# Model of Weak Interactions with a 300-BeV Cutoff*

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A model of weak interactions is displayed which preserves all the observed symmetries of weak interactions in lowest order. It is mediated by three intermediate vector bosons with their respective antiparticles, and also contains a massive neutral muon-type lepton and electron-type lepton, so altogether there are two lepton triplets: $e$, $\nu_e$, $\lambda_e$ and $\mu$, $\nu_\mu$, $\lambda_\mu$. Higher-order corrections diverge, but for a cutoff $\Lambda\sim300$ BeV, i.e., $G_F\Lambda^2\sim1$, the model is shown to be compatible with present experimental limits.

## I. INTRODUCTION

It has been known for a long time that corrections to lowest-order–perturbation-theory calculations of weak processes were, to say the least, ambiguous. When calculated perturbatively, they diverged in any theory for which the vector-vector nature of the interaction was considered to be fundamental. It was hoped for many years that these divergences were due to the fact that the hadrons and leptons were being treated as bare, elementary particles and that if electromagnetic and strong interactions were taken into account correctly, the divergence might be tempered or altogether eliminated. However, recently if has become clear that, under certain assumptions, the coefficients of the leading divergences are dictated by nonvanishing current commutators,[1] whose commutation relations hold even in the presence of strong interactions. The interpretation is relatively unambiguous for semileptonic interactions and a cutoff in the neighborhood of 50–100 BeV is necessary in order to account for the absence of $K_L^0\to\mu^+\mu^-$.[1,2] For nonleptonic weak interactions the $\Delta S=2$ $K_L$-$K_S$ mass difference suggests[2,3] a cutoff $\Lambda\sim4$ BeV in a theory mediated by an intermediate vector boson (IVB). In particular, even the lowest-order nonleptonic weak interaction is quadratically divergent[4] for such a theory. Recently, several papers have appeared either attempting to exploit this divergence[5] or to show how in certain cases it may not be present and what consequences this has.[6,7]

The upshot of all this is that the value of the cutoff has to be small (of the order of a few BeV) and, if these calculations are to make sense, the value of the IVB mass even smaller, nor does using an elementary current-current interaction improve matters. Since such a vector boson has not yet been observed experimentally,[8] one may ask what are the possible remedies to such a state of affairs. The first one is to say that we may just not be able to deduce anything meaningful from perturbation-theory calculations done to a few orders with a cutoff and that, in fact, the perturbation expansion is meaningless or at best misleading unless the whole series is summed.[9] A second possible solution is to say that the observed vector-vector nature of the weak interactions is accidental and the weak interactions are really mediated by scalar bosons.[10–12] A third possibility[13] is to introduce, in addition to IVB's, scalar bosons coupled by derivatives to the vector currents, such as to remove the divergences which would lead to discrepancies with the observed selection rules of the weak interactions if the cutoff $\Lambda$ were of the order of several hundred BeV. At that point a natural length, namely, $1/\sqrt{G_F}$, is introduced, which may provide a scale for the cutoff.

We would like to propose a model for weak interactions similar to a schizon model we already considered,[14] which is in the spirit of the third possibility discussed previously. It is neither as ambitious nor as general as the schemes of Ref. 13, its only virtue being relative simplicity in that it involves in addition to the usual hadrons and leptons, three IVB's and their antiparticles and two massive neutral leptons and their antiparticles.

In Sec. II, we shall present the model and show how it reproduces to lowest order the conventional results. In Sec. III, we will discuss the question of divergences in nonleptonic weak interactions, in Sec. IV, the divergences in semileptonic and leptonic processes, and finally in Sec. V, possible extensions of the model and some experimental consequences.

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## II. MODEL OF WEAK INTERACTIONS

To begin with, we assume the existence of a massive neutral lepton, which we call $\lambda_\mu$, having the same quantum numbers as the muon's neutrino $\nu_\mu$ and an analogous heavy $\nu_e$, which we call $\lambda_e$. The masses of $\lambda_\mu$ and $\lambda_e$ are taken to be greater than that of the $K$ meson so that they would not have been seen in strange particle decays. They may naturally be combined to form a pair of triplets $(e,\nu_e,\lambda_e)$ and $(\mu^-, \nu_\mu, \lambda_\mu)$ analogous to quarks or another triplet model of hadrons, though we shall not say anything yet about the strangeness of the leptons. In addition to the usual lepton current

$$j_\sigma{}^+ = i\bar{e}\gamma_\sigma(1-\gamma_5)\nu_e + i\bar{\mu}\gamma_\sigma(1-\gamma_5)\nu_\mu\,, \qquad (1)$$

we introduce a new current involving $\lambda_\mu$ and $\lambda_e$

$$\bar{j}_\sigma{}^+ = i\bar{e}\gamma_\sigma(1-\gamma_5)\lambda_e + i\bar{\mu}\gamma_\sigma(1-\gamma_5)\lambda_\mu\,. \qquad (2)$$

In a conventional IVB theory, one takes the weak interactions to be described by a Lagrangian of the form

$$\mathcal{L}_{\text{int}} = g(J_\sigma{}^+ + j_\sigma{}^+)W_\sigma{}^- + \text{H.c.}, \qquad (3)$$

where $J_\sigma$ is the hadronic Cabibbo current,[15] $W$ is the IVB (of mass $M_W$) field and $g$ is fixed by the relation

$$g^2/M_W{}^2 = G_F/\sqrt{2}\,, \qquad (4)$$

where $G_F = 10^{-5}/M_N{}^2$. Since $J_\sigma{}^+$ is a member of an octet, the effective current-current Lagrangian describing nonleptonic weak interactions contains both octet and 27 representation components. To obtain the observed octet transformation properties of the strangeness-changing nonleptonic effective Lagrangian, one must invoke a dynamical octet enhancement.[16] Alternatively, one may start from a fundamental Lagrangian involving more vector bosons such that the current-current strangeness-changing effective Lagrangian has octet transformation properties. The minimum number of vector bosons with which one may accomplish this feat is four,[17] i.e., two with their respective antiparticles. The model we would like to describe involves three such bosons (and their antiparticles—so altogether six) of which two are charged and one is neutral. The Lagrangian is

$$\begin{aligned}\mathcal{L}_{\text{int}} = g[&J_\sigma{}^{\pi^+}W_{a,\sigma}{}^- + J_\sigma{}^{K^+}W_{b,\sigma}{}^- \\ &+ (J_\sigma{}^{K^0}+\gamma V_\sigma{}^0)W_{c,\sigma}{}^0 + j_\sigma{}^+(W_{a,\sigma}{}^-\cos\theta + W_{b,}{}^-\sin\theta) \\ &+ \bar{j}_\sigma{}^+(-W_{a,\sigma}{}^-\sin\theta + W_{b,\sigma}{}^-\cos\theta)] + \text{H.c.}\end{aligned} \qquad (5)$$

Here $J_\sigma$ is a $V-A$ hadron current, whose transformation properties under $SU(3)$ are assumed to be those of the particles in its superscript label; $V_\sigma{}^0$ is the baryonic number current, which is of course conserved and an $SU(3)$ scalar; $\theta$ is the Cabibbo[15] angle; and $\gamma$ is a constant to be determined by fitting to experiment. The results of the Cabibbo theory for semileptonic and leptonic processes are immediately obtained if we take all $W$ mesons to have the same mass $M_W$, namely, we obtain lowest-order Lagrangian for processes not involving $\lambda_e$ or $\lambda_\mu$.

$$\begin{aligned}\mathcal{L}_{\text{eff}} = G_F[&J_\sigma{}^{\pi^+}J_\sigma{}^{\pi^-} + J_\sigma{}^{K^+}J_\sigma{}^{K^-} \\ &+ (J_\sigma{}^{K^0}+\gamma V_\sigma{}^0)(J_\sigma{}^{\bar{K}^0}+\gamma V_\sigma{}^0) + J_\sigma{}^{\pi^+}j_\sigma{}^-\cos\theta \\ &+ J_\sigma{}^{K^+}j_\sigma{}^-\sin\theta + j_\sigma{}^+j_\sigma{}^-] + \text{H.c.}\end{aligned} \qquad (6)$$

We notice also that, since $V_\sigma{}^0$ is an $SU(3)$ scalar, the $\Delta S=1$ nonleptonic effective Lagrangian has octet transformation properties. Furthermore, in the roughest of approximations, namely, that all current-current matrix elements are of order unity, it predicts that the rate for $\Delta S=1$ nonleptonic decays should be of order of magnitude $\gamma^2/\sin^2\theta$ compared to semileptonic decay rates. We shall say more about the parameter $\gamma$ in Sec. III.

