gamma_{1-1}{}^P(t)$, contains a factor of $[\alpha_P(t)-1]$—one $[\alpha_P(t)-1]^{1/2}$ from the d function combined with a $[\alpha_P(t)-1]^{1/2}$ from the nonsense $\gamma\gamma P$ vertex. The full set of pseudothreshold and threshold (PT) relations lead to the following constraints on the residues:

$$\gamma_{11}(0)-\gamma_{1-1}(0)+\gamma_{00}(0)=0,$$
$$\gamma_{11}(4m_\gamma{}^2)=\gamma_{1-1}(4m_\gamma{}^2),$$
$$\sqrt{2}m_\gamma\gamma_{10}(4m_\gamma{}^2)=\gamma_{1-1}(4m_\gamma{}^2),$$
$$\gamma_{00}(4m_\gamma{}^2)=-2\gamma_{1-1}(4m_\gamma{}^2), \quad (26)$$
$$\gamma_{1-1}(4m_\gamma{}^2)+2m_\gamma{}^2[-\gamma_{11}'(4m_\gamma{}^2)+4\sqrt{2}m_\gamma\gamma_{10}'(4m_\gamma{}^2)+\gamma_{00}'(4m_\gamma{}^2)-\gamma_{1-1}'(4m_\gamma{}^2)]=0.$$

The primes denote the derivative of the residues with respect to t.

By expanding the Trueman-Wick crossing matrix for large s and substituting the expression for a t-channel Regge pole, we can compute the leading contribution to the s-channel amplitudes. The results for all the s-channel amplitudes are given in the Appendix. Here we are primarily interested in the asymptotic behavior of $M_{1;1}{}^s$ at $t=0$:

$$M_{1;1}{}^s=\frac{-s^\alpha f(\alpha)}{(t-4m_\gamma{}^2)^2}[-2m_\gamma{}^2\gamma_{11}+2\sqrt{2}m_\gamma t\gamma_{10}+2m_\gamma{}^2\gamma_{00}-(t-2m_\gamma{}^2)\gamma_{1-1}], \quad (27)$$

where $f(\alpha)=(1\pm e^{-i\pi\alpha})/\sin\pi\alpha$. From Eq. (27) we see that the asymptotic expression for $M_{1;1}{}^s(t=0)$ with zero-mass photons can be obtained by taking a number of different limits:

$$\lim_{m_\gamma\to 0} M_{1;1}{}^s(t=0)=-s^{\alpha(0)}f(\alpha(0))\lim_{m_\gamma\to 0}\frac{\gamma_{00}(0)}{4m_\gamma{}^2},$$
$$\lim_{t\to 0}\lim_{m_\gamma\to 0} M_{1;1}{}^s(t)=s^{\alpha(0)}f(\alpha(0))\lim_{t\to 0}\frac{\gamma_{1-1}(t)}{t}, \quad (28)$$
$$\lim_{m_\gamma\to 0} M_{1;1}{}^s(t=t^+=4m_\gamma{}^2)=s^{\alpha(0)}f(\alpha(0))\lim_{m_\gamma\to 0}\gamma_{1-1}'(4m_\gamma{}^2).$$

We will show that the PT relations imply that these three expressions are all equivalent. Note that the last expression contains the derivative of the residue and, barring accidents, is apparently finite for all trajectories, including the Pomeranchuk with $\gamma_{1-1}{}^P(t)=[\alpha_P(t)-1]\bar{\gamma}_{1-1}{}^P(t)$. To calculate these limits, we first expand the above residues about $t=0$,

$$\gamma_{\lambda_1\lambda_2}(t)=a_{\lambda_1\lambda_2}+b_{\lambda_1\lambda_2}t+\cdots, \quad (29)$$

and substitute into the PT relations. With the smoothness assumption that $a_{\lambda_1\lambda_2}(m_\gamma)$, $b_{\lambda_1\lambda_2}(m_\gamma)$, $\cdots$, are bounded as $m_\gamma \to 0$, we obtain the following set of constraints on these coefficients:

$$\begin{aligned} &a_{11}+a_{00}=a_{1-1}\,,\\ &b_{11}=O(m_\gamma)\,,\\ &a_{1-1}=-2m_\gamma^2(b_{00}+b_{1-1})+O(m_\gamma^3)\,,\\ &a_{00}=-4m_\gamma^2 b_{1-1}+O(m_\gamma^3)\,,\\ &a_{10}=-\sqrt{2}m_\gamma(-b_{1-1}+b_{00})+O(m_\gamma^3)\,. \end{aligned} \tag{30}$$

