One-Boson-Exchange Contributions to High-Energy p-$\bar{p}$ Elastic Scattering in a Nonlocal Field Theory

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The p-$\bar{p}$ elastic scattering in the GeV region is studied through the exchange of heavy bosons. Within the framework of a local field theory, the experimental results cannot be reproduced satisfactorily. Better agreement is obtained when suitable vertex functions representing a nonlocal Lagrangian are introduced. The fundamental length contained in the vertex function is found to be the nucleon Compton wavelength.

1. INTRODUCTION

The problem of the p-$\bar{p}$ interaction at energies below 350 MeV has been extensively studied, and a full review of the problem has been given by Phillips.[1] At these energies the experimental data are quite satisfactorily understood and reproduced in theory.

At higher energies (GeV region), as stressed by Taketani *et al.*,[2] the colliding nucleons can come so close that many-pion-exchange processes are of primary importance. Since we still do not have any detailed knowledge about the interaction of pions, we can, following Sakurai,[3] suppose that the exchange of heavy bosons, namely, pion resonances, could account for the major contribution of such a process. Unfortunately, the p-$\bar{p}$ elastic scattering data are still scanty in the high-energy region, and, despite the plethora of theories, none can completely account for the situation. In this paper the differential cross section for p-$\bar{p}$ elastic scattering is calculated at a laboratory momentum in the GeV region, considering the contributions to $d\sigma/d\Omega$ coming from the exchanges of the pseudoscalar boson η(958 MeV); the vector bosons ρ(765 MeV), ω(783 MeV), and ϕ(1019 MeV); and the tensor boson f^0(1260 MeV). Masses and parities of these particles are taken from Ref. 4.

As shown in Figs. 2 and 3, the contributions to $d\sigma/d\Omega$ from f^0 and ω mesons are much greater than those from the η, ρ, or ϕ mesons. In forward scattering, where the differential cross section reaches its largest value, the contributions of the ρ and ϕ mesons are about 10^2 times less than the f^0 and ω contributions, while the η-meson contributions are about 10^5 times less than those of f^0 and ω. Therefore, it can be argued that the most important role is played by the f^0 and ω mesons. However, comparing theoretical results with experimental data,[5–8] it turns out that the theoretical f^0 and ω contributions are too high. A similar situation occurs in considering the $p+\bar{p}\to\pi^++\pi^-$ process by means of an intermediate f^0 meson.[9]

2. CROSS SECTION

Since the annihilation of the η, ρ, ω, ϕ, and ρ^0 mesons into a nucleon and an antinucleon has not been observed, only the contributions to the direct diagram (given in Fig. 1) have been computed. As a starting point, the following interaction Lagrangians are considered[10]:

(a) pseudoscalar coupling

$$\mathcal{L}_P=g_P\bar{\psi}\gamma_5\psi\phi\,, \tag{1}$$

(b) vector coupling

$$\mathcal{L}_V=g_V\bar{\psi}\gamma_\mu\psi\phi_\mu+(f_V/4m_V)\bar{\psi}\sigma_{\mu\nu}(\partial_\nu\phi_\mu-\partial_\mu\phi_\nu)\,, \tag{2}$$

(c) tensor coupling[11]

$$\mathcal{L}_T=(g_{\rm I}/2M)[\bar{\psi}i\gamma_\mu(\partial_\nu\psi)-(\partial_\nu\bar{\psi})i\gamma_\mu\psi]\phi^{\mu\nu}+(g_{\rm II}/M^2)[(\partial_\mu\bar{\psi})(\partial_\nu\psi)]\phi^{\mu\nu}. \tag{3}$$

Average values of the η, ρ, and ω coupling constants are used, as determined by the low-energy data[12]

$$g_\eta^2/4\pi=5\,,\quad g_\rho^2/4\pi=2\,,\quad g_\omega^2/4\pi=22\,,\quad f_\rho/g_\rho=2.8\,,\quad f_\omega/g_\omega=0. \tag{4}$$