## III. DIVERGENCES IN NONLEPTONIC PROCESSES

(A) Let us begin by considering the amplitude for a nonleptonic weak transition $A \to B$, where $A$ and $B$ are hadron states. The matrix element for this process is most easily analyzed if we write the hadronic part of the Lagrangian (5) in a Cartesian $SU(3)$ basis, where we relabel the $W$ fields

$$\begin{aligned}W_a{}^- &= W^1 - iW^2\,,\\ W_b{}^- &= W^4 - iW^5\,,\\ W_c{}^0 &= W^6 - iW^7\,;\end{aligned} \qquad (7)$$

the hadronic part of the Lagrangian (5) becomes

$$\mathcal{L}^{\text{hadr}}{}_{\text{int}} = g \sum_{i=1,2,4,5,6,7} (J_\sigma{}^iW_\sigma{}^i + V_\sigma{}^0W_\sigma{}^6)\,. \qquad (8)$$

The amplitude for the transition $A \to B$ is then given by

$$\begin{aligned}T_{A\to B} = \frac{-ig^2}{2(2\pi)^4}\int\int d^4q d^4x\, e^{-iq\cdot x}\frac{\delta_{\sigma\tau}-q_\sigma q_\tau/M_W{}^2}{q^2-M_W{}^2} \\ \times\langle A|T\{\sum_{i=1,2,4,5,6,7} J_\sigma{}^i(x)J_\tau{}^i(0) + V_\sigma{}^0(x)V_\tau{}^0(0) \\ + V_\sigma{}^0(x)J_\tau{}^6(0) + J_\sigma{}^6(x)V_\tau{}^0(0)\}|B\rangle\,.\end{aligned} \qquad (9)$$

$T_{A\to B}$ is, in general, quadratically divergent because of the $q_\mu q_\nu$ term in the $W$-meson propagator which leads to an expression of the form

$$\begin{aligned}T_{A\to B}|_{\text{quad div}} = -i\frac{G_F}{2\sqrt{2}(2\pi)^4}\int\int \frac{d^4q\, d^4x\, e^{-iq\cdot x}}{q^2-M_W{}^2} \\ \times\delta(x_0)\langle A|\{\sum_{i=1,2,4,5,6,7}[J_0{}^i(x),\partial_\tau J_\tau{}^i(0)] \\ + [V_0{}^0(x),\partial_\tau J_\tau{}^6(0)]\}|B\rangle\,,\end{aligned} \qquad (10)$$

[15] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).
[16] R. Dashen, S. Frautschi, M. Gell-Mann, and Y. Hara, in *The Eightfold Way*, edited by M. Gell-Mann and Y. Ne'eman (W. A. Benjamin, Inc., New York, 1964), p. 254.
[17] S. Okubo, Phys. Letters **8**, 362 (1964).

where we have of course made use of the fact that the baryonic number current $V_\mu^0$ is conserved.[18] To go further, we must have a model for the strong interaction Hamiltonian and from it obtain an expression for the divergences of the currents. We shall describe one such model here, leaving others to Sec. V. The simplest one, as proposed by Gell-Mann,[19] is

$$H=\bar{H}+\epsilon_0 u_0+\epsilon_8 u_8\,, \tag{11}$$

where $\bar{H}$ is invariant under chiral $SU(3)\times SU(3)$ and $u_0$, $u_8$ are scalar densities belonging to the $(3,\bar{3})$ and $(\bar{3},3)$ representation of $SU(3)\times SU(3)$. The divergences of the vector and axial-vector currents are then given by

$$\begin{aligned} D^{V^i}&=\epsilon_8 f^{8ij}u^j\,,\\ D^{A^i}&=\epsilon_0 d^{0ij}v^j+\epsilon_8 d^{8ij}v^j\,, \end{aligned} \tag{12}$$

where $v^j$ are the pseudoscalar densities. From this, assuming the equal-time commutation relations of the fourth components of vector and axial-vector currents with the scalar and pseudoscalar densities to be as follows:

$$[V_0{}^i(x),u^j(0)]_{x_0=0}=\quad if^{ijk}u^k(0)\delta(\mathbf{x})\,, \tag{13a}$$

$$[V_0{}^i(x),v^j(0)]_{x_0=0}=\quad if^{ijk}v^k(0)\delta(\mathbf{x})\,, \tag{13b}$$

$$[A_0{}^i(x),v^j(0)]_{x_0=0}=\quad id^{ijk}u^k(0)\delta(\mathbf{x})\,, \tag{13c}$$

$$[A_0{}^i(x),u^j(0)]_{x_0=0}=-id^{ijk}v^k(0)\delta(\mathbf{x})\,. \tag{13d}$$

The only term in (10) which leads to $\Delta S=1$ transitions vanishes as

$$[V_0{}^0(x),\partial_\tau J_\tau{}^\sigma(0)]_{x_0=0}=0 \tag{14}$$

by the above relations, and, for the others, we find that

$$\sum_{i=1,2,4,5,6,7}[V_0{}^i(x),D^{V^i}(0)]_{x_0=0}=3i\epsilon_8 u_8(0)\delta(\mathbf{x})\,, \tag{15a}$$

$$\sum_{i=1,2,4,5,6,7}[A_0{}^i(x),D^{V^i}(0)]_{x_0=0}=0\,, \tag{15b}$$

$$\sum_{i=1,2,4,5,6,7}[V_0{}^i(x),D^{A^i}(0)]_{x_0=0}=0\,, \tag{15c}$$

$$\sum_{i=1,2,4,5,6,7}[A_0{}^i(x),D^{A^i}(0)]_{x_0=0}=i[4\epsilon_0 u_0(0)+\epsilon_8 u_8(0)]\delta(\mathbf{x})\,. \tag{15d}$$

The vanishing of the commutators in (15b), (15c) is true for all indices $i$, while the vanishing of the coefficient of $u_3$ on the right-hand side of (15a), (15d) occurs only after summing over $i$. Remembering now that $J=V-A$, we have

$$\begin{aligned} T_{A\to B}&=\frac{-4iG_F}{2\sqrt{2}(2\pi)^4}\int\frac{d^4q}{q^2-M_W{}^2}\langle A\,|\,\epsilon_0u_0(0)+\epsilon_8u_8(0)\,|\,B\rangle\\ &=\frac{G_F}{\sqrt{2}}\frac{\Lambda^2}{4\pi^2}\langle A\,|\,\epsilon_0u_0(0)+\epsilon_8u_8(0)\,|\,B\rangle\,, \end{aligned} \tag{16}$$

so that even for an enormous value of $\Lambda^2$, $G_F\Lambda^2/4\pi^2\sim 1$, the only effect is to have a contribution to $T_{A\to B}$ proportional to $\epsilon_0u_0+\epsilon_8u_8$, i.e., an $SU(3)\times SU(3)$ breaking term proportional to the original $SU(3)\times SU(3)$ breaking. Note that, in principle, this value of the cutoff could lead to violations of the strong interaction selection rules such as isospin invariance if any of the commutators in (15) had led to a $u_3$ term. This violation of isospin invariance would have shown up as an apparently large splitting of the masses of particles belonging to the same isomultiplet. If the second and third commutators in (15) had not vanished, we would have obtained a strong violation of parity. Note also that the vanishing of (15b) and (15c) for all values of $i$ means that even in a conventional weak-interaction model, in which there is only one charged $W$ meson coupled to a Cabibbo-type hadron current, there is no quadratically divergent parity-violating term to order $G_F$ with the assumed commutation relations (13a)–(13d).

As stressed in Ref. 13, a value of $\Lambda$ as large as $G_F\Lambda^2\sim 16\pi^2$ violates unitarity, so we must have a damping mechanism occurring before then. In our model, we shall only attempt to require consistency with experiment for a smaller value of $\Lambda$, namely, $G_F\Lambda^2\sim 1$.