With the help of these threshold conditions we find that the different limits given in Eq. (28) are indeed the same and that when $m_\gamma=0$ we have

$$M_{1;1}{}^s(t=0)=b_{1-1}\frac{1\pm e^{-i\pi\alpha(0)}}{\sin\pi\alpha(0)}s^{\alpha(0)}\,. \tag{31}$$

This result is independent of whether or not the trajectory actually goes through $J=1$ at $t=0$, so that the nonsense factor $(a_{1-1}{}^P=0)$ in no way forces the contribution of the Pomeranchuk trajectory to vanish. Moreover, the full set of PT relations are satisfied in the final limit with no constraints on b_{1-1}. In other words in the limit of zero mass, the first two terms in the expansion of $\gamma_{1-1}(t)$ around $t=0$, $a_{1-1}+b_{1-1}t$, go into a constant so that

$$\bar{\gamma}_{1-1}(t)\equiv \lim_{m_\gamma\to 0}\frac{\gamma_{1-1}(t)}{t-4m_\gamma^2}=b_{1-1}+O(t)\,.$$

We may thus say that the nonsense factor $\alpha_P(t)-1$ has been canceled by a kinematical pole just as in the case of the nonsense factor $\alpha_\pi(t)$ in pion photoproduction. On the other hand, an important distinction between the two results is that in the π exchange, the constant $b_1{}^\pi$ is determined by the charge of the pion, while there is no easy determination of $b_{1-1}{}^P$. In this discussion $b_{1-1}{}^P$ can accidentally be zero but this is not required by any of the kinematical constraints (PT relations and nonsense factors). However, the smooth dependence of $M_{1;1}{}^s$ on the mass suggests the application of the vector dominance model. This application has already been discussed by other authors,[5] and we expect the vector-dominance value which is clearly nonzero to give an approximate value for b_{1-1}.

IV. CONCLUSIONS

We have presented an alternative method of dealing with the kinematics and gauge invariance of the zero-mass photon, which clearly can be extended to more complex processes involving higher-spin hadrons. In the Reggeized form, this includes the elimination of the nonsense factors associated with the pion pole of π photoproduction. In this light, it is significant that this same procedure apparently eliminates the nonsense factor from the Pomeranchuk residue to Compton scattering.

Clearly some dynamical scheme is necessary to determine the actual strength of the coupling of the Pomeranchuk trajectory. The most obvious scheme is the vector dominance model. Abarbanel *et al.*[3] give a different estimate in terms of an N/D model that unitarizes in the t channel the contribution of the s- and u-channel pole terms. This singular-residue model has the feature of relating the Pomeranchuk coupling to the charge of the target particle. We suggest that a more realistic model would consider direct-channel unitarity as in the multiperipheral model, particularly since this unitarity condition requires, as pointed out in the Introduction, that the Pomeranchuk exchange couple to forward elastic Compton scattering amplitudes, unless the sense amplitudes for ρ^0 photoproduction vanish.

As to the application of the ρ-dominance model to the determination of b_{1-1}, we note that a smooth connection in the mass between a photon process and an analogous ρ process is only indirectly related to vector dominance. For example, both $M_{1;1}{}^s(\rho^0\pi\to\rho^0\pi)$ and $M_{1-1}{}^t(\rho^0\rho^0\to\pi\pi)$ go over smoothly to $M_{1;1}{}^s(\gamma\pi\to\gamma\pi)$ as the mass m_ρ is taken to zero, but $M_{1;1}{}^s(\rho^0\pi\to\rho^0\pi)$ is a more suitable amplitude for high-energy comparison with $M_{1;1}{}^s(\gamma\pi\to\gamma\pi)$ because it lacks the threshold singularity at $t=4m_\rho^2$. The important point is that our smoothness assumption gives $M_{1;1}{}^s$ $(\gamma\pi\to\gamma\pi)$ and $M_{1;1}{}^s$ $(\rho^0\pi\to\rho^0\pi)$ more nearly the same t dependence for the Pomeranchuk exchange. Similar remarks can be made in the experimentally more interesting comparison of $\gamma p\to\rho^0 p$ and $\gamma p\to\gamma p$ by repeating our analysis for the nucleon target.

