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Low-Energy Pion-Nucleon Scattering Phase Shifts

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Low-energy pion-nucleon scattering phase shifts have been calculated using a model which includes only N, N^*, ρ, and ϵ intermediate states. This remarkably simple model yields good agreement with phase shifts determined from experimental data. There are two adjustable parameters in the model. The even s-wave scattering length fixes one of these, and the other is determined by the ρ-nucleon magnetic moment coupling. The phase shifts are not sensitive to the choice of this coupling.

I. INTRODUCTION

Pion-nucleon scattering has been studied extensively for almost 20 years. Despite the large amount of data which has been accumulated during this time, there still does not exist a simple picture of the low-energy pion-nucleon scattering process.

It is well known that perturbation calculations using only pseudoscalar coupling of pions and nucleons give incorrect predictions.[1] Attempts to include the contributions from low-lying pion-nucleon and pion-pion resonances have improved the theoretical results.[2] The choice of which states to include has been based on a variety of criteria.

In this paper we present a calculation of the s- and p-wave pion-nucleon phase shifts for elastic scattering based on a simple model which includes contributions from pseudoscalar pion coupling to the nucleon ($I=J=\frac{1}{2}$) plus contributions from the N^* ($I=J=\frac{3}{2}$) and from two pion-pion resonances—the ρ ($I=J=1$) and the ϵ ($I=J=0$).[3] We have included only the lowest energy states in each isotopic spin channel which could significantly contribute to the scattering.

The calculation is presented in the spirit of a phenomenological theory. Its main virtue is that it reproduces with good accuracy all the s- and p-wave phase shifts while being extremely simple. There are no adjustable constants once the scattering lengths and ρ-nucleon coupling are fixed. We do not adjust the parameters associated with the masses and coupling constants of the various particles which enter the calculation—these are taken from experiment. Thus our results are fixed aside from the sometimes large uncertainties in the experimental data.

The contributions to pion-nucleon scattering are obtained from the diagrams shown in Fig. 1. We have assumed the "narrow resonance" approximation in dealing with the propagators for unstable particles. Furthermore, we have adopted the view of Amati and Fubini[2] in setting the value of $s=m^*$ to fix the off-the-mass-shell continuation of the N^* propagator. This procedure is ambiguous in that we could add a polynomial in s to the propagator and obtain the same on-the-mass-shell result.

II. MODEL

In a previous paper[4] the authors have obtained the s- and p-wave scattering lengths for pion-nucleon scattering by taking into account the contributions from the exchanges of the lowest-lying meson and baryon states, i.e., N, N^*, ρ, and ϵ.

The effective Lagrangian of our model is given by

$$L_{\text{int}}=ig_r\bar{N}\gamma_5\boldsymbol{\tau}N\cdot\boldsymbol{\pi}+g_{\epsilon pp}\bar{N}N\epsilon+g_{\epsilon\pi\pi}\epsilon\boldsymbol{\pi}\cdot\boldsymbol{\pi}+ig_{\rho NN}\bar{N}[\gamma_\mu+(i\kappa/2m_N)\sigma_{\mu\nu}\overleftrightarrow{\partial}_\nu]\boldsymbol{\tau}N\cdot\boldsymbol{\rho}_\mu+ig_{\rho\pi\pi}\boldsymbol{\rho}_\mu\cdot\boldsymbol{\pi}\times\overleftrightarrow{\partial}_\mu\boldsymbol{\pi}+(g^*/m_\pi)\bar{N}_\mu{}^*N\partial_\mu\pi. \quad (1)$$

The coupling constants are given as follows: $g_r{}^2/4\pi=14.64$, $g_{\rho\pi\pi}{}^2/4\pi=2.2$, $g^{*2}/4\pi m_\pi{}^2=0.37$, $\kappa=3.7$, and $g_{\epsilon\pi\pi}{}^2/4\pi=11$. To obtain $g_{\rho NN}$ we have made use of the universality of the ρ coupling to the isotopic spin cur-

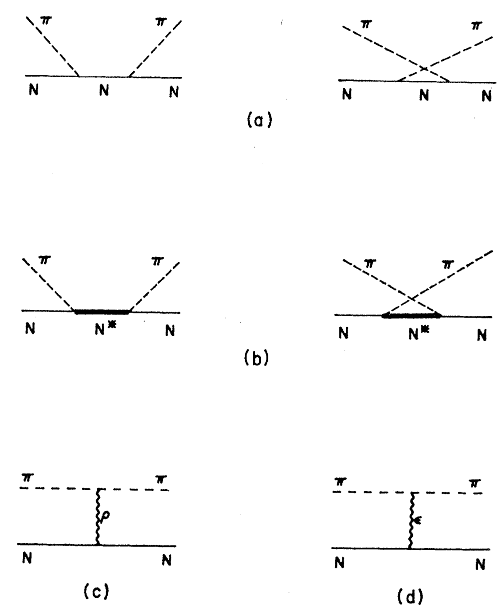

Fig. 1. Feynman diagrams for pion-nucleon scattering.