(B) We must now go on to look at higher-order terms in $G_F$. Let us begin by considering a general $G_F{}^2$ term which would have the form

$$g^4\int\int\int d^4x\,d^4y\,d^4z\,\Delta_{\mu\nu}(x-y)\Delta_{\lambda\sigma}(z)\times\langle A\,|\,T\{J_\mu{}^i(x)J_\nu{}^j(y)J_\lambda{}^k(z)J_\sigma{}^l(0)\}\,|\,B\rangle\,, \tag{17}$$

where $\Delta_{\mu\nu}(x-y)$ is the $W$-meson propagator. The most divergent part of this integral, in momentum space, comes from the $q_\mu q_\nu/M_W{}^2$ part of the $W$ propagator and gives

$$g^4\int\cdots\int\frac{d^4x\,d^4y\,d^4z\,d^4q\,d^4q'}{(q^2-M_W{}^2)(q'^2-M_W{}^2)}e^{-iq\cdot(x-y)}e^{-iq'\cdot z}\times\frac{q_\mu q_\nu q_\lambda' q_\sigma'}{M_W{}^4}\langle A\,|\,T\{J_\mu{}^i(x)J_\nu{}^j(y)J_\lambda{}^k(z)J_\sigma{}^l(0)\}\,|\,B\rangle\,. \tag{18}$$

If we take a value for the cutoff $\Lambda$ of the order of $G_F\Lambda^2\sim 1$, we might at first expect a large $K_L-K_S$ mass difference, namely, one of order $G_F$ rather than $G_F{}^2$ in the limit of conserved currents and, in general, of order one rather than $G_F{}^2$, which would contradict experi-

[18] Strictly speaking, Eq. (9) is incorrect if the time-ordered product is not covariant. In calculating the quadratically divergent term, however, we will neglect Schwinger terms, assuming that they are cancelled by the divergence of the term one would have to add to (9) to make it covariant. For a discussion of this point and further references, see S. Adler and R. Dashen, *Current Algebra* (W. A. Benjamin, Inc., New York, 1968), Chap. 3.

[19] M. Gell-Mann, Physics **1**, 63 (1964). I would like to thank Dr. I. Gerstein for a helpful discussion of this model.

ment. That this is not so in this model can be seen as follows: To have a $\Delta S=2$ transition, one must have a double exchange of $W_c^0$ since $W_a$ and $W_b$ may be imagined as conserving strangeness in their coupling to hadrons so, inserting the correct current indices in (18) for a $\Delta S=2$ transition, we would have

$$\langle A|T\{J_\mu^{K^0}(x)V_\nu^0(y)J_\lambda^{K^0}(z)V_\sigma^0(0)\}|B\rangle. \quad (19)$$

Inserting (19), as labeled, into the expression (18) for the most divergent part, to order $G_F^2$ of the amplitude, we find a vanishing result since the equal-time commutators obtained by contracting $q_\mu q_\nu q_\lambda' q_\sigma'$ into the time-ordered product all vanish.

$$\begin{aligned}[][V_0^0(x),J_\mu^{K^0}(0)]_{x_0=0}&=[V_0^0(x),\partial_\mu J_\mu^{K^0}(0)]_{x_0=0}\\ &=[J_0^{K^0}(x),J_\mu^{K^0}(0)]_{x_0=0}=[J_0^{K^0}(x),\partial_\mu J_\mu^{K^0}(0)]_{x_0=0}\\ &=0. \end{aligned} \quad (20)$$

This ensures that even the next most divergent terms, proportional to $\delta_{\mu\nu}q_\lambda' q_\sigma'$ or $q_\mu q_\nu \delta_{\lambda\sigma}$ vanish, when we contract $q_\lambda' q_\sigma'$ or $q_\mu q_\nu$ into the time-ordered product of currents. We are taking for the product $W$-mesons propagators

$$\Delta_{\mu\nu}(q)\Delta_{\lambda\sigma}(q')=\left(\frac{\delta_{\mu\nu}-q_\mu q_\nu/M_W^2}{q^2-M_W^2}\right)\left(\frac{\delta_{\lambda\sigma}-q_\lambda' q_\sigma'/M_W^2}{q'^2-M_W^2}\right). \quad (21)$$

So the matrix element, which gives the $K_L-K_S$ mass difference to order $G_F^2$, is proportional to

$$g^4\int\cdots\int\frac{d^4x\,d^4y\,d^4z\,d^4q\,d^4q'\,e^{-iq\cdot(x-y)-iq'\cdot z}}{(q^2-M_W^2)(q'^2-M_W^2)} \times[\langle K_0|T\{J_\mu^{K^0}(x)V_\mu^0(y)J_\lambda^{K^0}(z)V_\lambda^0(0)\}|\bar{K}_0\rangle]. \quad (22)$$

By a very liberal use of techniques for finding the large-momentum-transfer behavior of time-ordered products, along the lines of Bjorken's[20] work, we would conclude that the integrals over $q$ and $q'$ diverge only logarithmically. It is of course an assumption, and a highly questionable one at that, that these techniques can be applied to the time-ordered product of four or more currents. If they are not valid, our argument fails.

This feature of reducing the degree of divergence of $\Delta S=2$ transitions holds even in higher orders: For an $n$th order matrix element, i.e., proportional to $(G_F)^n$, one would have to exchange two $W_c$ mesons in order to obtain a $\Delta S=2$ transition and one may easily see, since the equal-time commutation relations of $V_0^0$ with other currents or divergences of currents are zero, the maximum degree of divergence is $g^4(G_F\Lambda^2)^{n-2}$. We thus have the following picture for the $K_L-K_S$ mass difference, which we shall call $\Delta M$:

$$\Delta M=m_0\sum_{n=2}^{\infty}a_n(G_F\Lambda^2)^n,$$ (conventional $W$ theory with nonconserved currents and nonvanishing commutators)

$$\Delta M=m_0^3G_F\sum_{n=1}^{\infty}b_n(G_F\Lambda^2)^n,$$ (conventional $W$ theory with conserved currents)

$$\Delta M=m_0g^4\sum_{n=0}^{\infty}c_n(G_F\Lambda^2)^n,$$ (our model) (23)

where $a_n$, $b_n$, $c_n$ are numerical coefficients and $m_0$ is a mass which has been inserted for dimensional purposes and depends only on the hadron properties. In a model such as that of Ref. 13, $\Delta M$ would be of the same form as in our model with the divergences being due to what they call the diagonal interaction. Of course, logarithmic divergences are being incorporated into the coefficients $a_n$, $b_n$, $c_n$: the quantity $c_0$, for instance, is proportional to $\ln\Lambda^2$ or $(\ln\Lambda^2)^2$. In addition, the models differ in that, whereas in the conventional theories the lowest-order contributions to $\Delta S=2$ transition are proportional to $\sin^2\theta$, in our model we instead have the factor $\gamma^2$. One might at first be tempted to have $\gamma\sim1$ so that, in the roughest of approximations, namely, that all current-current matrix elements had the same magnitude, one would obtain an effective enhancement of $\Delta S=1$ nonleptonic decays over $\Delta S=1$ semileptonic decays by a factor of $1/\sin^2\theta$. Such an enhancement seems unnecessary, however; at least in a conventional model this roughest of approximations appears to be wrong and one can obtain a moderately good fit to nonleptonic weak decays without additional enhancement.[21] Furthermore, it appears that $\gamma^2\sim1$ would lead to a serious overestimate of the $K_L-K_S$ mass difference unless $g^2$ were very small, but $g^2$ is constrained by $g^2/M_W^2=10^{-5}/M_n^2$. The question of an optimum value for $\gamma$ is presently under examination, but probably it should be of the order of magnitude of $\sin\theta$. Perhaps it is a good time to remind the reader that we are not attempting to obtain a full theory of weak interactions, but rather a model which does not contradict experiment with a cutoff $\Lambda$ as large as 300 BeV. This does not mean that we can choose $M_W$ as large as 300 BeV for $\Delta M$ must be effectively second-order weak and $g^2$, as we said before, is constrained by $g^2/M_W^2=10^{-5}/M_n^2$. In order for $\Delta M$ to be small, we must have $M_W$ of the order of magnitude of a few BeV. The effective cutoff, how-

[20] J. D. Bjorken, Phys. Rev. **148**, 1467 (1966).

[21] C. Itzykson and M. Jacob, Nuovo Cimento **48A**, 655 (1967); Y. Hara, Progr. Theoret. Phys. (Kyoto) **37**, 710 (1967); S. Nussinov and G. Preparata, Phys. Rev. **175**, 2180 (1968).

ever, occurs at a much higher value, where, in fact, the contributions of higher-order perturbations are of a possibly comparable order of magnitude with respect to each other.

