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Possibility of Non-Octet Axial-Vector Currents*

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Three models of axial-vector currents which contain both octet and non-octet components are examined. The first model is based upon the $\Delta U=\frac{1}{2}$ rule, and it forbids decays with $\Delta S=-\Delta Q$ and $\Delta S=2$. The second model conserves a "weak hypercharge," and introduces $\Delta S=-\Delta Q$ and $\Delta S=2$ terms in a specific way. The third model assumes that the usual $\Delta T=1$ and $\Delta T=\frac{1}{2}$ rules are valid even when the octet rule is not. Although present data on semileptonic hyperon decay are not accurate enough to determine the precise admixture of non-octet components, they seem to indicate that such currents are restricted to the **27** representation, and that they may form about 10% of the total current. Throughout the discussion the conserved-vector-current hypothesis is assumed to hold. Some of the general theoretical consequences that would follow from the existence of non-octet axial-vector currents are also considered.

1. INTRODUCTION

Although present data on semileptonic hyperon decay are consistent with the hypothesis of octet dominance,[1] they are not so accurate as to confirm the hypothesis beyond reasonable doubt. For example, a recent measurement of the branching ratio for $\Xi^-\rightarrow\Lambda e^-\nu^-$ is within one standard deviation of the predicted value, but it is also within one standard deviation of a value twice as large as that predicted.[2] The resolution of this uncertainty and others like it must await more accurate experiments; but in the meantime it may be useful to consider alternatives to octet dominance for the weak hadronic current.

As far as the vector part of the current is concerned, there are attractive arguments in favor of octet transformation properties. The conserved vector current (CVC) hypothesis, which agrees well with experiment,[3] requires the weak vector current and the electromagnetic current to lie in the same multiplet[4]; the latter, through its relationship with the charge operator, transforms as a member of an octet, and so the former must do likewise. Furthermore, the Ademollo-Gatto theorem[5] suggests that medium-strong, $SU(3)$-violating interactions do not distort the properties of the vector current in any serious way. In the case of the axial-vector current, however, the arguments are not so strong. It is true that the hypothesis of partial conservation of axial-vector current (PCAC)[6] and the Adler-Weisberger relation[7] point towards octet transformation properties, but their analogs for the strangeness-violating current are not well established.[8] We shall, therefore, assume that if there is a serious breakdown of octet dominance, it will occur in the axial-vector current but not in the vector one.

There are two possible ways in which such a breakdown can take place; either the current transforms according to an irreducible representation other than the octet, or it is an admixture of several representations, i.e., a reducible representation. The axial-vector current, besides contributing to hyperon decay, also gives rise to the meson decays $\pi\rightarrow\mu\nu$ and $K\rightarrow\mu\nu$, and so it must contain an octet component. Consequently, we can dismiss the first type of breakdown and assume the current to be a reducible representation consisting of the octet and at least one other representation. For hyperons, these additional representations are contained in the direct product $8\otimes 8$.

The introduction of non-octet components provides us with much latitude in constructing the current, and it is necessary to find models which will limit our scope. Since there is no analog of the Ademollo-Gatto theorem[5] for the axial-vector current, it may happen that the "bare" current is pure octet, and that significant non-octet components are induced by medium-strong, $SU(3)$-violating interactions. If this is the case, the charge independence of medium-strong interactions will ensure that the usual strangeness and isospin selection rules are maintained, and hence it will restrict the number of components in the current. If, on the other hand, renormalization effects are relatively small, then non-

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[1] W. Willis, H. Courant, H. Filthuth, P. Franzini, A. Minguzzi-Ranzi, A. Segar, R. Engelmann, V. Hepp, E. Kluge, R. A. Burnstein, T. B. Day, R. G. Glasser, A. J. Herz, B. Kehoe, B. Sechi-Zorn, N. Seeman, and G. A. Snow, Phys. Rev. Letters **13**, 291 (1964); N. Brene, L. Veje, M. Roos, and C. Cronstrom, Phys. Rev. **149**, 1288 (1966); C. E. Carlson, *ibid.* **152**, 1433 (1966); H. T. Nieh and M. M. Nieto, *ibid.* **172**, 1694 (1968).