* On leave of absence from the Saha Institute of Nuclear Physics, Calcutta, India during 1967–1968.

[1] See, e.g., K. Nishijima, *Fundamental Particles* (W. A. Benjamin, Inc., New York, 1963).

[2] P. Amati and S. Fubini, Ann. Rev. Nucl. Sci. **12**, 359 (1962).

[3] B. Dutta-Roy and I. R. Lapidus, Phys. Rev. **169**, 1357 (1968).

[4] B. Dutta-Roy, I. R. Lapidus, and M. J. Tausner, Phys. Rev. **177**, 2529 (1969).

rent. The coupling of the ϵ to the nucleon is determined from the even s-wave scattering length for pion-nucleon scattering and is relatively insensitive to the variations of $a_s^{(+)}$ which appear in the literature.[4]

The amplitude for pion-nucleon scattering $\pi(k)+N(p)\to\pi(k')+N(p')$ is given by[5]

$$M=(m_N/4\pi W)\bar{u}(p')[-A+i(\mathbf{k}+\mathbf{k}')\cdot\gamma B/2]u(p)\,, \quad (2)$$

where W is the c.m. energy. We then obtain the contributions from the various exchanges:

$$A_N^{(+)}=A_N^{(-)}=0\,, \quad (3a)$$

$$B_N^{(\pm)}=g_r^2\left(\frac{1}{m_N^2-s}\mp\frac{1}{m_N^2-u}\right); \quad (3b)$$

$$A_{N^*}^{(\pm)}=\binom{1}{\frac{1}{2}}\left(\frac{1}{m^{*2}-s}\pm\frac{1}{m^{*2}-u}\right)(\alpha_1 t+\alpha_2)\,, \quad (4a)$$

$$B_{N^*}^{(\pm)}=\binom{1}{\frac{1}{2}}\left(\frac{1}{m^{*2}-s}\mp\frac{1}{m^{*2}-u}\right)(\beta_1 t+\beta_2)\,, \quad (4b)$$

where

$$\alpha_1=\tfrac{2}{3}(g^{*2}/m_\pi^2)\tfrac{1}{2}(m_N+m^*)=\beta_1(m_N+m^*)\,, \quad (4c)$$

$$\alpha_2=\frac{2}{3}\frac{g^{*2}}{m_\pi^2}q^{*2}\left[m_N+m^*+\frac{1}{3}\frac{E^*+m_N}{E^*-m_N}(m^*-m_N)\right], \quad (4d)$$

$$\beta_2=\frac{2}{3}\frac{g^{*2}}{m_\pi^2}q^{*2}\left[1-\frac{1}{3}\frac{E^*+m_N}{E^*-m_N}\right]. \quad (4e)$$

q^* and E^* are the c.m. momentum and energy of the nucleon when $W=m^*$;

$$A_\rho^{(+)}=B_\rho^{(+)}=0\,, \quad (5a)$$

$$A_\rho^{(-)}=-g_{\rho\pi\pi}g_{\rho pp}(\kappa/2m_N)[(s-u)/(m_\rho^2-t)]\,, \quad (5b)$$

$$B_\rho^{(-)}=g_{\rho\pi\pi}g_{\rho pp}[2(1+\kappa)/(m_\rho^2-t)]; \quad (5c)$$

$$A_\epsilon^{(-)}=B_\epsilon^{(+)}=B_\epsilon^{(-)}=0\,, \quad (6a)$$

$$A_\epsilon^{(+)}=g_{\epsilon\pi\pi}g_{\epsilon pp}/(m_\epsilon^2-t)\,. \quad (6b)$$

s, t, and u are the Mandelstam variables, and the superscripts designate the even (+) or odd (−) amplitudes. The advantage of using the even and odd amplitudes rather than the isotopic spin amplitudes is that it enables us to isolate the ϵ and ρ contributions, respectively. We shall present our final results in both representations.