The next possible source of conflict with experiment is an effectively strong $\Delta S=0$ parity violation. We have already seen, to order $G_F\Lambda^2$, that there was no such parity violation in this model, but one may ask is there one to order $(G_F\Lambda^2)^2$, which would still of course lead to an effectively large parity violation. To this end we must examine the maximally divergent term (to order $G_F^2$) with $\Delta S=0$, which is of the form

$$G_F^2\int\cdots\int d^4x\, d^4y\, d^4z\, d^4q\, d^4q' \times\frac{e^{-iq\cdot(x-y)-iq'\cdot z}}{(q^2-M_W^2)(q'^2-M_W^2)}q_\mu q_\nu q_\lambda' q_\sigma' \times\sum_{i,j=1,2,4,5,6,7}\langle A|T\{J_\mu^i(x)J_\nu^i(y)J_\lambda^j(z)J_\sigma^j(0)\}|B\rangle. \quad (24)$$

The maximally divergent term, which behaves like the matrix element of the divergence of a current between states $A$ and $B$ and typically has a magnitude of $(G_F\Lambda^2/4\pi^2)^2$, is obtained by successive contractions of the $q$'s and $q'$'s into the time-ordered product. When the operation was performed, we found no parity violation, namely, the final matrix element was only of the divergence of the vector current, not of the axial-vector current. We have no reason for this, nor can we make an argument of why it should be true for higher orders of $G_F$; it may well not be true in higher orders in fact, though the question is still under examination. As of now, our calculation appears to lead accidentally to no parity violation in order $(G_F\Lambda^2)^2$.

As an example, we show one of the terms obtained by contracting the $q$'s and the $q'$'s. It is

$$G_F^2\int\int\int\frac{d^4q\, d^4q'\, d^4z\, e^{iq\cdot z}}{(q^2-M_W^2)(q'^2-M_W^2)}\sum_{i,j=1,2,4,5,6,7}\sum_{k,k'}f^{ijk} \times f^{ijk'}q_\sigma'\langle A|T\{\partial_\lambda J_\lambda^k(0)J_\sigma^{k'}(-z)\}|B\rangle. \quad (25)$$

The indices $k$, $k'$ are either equal to one another, in which case the calculation is identical to the one for the maximally divergent parity-violating term to order $G_F$, or else one of them is equal to three and the other to eight. If that is the case, the maximally divergent part is still only parity-conserving since the equal-time commutator of $V_0^{3,8}$ with $\partial_\lambda A_\lambda^{8,3}$ is equal to zero.

As we stated earlier, we have not been able to show that the maximally divergent parity-violating term vanishes to higher order. However, since the characteristic expansion involves

$$\frac{1}{\pi}\frac{g^2}{4\pi}\frac{\Lambda^2}{M_W^2}=\frac{G_F\Lambda^2}{4\pi^2\sqrt{2}},$$

we see that even for $G_F\Lambda^2\sim1$, in addition to a damping factor due to the fact that the maximally divergent part is proportional to the breaking of $SU(3)\times SU(3)$, we have in third order a typical factor of $(1/4\pi^2)^3$ which is sufficient to make the parity violation quite small.

There are other schemes, one of which we will mention in Sec. V, involving doubling the number of vector bosons, to alleviate this problem, but we admit they are all rather unattractive.

These arguments have a certain unaesthetic quality, but if our calculation of the $(G_F\Lambda^2)^2$ parity violation is correct, are sufficient to not rule out the model for $G_F\Lambda^2\sim1$. We believe this is probably the weakest point of this model as, curiously enough, also of the model of Ref. 11.

## IV. DIVERGENCES IN LEPTONIC AND SEMILEPTONIC PROCESSES

(a) As we have seen, the model is compatible for nonleptonic processes with a 300-BeV cutoff, a value ruled out in a conventional $W$ model by the upper bound for $K_L\to\mu^+\mu^-$ which sets a limit of $\Lambda\sim50$ BeV. The diagram which sets this limit is depicted in Fig. 1(a) and in a conventional model has a magnitude proportional to

$$G_F\Lambda^2\bar{\mu}\gamma_\sigma(1-\gamma_5)\mu\langle K_L|J_\sigma|0\rangle. \quad (26)$$

In our model, there is an additional diagram, to order $G_F^2$, depicted ln Fig. 1(b), and to leading order the two diagrams cancel one another, that is, the quadratic divergences of Figs. 1(a) and 1(b) cancel one another. The logarithmic divergences do not and are typically of the form

$$G_F^2\bar{\mu}\gamma_\sigma(1-\gamma_5)\mu\langle K_L|J_\sigma|0\rangle m_{\lambda_\mu}^2\ln\frac{\Lambda^2}{M_W^2}, \quad (27)$$

where $m_{\lambda_\mu}$ is the mass of the neutral lepton $\lambda_\mu$. Since we shall assume this mass is not too large, e.g., of the order of 0.5–1.0 BeV, we clearly may have $G_F\Lambda^2\sim1$ without contradicting experiment. The reason for the cancellation of the quadratic divergences is that to describe $K_L\to\mu^+\mu^-$ one must exchange one $W_a$ and one $W_b$ in order to have $\Delta S=1$. The Lagrangian (5) says however that the $\nu_\mu$ contribution [Fig. 1(a)] is proportional to $\sin\theta\cos\theta$ while the $\lambda_\mu$ contribution [Fig. 1(b)] is proportional to $-\sin\theta\cos\theta$, so the leading divergences, which are independent of the masses of the leptons, cancel. One might worry about higher-order graphs such as the one depicted in Fig. 1(c), which involves a $K_L\to n\pi$ transition, followed by $n\pi\to\mu^+\mu^-$ since the $n\pi\to\mu^+\mu^-$ transition is, in fact, quadratically divergent. We have already seen, however, that the amplitude for $K_L\to n\pi$ is only logarithmically divergent so that the over-all rate is proportional to $G_F\ln(\Lambda^2/M_W^2)\times G_F^2\Lambda^2$ and hence of the same order of magnitude as (27).

To see why the quadratic divergence vanishes, from a slightly more fundamental point of view, observe that if $m_{\lambda_\mu}=0$, we would not be able to distinguish between

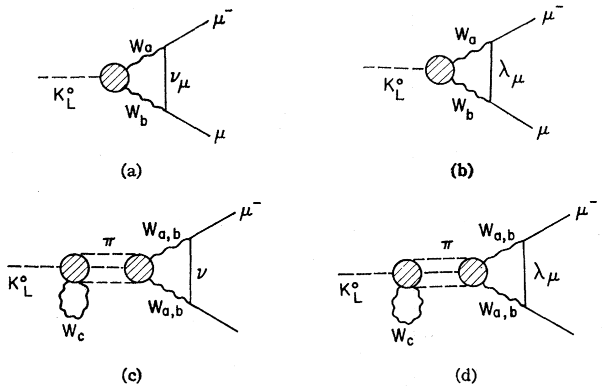


FIG. 1. (a) and (b) Lowest-order weak graphs contributing to $K_L{}^0 \to \mu^+\mu^-$. (c) and (d) Higher-order weak graphs contributing to $K_L{}^0 \to \mu^+\mu^-$.

$\nu_\mu$ and $\lambda_\mu$. In particular, if one defined $\nu_\mu'$ and $\lambda_\mu'$ by

$$\begin{aligned} \nu_\mu &= \nu_\mu' \cos\theta + \lambda_\mu' \sin\theta\,, \\ \lambda_\mu &= -\nu_\mu' \sin\theta + \lambda_\mu' \cos\theta\,, \end{aligned} \tag{28}$$

and similarly $\nu_e'$ and $\lambda_e'$, the coupling of leptons to intermediate bosons in the Lagrangian (5) could be rewritten, with $j'$ equal to $j$ with $\nu$, $\lambda$ replaced by $\nu'$, $\lambda'$, as

$$\mathcal{L}_{\text{leptonic}} = g(j_\sigma{}^{+\prime} W_{a,\sigma}{}^- + \bar{j}_\sigma{}^{+\prime} W_{b,\sigma}{}^-) + \text{H.c.}, \tag{29}$$

so that, imagining $W_a$, $\nu_\mu'$, $\nu_e'$, and of course $e$ and $\mu$ to have strangeness zero while $W_b{}^+$ and $\lambda_\mu'$, $\lambda_e'$ have strangeness one, we see that the interactions of leptons and hadrons with $W$'s conserve strangeness, and hence $K_L \to \mu^+\mu^-$ is forbidden.[22] Strangeness nonconservation is introduced by higher-order diagrams involving $W_c$ and by having $m_\lambda \neq 0$, in that the term $m_{\lambda_\mu} \bar{\lambda}_\mu \lambda_\mu$ in the free-lepton Lagrangian is not invariant under the rotation defined by (28).