[2] J. R. Hubbard, J. P. Berge, and P. M. Dauber, Phys. Rev. Letters **20**, 465 (1968).

[3] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958); S. Gershstein and J. Zeldovich, Zh. Eksperim. i Teor. Fiz. **29**, 698 (1955) [English transl.: Soviet Phys.—JETP **2**, 576 (1956)]. For a review of the experimental evidence in favor of CVC see J. Bernstein, *Elementary Particles and Their Currents* (W. H. Freeman and Co., San Francisco, 1968), Chap. 10.

[4] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[5] M. Ademollo and R. Gatto, Phys. Rev. Letters **13**, 264 (1964).

[6] Y. Nambu, Phys. Rev. Letters **4**, 380 (1960); M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960).

[7] S. L. Adler, Phys. Rev. Letters **14**, 1051 (1965); W. I. Weisberger, *ibid.* **14**, 1047 (1965).

[8] C. H. Chan and F. T. Meiere, Phys. Rev. (to be published).

octet components may be intrinsic to the current, and it will be necessary to construct other models.

One approach to the problem is suggested by the Cabibbo theory.[4] In its original form, this theory requires that the strangeness-conserving and strangeness-violating currents belong to the same octet; as a result, their matrix elements for hyperon decay should be governed by the same D/F ratio. The obvious extension to our case is to require the $\Delta S=0$ and $\Delta S=1$ currents to belong to the *same reducible multiplet*. Is then follows that the two currents can be "rotated" into one another by means of $SU(3)$ transformations. Because the currents have equal electric charges, these transformations must belong to the U-spin subgroup, and it is convenient to use U spin, rather than isospin, as an analytical tool.

In terms of U spin, the selection rules $\Delta S=0$ and $\Delta S=\Delta Q=1$ are equivalent to

$$\Delta U_3=\mp\tfrac{1}{2}, \quad \Delta Q=1, \tag{1}$$

and the rules $\Delta S=2$ and $\Delta S=-\Delta Q=-1$ become

$$\Delta U_3=\pm\tfrac{3}{2}, \quad \Delta Q=1. \tag{2}$$

The simplest way to forbid decays governed by Eq. (2) and to implement a modified Cabibbo hypothesis is assume the selection rule

$$\Delta U=\tfrac{1}{2}, \quad \Delta Q=1 \tag{3}$$

and to write the axial-vector current in the form

$$A=\cos\theta A_{-1/2}+\sin\theta A_{+1/2}, \tag{4}$$

where $A_{\pm 1/2}$ are the two members of a U-spin doublet. Equation (3) automatically excludes a $\mathbf{10}^*$ component from A because $\Delta U=1+\tfrac{1}{2}\Delta Q$ in that representation. As far as isospin selection rules are concerned, the octet components of A satisfy the usual $\Delta T=1$ and $\Delta T=\tfrac{1}{2}$ rules; the $\mathbf{10}$ components satisfy $\Delta T=1$ and $\Delta T=\tfrac{3}{2}$; and the $\mathbf{27}$ components correspond to $\Delta T\leq 2$ for strangeness-conserving decays, and to $\Delta T\leq\tfrac{3}{2}$ for strangeness-violating ones.