On the other hand, there are amplitudes in which vector dominance is expected to fail. For example, if there is a more massive pion π', then the comparison of $\rho_{\rm tr}{}^0 p\to\pi'^+ n$ and $\gamma p\to\pi'^+ n$ will run into difficulties, since only the amplitude with ρ^0 has a pion pole. At high energies, the amplitudes for zero helicity ρ^0 in the t channel will contribute a pole term proportional to $[m_\rho(-t)^{1/2}/(t-m_\pi^2)]g_{\rho\pi\pi'}g_{\pi\bar{p}n}$. Unless this exchange accidentally decouples, we see that the ratio m_ρ/m_π causes there to be an appreciable contribution in the region $-m_\rho^2<-t<-m_\pi^2$ in only one of the processes. We feel that it is generally true that the validity of vector dominance is dependent on the smallness of the ρ mass compared to an appropriate scale based on nearby singularities.

Finally we note that the fixed-pole models due to Abarbanel *et al.*[3] and to Mandelstam[4] presently give no estimates of the Pomeranchuk coupling at $t=0$. However, as we emphasized in the Introduction, our results are consistent with fixed poles[11] and our analysis

---

[11] The only effect on the physical amplitude of a fixed pole at a wrong-signature nonsense point ($J=1$; $I_t=0$, 2; $I_t=1$ cannot contribute to chargeless photons in $\gamma\pi\to\gamma\pi$) is to cancel to nonsense factor by introducing a singularity $1/[\alpha(t)-1]$ in a Regge residue. The identical result is achieved here by the cancellation of the nonsense factor against a kinematic singularity.

may suggest that they are only important when zero-mass particles are involved.

ACKNOWLEDGMENTS

We would like to thank Professor Geoffrey F. Chew for his valuable advice and encouragement. We would also like to thank Professor J. D. Jackson, Professor S. Mandelstam, Professor I. J. Muzinich, and J. H. Weis for many illuminating discussions.

APPENDIX

Here we give the expressions for the contribution of a Regge pole to all the s-channel amplitudes for the process $\gamma\pi \to \gamma\pi$. It is the behavior of these s-channel amplitudes at $t=4m_\gamma^2$ and $t=0$ that gives the PT relations of Eq. (26). We also give a short discussion of the more general limiting procedure, namely, one in which the masses of the two photons are different and taken to zero independently.

The full set of crossing relations result in the following set of equations:

$$\begin{aligned} M_{1;1}{}^s &= \frac{-f(\alpha)s^\alpha}{(t-4m_\gamma^2)^2}[-2m_\lambda^2\gamma_{11}+2\sqrt{2}m_\gamma t\gamma_{10} \\ &\qquad +2m_\gamma^2\gamma_{00}-(t-2m_\gamma^2)\gamma_{1-1}], \\ M_{1;0}{}^s &= \frac{(-t)^{1/2}f(\alpha)s^\alpha}{(t-4m_\gamma^2)^2}(-\sqrt{2}m_\gamma\gamma_{11}+2t\gamma_{10} \\ &\qquad +\sqrt{2}m_\gamma\gamma_{00}-\sqrt{2}m_\gamma\gamma_{1-1}), \\ M_{1;-1}{}^s &= \frac{-f(\alpha)s^\alpha}{(t-4m_\gamma^2)^2}[(t-2m_\gamma^2)\gamma_{11}-2\sqrt{2}m_\gamma t\gamma_{10} \\ &\qquad -2m_\gamma^2\gamma_{00}+2m_\gamma^2\gamma_{1-1}], \\ M_{0;0}{}^s &= \frac{f(\alpha)s^\alpha}{(t-4m_\gamma^2)^2}(4m_\gamma^2\gamma_{11}-4\sqrt{2}m_\gamma t\gamma_{10} \\ &\qquad -t\gamma_{00}+4m_\gamma^2\gamma_{1-1}), \\ f(\alpha) &= (1\pm e^{-i\pi\alpha})/\sin\pi\alpha. \end{aligned} \tag{A1}$$