(b) There are other semileptonic processes which are quadratically divergent in order $G_F{}^2$ and we shall now comment on them.

(i) $K^+ \to \pi^+ \nu\bar{\nu}$: In terms of primed leptons, this would be of the form $K^+ \to \pi^+ + \lambda_\mu' + \nu_\mu'$.

(ii) $\nu + p \to \nu + p$.

In both these cases, the amplitudes are smaller than the amplitude for the principal semileptonic mode, namely, $K^+ \to \pi^0 + e^+ + \nu$ and $\nu_\mu + p \to \mu^- n$ by a factor of $G_F \Lambda^2 / 4\pi^2 \sim 1/40$, which implies the rates should be smaller by a factor of $10^{-4}$. Since the experimental limits are that the rates for (a) and (b) are less than $\sim 5\%$ of the principal semileptonic mode,[23] we see no contradiction. Similarly, the amplitude for electron-neutrino scattering is quadratically divergent to order $G_F{}^2$.

[22] The idea of having a model in which strangeness conservation is broken by the mass of the heavy neutral leptons first appears in Ref. 11.

[23] L. B. Auerbach *et al.*, Phys. Rev. **155**, 1505 (1967).

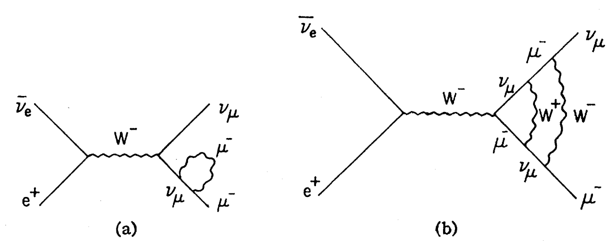


FIG. 2. (a) Lowest-order weak correction to $\mu$ decay. (b) Higher-order weak correction to $\mu$ decay.

Let us now turn to the question of universality, namely, the equality of the constants characterizing $\mu$ decay and neutron $\beta$ decay. Since the leptons couple exclusively to charged $W$ mesons, one may readily see that the only diagrams contributing to $\mu$ decay of order $G_F{}^2$ are external line corrections as depicted in Fig. 2(a). To higher order, vertex corrections may also appear; an example of this is given in Fig. 2(b). The structure of the corrections to a hadron vertex is of course much more complicated. As emphasized by Gell-Mann *et al.*[13] in their analysis of the same problem, though they dealt only with regard to what they call "diagonal interactions," there are several possibilities. One is that conventional weak couplings are to be interpreted as the values after the renormalization has been carried out. A second possibility is to have the renormalizations be universal: This will not quite work out in our scheme. We postpone a discussion of this point to Sec. V, where we shall discuss possible extensions of the present model. Let us just say now that even for $G_F \Lambda^2 \sim 1$, deviations from universality are of the order of $3\%$, which is probably compatible with experiment, given the uncertainties in radiative corrections and in the determination of the Cabibbo angle. Intuitively this result comes from the comparison of the lowest-order term, equal to say $G_F$, with the first correction which has an order of magnitude of $G_F{}^2 \Lambda^2 / 4\pi^2 \sim G_F \times 1/4\pi^2$. Graphically we depict in Figs. 3(a), 3(b), and 3(c) the first

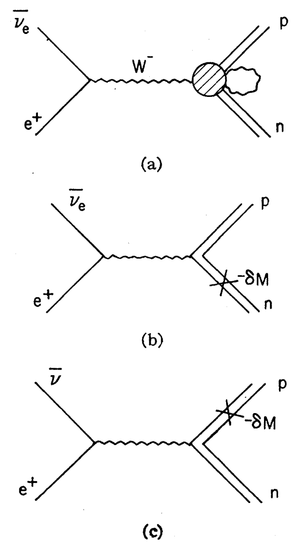


FIG. 3. Lowest-order weak corrections to neutron $\beta$ decay.

corrections to the hadronic vertex. The 3% figure quoted above was obtained by considering

$$\langle ne^{+}\nu_e|H_{\text{weak}}|p\rangle_{\text{weak hadronic corrections}}=i\frac{G_F}{\sqrt{2}}\bar{e}\gamma_\lambda(1-\gamma_5)\nu_e\times\frac{g^2}{2(2\pi)^4}\int d^4q\frac{(\delta_{\mu\nu}-q_\mu q_\nu/M_W^2)}{q^2-M_W^2}[T_{\lambda\mu\nu}(k,q,p',p)-\delta M_{\lambda\mu\nu}], \quad (30)$$

where $k$ is the momentum transfer to the leptons, which we take equal to zero, $p'$ and $p$ are the neutron and proton momenta, which are equal for $k\to 0$:

$$T_{\lambda\mu\nu}(k,q,p',p)=\int\int d^4x\, d^4y\, e^{ik\cdot x}e^{-iq\cdot y}\sum_{i=1,2,4,5,6,7}\langle n|T\{[\cos\theta\,(J_\lambda{}^1(x)+iJ_\lambda{}^2(x))$$
$$\times(J_\mu{}^i(y)J_\nu{}^i(0)+V_\mu{}^0(y)V_\nu{}^0(0))]+\sin\theta\,(J_\lambda{}^4+iJ_\lambda{}^5)(J_\mu{}^{K^0}V_\nu{}^0+V_\mu{}^0J_\nu{}^{K^0})\}|p\rangle, \quad (31)$$

where $\delta M_{\lambda\mu\nu}$ cancels the external line pole terms in $T_{\lambda\mu\nu}$ contributing only to the weak mass renormalization of the hadrons. (See Appendix A for a treatment of the problem in a conventional theory.) The $q_\mu q_\nu$ term in (30) leads to a quadratic divergence, which we evaluate once again by use of current commutators. The term in (31) proportional to $\sin\theta$ does not contribute to the quadratic divergence; the term proportional to $\cos\theta$ leads to a modification of the lowest-order term, which we write as

$$\langle ne^{+}\nu_e|H_w|p\rangle_0=(G_F/\sqrt{2})\bar{e}\gamma_\lambda(1-\gamma_5)\nu\times\cos\theta\,\langle n|J_\lambda{}^1(0)+iJ_\lambda{}^2(0)|p\rangle. \quad (32)$$

This modification is of magnitude

$$\langle ne^{+}\nu_e|H_w|p\rangle_0[2G_F\Lambda^2/\sqrt{2}(4\pi^2)]. \quad (33)$$

This result has been obtained, using as normalization for the current commutator

$$[J_0{}^i(x),J_\mu{}^j(0)]_{x_0=0}=2if^{ijk}J_\mu{}^k(0)\delta(\mathbf{x}), \quad (34)$$

where the factor of 2 arises because the currents are $V-A$. We see from (33) that the correction to the lowest-order coupling $\langle|H_w|\rangle_0$ are of the order of 3% for $G_F\Lambda^2\sim 1$. This would imply that we are skating on thin ice with such a large value of the cutoff; reducing it by a factor of 2 would of course ensure no contradiction with experiment. However, we have not yet calculated the weak radiative corrections to $\mu$ decay. In Sec. V, we will show how, in a model, the deviations from universality are smaller than the figure we have just suggested.

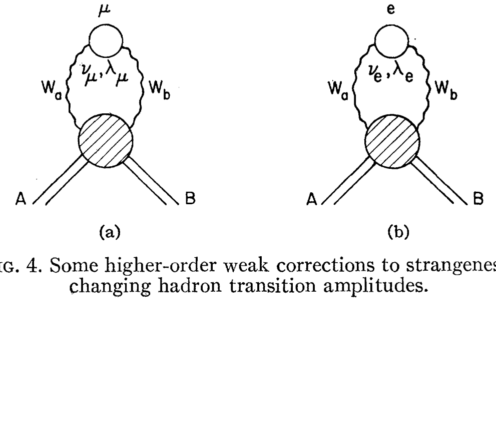


FIG. 4. Some higher-order weak corrections to strangeness-changing hadron transition amplitudes.