The values of the ϕ-meson coupling constants[13] and the f^0-meson coupling constants[11] are

$$g_\phi^2/4\pi=2.72\,,\qquad f_\phi/g_\phi=0\,, \tag{5}$$

$$g_{\rm I}^2/4\pi=15\,,\qquad g_{\rm II}^2/4\pi=0. \tag{6}$$

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The form of the propagator for a spin-2^+ particle with mass m has been investigated by Rivers.[14] It can be written

$$\langle T\phi^{\mu\nu}(x_1)\phi_{\mu'\nu'}(x_2)\rangle_0 = i\int \frac{d^4k}{(2\pi)^4}\frac{D_{\mu'\nu'}{}^{\mu\nu}(k)}{k^2+m^2}e^{-ik\cdot(x_1-x_2)}, \tag{7}$$

with

$$D_{\mu'\nu'}{}^{\mu\nu}(k)=\tfrac{1}{2}[\theta_{\mu'}{}^{\mu}(k)\theta_{\nu'}{}^{\nu}(k)+\theta_{\nu'}{}^{\mu}(k)\theta_{\mu'}{}^{\nu}(k)]-\tfrac{1}{3}\theta^{\mu\nu}(k)\theta_{\mu'\nu'}(k), \tag{8}$$
$$\theta^{\mu\nu}(k)=g^{\mu\nu}+k^{\mu}k^{\nu}/m^2,$$

where k is the intermediate-boson momentum.[15]

Here the metric $g_{\mu\nu}$ is $++++$, and the fourth component of the four-momentum is taken to be imaginary. Using standard field-theoretical methods, the second-order differential cross sections $(d\sigma/d\Omega)_i$ $(i=\eta, \rho, \omega, \phi, f^0)$ are given by[16]

$$\left(\frac{d\sigma}{d\Omega}\right)_P=\left(\frac{g_P{}^2}{4\pi}\right)^2\frac{p^4(1-\cos\theta)^2}{E_c{}^2[2p^2(1-\cos\theta)+m_P{}^2]^2}, \tag{9}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_V=\left(\frac{g_V{}^2}{4\pi}\right)^2\frac{2(p^2+E^2)^2+2(p^2\cos\theta+E^2)^2+4p^2(1-\cos\theta)(p^2-E^2)}{E_c{}^2[2p^2(1-\cos\theta)+m_V{}^2]^2}, \tag{10}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_T=\left(\frac{g_T{}^2}{4\pi}\right)^2\frac{H(p,E,\cos\theta)}{16M^4E_c{}^2[2p^2(1-\cos\theta)+m_T{}^2]^2}, \tag{11}$$

where

$$\begin{aligned}H(p,E,\cos\theta)=\;&2[(p^2+E^2)+(p^2\cos\theta+E^2)]^2[3(p^2+E^2)^2+3(p^2\cos\theta+E^2)^2\\&+3(p^2-E^2)^2-3(p^2\cos\theta-E^2)^2+p^4(1-\cos\theta)^2]\\&+\{[(p^2+E^2)+(p^2\cos\theta+E^2)]^2+p^2(1-\cos\theta)[(p^2-E^2)+(p^2\cos\theta-E^2)]\}^2-(32/3)\\&\times[(p^2+E^2)+(p^2\cos\theta+E^2)]^2(p^2-E^2)^2+(16/9)[(p^2-E^2)+(p^2\cos\theta-E^2)]^2(p^2-E^2)^2.\end{aligned} \tag{12}$$

The symbols are p (c.m. momentum), θ (c.m. scattering angle), E_c (c.m. energy), $E=\frac{1}{2}E_c$, M (nuclear mass), m_P (pseudoscalar-meson rest mass), m_V (vector-meson rest mass), and m_T (tensor-meson rest mass).

The values of theoretical cross sections, Eqs. (9)–(11), have been calculated for $p_L=2.7$,[6] 3.66,[7] 4,[8] and 6.9 [5] GeV/c as a function of $\cos\theta$. (p_L is the incoming antiproton laboratory momentum $p_L=2(p/M)(p^2+M^2)^{1/2}$.)

At $p_L=2.7$ GeV/c (Fig. 2), the η-meson contribution increases with the scattering angle, in contrast with the experimental data. The ρ and ϕ contributions are smaller than the experimental value in the forward scattering region, but greater for most θ; and the ω and f^0 contributions are always much greater than the experimental value. The situation is similar at $p_L=3.66$ GeV/c (Fig. 3). Here, because the incoming antiproton momentum has increased, the ρ and ϕ contributions have become greater than the experimental value even in the forward scattering region.

At higher energies, namely, at $p_L=4$ and 6.9 GeV/c, the results are analogous to the ones discussed previously, and there is no point in entering in further details.

