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Mirage Trajectories*

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A mirage trajectory is a Regge trajectory which has no physical particle lying on it. We discuss both its possible dynamic origin and its peculiar properties. In particular, we consider the possibility that the Pomeranchon is a mirage trajectory. This conjecture (1) simplifies the structure of Mandelstam-type cuts, (2) gives a simple physical interpretation of the Harari hypothesis, and (3) indicates a possible explanation for the smallness of the Pomeranchon slope near $t=0$, while still allowing the Pomeranchon to be a moving pole.

I. INTRODUCTION

THE existence of the Pomeranchuk trajectory was first conjectured by Chew and Frautschi[1] and by Gribov.[2] It is assumed to be an even-signatured Regge trajectory with the quantum numbers of the vacuum, which passes through the angular-momentum value $j=1$ at precisely zero energy. If no other j singularities have these properties, one is able to readily understand many features of high-energy scattering of strongly interacting particles. However, both theoretical and experimental uncertainties[3] have kept physicists from establishing the true nature of the Pomeranchon. One particularly puzzling feature of the Pomeranchuk trajectory is its unusually small slope as indicated by fitting the high-energy πN and NN scattering data using Regge poles.[4] Recently, an interesting observation on the difference between Pomeranchon and other ordinary Regge trajectories has been made by Harari.[5] Within the context of finite-energy sum-rule bootstrap, Harari conjectured that the Pomeranchon is mostly built by the nonresonating background, while the ordinary trajectories are built by the low-energy resonances. Although this conjecture does not preclude the dynamical equivalence of all hadrons, it does indicate a special characteristic associated with the Pomeranchon, not shared by most other trajectories.

Another puzzling feature of hadron physics is the existence of branch points (cuts) in the j plane. They were first shown to be present by Mandelstam[6] in a perturbation-theory model; and they have the effect of removing the Gribov-Pomeranchuk[7] essential singularities. However, if the Pomeranchon has a zero-energy intercept of 1, one finds that an accumulation of indefinitely many cuts will occur. This occurrence does not necessarily violate any obvious S-matrix principle, yet it is considered by many[8] as a defect of the present model.

Many attempts[9] in trying to understand and resolve these and other related puzzling problems have been made, with no satisfactory explanation having yet been reached. We would like to suggest a new possibility which (i) allows the possibility of simplifying the structure of Mandelstam-type cuts, (ii) gives a simple physical interpretation to the Harari conjecture, and (iii) indicates why the slope of the Pomeranchuk trajectory may be small near t (the total energy)$=0$, while still allowing the Pomeranchon to be a moving pole. We shall consider in this paper the possibility that the Pomeranchon is a mirage Regge trajectory. A mirage trajectory is a moving Regge pole in the j plane, which "decouples" from all physical communicating channels whenever the pole moves through a physical value ($j=0$, 2, 4 $\cdots$ in the case of the Pomeranchon). Consequently, no "physical" particle poles (stable or

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[1] G. F. Chew and S. C. Frautschi, Phys. Rev. Letters **7**, 394 (1961).
[2] V. N. Gribov, Zh. Eksperim. i Teor. Fiz. **41**, 667 (1961); **41**, 1962 (1961) [English transl.: Sov. Phys.—JETP **14**, 478 (1962); **14**, 1395 (1962)].
[3] See, for example, G. F. Chew, Comments Nucl. Particle Phys. **1**, 121 (1967).
[4] See, for example, W. Rarita, R. Riddell, Jr., C. Chiu, and R. Phillips, Phys. Rev. **165**, 1615 (1968).
[5] H. Harari, Phys. Rev. Letters **20**, 1395 (1968).
[6] S. Mandelstam, Nuovo Cimento **30**, 1148 (1963). See also V. N. Gribov, I. Ya. Pomeranchuk, and K. A. TerMartirosyan, Phys. Rev. **139**, B184 (1965).
[7] V. N. Gribov and I. Ya. Pomeranchuk, Phys. Letters **2**, 239 (1962).
[8] See, for example, J. Schwarz, Phys. Rev. **167**, 1342 (1968). We shall not consider here the possibility that the Pomeranchon intercept is less than one.
[9] R. Oehme, Phys. Rev. Letters **18**, 1222 (1967); J. Finkelstein and C. I. Tan, *ibid.* **19**, 1061 (1967). See also L. Van Hove, in *Proceedings of the Thirteenth Annual International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967), p. 253.

unstable) lie on a mirage trajectory. We shall show in what follows how this possibility can greatly simplify these mysteries surrounding the Pomeranchon.