The scattering amplitudes and phase shifts are then defined as follows:

$$f_1=[(E+m_N)/8\pi W][A+(W-m_N)B]\,, \quad (7)$$

$$f_2=[(E-m_N)/8\pi W][-A+(W+m_N)B]\,, \quad (8)$$

[5] J. Hamilton and W. Woolcock, Rev. Mod. Phys. 35, 737 (1963).

$$\begin{aligned}f_{l\pm}&=\tfrac{1}{2}\int_{-1}^{1}d\cos\theta\,[P_l(\cos\theta)f_1+P_{l\pm1}(\cos\theta)f_2]\\&=(1/8\pi W)\{(E+m_N)[A_l+(W-m_N)B_l]\\&\qquad+(E-m_N)[-A_{l\pm1}+(W+m_N)B_{l\pm1}]\}\\&=(\tan\delta_{l\pm})/q\,,\end{aligned} \quad (9)$$

where

$$A_l=\tfrac{1}{2}\int_{-1}^{1}AP_l(\cos\theta)d\cos\theta\,, \quad (10a)$$

$$B_l=\tfrac{1}{2}\int_{-1}^{1}BP_l(\cos\theta)d\cos\theta\,, \quad (10b)$$

and E and q are the energy and momentum of the nucleon in the c.m. system with total energy W.

In order to evaluate the phase shifts we project out the lowest partial waves corresponding to angular momentum $l=0$, 1, and 2. We then obtain the following:

Nucleon exchange

$$A_0^{(\pm)}=A_1^{(\pm)}=A_2^{(\pm)}=0\,, \quad (11)$$

$$B_0^{(\pm)}=g_r^2\left[\frac{1}{m_N^2-s}\pm\frac{1}{2q^2}Q_0(x_N)\right], \quad (12a)$$

$$B_1^{(\pm)}=\pm(g_r^2/2q^2)Q_1(x_N)\,, \quad (12b)$$

$$B_2^{(\pm)}=\pm(g_r^2/2q^2)Q_2(x_N)\,, \quad (12c)$$

where the $Q_n(x)$ are Legendre functions of the second kind given by

$$Q_0(x)=\tfrac{1}{2}\ln\left(\frac{x+1}{x-1}\right), \quad (13a)$$

$$Q_1(x)=\tfrac{1}{2}x\ln\left(\frac{x+1}{x-1}\right), \quad (13b)$$

$$Q_2(x)=\tfrac{1}{4}(3x^2-1)\ln\left(\frac{x+1}{x-1}\right)-\tfrac{3}{2}x\,, \quad (13c)$$

and

$$x_N=1-(s-m_N^2-2m_\pi^2)/2q^2\,, \quad (14)$$

$$q^2=[s-(m_N+m_\pi)^2][s-(m_N-m_\pi)^2]/4s\,. \quad (15)$$

N exchange*

$$A_0^{(+)}=(\alpha_2-2q^2\alpha_1)/(m^{*2}-s)+\alpha_1-[2q^2\alpha_1(x_s-1)+\alpha_2]\times Q_0(x_s)/2q^2\,, \quad (16a)$$

$$A_1^{(+)}=\tfrac{2}{3}q^2\alpha_1(m^{*2}-s)^{-1}-[2q^2\alpha_1(x_s-1)+\alpha_2]Q_1(x_s)/2q^2\,, \quad (16b)$$

$$A_2^{(+)}=-[2q^2\alpha_1(x_s-1)+\alpha_2]Q_2(x_s)/2q^2\,, \quad (16c)$$

$$B_0^{(+)}=(\beta_2-2q^2\beta_1)/(m^{*2}-s)-\beta_1+[2q^2\beta_1(x_s-1)+\beta_2]Q_0(x_s)/2q^2\,, \quad (17a)$$

$$B_1{}^{(+)}=\tfrac{2}{3}q^2\beta_1(m^{*2}-s)^{-1}+[2q^2\beta_1(x_s-1)+\beta_2]Q_1(x_s)/2q^2\,, \quad (17b)$$

$$B_2{}^{(+)}=[2q^2\beta_1(x_s-1)+\beta_2]Q_2(x_s)/2q^2\,, \quad (17c)$$

$$A_0{}^{(-)}=\tfrac{1}{2}(2q^2\alpha_1-\alpha_2)/(m^{*2}-s)+\tfrac{1}{2}\alpha_1-[2q^2\alpha_1(x_s-1)+\alpha_2]Q_0(x_s)/4q^2\,, \quad (18a)$$