The situation is just as in Ref. 9, where it was shown that better agreement with the experimental data is obtained by introducing a relativistic cutoff factor, whose theoretical basis and justification have been given by Wataghin in a series of papers.[17] Following the

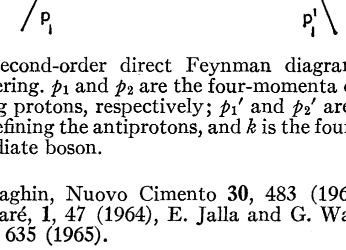

FIG. 1. Second-order direct Feynman diagram for the p-$\bar{p}$ elastic scattering. p_1 and p_2 are the four-momenta of the incoming and outgoing protons, respectively; p_1' and p_2' are the analogous quantities defining the antiprotons, and k is the four-momentum of the intermediate boson.

[14] R. J. Rivers, Nuovo Cimento **34**, 386 (1964).

[15] A different way of deriving Eq. (8) has been suggested by M. Zaidi and can be found in the Appendix.

[16] The behavior of $(d\sigma/d\Omega)_V$ due to the purely derivative part of the ρ-meson interaction Lagrangian increases with scattering angle, in contradiction to experiment. This situation is not improved even if the relativistic cutoff (to be considered later) is taken into account. This term is henceforth to be neglected.

[17] G. Wataghin, Nuovo Cimento **30**, 483 (1963); Ann. Inst. Henri Poincaré, **1**, 47 (1964), E. Jalla and G. Wataghin, Nuovo Cimento **39**, 635 (1965).

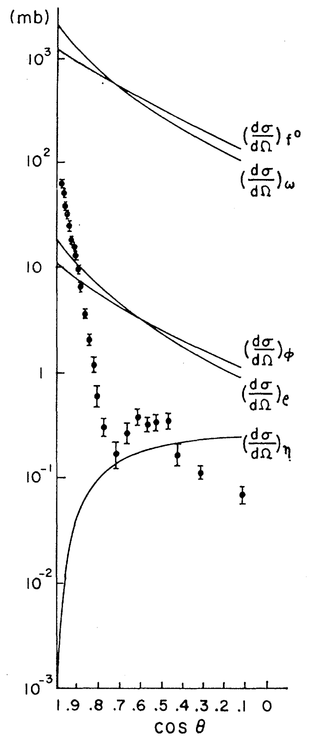

FIG. 2. Predicted contributions to the p-$\bar{p}$ elastic differential cross section from the η, ρ, ϕ, ω, and f^0 mesons, compared with the experimental data at an incoming antiproton laboratory momentum of 2.7 GeV/c (Ref. 6).

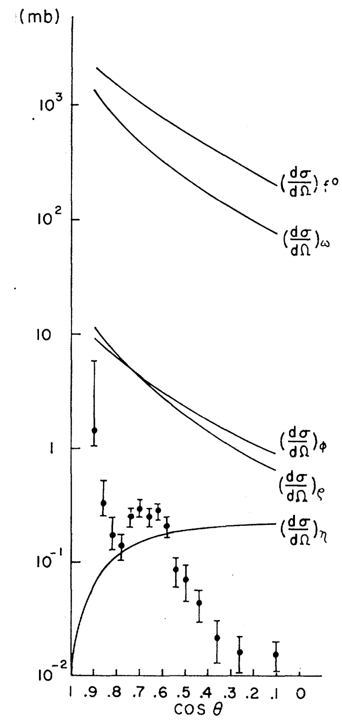

FIG. 3. Predicted contributions to the p-$\bar{p}$ elastic differential cross section from the η, ρ, ϕ, ω, and f^0 mesons, compared with the experimental data at an incoming antiproton momentum of 3.66 GeV/c (Ref. 7).