In Sec. II, we discuss the dynamical nature of a mirage trajectory. In particular, we shall construct a model to exhibit the ingredients which are required for a mirage trajectory to be present. We next turn to the discussion on consequences of our conjectures. In Sec. III, we review the argument for the necessity of Mandelstam cuts. We show that there is an intimate connection between the cuts of Mandelstam type and the normal threshold singularities of scattering amplitudes. As a consequence, we argue that it is possible for a mirage trajectory not to participate in producing Mandelstam cuts. In Sec. IV, we examine the conjecture of Harari and show that it can have a simple physical interpretation if the Pomeranchon is a mirage trajectory. We add in Sec. V further remarks concerning our conjecture including a possible explanation for the smallness of the Pomeranchon slope near zero total energy.

II. DYNAMICAL CONSIDERATIONS

When a mirage trajectory passes through a physical value at $s=s_0$, the residue of a physical channel will have the behavior $\beta_{ss}\approx C_1^2(s-s_0)$ where $C_1=0$ or finite. Consequently, no physical particle with spin j will be present at s_0. We know there are three kinds of pole-elimination mechanisms that will give this kind of behavior: Chew mechanism, Gell-Mann mechanism, and noncompensation mechanism.[10] In order to be more specific, let us construct a model based on the Gell-Mann mechanism. As is well known in this case, the residues behave in the following manner:

$$\begin{aligned}&\beta_{ss}\approx C_1^2(s-s_0)\,,\\&\beta_{nn}\approx C_2^2\,,\\&\beta_{sn}\approx C_1C_2(s-s_0)^{1/2}.\end{aligned}\tag{2.1}$$

We shall first explain how this is possible from the dynamical point of view and then generalize to the case of a mirage trajectory which chooses "nonsense" at every physical j value ($j=0,\ 2,\ 4,\ \cdots$, for Pomeranchon).

A. Gell-Mann Mechanism

Consider two spin-$\frac{1}{2}$ particles interacting with a local spherically symmetric potential. The most general form of potential $V(\mathbf{r})$[11] can be written as

$$V(\mathbf{r})=V_1(\mathbf{r})\Lambda^{(0)}+V_2(\mathbf{r})\Lambda^{(1)}+V_3(\mathbf{r})S_{12}\,,\tag{2.2}$$

where $\Lambda^{(0)}$ and $\Lambda^{(1)}$ are projection operators on singlet- and triplet-spin states, and S_{12} is the tensor operator. The V_1, V_2, and V_3 are functions of $\mathbf{r}$ only.

[10] See, for example, C. Chiu, S. Chu, and L. Wang, Phys. Rev. **161**, 1563 (1967).

[11] See, for example, M. Goldberger and K. Watson, *Collision Theory* (Wiley-Interscience, Inc., New York, 1964).

Let us concentrate on the triplet states with parity $(-1)^{J+1}$, and labeling states by $|j,l=j\pm1\rangle$, the partial-wave analysis leads to a set of ordinary coupled radial Schrödinger equations. Once the potential is given, the Jost functions can be obtained from the solution of this set of differential equations. Let us assume that the solution contains Regge poles such that the leading Regge trajectory $\alpha(s)$ passes through $j=0$ at $s=s_0$ below threshold, and we shall investigate the behavior of Regge residues associated with this pole.

At $j=0$, the state $|j,l=j-1\rangle$ has become "nonsense." The set of equations will decouple as j approaches this value. They have the form

$$\left(-\frac{d^2}{dr^2}+V_1+2V_3\right)\psi_n=k^2\psi_n\,,\tag{2.3}$$

$$\left(-\frac{d^2}{dr^2}+\frac{2}{r^2}+V_1-4V_3\right)\psi_s=k^2\psi_s\,,\tag{2.4}$$