$$A_1{}^{(-)}=-\tfrac{1}{3}q^2\alpha_1(m^{*2}-s)^{-1}-[2q^2\alpha_1(x_s-1)+\alpha_2]Q_1(x_s)/4q^2\,, \quad (18b)$$

$$A_2{}^{(-)}=-[2q^2\alpha_1(x_s-1)+\alpha_2]Q_2(x_s)/4q^2\,, \quad (18c)$$

$$B_0{}^{(-)}=\tfrac{1}{2}(2q^2\beta_1-\beta_2)/(m^{*2}-s)-\tfrac{1}{2}\beta_1+[2q^2\beta_1(x_s-1)+\beta_2]Q_0(x_s)/4q^2\,, \quad (19a)$$

$$B_1{}^{(-)}=-\tfrac{1}{3}q^2\beta_1(m^{*2}-s)^{-1}+[2q^2\beta_1(x_s-1)+\beta_2]Q_1(x_s)/4q^2\,, \quad (19b)$$

$$B_2{}^{(-)}=[2q^2\beta_1(x_s-1)+\beta_2]Q_2(x_s)/4q^2\,, \quad (19c)$$

where

$$x_s=1-(m^{*2}-2m_N{}^2-m_\pi{}^2+s)/2q^2\,. \quad (20)$$

ρ exchange

$$A_0{}^{(+)}=A_1{}^{(+)}=A_2{}^{(+)}=B_0{}^{(+)}=B_1{}^{(+)}=B_2{}^{(+)}=0\,, \quad (21)$$

$$A_0{}^{(-)}=-(g_{\rho\pi\pi}g_{\rho pp}\kappa/2m_N)\times[2s-2m_N{}^2-2m_\pi{}^2+m_\rho{}^2)Q_0(x_\rho)/2q^2-1]\,, \quad (22a)$$

$$A_1{}^{(-)}=-(g_{\rho\pi\pi}g_{\rho pp}\kappa/2m_N)\times(2s-2m_N{}^2-2m_\pi{}^2+m_\rho{}^2)Q_1(x_\rho)/2q^2\,, \quad (22b)$$

$$A_2{}^{(-)}=-(g_{\rho\pi\pi}g_{\rho pp}\kappa/2m_N)\times(2s-2m_N{}^2-2m_\pi{}^2+m_\rho{}^2)Q_2(x_\rho)/2q^2\,, \quad (22c)$$

$$B_0{}^{(-)}=2g_{\rho\pi\pi}g_{\rho pp}(1+\kappa)Q_0(x_\rho)/2q^2\,, \quad (23a)$$

$$B_1{}^{(-)}=2g_{\rho\pi\pi}g_{\rho pp}(1+\kappa)Q_1(x_\rho)/2q^2\,, \quad (23b)$$

$$B_2{}^{(-)}=2g_{\rho\pi\pi}g_{\rho pp}(1+\kappa)Q_2(x_\rho)/2q^2\,, \quad (23c)$$

where

$$x_\rho=1+m_\rho{}^2/2q^2\,. \quad (24)$$

ϵ exchange

$$A_0{}^{(-)}=A_1{}^{(-)}=A_2{}^{(-)}=B_0{}^{(+)}=B_1{}^{(+)}=B_2{}^{(+)}=B_0{}^{(-)}=B_1{}^{(-)}=B_2{}^{(-)}=0\,, \quad (25)$$

$$A_0{}^{(+)}=g_{\epsilon\pi\pi}g_{\epsilon pp}Q_0(x_\epsilon)/2q^2\,, \quad (26a)$$

$$A_1{}^{(+)}=g_{\epsilon\pi\pi}g_{\epsilon pp}Q_1(x_\epsilon)/2q^2\,, \quad (26b)$$

$$A_2{}^{(+)}=g_{\epsilon\pi\pi}g_{\epsilon pp}Q_2(x_\epsilon)/2q^2\,, \quad (26c)$$

where

$$x_\epsilon=1+m_\epsilon{}^2/2q^2\,. \quad (27)$$

III. RESULTS

The phase shifts may be obtained from Eq. (9). It is important to note that the contributions from different intermediate states are individually quite large. Table I illustrates the cancellations which take place between the contributions from different intermediate states.

TABLE I. Contributions of intermediate states to the phase shifts.