If we prefer not to forbid decays satisfying Eq. (2) but merely to suppress them relative to the common modes of semileptonic decay, we can adopt the point of view that the hadronic current conserves a "weak hypercharge."[9,10] The complete current is then obtained by rotating the $T=T_3=1$ component of each representation through an angle 2θ about the second axis of U-spin space. The octet and $\mathbf{10}$ components still satisfy $\Delta U=\tfrac{1}{2}$, but the $\mathbf{10}^*$ and $\mathbf{27}$ components give rise to $\Delta U=\tfrac{3}{2}$ terms and therefore generate all possible strangeness section rules.[10]

Neither of these two modified forms of the Cabibbo theory includes as a special case the type of breakdown of octet dominance engendered by medium-strong interactions. This type requires that the isospin rules $\Delta T=1$ and $\Delta T=\tfrac{1}{2}$ be preserved; but for all representations except the octet, both the $\Delta U=\tfrac{1}{2}$ rule and "weak hypercharge" conservation give rise to strangeness-violating currents with $\Delta T>\tfrac{1}{2}$. Thus we have three distinct approaches to the problem of non-octet components in the axial-vector current.

In this paper we shall apply these three approaches to the phenomenological analysis of semileptonic hyperon decay. Where possible, we shall use the existing data to determine the specific form of the axial-vector current and to estimate the admixture of non-octet components. For the vector current, we adopt the CVC hypothesis as extended by Cabibbo.[4]

The $\Delta U=\tfrac{1}{2}$ rule is discussed in Sec. 2, and "weak hypercharge" conservation in the third. The case in which the usual isospin rules are preserved is discussed in Sec. 4, and the three approaches are compared with one another in the concluding section.

2. $\Delta U=\tfrac{1}{2}$ RULE

We write the matrix element for the semileptonic hyperon decay

$$\alpha\rightarrow\beta+e+\nu \tag{5}$$

in the form

$$M(\alpha\rightarrow\beta)=(G/\sqrt{2})\langle\beta|i\gamma_\mu(V+A\gamma_5)|\alpha\rangle L_\mu, \tag{6}$$

where L_μ is the usual lepton factor and G is the weak-interaction coupling constant. In Eq. (6) we neglect both the dependence of V and A upon momentum transfer, and additional terms induced by strong interactions (for example, "weak magnetism"). These corrections are of order 10% and we shall comment on them at a later stage. We also neglect corrections due to CP violation and treat V and A as real constants. Our sign convention is such that in neutron β decay

$$(A/V)_{n\rightarrow p}\approx +1.2. \tag{7}$$

If we now assume the $\Delta U=\tfrac{1}{2}$ rule for the axial-vector current, the coupling constants for strangeness-conserving and strangeness-violating decays, denoted by $A^{(0)}$ and $A^{(1)}$, respectively, can be expressed in tensor notation as (see Appendix)

$$A^{(0)}=\{F(F_1{}^2)+D(D_1{}^2)+G[10]_{11}{}^{21}+H[27]_{11}{}^{21}\}\cos\theta_A, \tag{8a}$$

$$A^{(1)}=\{F'(F_1{}^3)+D'(D_1{}^3)+G'[10]_{11}{}^{31}+H'[27]_{11}{}^{31}\}\sin\theta_A. \tag{8b}$$

The requirement that $A^{(0)}$ and $A^{(1)}$ belong to the same reducible $SU(3)$ multiplet implies that

$$F'/F=D'/D=G'/G=H'/H=1. \tag{9}$$

[9] N. Cabibbo, Phys. Rev. Letters **12**, 62 (1964).

[10] B. de'Espagnat and M. K. Gaillard, Nuovo Cimento **42**, 1035 (1966). See also E. de Rafael and M. Goldhaber, Phys. Rev. Letters **20**, 522 (1968); R. M. Delaney and D. J. Welling, Phys. Rev. **176**, 1841 (1968).