where ψ_n, ψ_s are wave functions for the nonsense and sense states, respectively. The analytically continued partial-wave amplitude $a_{ss}(j,s)$ should then coincide, at $j=0$, with that calculated from (2.4) alone. Now, it is easy to see that in order for the pole to be absent in $a_{ss}(j,s)$ at $j=0$, $s=s_0$, the potential (V_1-4V_3) must not produce a P-wave bound state there. This can always be done by adjusting functions $V_1(\mathbf{r})$ and $V_3(\mathbf{r})$. For instance, by choosing $-2V_3(r)\gg V_1(r)>0$, the combination (V_1-4V_3) will correspond to a repulsive potential. Consequently the Regge pole $\alpha(s)$ will remain only in the amplitude $a_{nn}(j,s)$, not in $a_{ss}(j,s)$. The residues β_{ss}, β_{sn}, and β_{nn} will behave as in (2.1).

To summarize, the Gell-Mann mechanism occurs if the "force" that is responsible for producing a Regge pole comes completely from the nonsense channel. At the $j=0$, the sense and nonsense channels decouple from each other. This pole will only remain in the nonsense-nonsense amplitude, and we say this pole has chosen "nonsense." However, as one moves away from $j=0$, the distinction between the sense channel and the nonsense channel is lost. This Regge pole will couple to all analytically continued partial-wave channels.

It is obvious that the above model is not adequate to produce a mirage trajectory. As one moves to the next integer $j=2$, both channels, $|l=j+1\rangle$ and $|l=j-1\rangle$, are physical there (we shall ignore signature for now). Consequently, this trajectory $\alpha(s)$ cannot choose nonsense. In order to build a mirage trajectory, it is clear that additional structure[12] has to be introduced into the model.

[12] It is not sufficient for our purpose to have the trajectory "turn over" as s increases along the real axis, thus, never reach the line $\mathrm{Re}(j)=1$. When this happens, there will always be a point s_1, on the unphysical sheet, where $\alpha(s_1)=1$, at $s=s_1$, the pole has to choose sense.

B. Mirage Trajectory by Choosing Nonsense at all Physical J Values

What we need is a model that allows a Regge trajectory to choose nonsense at all physical j values, regardless of how large the spin is. Dynamics involving high-spin particles or multiparticle channels is obviously required. The model[13] of "nearby dominance," used for discussing rising Regge trajectories, seems most appropriate here. The model asserts that one could study the dynamics of a Regge trajectory at one small energy region at a time. At any particular energy region, the dominant forces that bind this Regge trajectory come from the high-spin channels whose thresholds lie in that region. With respect to these dominant channels, the "orbital" angular momenta could thus remain small. The physical justification of this model is discussed in Ref. 13, and we shall only concentrate on discussing how a mirage trajectory can be formed within this model.

We shall generalize the case of two spin-$\frac{1}{2}$ particles to the case of two particles with arbitrary spin σ_1 and σ_2. We assume that a Regge trajectory is formed, in a particular energy region, by the interaction between these two particles. The interaction potential has the form $V(\mathbf{r})=\sum V_i(r)\Lambda_i$, where each $V_i(r)$ is a scalar potential, and each Λ_i is a tensorial operator in the spin space. In analogy to the triplet states, we shall consider those states corresponding to a total spin $|\boldsymbol{\sigma}|=|\boldsymbol{\sigma}_1|+|\boldsymbol{\sigma}_2|$. The states in this subspin space can be labeled by $|j, l=j\pm(\sigma-n)\rangle$, $n=0, 1, \cdots \sigma$. This model of nearby dominance asserts that the produced Regge trajectory $\alpha(s)$ will have a value $\alpha(s)\approx\sigma$ in the neighborhood of the threshold of these two particles. Let us first adjust the potentials so that $\alpha(s_\sigma)=\sigma-1$, at a point s_σ near this threshold; and, in particular, we choose the $V_i(r)$ in such a way that the dominant force for binding this trajectory comes from the channel labeled by $|j, l=j-\sigma\rangle$. However, this channel is nonsense at $j=\sigma-1$. As one moves toward this j value, this Regge trajectory will be choosing nonsense.

As one goes up in energy, one just has to keep on changing the spin σ_1 and σ_2 of the dominant channel. By adjusting the potential appropriately, the output trajectory will be able to choose nonsense at every physical angular-momentum value. Consequently, there will be no physical particles lying on this Regge trajectory, and, by definition, it is a mirage trajectory.