Channel	$T_{\rm lab}$ (MeV)	tanδ N	N^*	ρ	ϵ	δ (deg)
$S^{(+)}$	60	−1.54	1.96	0.0	−0.40	0.7
	110	−2.08	2.63	0.0	−0.51	1.8
	200	−2.78	3.49	0.0	−0.63	4.2
$S^{(-)}$	60	0.14	−0.12	0.06	0.0	5.1
	110	0.22	−0.19	0.10	0.0	7.5
	200	0.36	−0.33	0.15	0.0	10.8
$P_{1/2}{}^{(+)}$	60	−0.04	0.01	0.0	−0.01	−2.0
	110	−0.09	0.03	0.0	−0.01	−4.2
	200	−0.19	0.08	0.0	−0.03	−7.8
$P_{1/2}{}^{(-)}$	60	−0.02	0.01	0.01	0.0	0.1
	110	−0.04	0.02	0.03	0.0	0.8
	200	−0.07	0.05	0.08	0.0	3.4
$P_{3/2}{}^{(+)}$	60	0.02	0.06	0.0	−0.01	4.3
	110	0.05	0.24	0.0	−0.02	15.3
	200	0.11	[a]	0.0	−0.04	94.0
$P_{3/2}{}^{(-)}$	60	−0.02	−0.03	−0.00	0.0	−3.0
	110	−0.05	−0.11	−0.01	0.0	−9.6
	200	−0.11	[a]	−0.01	0.0	−98.0[a]

[a] N^* resonates at $m^*=1236$ MeV.

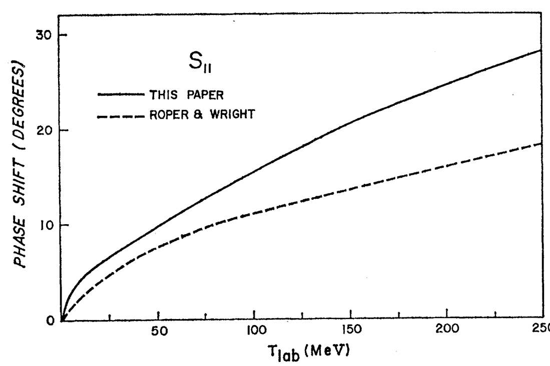

FIG. 2. Phase shifts for pion-nucleon scattering in the S_{11} channel.

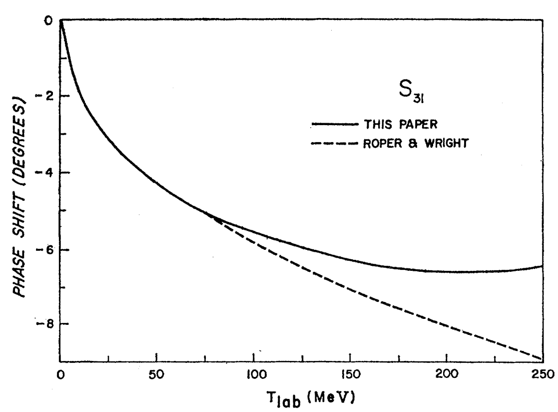

FIG. 3. Phase shifts for pion-nucleon scattering in the S_{31} channel.

It is more useful to display our results in terms of the $L(T,J)$ representation. Figures 2–7 show a comparison between our phase-shift predictions and fits to the experimental data.[6,7] The "experimental" results quoted

[6] L. D. Roper, R. M. Wright, and B. T. Feld, Phys. Rev. **138**, B190 (1965).

[7] L. D. Roper and R. M. Wright, Phys. Rev. **138**, B921 (1965).

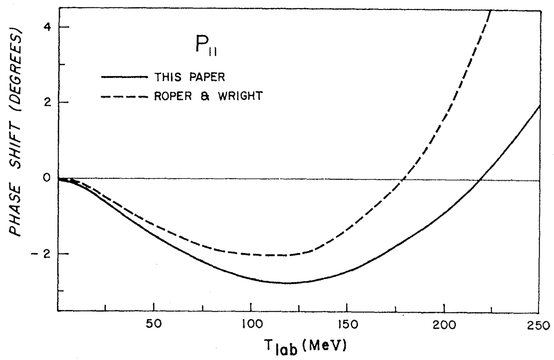

FIG. 4. Phase shifts for pion-nucleon scattering in the P_{11} channel.