Amplitudes for decays of interest are then given by

$$\begin{aligned} A(n\to p)&=(F+D-\sqrt{2}H-\sqrt{2}G)\cos\theta_A\,,\\ A(\Xi^-\to\Xi^0)&=(F-D+\sqrt{2}H-\sqrt{2}G)\cos\theta_A\,,\\ \sqrt{3}A(\Sigma^-\to\Lambda)&=(-\sqrt{2}D-3H-3G)\cos\theta_A\,,\\ \sqrt{3}A(\Sigma^+\to\Lambda)&=(\sqrt{2}D+3H-3G)\cos\theta_A\,,\end{aligned}\tag{10a}$$

$$\begin{aligned} (\sqrt{6})A(\Lambda\to p)&=(-3F-D+6\sqrt{2}H)\sin\theta_A\,,\\ (\sqrt{6})A(\Xi^-\to\Lambda)&=(3F-D+6\sqrt{2}H)\sin\theta_A\,,\\ A(\Xi^0\to\Sigma^+)&=A(n\to p)\tan\theta_A\,,\\ A(\Sigma^-\to n)&=A(\Xi^-\to\Xi^0)\tan\theta_A\,,\end{aligned}\tag{10b}$$

and amplitudes for other decay modes can be obtained either from Eq. (8) or from Eq. (10) and the $\Delta U=\frac{1}{2}$ rule.

The first point to notice about Eq. (10) is that the equality

$$A(\Xi^0\to\Sigma^+)/A(\Sigma^-\to n)=-A(n\to p)/A(\Xi^-\to\Xi^0)\tag{11}$$

holds for all values of θ_A. If we assume the conserved-vector-current hypothesis, namely,

$$V=(F_1{}^2\cos\theta_V+F_1{}^3\sin\theta_V)\,,\tag{12}$$

then a corresponding equality will hold for the vector amplitudes. It then follows that the A/V ratios satisfy

$$\left(\frac{A}{V}\right)_{\Xi^0\to\Sigma^+}\bigg/\left(\frac{A}{V}\right)_{\Sigma^-\to n}=\left(\frac{A}{V}\right)_{n\to p}\bigg/\left(\frac{A}{V}\right)_{\Xi^-\to\Xi^0}\tag{13}$$

irrespective of the relationship between the Cabibbo angles θ_V and θ_A. As a measure of these angles we find that

$$\left(\frac{A}{V}\right)_{\Sigma^-\to n}=\frac{\tan\theta_A}{\tan\theta_V}\left(\frac{A}{V}\right)_{\Xi^-\to\Xi^0}.\tag{14}$$

Equations (13) and (14) do not depend upon the choice of coupling constants F, D, G, and H in Eq. (8), and hence they are independent of the admixtures of **8**, **10**, and **27** in the axial-vector current. The only assumptions upon which they do depend are the $\Delta U=\frac{1}{2}$ rule and the Cabibbo-type hypothesis of Eqs. (9) and (12). In order to make further predictions, we must now introduce additional assumptions.

As a guide for these assumptions we use the experimental result that[11]

$$|A(\Sigma^+\to\Lambda)|=|A(\Sigma^-\to\Lambda)|\approx(\sqrt{\tfrac{2}{3}})(0.76\pm0.07).\tag{15}$$

From Eq. (10a) we find two simple choices of coupling constants that give rise to Eq. (15). In one case the $\mathbf{8}_D$ and **27** components are absent, i.e.,

$$D=H=0\,,\quad G\neq0\tag{16a}$$

and the axial-vector current is of the form

$$A\sim\mathbf{8}_F+\mathbf{10}.\tag{16b}$$

In the other case the **10** is absent, i.e.,

$$G=0\tag{17a}$$

and the current is

$$A\sim\mathbf{8}_F+\mathbf{8}_D+\mathbf{27}.\tag{17b}$$

If the axial-vector current is given by Eq. (16), then we find that

$$\left(\frac{A}{V}\right)_{\Sigma^-\to n}=\frac{\tan\theta_A}{\tan\theta_V}\left(\frac{A}{V}\right)_{n\to p}.\tag{18}$$

Since the angles θ_A and θ_V are expected to be roughly equal to one another, it follows that the (A/V) ratio for $\Sigma^-\to n$ should be approximately equal to that for neutron β decay. A recent measurement, however, yields[12]

$$(A/V)_{\Sigma^-\to n}=-0.05_{-0.23}{}^{+0.32}\tag{19}$$

and indicates that this equality is not likely to be fulfilled. Furthermore, the branching ratio for $\Sigma^-\to n$ predicted on the basis of Eq. (16) is an order of magnitude larger than the observed one.[13] Therefore, we can discard the possibility of an axial-vector current consisting of the $\mathbf{8}_F$ and **10** representations.