We have demonstrated one type of mirage trajectory to show that the existence of these trajectories is really not so unlikely as it would seem at first sight. In fact, other models can readily be constructed. One can easily convince oneself that they do not violate any accepted property of scattering amplitudes, and their presence will lead to many interesting consequences. We shall next consider some of them.

[13] S. Y. Chu, C. I. Tan and P. D. Ting (unpublished); S. Y. Chu and C. I. Tan, Lawrence Radiation Laboratory Report No. UCRL-17511, 1967 (unpublished).

III. MANDELSTAM CUTS AND MIRAGE TRAJECTORIES

By Mandelstam cuts, we mean those moving branch points in the j plane which owe their existence to the presence of Gribov-Pomeranchuk fixed poles.[6–8] In the following, we shall review first, the necessity for Mandelstam cuts and their relation with the normal threshold singularities in the energy plane. Then, we discuss the case where mirage trajectories are involved and show that it is possible for them not to participate in producing Mandelstam cuts.

It has become a well-known fact that, if we are to avoid essential singularities in the j plane, the existence of Gribov-Pomeranchuk fixed poles, at wrong-signature nonsense-j values, forces the presence of moving cuts. These essential singularities, which are due to the accumulation of poles, would violate crossed-channel unitarity when high-spin particles are present.

For the even-signatured partial-wave scattering amplitude of two particles with spins σ_A and σ_B, the nonsense wrong-signature points occur at

$$J=\sigma_A+\sigma_B-n\,,\quad n=1, 3, 5, \cdots. \tag{3.1}$$

They correspond to the system having a nonsensical orbital angular momentum $l=-1, -3, -5, \cdots$. To avoid essential singularities, the moving cuts will have to have the special property that they coincide with the normal threshold $s=(m_A+m_B)^2$ at these j values, so that the "elastic" discontinuity formula would no longer be applicable. In terms of the position of the nth cut in the j plane, the above condition corresponds to

$$\alpha_{[A,B]}{}^{c,n}[s=(m_A+m_B)^2]=\sigma_A+\sigma_B-n\,,\qquad n=1, 3, 5, \cdots. \tag{3.2}$$

Mandelstam, by investigating those perturbation diagrams containing Gribov-Pomeranchuk fixed poles, has found cuts that satisfy [Eq. (3.2)]. Their positions are given by

$$\begin{aligned}\alpha_{[A,B]}{}^{c,n}(s)&=\alpha_A((s^{1/2}-m_B)^2)+\sigma_B-n\,,\\ \alpha_{[B,A]}{}^{c,n}(s)&=\sigma_A+\alpha_B((s^{1/2}-m_A)^2)-n\,,\end{aligned} \tag{3.3}$$

where $\alpha_A(s)$ and $\alpha_B(s)$ are Regge trajectories on which particles A and B lie, respectively.

We would like to point out that in the case where fixed poles are absent, the necessity for the existence of moving cuts is removed. One such case is when α_A or α_B (or both) is a mirage trajectory. Since particle A really does not exist, there will be no normal threshold at the point $s=(m_A+m_B)^2$. Consequently, moving cuts $\alpha_{[A,B]}{}^{c,n}(s)$ and $\alpha_{[B,A]}{}^{c,n}(s)$ are not required.

It is not surprising to find that Mandelstam's proof for the existence of moving cuts also fails in this case. The proof made use of the existence of Gribov-

Pomeranchuk fixed poles to show that the discontinuity across "formal" branch cuts cannot vanish. However, if one of the trajectories involved is a mirage trajectory, the existence of a Gribov-Pomeranchuk fixed pole can no longer be ascertained. Consequently, the proof for the existence of Mandelstam cuts will no longer hold.

We have just seen that the type of cuts described by Eq. (3.3) does not have to be present if one of the trajectories involved is a mirage trajectory, in contrast with that associated with ordinary Regge trajectories. In all the above arguments, we implicitly assume that amplitudes with one or more external Reggeons can be defined. If this can be done,[14] than a great simplification[15] on high-energy Regge representation can be achieved.

IV. HARARI'S CONJECTURE AND MIRAGE TRAJECTORY

An interesting observation[5] has been made by Harari on the different roles played by the Pomeranchon and other Regge trajectories in relating the high-energy behavior of scattering amplitudes to their low-energy behavior. It has led him to conjecture that, within the context of finite-energy sum rules, the Pomeranchon is mostly built by the nonresonating background, while "ordinary" Regge trajectories can be bootstrapped by using the resonance approximation. Preliminary tests[16] of this conjecture have been encouraging.