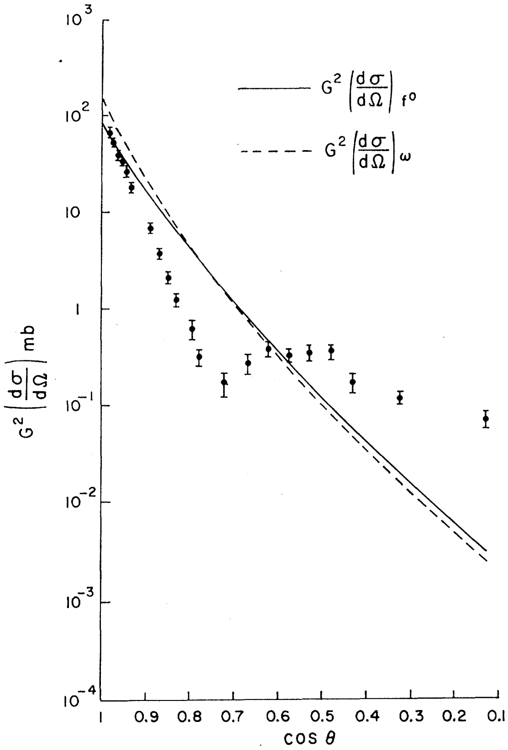

FIG. 4. Predicted nonlocal theory contributions to the p-$\bar{p}$ elastic differential cross section from the ω and f^0 mesons, compared with the experimental data at an incoming antiproton laboratory momentum of 2.7 GeV/c (Ref. 6). G is given by Eq. (13), with $l=1/M$.

prescriptions of Ref. 17, a cutoff factor G is introduced at every vertex of the graph in Fig. 1, namely,

$$G=[1+I_s^2(p)]^{-4}[1+I_t^2(p)]^{-2}[1+I_s^2(k)]^{-2}, \quad (13)$$

where

$$\begin{aligned} I_s^2(p)&=l^2p^2\sin^2\tfrac{1}{2}\theta,\\ I_t^2(p)&=l^2[p^2\sin^2\tfrac{1}{2}\theta+M^2], \quad (14)\\ I_s^2(k)&=l^2k^2=l^2 4p^2\sin^2\tfrac{1}{2}\theta. \end{aligned}$$

The quantity l, in Eqs. (14), is called a fundamental length, and is considered a free parameter.

Calculations of $G^2(d\sigma/d\Omega)_i$ have been made with $l=1/M$, $1/2M$, and $1/m_i$, where m_i is the mass of the meson under consideration.

The general result is the following: In the nonlocal theory, the contributions to the differential cross section from the η, ρ, and ϕ mesons are negligible compared to the ω and f^0 contributions for any value of l, and extremely small compared with the experimental data.

For $l<1/M$, the values of $G^2(d\sigma/d\Omega)_{\omega,f^0}$ become too high compared to the experimental data: In the case of $l=1/2M$, for example, they are about 10 times greater than the data in the forward scattering region. For $l>1/M$ the situation is reversed. $G^2(d\sigma/d\Omega)_{\omega,f^0}$ are too small with respect to the data. Therefore, it seems that the value $l=1/M$ is the most favorable to obtain differential cross sections comparable with the experimental data. In Figs. 4 and 5 the present theory's results for $G^2(d\sigma/d\Omega)_\omega$ and $G^2(d\sigma/d\Omega)_{f^0}$ are plotted together with the data at $p_L=2.7$ and 3.66 GeV/c. In both figures it can be seen that the ω and f^0 contributions are of the same order of magnitude, and that the f^0 contribution becomes prevalent as the energy of the incoming antiproton increases.

At $p_L=4$ GeV/c, Czyzewski and collaborators give the experimental differential cross section normalized to the forward scattering cross section, under the assumption that $(d\sigma/d|t|)_{t=0}$ has no real part. In Fig. 6, a comparison is made between the theoretical $G^2(d\sigma/d|t|)_{\omega,f^0}$ and the experimental values, both normalized to the zero-momentum-transfer differential cross section.

In Fig. 7, a comparison is made between the nonlocal theory contributions of the ω and f^0 mesons and the experimental data at an incoming antiproton laboratory momentum of 6.9 GeV/c.[5]

In conclusion, the nonlocal theory used above succeeds reasonably well in reproducing the experimental behavior, especially in the zero-momentum-transfer region, but it does not exhibit the dips, observed in almost all experiments, at $t\simeq 0.5-0.6$ (GeV/c)^2. Moreover, all the results given above are very sensitive to any change of the meson coupling constants. Here, the coupling constant values have been assumed, because, as mentioned before, they give rise to results which reproduce the low-energy data reasonably well.