Assuming that the axial-vector current satisfies Eq. (17), we find that

$$\frac{\tan\theta_V}{\tan\theta_A}\left(\frac{A}{V}\right)_{\Lambda\to p}=\left(\frac{A}{V}\right)_{n\to p}+(\sqrt{\tfrac{2}{3}})\frac{A(\Sigma^-\to\Lambda)}{\cos\theta_V}.\tag{20}$$

From Eqs. (7) and (15) and the approximation

$$\theta_A\approx\theta_V\approx0.26\,,\tag{21}$$

we then obtain

$$\left(\frac{A}{V}\right)_{\Lambda\to p}\approx0.7\quad\text{or}\quad1.7\,.\tag{22}$$

The two values in Eq. (22) correspond to the uncertainty in the absolute sign of $A(\Sigma^-\to\Lambda)$; the first ones seems closer to the experimental value[14] than the second and it also agrees with the prediction of the usual Cabibbo theory.[1] In contrast to the case of Eq. (16), the A/V ratio for $\Sigma^-\to n$ is now independent of that for neutron β decay; it still satisfies Eq. (14), and is related to the (A/V) ratio for $\Xi^-\to\Lambda$ by means

[11] N. Barash, T. B. Day, R. G. Glasser, B. Kehoe, R. Knop, B. Sechi-Zorn, and G. A. Snow, Phys. Rev. Letters **19**, 181 (1967).

[12] L. K. Gershwin, M. Alston-Garnjost, R. O. Bangerter, A. Barbaro-Galtieri, F. T. Solmitz, and R. D. Tripp, Phys. Rev. Letters **20**, 1270 (1968).

[13] N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, A. H. Rosenfeld, P. Söding, C. G. Wohl, and M. Roos, Lawrence Radiation Laboratory Report No. UCRL-8030, Pt. 1, 1968 (revised) (unpublished).

[14] J. Barlow *et al.*, Phys. Letters **18**, 64 (1965). The sign convention for (A/V) in this paper is the opposite of ours.

of the equation

$$\left(\frac{A}{V}\right)_{\Xi^-\to\Lambda}=\left(\frac{A}{V}\right)_{\Sigma^-\to n}-(\sqrt{\tfrac{2}{3}})\frac{\tan\theta_A}{\sin\theta_V}A(\Sigma^-\to\Lambda)\,. \quad (23)$$

It should be noted that Eq. (17) includes as a special case the absence of a **27** component from the axial-vector current [i.e., $H=0$ in Eqs. (10) and (17)]. Equations (20), (22), and (23) are therefore valid for both a pure octet current and for an **8**+**27** one. In the pure octet case, we obtain an additional relation

$$\left(\frac{A}{V}\right)_{\Sigma^-\to n}=\frac{\tan\theta_A}{\tan\theta_V}\left(\frac{A}{V}\right)_{n\to p}+(\sqrt{6})\frac{\tan\theta_A}{\sin\theta_V}A(\Sigma^-\to\Lambda) \quad (24)$$

which, on the basis of Eqs. (7), (15), and (21) together with a negative sign for $A(\Sigma^-\to\Lambda)$, predicts that[1]

$$\left(\frac{A}{V}\right)_{\Sigma^-\to n}\approx -0.3\,. \quad (25)$$

The measured value of this parameter [see Eq. (19)] is not accurate enough for us to conclude that Eq. (24) is satisfied.

A rough analysis of the branching ratios for semi-leptonic hyperon decay[13] suggests that when Eq. (17) holds a possible set of coupling constants might be

$$F\approx 0.4\,,\quad D\approx 0.7\,,\quad H\approx 0.04\,. \quad (26)$$

These numbers are subject to a fair amount of uncertainty because of the rather large errors in the data. Although H is much smaller than F and D in Eq. (26), it should be borne in mind that the normalization of the **27** currents in Eq. (8) is roughly four times larger than that of the F-type current (see Appendix). When this is taken into account, the admixture of **27** could be about 20% of the total current.