The failure of resonance saturation in the case of the Pomeranchon does not necessarily indicate doubts on the "bootstrap" nature of the Pomeranchon; however, it shows signs of having certain peculiar properties which ordinary trajectories do not possess. We would like to suggest that a mirage trajectory would lead to the phenomenon observed by Harari. This phenomenon arises because a mirage trajectory fails to give rise to long-range attractive forces when being exchanged in any allowed reaction.

In order to introduce the notion of forces, we can, for instance, adopt the method of Charap and Fubini,[17] who demonstrated that an energy-independent local potential could be given a meaning in a relativistic theory at low energies. In particular, the long-range part of this potential is shown to correspond exactly to poles in small t region. When a Regge trajectory with low-mass particles is exchanged, it will give rise to forces that are of long range (either attractive or repulsive). Conversely, if Pomeranchon is a mirage trajectory, it will not give rise to a long-range Yukawa force, because Pomeranchon does not correspond to poles of the full amplitude in the t plane. It is a well-known fact that in nonrelativistic-potential-theory problems without attractive long-range force, no resonances can be found. Since our example is approximately given by potential theory in the low-energy region, it is then natural to expect that no resonances are being formed due to the exchange of the Pomeranchon. Furthermore, if one is allowed to interchange the "cause" and "effect" in the discussion of finite-energy sum-rule bootstrap the part of the Harari conjecture on the Pomeranchon then follows from the argument given above.

We would like to add that there might be other Regge trajectories which in most instances behave as ordinary trajectories, but not in some other instances. These include trajectories whose first few recurrences are missing, or whose coupling to specific channels[13] is very weak. If this is true, a generalization of Harari's conjecture is required.

V. CONCLUDING REMARKS

We have discussed in this paper several interesting consequences of the assumption that the Pomeranchon is a mirage Regge trajectory. These include the explanation of the Harari hypothesis, and the possible removal of Pomeranchon cuts. We have also pointed out the possible dynamical origin of mirage trajectories and have illustrated a special model in which a mirage trajectory is formed by choosing nonsense at every physical value of angular momentum.

The mysteries surrounding the Pomeranchon are by no means solved. Our conjecture perhaps provides a new route of attacking these puzzling problems. One unique property of the Pomeranchon is the possibility of its small slope near $t=0$. Let us consider the low-threshold two-particle channels that communicate with the Pomeranchon. We find that all the lowest ones $\pi\pi$, $N\bar{N}(939)$, etc., cannot be the dominating channels in producing the mirage Pomeranchon, because at $j=2$ all these channels have $\sigma \leq 2$, and thus are sense channels. In fact, the lowest channels that can be nonsense channels at $j=2$ are $N^*\bar{N}^*(1238)$ or $N(939)\bar{N}^*(1688)$. They have thresholds around 2.5 GeV. If all hadron bound states are not very deeply bounded due to the finite range and strength of the strong interaction force, then in our model as mentioned in Sec. II B, $\alpha_P(s_\sigma)$ equals to $\sigma-1$, with s_σ near the

[14] N. F. Bali, G. F. Chew, and A. Pignotti, Phys. Rev. **163**, 1572 (1967).

[15] We have here considered only cuts of the type described by Eq. (3.2). There is also the type called trajectory-trajectory cuts, which, in the case of two identical trajectories, has the position $\alpha_A{}^c(s)=2\alpha_A(s/4)-1$. These are the ones which become important at negative s. Since this type of cut is also intimately connected with the existence of Gribov-Pomeranchuk poles (see Ref. 6), we believe that a mirage trajectory will also not produce this type of cut if it does not produce the type of cuts described by Eq. (3.2).

[16] F. J. Gilman, H. Harari, and Y. Zarmi, Phys. Rev. Letters **21**, 323 (1968).