As the reader may verify, the condition at $t=0$ in Eq. (26) is a pseudothreshold relation necessary to allow $M_{1;-1}{}^s$ to vanish like t in the forward direction, and the threshold conditions at $t=4m_\gamma^2$ are necessary to remove the double pole from the s-channel amplitudes. The Regge contributions at the point $t=0$ to $M_{1;1}{}^s$ and $M_{0;0}{}^s$ are

$$\begin{aligned} M_{1;1}{}^s &= \frac{1}{8m_\gamma^2}[\gamma_{11}(0)-\gamma_{00}(0)-\gamma_{1-1}(0)]f(\alpha)s^\alpha, \\ M_{0;0}{}^s &= \frac{1}{4m_\gamma^2}[\gamma_{11}(0)+\gamma_{1-1}(0)]f(\alpha)s^\alpha, \end{aligned} \tag{A2}$$

and for the Pomeranchuk trajectory

$$\gamma_{1-1}(0)=[\alpha_P(0)-1]\bar{\gamma}_{1-1}(0)=0.$$

Note that at $t=0$ the pseudothreshold relation yields Hara's theorem, $(M_{1;1}{}^s/M_{0;0}{}^s)_{t=0}\to 1$ as $s\to\infty$.

We have also studied the more general case with $m_{\gamma_1}\neq m_{\gamma_2}$ in order to understand the over-all consistency of our scheme. If the helicity amplitudes are Reggeized by

$$M_{\lambda_1\lambda_2}{}^t=\frac{(\phi^{1/2})^{|\lambda_1-\lambda_2|}}{(t-t^+)(t-t^-)}\sum\frac{\gamma_{\lambda_1\lambda_2}{}^k(t)}{\sin\pi\alpha_k}\times(1\pm e^{-i\pi\alpha_k})s^{\alpha_k-|\lambda_1-\lambda_2|}, \tag{A3}$$

where the γ^k are free of kinematic singularities, the PT relations at $t^\pm=(m_{\gamma_1}\pm m_{\gamma_2})$ can be written as[12]

$$U_{\tau\lambda_2}\phi^{1/2|\lambda_1-\lambda_2|}\gamma_{\lambda_1\lambda_2}(t)\propto(t-t_\pm)^{2+\tau_1\pm\tau_2} \tag{A4}$$

with

$$U_{\tau\lambda}=D_{\lambda\tau}{}^1(\tfrac{1}{2}\pi,\tfrac{1}{2}\pi,-\tfrac{1}{2}\pi).$$

Note that in the limit $m_{\gamma_1}=m_{\gamma_2}$ these relations imply the additional factors of t present in the kinematical singularities of the equal-mass case, Eq. (13), and that they also reduce to the set of relations given in Eq. (26).

Using these relations we find that the results of Sec. III B hold in general when m_{γ_1} and m_{γ_2} go to zero simultaneously with an arbitrary nonzero ratio. However, in the special case of $m_{\gamma_1}/m_{\gamma_2}\to 0$ the nonsense factor $\alpha_p(0)-1$ is not automatically eliminated. More specifically, if we consider the process $\gamma\pi\to\rho^0\pi$ where $m_\gamma=0$ and $m_\rho\neq 0$ and then take the limit $m_\rho\to 0$, we find that the factor $\alpha_P(0)-1$ will remain in the final zero-mass limit if it exists in the $\gamma\pi\to\rho\pi$ amplitude. Thus the over-all consistency of our scheme requires the nonsense factor to be absent in both Compton scattering and photoproduction of vector particles. This fact may prove beneficial to the comparison of $\gamma\pi\to\gamma\pi$ and $\gamma\pi\to\rho^0\pi$ by the vector-dominance model.

[12] G. Cohen-Tannoudji, A. Morel, and H. Navelet, Ann. Phys. (N. Y.) **46**, 239 (1968).