To summarize, we see that if the axial-vector current obeys the $\Delta U=\frac{1}{2}$ rule but does not transform as an octet, the most attractive possibility is given by Eq. (17). The most important test of this possibility is an accurate measurement of A/V ratio for $\Sigma^-\to n$. The corresponding ratios for $\Lambda\to p$ and $\Xi^-\to\Lambda$ provide tests of the $\Delta U=\frac{1}{2}$ assumption, and accurate data on $\Xi\to\Sigma$ leptonic decay would help test the consistency of the scheme.

3. CONSERVED WEAK HYPERCHARGE

According to the conserved "weak-hypercharge" hypothesis,[9,10] the axial-vector current points along a definite direction in unitary symmetry space, and it commutes with a particular generator of $SU(3)$. This generator is obtained by rotating the usual hypercharge operator through an angle 2θ about the F_7 axis,

$$Y'=e^{2i\theta F_7}Ye^{-2i\theta F_7}\,, \quad (27)$$

and the corresponding current is obtained by applying the same rotation to the $T=T_3=1$, $Y=0$ components of the **8**, **10**, **10***, and **27** representations.[15] Since the F_7 axis of $SU(3)$ coincides with the second axis of U-spin space, the effects of the rotation upon these components are governed by their U-spin properties.

In the **8** and **10** representations, the $T=T_3=1$, $Y=0$ component is the $U_3=-\frac{1}{2}$ member of a doublet; in the **10***, it belongs to a quartet; and in the **27** it is part doublet and part quartet. Under the rotation of Eq. (27), doublet terms are transformed into the admixture of $U_3=\pm\frac{1}{2}$ components given in the right-hand side of Eq. (4), and quartet terms become an admixture of all four U_3 substates with a more complicated dependence on the angle θ. Consequently the **8** and **10** representations play exactly the same roles in the conserved weak hypercharge model as they do in the $\Delta U=\frac{1}{2}$ model (see Sec. 2). The **10*** and **27**, however, do not: they are the only representations giving rise to $\Delta S=-\Delta Q$ and $\Delta S=2$ decays, and, because of the presence of $\Delta U=\frac{3}{2}$ components, their contributions to the usual $\Delta S=0$ and 1 decays no longer satisfy the $\Delta U=\frac{1}{2}$ rule.

The **8** and **10** components of the current are given in Eqs. (8) and (9), and the **10*** and **27** ones are

$$A^0=\cos\theta\{K(1-3\sin^2\theta)[10^*]_{13}{}^{23}+H''((1-2\sin^2\theta)\times[27]_{13}{}^{23}+\sin^2\theta[27]_{12}{}^{22})\}\,, \quad (28a)$$

$$A^1=\sin\theta\{K(3\sin^2\theta-2)[10^*]_{12}{}^{32}+H''((2\sin^2\theta-1)\times[27]_{12}{}^{32}+\cos^2\theta[27]_{13}{}^{33})\}\,, \quad (28b)$$

$$A^{-1}=-\cos^2\theta\sin\theta\{K[10^*]_{13}{}^{22}+H''[27]_{13}{}^{22}\}\,, \quad (28c)$$

$$A^2=-\sin^2\theta\cos\theta\{K[10^*]_{12}{}^{33}+H''[27]_{12}{}^{33}\}\,. \quad (28d)$$

In Eq. (28), A^0 and A^1 represent $\Delta S=0$ and $\Delta S=1$ currents, respectively; A^{-1} is a current with $\delta S=-\Delta Q$; and A^2 is one with $\Delta S=2$. The last two currents both obey the isospin selection rule $\Delta T=\frac{3}{2}$, and in A^0 and A^1, the **10*** components obey $\Delta T=1$ and $\Delta T=\frac{1}{2}$, respectively; for the **27** components of A^0 and A^1, however, the selection rules are $\Delta T\leq 2$ and $\Delta T\leq\frac{3}{2}$, respectively.