[17] J. Charap and S. Fubini, Nuovo Cimento **14**, 540 (1959). For one to use the method of Charap and Fubini directly, the scattering amplitude has to satisfy Mandelstam representation, or its equivalent. The complications coming from the possible indefinitely rising Regge trajectories are discussed in Ref. 13. By adopting the model of "nearby dominance," these difficulties can be avoided. See also Jerome Finkelstein, Lawrence Radiation Laboratory Report No. UCRL-17311, 1967 (unpublished).

threshold of the external particles $[\sim(2.5)^2 \text{ GeV}^2]$ and $\sigma\approx 3$ in this case. The slope will be $\alpha_P'\approx 0.2 \text{ GeV}^{-2}$. Of course these are only very crude arguments, the essential point is: The fact that Pomeranchon chooses nonsense at $j=2$ requires that it can couple strongly only to those channels with thresholds higher than the thresholds of the channels that coupled to ordinary meson trajectories.

Other problems which our conjecture might shed some light on include: Why the diffractive dissociation cross section is small compared with the elastic cross section and the possibility of a vanishing residue of Pomeranchon-Pomeranchon coupling in the multi-Regge model.[18]

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[18] G. Chew and A. Pignotti, Phys. Rev. **176**, 2112 (1968).

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Current Algebra, $\pi\pi$ Scattering, and the ρ Leptonic-Decay Branching Ratio

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Following Weinberg, we use linearity, current algebra, and the hypothesis of partially conserved axial-vector current to restrict the form of the $\pi\pi$ amplitude below threshold. However, we do *not* assume isoscalarity of the σ term. Instead, we propose a model in which nonpole scattering diagrams are constant below threshold. Then, the current-algebra restriction leads to a consistency condition from which the ρ leptonic-decay branching ratio can be calculated. The prediction is $B(l^+l^-)\approx(0.68\pm0.18)\times10^{-4}$. This agrees well with recent experimental data: $B(e^+e^-)_{\text{expt}}=(0.56\pm0.06)\times10^{-4}$, $B(\mu^+\mu^-)_{\text{expt}}=(0.66\pm0.15)\times10^{-4}$. In addition, the $\pi\pi$ scattering lengths are determined to be in agreement or nearly in agreement with Weinberg's. The model suggests that the small scattering lengths result from a cancellation of large nonpole and ϵ-exchange contributions.

I. INTRODUCTION

The current-algebra calculation of $\pi\pi$ scattering lengths by Weinberg[1] assumes isoscalarity of the so-called σ term. In this paper we replace the isoscalarity hypothesis with a more dynamical one; in particular, we assume that the nonpole part of the scattering amplitude is constant below threshold. This conjecture is strong enough to imply a consistency condition for the $\pi\pi$ coupling constants (especially the $\rho\pi\pi$ and $\epsilon\pi\pi$ vertices) as well as to determine the $\pi\pi$ scattering lengths.

The consistency condition can be compared with experimental data in two ways. By using the results of vector dominance we can predict a value for the ρ leptonic-decay branching ratio which is in good agreement with experiment. We can also find a correction to the usual ρ-dominance value of the pion electromagnetic radius which has the correct sign and possibly the correct magnitude.

The $\pi\pi$ scattering lengths are determined to be close to those of Weinberg; thus the theory is compatible with an isoscalar σ term. The model suggests that the small scattering lengths are produced by a cancellation of large contributions from nonpole diagrams and ϵ-exchange diagrams.

In Sec. II, we use linearity, current algebra, and the hypothesis of partially conserved axial-vector current (PCAC) to restrict the scattering amplitude. Section III describes and "solves" the model for $\pi\pi$ scattering. In Sec. IV, the consistency condition is compared with experimental data. The $\pi\pi$ scattering lengths are derived in Sec. V.

II. LINEARITY, CURRENT ALGEBRA, AND PCAC RESTRICTIONS

Following Weinberg,[1] we define an invariant off-mass-shell amplitude[2]:

$$\langle ld,qb|M|pc,ka\rangle$$
$$=(-i)\left[\frac{(l^2-m_\pi^2)(q^2-m_\pi^2)(p^2-m_\pi^2)(k^2-m_\pi^2)}{(m_\pi^2F_\pi)^4}\right]$$
$$\times\int d^4xd^4yd^4ze^{-ipz+ilx+iqy}$$
$$\times\langle 0|T\{\partial A^d(x)\partial A^b(y)\partial A^c(z)\partial A^a(0)\}|0\rangle,\quad(2.1)$$

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[1] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).

[2] The metric and propagators are those of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill Book Co., New York, 1965).