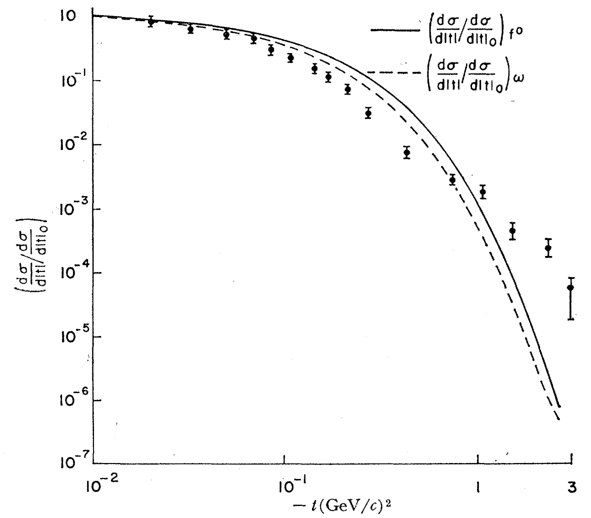

FIG. 6. Predicted ω and f^0 nonlocal contributions to the p-$\bar{p}$ elastic differential cross section, normalized to zero momentum transfer, compared with the experimental data at an incoming antiproton laboratory momentum of 4 GeV/c (Ref. 8). G has the same meaning as in Fig. 4.

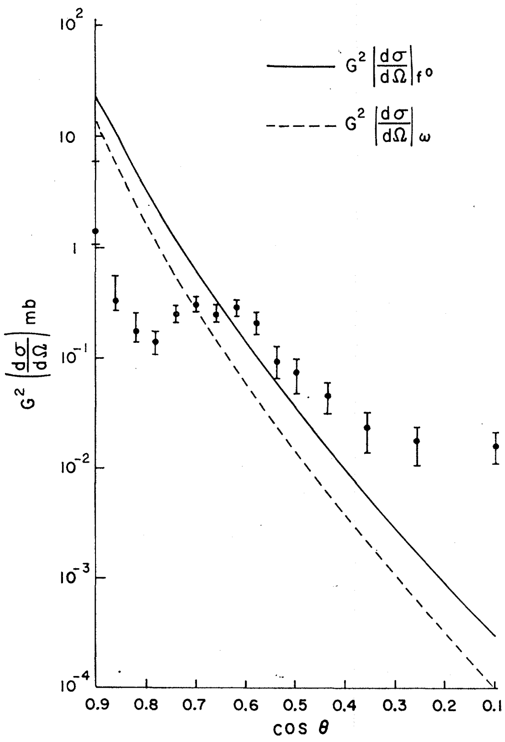

FIG. 5. Predicted nonlocal theory contributions to the p-$\bar{p}$ elastic differential cross section from the ω and f^0 mesons, compared with the experimental data at an incoming antiproton laboratory momentum of 3.66 GeV/c (Ref. 7). G has the same meaning as in Fig. 4.

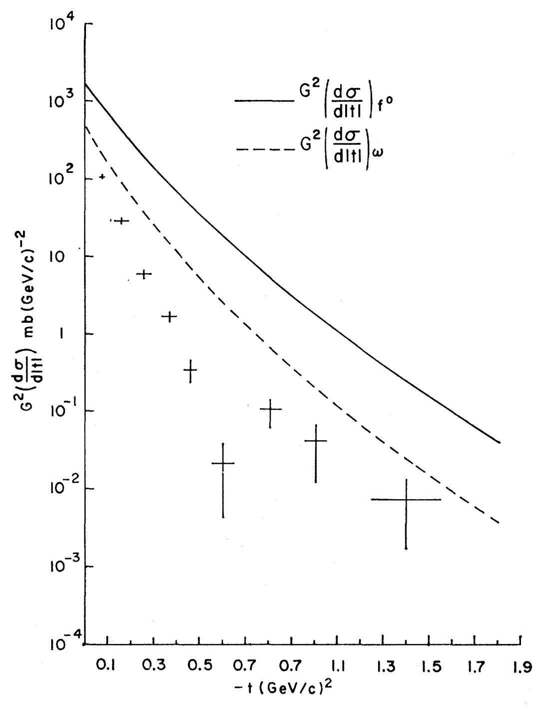

FIG. 7. Predicted ω and f^0 nonlocal contributions to the p-$\bar{p}$ elastic differential cross section, compared with the experimental data at an incoming antiproton laboratory momentum of 6.9 GeV/c (Ref. 5). G has the same meaning as in Fig. 4.