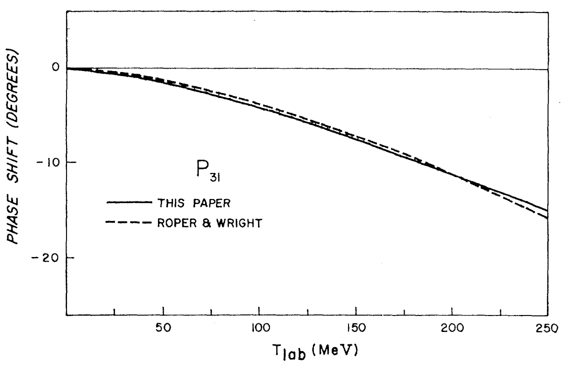

FIG. 5. Phase shifts for pion-nucleon scattering in the P_{31} channel.

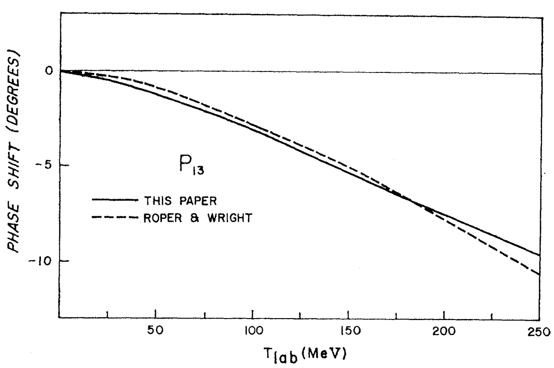

FIG. 6. Phase shifts for pion-nucleon scattering in the P_{13} channel.

from Roper and Wright[7] are their solution A, other fits to the data contained in this apper have the same qualitative features. Only for the P_{11} phase shift does the A solution differ significantly from ours. For this phase shift we have quoted the B solution.

IV. DISCUSSION

In spite of the simplicity of the model which we have used, we have obtained good agreement between our predicted low-energy phase shifts and experimental

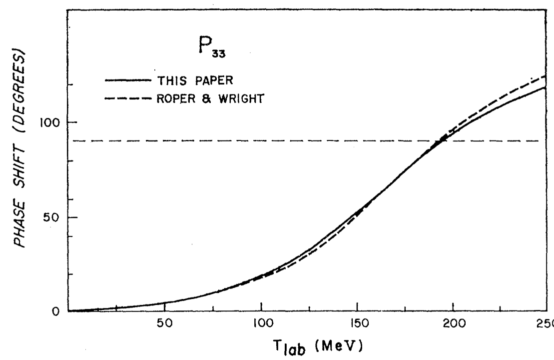

FIG. 7. Phase shifts for pion-nucleon scattering in the P_{33} channel.

results. It is of interest to explore the dependence of the predictions on the various parameters which enter the calculation.

The N^*, ρ, and ϵ widths[8] determine the $N^*N\pi$, $\rho\pi\pi$, and $\epsilon\pi\pi$ coupling constants. The "electric" ρNN coupling is obtained from the assumption of the universal coupling of the ρ to the isovector current. On the other hand, the "magnetic" coupling is adjustable. We have used the value $\kappa = 3.7$ based on the anomalous magnetic moments of the nucleons.

As a check we have also calculated the phase shifts for $\kappa = 1.0$ corresponding to no anomalous magnetic moment. The results obtained in this case are essentially unchanged (differing by no more than 2°), except for the already small P_{11} phase shift which turns over and becomes positive at a much higher energy. Since this disagrees with experiment, it is suggested that this provides additional support for the use of $\kappa = 3.7$ as opposed to $\kappa = 1.0$.

The ϵNN coupling is determined from the $S_{1/2}^{(+)}$ scattering length. As shown in Ref. 4, the coupling constant is insensitive to the exact value of this scattering length as long as it is small.

Our model is not expected to be good much above the inelastic threshold ($T_{lab} = 170$ MeV). We have carried out the calculation to high energies and, as anticipated, we do not get agreement with fits to the experimental data.[9]

A more elaborate picture of low-energy scattering might also include the nonresonant background in each channel, finite-width effects, and higher-energy resonances which are known to be present in these channels. However, this would introduce additional adjustable parameters into the model. Our results suggest that these are not really needed at low energies and instead that a simple model does given an adequate picture of low-energy pion-nucleon scattering.

[8] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).

[9] C. Lovelace, in *Proceedings of the Heidelberg International Conference on Elementary Particles*, edited by H. Filthuth (Wiley-Interscience, Inc., New York, 1968), p. 79.