It follows from Eqs. (28c) and (28d) that the $\Delta S=2$ current is suppressed relative to the $\Delta S=-\Delta Q$ current by a factor $\tan\theta$. As a result, the relations

$$|A(\Xi^-\to n)|=|A(\Xi^0\to\Sigma^-)|\tan\theta\,,\qquad |A(\Xi^0\to p)|=|A(\Sigma^+\to n)|\tan\theta \quad (29)$$

are valid for all values of the coupling constants K and H''.[10] This suppression is counterbalanced by the fact that the rate for an electronic decay mode is roughly proportional to the fifth power of the mass difference between parent and daughter baryon.[16] Taking this into

[15] The $T=2$ member of the **27** could also be used to generate part of the current, but we regard it as an additional complication to be included only if absolutely necessary.

[16] See, for example, L. Wolfenstein, Phys. Rev. **135**, B1436 (1964).

account, and using $\tan\theta \approx \frac{1}{4}$, we find from Eq. (29) that

$$\Gamma(\Xi^- \to pe^-\nu^-) \approx 15\Gamma(\Xi^0 \to \Sigma^- e^+\nu), \quad \Gamma(\Xi^0 \to pe^-\nu^-) \approx 0.5\Gamma(\Sigma^+ \to ne^+\nu). \tag{30}$$

For muonic decay modes the corresponding ratios are slightly different.

Turning to $\Delta S=0$ and 1 decays, we find that, because of the complicated θ dependence in the **10*** and **27** currents [Eqs. (28a) and (28b)], there are no simple or useful relations amongst decay amplitudes. To simplify matters, we make use of the approximate equality between the absolute values of the amplitudes for $\Sigma^+ \to \Lambda$ and $\Sigma^- \to \Lambda$ [see Eq. (15)]. It turns out that there are two ways in which this can be achieved: One way is to omit the $\mathbf{8}_D$ and **27** terms of Eqs. (8) and (28) and assume an axial-vector current of the form

$$A \sim \mathbf{8}_F + \mathbf{10} + \mathbf{10}^*, \tag{31}$$

and the other is to take a current with no **10** and **10*** components, i.e.,

$$A \sim \mathbf{8}_F + \mathbf{8}_D + \mathbf{27}. \tag{32}$$

The interesting features of Eqs. (31) and (32) is that in both cases, the $\Delta S=-\Delta Q$ and $\Delta S=2$ currents belong to a single representation [see Eq. (28)].

If we assume the CVC hypothesis for the vector current [see Eq. (12)], then the axial-vector current of Eq. (31) provides a reasonable fit to the data on $\Delta S=0$ and $\Delta S=1$ decays[13] for the following choice of coupling constants in Eqs. (8) plus (28):

$$D=H''=0, \quad F\approx -0.3, \quad G\approx -0.7, \quad K\approx -0.21. \tag{33}$$

The corresponding fit for the current of Eq. (32) is

$$G=K=0, \quad F\approx 0.4, \quad D\approx 0.8, \quad H''\approx 0.01. \tag{34}$$

In both cases we have taken $\theta_V=\theta_A=0.26$. It is apparent from the values of G, K, and H'' in Eqs. (33) and (34) that the admixture of non-octet current in Eq. (31) is substantially greater than the corresponding admixture for Eq. (32). Consequently, we expect that Eq. (31) will predict a larger rate for $\Delta S=-\Delta Q$ transitions than will Eq. (32).

To show that this is indeed the case, we consider the ratio

$$R=\Gamma(\Sigma^+ \to ne^+\nu)/\Gamma(\Sigma^- \to ne^-\nu^-). \tag{35}$$

From Eq. (33) we predict that $R\approx 2$, and from Eq. (34) we obtain $R\approx 0.03$. Since the experimental upper limit on R is 0.05,[13] we conclude that Eq. (31) must be rejected and that Eq. (32) gives the form of the axial-vector current in the conserved-hypercharge model.