## V. EXTENSIONS AND VARIATIONS OF THE MODEL

(a) The first question we should ask is whether or not it is possible to have a model with fewer particles for which the cutoff may still be large. There is one; it is described by the Lagrangian of (5) with no $W_c$ mesons,[24] i.e., with altogether only four intermediate vector mesons. The difficulty with such a model is that nonleptonic strangeness-changing hadron decays $A\to B$ occur only to order $G_F{}^2$. The simplest graphs which allow $A\to B$ with $\Delta S=1$ are those of Fig. 4. *A priori* one might believe them to be quartically divergent, but remembering the transformation (28), embodying the fact that the semileptonic weak interactions are strangeness-conserving to order $G_F{}^2$ for $m_{\lambda_\mu}=m_{\lambda_e}=0$, we realize that the graphs in Fig. 4 are not quartically divergent, but only quadratically divergent and proportional to $m_{\lambda_\mu}$ and $m_{\lambda_e}$, respectively. The fact that we do two loop integrals introduces a factor of $(1/4\pi^2)^2$ so that with $G_F\Lambda^2\sim 1$, the amplitude described in Fig. 4 is too small by several orders of magnitude to account for nonleptonic $\Delta S=1$ transitions. It is, of course, nonzero and present even in our model and represents a perturbation of the principal term caused by the emission and absorption of a $W_c$ meson.

(b) If the commutation relations (15b), (15c) are changed, one would get large parity violations. Supposing then that (13b) and (13c) do not hold, what can we do? The simplest thing would be to double the number of IVB's and label them as $W^i$ and $W_5{}^i$, where $i=1, 2, 4, 5, 6, 7$. Instead of (8), one would have a hadronic interaction Lagrangian of the form

$$\mathcal{L}_{\text{hadronic}}=g[\sum_{i=1,2,4,5,6,7}(V_\mu{}^iW_\mu{}^i-A_\mu{}^iW_{5,\mu}{}^i)+V_\mu{}^0(W_\mu{}^6+W_{5,\mu}{}^6)] \quad (35)$$

and the lepton currents, of course, would be coupled to $W$ and $W_5$. Now, aside from graphs like Fig. 4, it is the coupling of $V_\mu{}^0$ which accounts for parity violation as well as strangeness nonconservation.

[24] Dr. N. Christ and Dr. N. Kroll have independently considered such a model. I would like to thank Dr. N. Christ for a discussion of their work.

(c) Another possible extension is to make the model more symmetrical looking by coupling neutral lepton currents to $W_c$. The most symmetrical looking coupling one could introduce would be an addition to (29) of the form

$$\mathcal{L}_{\text{add}} = g\tilde{j}_\sigma' W_{c,\sigma} + \text{H.c.}, \tag{36}$$

where

$$\tilde{j}_\sigma' = i\bar{\lambda}_\mu'\gamma_\sigma(1-\gamma_5)\nu_\mu' + i\lambda_e'\gamma_\sigma(1-\gamma_5)\nu_e', \tag{37}$$

with $\nu'$, $\lambda'$ defined by (28). Rewriting this in terms of $\nu$ and $\lambda$, one sees that one has introduced a direct neutrino-neutrino current which is not allowed. The next best thing is to introduce a coupling of the form

$$\mathcal{L}_{\text{add}} = g\tilde{j}_\sigma W_{c,\sigma}, \tag{38}$$

where $j_\sigma$ is obtained from (37) by replacing $\nu'$, $\lambda'$ with $\nu$, $\lambda$. This leads, however, to a contribution of order $(1/4\pi^2)(G_F\Lambda^2)^3 \sin\theta$ to nonleptonic $\Delta S=1$ decays, which is both too large numerically and against the general point of view we have adopted, namely, of reducing the degree of divergence of strangeness-changing transitions. It appears, therefore, that we may not add neutral lepton currents in any simple way.

(d) There are of course an infinite number of models one can construct along the same lines by introducing more particles. For instance, instead of combining $\lambda_\mu$ and $\lambda_e$ with $\mu^-$, $\nu_\mu$ and $e^-$, $\nu_e$ to form triplets of leptons, one could introduce massive charged leptons, which we might call $\lambda_\mu^\pm$, $\lambda_e^\pm$ so that instead of two triplets of leptons, we have four doublets. Relabeling $\lambda_\mu$, $\lambda_e$ now as $\lambda_\mu^0$, $\lambda_e^0$, we could then form the doublets either as $(\lambda_\mu^-, \lambda_\mu^0)$ or $(\lambda_\mu^+, \lambda_\mu^0)$. If we took the latter choice, it is easy to see that one may construct a model in which, e.g., the $G_F^2\Lambda^2$ divergent term in the scattering of $\nu_\mu + p \to \nu_\mu + p$ vanishes. The relevant bare diagrams are displayed in Fig. 5. However, since there are no models involving vector mesons for which all the divergences are removed, we do not believe it is worthwhile discussing models with additional charged leptons at the moment. Of course, if such particles were found experimentally, one might look at models of the type we have described with an eye to filling them in.

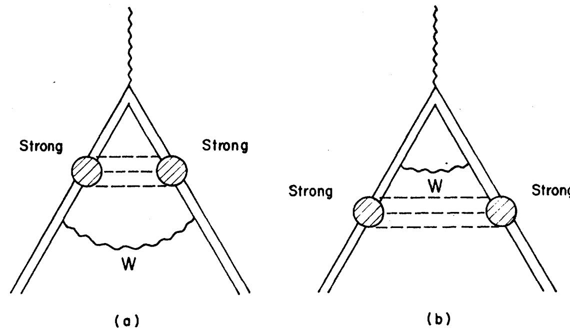


FIG. 5. Lowest-order weak correction to a weak hadron vertex.

(e) Before going on to the conclusions let us now show a simple model which illustrates the main features of our previous considerations. Consider the hadrons to be bound states of an elementary triplet, which we may take to be quarks and label them as $q_i$. The leptons are similarly classified into two triplets. The total Lagrangian, neglecting electromagnetism and summing over quark and lepton indices, may be written as

$$\begin{aligned}\mathcal{L} = (\bar{q}\gamma_\sigma\partial_\sigma q + \bar{l}_{e,\mu}\gamma_\sigma\partial_\sigma l_{e,\mu}) + \mathcal{L}_0(W) + (q,l \text{ mass terms}) + (q \text{ strong interactions})\\ + ig\{[\bar{q}\gamma_\sigma(1-\gamma_5)\lambda^{\pi^+}q + \bar{l}_{\mu,e}\gamma_\sigma(1-\gamma_5)(\lambda^{\pi^+}\cos\theta + \lambda^{K^+}\sin\theta)l_{\mu,e}]W_{a,\sigma}^-\\ + [\bar{q}\gamma_\sigma(1-\gamma_5)\lambda^{K^+}q + \bar{l}_{\mu,e}\gamma_\sigma(1-\gamma_5)(-\lambda^{\pi^+}\sin\theta + \lambda^{K^+}\cos\theta)l_{\mu,e}]W_{b,\sigma}^-\\ + [\bar{q}\gamma_\sigma(1-\gamma_5)\lambda^{K^0}q + \bar{q}\gamma_\sigma q]W_{c,\sigma}^0\},\end{aligned} \tag{39}$$

where the $\lambda^i$ are just the $3\times3$ $SU(3)$ matrices as defined in the texts referred to in Refs. 16 and 18. The final term is of course the coupling of the baryonic number current to $W_c^0$ and $\mathcal{L}_0(W)$ is the free $W$ Lagrangian.