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APPENDIX

The polarization tensor $e_{\mu\nu}^{(\lambda)}$ of the spin-2 particle field, in the rest frame of the particle, should be

$$\sum_\lambda e_{ij}^{(\lambda)} e_{lm}^{(\lambda)} = a\delta_{il}\delta_{jm} + b\delta_{im}\delta_{jl} + c\delta_{ij}\delta_{lm}. \quad \text{(A1)}$$

The condition that Eq. (A1) should be invariant under the interchange of the i and j indices implies that

$$a = b. \quad \text{(A2)}$$

At the same time, $e_{ij}^{(\lambda)}$ should be traceless, so that

$$c = -\tfrac{2}{3}a \quad \text{(A3)}$$

and

$$5 = \sum_\lambda e_{ij}^{(\lambda)} e^{ij(\lambda)} = 10a. \quad \text{(A4)}$$

Substituting Eqs. (A2)–(A4) into Eq. (A1), we obtain

$$\sum_\lambda e_{ij}^{(\lambda)} e_{lm}^{(\lambda)} = \tfrac{1}{2}(\delta_{il}\delta_{jm} + \delta_{im}\delta_{jl}) - \tfrac{1}{3}\delta_{ij}\delta_{lm}. \quad \text{(A5)}$$

Therefore, in any reference frame, Eq. (A5) generalizes to

$$\sum_\lambda e_{\mu\nu}^{(\lambda)} e_{\alpha\beta}^{(\lambda)} = \tfrac{1}{2}(\theta_{\mu\alpha}\theta_{\nu\beta} + \theta_{\mu\beta}\theta_{\nu\alpha}) - \tfrac{1}{3}\theta_{\mu\nu}\theta_{\alpha\beta}, \quad \text{(A6)}$$

with

$$\theta_{\mu\nu} = g_{\mu\nu} + k_\mu k_\nu / m^2. \quad \text{(A7)}$$

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Quarks of Almost Any Spin

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Quark-model predictions are investigated for quarks of any half-integral spin. While quarks of spin $\frac{1}{2}$ must obey effective Bose statistics if they are to be in space-symmetric states, we show that quarks of spin greater than $\frac{1}{2}$ can obey either Fermi or Bose statistics. The generalization to quarks of any half-integral spin leads to essentially the same conclusions we found previously for spin-$\frac{3}{2}$ Fermi quarks. The quark-model predictions so far tested depend very little on the spin or statistics of quarks, but a measurement of the magnetic moment of the Ξ^- or Ω^- hyperon would test the spin and statistics of quarks.

I. INTRODUCTION

In a previous paper,[1] we showed that spin-$\frac{3}{2}$ quarks obeying the usual Fermi statistics could account for observed hadron properties as well as or better than spin-$\frac{1}{2}$ quarks obeying effective Bose statistics.[2] Here, we extend our consideration to quarks of any half-integral spin obeying either kind of statistics. We find that going to higher spins with either statistics makes very little change in the results we found for spin-$\frac{3}{2}$ Fermi quarks, although a more complicated dynamical assumption is required to limit the baryon multiplets and achieve quark saturation. We find a slight preference for quarks of spin $\frac{3}{2}$, $\frac{7}{2}$, $\frac{11}{2}$, $\cdots$, obeying Fermi statistics, but other combinations are not completely ruled out. The main experimental test of the spin and statistics of quarks would be determinations of the magnetic moments of the Ξ^- and Ω^- hyprons. The magnetic moment of the Σ^+ turns out to be almost independent of quark spin or statistics and in agreement with experiment.

The plan of this paper is to follow the format of A, generalizing to quarks of any spin. In Sec. II, we consider the baryon multiplet structure. In Sec. III, we consider baryon mass differences and, in Sec. IV, magnetic moments. Mesons are treated in Sec. V. In Sec. VI, we summarize the spin-statistics dependence of quark-model predictions.

II. BARYON STATES

In this section, we investigate how the baryon octet and decuplet [forming the 56-dimensional representation of $SU(6)$] can be obtained as s states with quarks of various spin-statistics combinations. For quarks of spin $\frac{1}{2}$, it is well known that a spin-$\frac{3}{2}$ decuplet can only be obtained with effective Bose statistics because the spin-$\frac{3}{2}$ combination of three spin-$\frac{1}{2}$ quarks is completely symmetric under spin interchange. For quarks of any half-integral spin greater than $\frac{1}{2}$, there is a

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[1] J. Franklin, Phys. Rev. **180**, 1583 (1969). We refer to this paper as A and generally follow its notation and use it for earlier references.

[2] By "effective Bose statistics" we mean that quarks in baryons combine like bosons, but the quark-antiquark pairs in mesons still act like fermions. This is, of course, different than true Bose statistics and could be achieved by the methods discussed in Refs. 6–8 of A.