If we now *assume* that the strong interactions are very rapidly convergent, that is, all loop integrals involving hadrons converge for values of (momentum transfer)$^2$ $<$ a few BeV$^2$, we see that the weak divergent corrections to $\beta$ decay will come primarily from the interactions of the bare quarks. To see this in terms of diagrams, the vertex correction divergence due to the $W$ will be present in Fig. 6(b), but effectively cut off by the strong interactions in Fig. 6(a). We may further clarify the nature of the divergence by writing the $W$ fields in terms of a vector and a scalar field, à la Stueckelberg[25]:

$$W_{j,\sigma} = A_{j,\sigma} + (\partial_\sigma/M_W)B_j \tag{40}$$

[25] E. C. G. Stueckelberg, Helv. Phys. Acta **11**, 225 (1938).

and the propagator in momentum space of the $W$ is as usual

$$\begin{aligned}\Delta_{ij;\sigma\tau}(q) &= \Delta_{ij;\sigma\tau}|_A(q) - (q_\sigma q_\tau/M_W^2)\Delta_{ij}|_B(q)\\ &= \delta_{ij}\left(\frac{\delta_{\sigma\tau} - q_\sigma q_\tau/M_W^2}{q^2 - M_W^2}\right).\end{aligned} \tag{41}$$

The divergences we have encountered so far are due to the $B$ fields; however, if the strong interactions of the

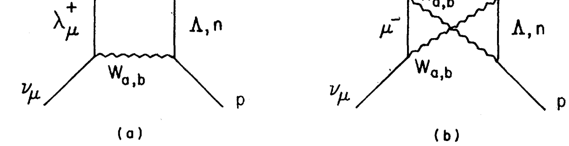


FIG. 6. Lowest-order graphs for $\nu_\mu + p \to \nu_\mu + p$ when new charged leptons are introduced.

quarks do not involve derivative couplings, under the transformation

$$q(x) \to e^{-i(g/M_W)B_c^0(x)} q(x)\,, \tag{42}$$

we find that only the kinetic energy term of the Lagrangian is changed:

$$\bar{q}\gamma_\sigma\partial_\sigma q \to \bar{q}\gamma_\sigma\partial_\sigma q - ig\bar{q}\gamma_\sigma q(\partial_\sigma B_c^0/M_W)\,, \tag{43}$$

which introduces an additional term, cancelling the coupling of the $B_c^0$ field to the baryonic number current in the interaction Lagrangian. We thus see, in an immediate sense, why $\Delta S=2$ nonleptonic transitions are less divergent in this model than they are in a conventional one.

Calculating the leading divergences, due to the couplings of the $B$ fields, we will now neglect the strong interactions of the $q$ particles for the reasons given previously and of course the coupling to the baryonic number current. We then see that, insofar as $B$-field couplings are concerned, the Lagrangian (39), is symmetrical under the interchange $q \leftrightarrow l_{\mu,e}$ for $\sin\theta \to 0$ except for mass terms and the term of the form

$$ig\bar{q}\gamma_\sigma(1-\gamma_5)\lambda^{K^0} q W_{c,\sigma}^0\,. \tag{44}$$

The deviations from universality, i.e., from $G_\beta = G_\mu \cos\theta = G_F \cos\theta$ are then only due to the above term in this model. For $G_F\Lambda^2 \sim 1$, we expect them to be less than 1%. [For instance, keeping only $i=6$, 7 in the summation of Eq. (31), we would find the 2 in (33) replaced by $\frac{1}{2}$, and hence the deviation from universality would be $\sim$0.75%.]

## VI. CONCLUSIONS

We have considered a model for weak interactions involving three IVB's, two neutral massive leptons, and their respective antiparticles in addition to the known hadrons and leptons. In lowest order, it incorporates all the usual phenomenology of weak interactions, with respect to rates as well as selection rules.

We then go on to consider the problem of higher-order weak corrections (it is here that the two new leptons are necessary to reduce divergences) and find that, whereas conventional theories require a cutoff of the order of a few BeV, our model does not lead to contradictions with experiment even for a cutoff of a few hundred BeV. This is of course true only if a large number of assumptions hold: Among them are (i) it makes sense to consider an expansion in powers of $G_F\Lambda^2$ where $\Lambda$ is the cutoff; (ii) one may actually apply Bjorken techniques[20] to the time-ordered product of more than two currents and correctly estimate the degree of divergence of the various pieces of diagrams involving the exchange of two or more vector bosons; (iii) the cancellation of Schwinger terms against the divergences of additional terms added to the time-ordered products of currents for covariance reasons (for possible difficulties along these lines see Appendix A); (iv) the validity of the particular model assumed for equal-time commutators of currents with divergences of currents; (v) the neglect of electromagnetic corrections; (vi) possible strong interactions of the $W$ mesons.

In closing, we would like to say a few words about the spirit with which the model was constructed: If one believes the $V-A$ nature of the weak interactions to be fundamental and does not accept the construction of Ref. 13, involving scalar as well as vector bosons, the limits on the cutoff in a conventional theory are very stringent, assuming of course the calculation of a quadratically divergent quantity such as the $K_L - K_S$ mass difference makes sense. The hope, of course, is that, even though each order of perturbation theory is increasingly divergent, the sum will somehow add up to something finite. Our aim was then twofold: first, to have a theory in which $\Lambda$, the cutoff, was large enough, i.e., of the order of a natural length $1/\sqrt{G_F}$, so that successive orders of perturbation theory were not too much smaller and, second, that even if the IVB mass limit is pushed up to say 5 BeV, one may still have a consistent theory which does not violate experimental limits. At the very least, even if some of our assumptions are wrong, our model probably allows the cutoff to be a factor of 10 larger than the conventional model. The fact that the cutoff in our model was so large does not allow us, however, to have a very large $W$ mass, as we said repeatedly. Rather than having $G_F\Lambda^2 \ll 1$ we require $g^2 \ll G_F\Lambda^2 \sim 1$; the $W$ mass must therefore be smaller than, e.g., 10–20 BeV, if our model is to be consistent with experiment.

## ACKNOWLEDGMENTS

We would like to thank the Physics Department of the University of California at Santa Barbara for their hospitality during part of this work, and Dr. N. Christ for some helpful discussions.

## APPENDIX A

We would like to calculate the contributions of the graphs in Fig. 3 in a conventional theory,

$$\langle ne^+\nu_e | H_W | p\rangle = \tfrac{1}{2}\sqrt{2}iG_F\bar{e}\gamma_\lambda(1-\gamma_5)\nu_e \frac{g^2}{2(2\pi)^4} \times \int d^4q \frac{\delta_{\sigma\tau} - q_\sigma q_\tau/M_W^2}{q^2 - M_W^2} T_{\lambda\sigma\tau}(q,0,p,p)\,, \tag{A1}$$

taking the momentum transfer to the leptons as vanishing.

$$T_{\lambda\sigma\tau}(q,0,p,p)=\int\int d^4x\, d^4y\, e^{-iq\cdot y}\sum_{i=1,2}\langle n|RT\{[J_\lambda^1(x)+iJ_\lambda^2(x)][J_\sigma^i(y)J_\tau^i(0)]\}R^{-1}|p\rangle$$
$$+i\delta^4(y)\langle n|RT\{[J_\lambda^1(x)+iJ_\lambda^2(x)]D_{\sigma\tau}^{ii}(y)\}R^{-1}|p\rangle+i\delta^4(x)\langle n|RT\{D_{\lambda\sigma}^{+i}(0)J_\tau^i(y)\}R^{-1}|p\rangle$$
$$+i\delta^4(x-y)\langle n|RT\{D_{\lambda\sigma}^{+i}(x)J_\tau^i(0)\}R^{-1}|p\rangle-\delta M_{\lambda\sigma\tau}, \quad \text{(A2)}$$

where the $D$ functions[26] enter to make the amplitude covariant. We assume that upon application of $q_\sigma q_\tau$ they cancel the Schwinger terms arising from the equal-time commutators obtained upon applying $q_\sigma q_\tau$ to the time-ordered product. $M_{\lambda\sigma\tau}$ is the part of $T_{\lambda\sigma\tau}$ that contributes only to the weak mass renormalization of the external lines and $R$ is the Cabibbo-angle rotation, $R=e^{-2i\theta F_7}$.

Assuming all the currents to be conserved, we find that the $q_\sigma q_\tau$ terms lead to a correction to the lowest-order graph, which is

$$\langle ne^+\nu_e|H_W|p\rangle_0=\tfrac{1}{2}\sqrt{2}G_F\bar{e}\gamma_\lambda(1-\gamma_5)\nu\cos\theta \times\langle n|[J_\lambda^1(0)+iJ_\lambda^2(0)]|p\rangle, \quad \text{(A3)}$$

of

$$\langle ne^+\nu_e|H_W|p\rangle_0\times\frac{g^2}{M_W^2}\times\frac{\Lambda^2}{4\pi^2}=\langle|H_W|\rangle_0\frac{G_F\Lambda^2}{\sqrt{2}4\pi^2}. \quad \text{(A4)}$$

The terms arising from nonconservation of the current are only logarithmically divergent if one makes generous use of Bjorken techniques.[20] Rather more troublesome, however, is the term in $T_{\lambda\sigma\tau}$ arising from the $\delta_{\sigma\tau}$ in the $W$ propagator and the second term in (A2), namely,

$$i\sum_{i=1,2}\int\int\frac{d^4q\, d^4x}{q^2-M_W^2} \times\langle n|RT\{[J_\lambda^1(x)+iJ_\lambda^2(x)]D_{\tau\tau}^{ii}(0)\}R|p\rangle, \quad \text{(A5)}$$

which in principle is quadratically divergent. In a model such as field algebra,[27] the Schwinger terms are $c$ numbers, which simplifies but does not solve these problems. In particular, the fact that the Schwinger term is a $c$ number ensures that this difficulty will not crop up in our model in strangeness-changing decays to the order we considered, nor in parity-violating terms, so the only place it is a real difficulty for the model is in our treatment of universality. However, here we have the simple Feynman-graph model. Sec. V (e), which leads us to suspect that these difficulties are only caused by an improper handling of the perturbation theory. The question is under investigation currently. Clearly, large operator Schwinger terms in the commutation relations could invalidate our model altogether.

[26] To see an example of the use of the $D$ functions for corrections to vertices, we refer to the appendices in the following paper: E. S. Abers, D. Dicus, R. E. Norton, and H. R. Quinn, Phys. Rev. **167**, 1461 (1968).

[27] T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 1029 (1967).

## APPENDIX B

We would like to discuss briefly the possible effect of a $WWW$ coupling on the preceding calculation. Since the coupling must be a derivative one, one might *a priori* worry about possible $G_F^2\Lambda^4$ terms which would destroy universality. A typical graph contributes to neutron $\beta$ decay and involving a $WWW$ coupling is drawn in Fig. 7. It is of the form

$$T=(\cdots)G_F\bar{e}\gamma_\lambda(1-\gamma_5)\nu g^2\int\int d^4q\, d^4x\, e^{-iq\cdot x}$$
$$\times\frac{\delta_{\mu\nu}-q_\mu q_\nu/M_W^2}{q^2-M_W^2}\frac{\delta_{\sigma\tau}-q_\sigma q_\tau/M_W^2}{q^2-M_W^2}$$
$$\times(-2q_\lambda\delta_{\sigma\nu}+q_\sigma\delta_{\nu\lambda}+q_\nu\delta_{\lambda\sigma})$$
$$\times\langle n|T\{J_\mu^{K^+}(x)J_\tau^{\bar{K}^0}(0)\}|p\rangle, \quad \text{(B1)}$$

where we have taken a minimal[28] $WWW$ coupling with coupling constant $g$ and assumed the momentum transfer to the leptons to be zero. Calling

$$a_{\lambda\sigma\nu}=(-2q_\lambda\delta_{\sigma\nu}+q_\sigma\delta_{\lambda\nu}+q_\nu\delta_{\lambda\sigma}), \quad \text{(B2)}$$

we find that

$$q_\mu q_\nu q_\tau q_\sigma a_{\lambda\sigma\nu}=0, \quad \text{(B3)}$$

which eliminates the potential quartic divergence. However,

$$q_\mu q_\nu\delta_{\sigma\tau}a_{\lambda\sigma\nu}=-q_\lambda q_\mu q_\tau+q^2q_\mu\delta_{\lambda\tau} \quad \text{(B4)}$$

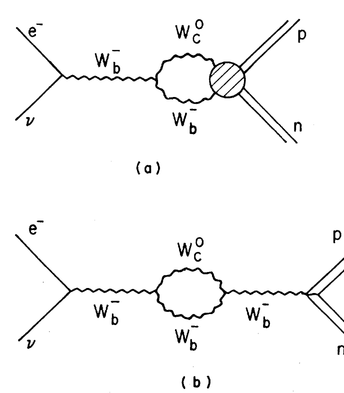


FIG. 7. Corrections to neutron $\beta$ decay if a $WWW$ coupling is allowed.

[28] See, e.g., J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill Book Co., New York, 1965).

and

$$\int \frac{d^4q}{q^2-M_W{}^2} d^4x\, e^{-iq\cdot x}(q^2 q_\mu \delta_{\lambda\tau} - q_\lambda q_\mu q_\tau) \times \langle n | T\{J_\mu{}^{K^+}(x) J_\tau{}^{\bar{K}^0}(0)\} | p \rangle$$

$$= \int \frac{d^4q}{(q^2-M_W{}^2)^2}(q^2\delta_{\lambda\tau}-q_\lambda q_\tau) \times \langle n | \sum_i f^{iK^+\bar{K}^0} J_\tau{}^i(0) | p \rangle \neq 0. \quad \text{(B5)}$$

The same reasoning may be applied to the term proportional to $q_\sigma q_\tau \delta_{\mu\nu} a_{\lambda\sigma\nu}$, so we have a quadratic divergence. There is also the graph of Fig. 7(b) which we must assume is cancelled by a mass-renormalization term. The diagrams of Fig. 7(a) could lead to a violation of universality. Of course, if we imagine $W_a$ to have strangeness zero, $W_b$ and $W_c$ to have strangeness one, and $WWW$ couplings to preserve strangeness, the diagrams of Fig. 7 are forbidden.

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# Solution of Nonrelativistic Partial-Wave Dispersion Relations*

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The partial-wave dispersion-relation (PWDR) problem is studied in the nonrelativistic elastic case. In particular, a set of conditions on the left-hand-cut discontinuity is obtained which is sufficient to guarantee that the PWDR problem has a solution. The nature of the solutions so obtained and the possible extensions are discussed.

## I. INTRODUCTION

ONE can introduce dynamical information about partial-wave amplitudes in a relatively consistent manner by requiring that the amplitude $f(k^2)$ be unitary, be analytic in the variable $k^2$ in the usual twice-cut plane, and have a prescribed left-hand-cut discontinuity which is regarded as input.

The problem consists of finding solutions to the singular, nonlinear integral equation (3.1) which are analytic in the desired region. The problem as stated is known not to lead to unique solutions in many cases as a result of the Castillejo-Dalitz-Dyson (CDD) ambiguity. The ancient and mysterious $N/D$ algorithm reduces the problem to one which is less formidable in appearance. In so doing, one considerably limits the types of solutions one can obtain as a result. However, the dynamical content (e.g., the nonlinearity) of the partial-wave dispersion relation (PWDR) is carefully disguised in the $N/D$ approach.

The rich dynamical content of the PWDR problem is indicated by the fact that although it in general has more than one solution, certain conditions must necessarily be fulfilled if it is to have any solution at all.[1] We will illustrate this here in a simple way.

In Secs. II and III we obtain and discuss conditions on $\Delta T$ which are sufficient to guarantee the existence of a certain type of solution to the PWDR problem. In Sec. IV we study the relative efficacy of these conditions by examining known solutions in simple cases. The $n$-pole case is discussed in Sec. V and distinct conditions are obtained there. Further discussion of the general problem is presented in Sec. VI.

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[1] A. Martin, Nuovo Cimento 38, 1326 (1965).

## II. PARTIAL-WAVE DISPERSION RELATIONS

Here we will study the construction of nonrelativistic, completely elastic partial-wave amplitudes from unitarity, analyticity in energy, and knowledge of the left-hand-cut discontinuity. That is, given the function $\Delta T(k^2)$ defined in $-\infty < k^2 < -\mu^2$, we wish to learn under what conditions there exists a function $f(k^2)$ with the following properties.

(i) $f(k^2)$ is a real analytic function of $k^2$ in the twice-cut $k^2$ plane, where the cuts lie along the real axis and extend over the domains $-\infty < k^2 < -\mu^2$ and $0 < k^2 < \infty$.

(ii) The function $f(k^2) \to 0$ as $k^2 \to \infty$ within the cut plane.

(iii) The discontinuity of $f(k^2)$ across the left-hand cut $-\infty < k^2 < -\mu^2$ is given by $\Delta T(k^2)$.

(iv) As one approaches the right-hand cut from above, $f(k^2+i\epsilon)$ satisfies the unitarity condition

$$\mathrm{Im} f(k^2+i\epsilon) = (k^2+i\epsilon)^{1/2} |f(k^2+i\epsilon)|^2 \quad (2.1)$$

or

$$f(k^2+i\epsilon) = (k^2+i\epsilon)^{-1/2} e^{i\delta(k^2)} \sin\delta(k^2), \quad \delta(k^2) \text{ real.} \quad (2.1')$$

A very simple type of necessary condition can be obtained as follows[2]:

Let $\phi(k^2)$ be any function analytic in the $k^2$ plane with only a right-hand cut. If $\phi$ is such that $k^{2+\epsilon} f \phi(k^2) \to 0$

[2] G. Tiktopoulos (unpublished).