Because of the normalization of the various tensors (see the Appendix), the value of H'' in Eq. (34) corresponds to a 5% admixture of **27** in the axial-vector current. It is also interesting to note that as far as $\Delta S=0$ and 1 decays are concerned, there is very little difference between this case and the $\Delta U=\frac{1}{2}$ model of Eqs. (7) and (26).

4. $\Delta T=1$ AND $\frac{1}{2}$ RULES

As a third model for non-octet currents, we consider the possibility that the usual isospin rules for $\Delta S=0$ and $\Delta S=1$ decays remain valid even though the octet rule does not. Currents corresponding to $\Delta S=-\Delta Q$ and $\Delta S=2$ are not expected to appear in this model, but if necessary they could be introduced on a purely phenomenological basis.

One difference between this model and the preceding ones is that there exists no $SU(3)$ transformation which, in the presence of non-octet components, will either rotate the $\Delta T=\frac{1}{2}$ current into, or generate it out of, the $\Delta T=1$ current. In the octet, for example, the $\Delta T=\frac{1}{2}$ component can be obtained from the $\Delta T=1$ component by a rotation of 180° about the second axis of U-spin space; however, when the same rotation is applied to the $\Delta T=1$ member of the **27** it yields an admixture of $\Delta T=\frac{1}{2}$ and $\frac{3}{2}$. This negative feature is caused by the varying U-spin properties of different representations, and it has as a practical consequence the lack of any natural relationship between all the coupling constants for $\Delta S=0$ and all those for $\Delta S=1$ decays.

To make the point in another way, we note that the equality between the two sets of coupling constants in the $\Delta U=\frac{1}{2}$ model comes about because we can make the $\Delta S=1$ current a particular U-spin transform of the $\Delta S=0$ current. A similar statement can be made for the conserved-weak-hypercharge model, but it cannot be made for the one we are considering now. This is not to say that we cannot postulate some relationship between both sets of coupling constants,[17] but it does imply that any such relationship will not have much meaning from the viewpoint of unitary symmetry.

Besides the usual isospin selection rules, we shall also assume that the axial-vector current is first-class.[18] Its general form is then given by

$$A^0=F(F_1{}^2)+D(D_1{}^2) +F([10]_{31}{}^{32}-[10^*]_{31}{}^{32})+H[27]_{31}{}^{32}, \tag{36a}$$

$$A^1=F'(F_1{}^3)+D'(D_1{}^3) +G'([10]_{31}{}^{33}-[10^*]_{31}{}^{33})+H'[27]_{31}{}^{33}. \tag{36b}$$

The **10** component of A^1 is actually zero, but we have put it down in order to emphasize the first-class property. As pointed out above, there is no $SU(3)$ argument which implies a simple relation between every coupling constant in Eq. (36a) and the corresponding one in Eq. (36b). If, however, we were to take the octet components of A^0 and A^1 as members of the same octet, then their F/D ratios would be equal:

$$F/D=F'/D'. \tag{37}$$

Except for this possibility, we regard all the coupling constants of Eq. (36) as independent parameters.

[17] L. M. Nath and S. D. Dhar, Nuovo Cimento 49, 459 (1967).
[18] S. Weinberg, Phys. Rev. 112, 1375 (1958).

Whatever the choice of coupling constants may be, the $\Delta T=1$ rule for strangeness-conserving decays and the $\Delta T=\frac{1}{2}$ rule for strangeness-violating ones are sufficient to give the following relations among decay amplitudes:

$$A(\Sigma^-\to\Sigma^0)=A(\Sigma^0\to\Sigma^+)\,, \tag{38a}$$

$$A(\Sigma^-\to n)=\sqrt{2}A(\Sigma^0\to p)\,, \tag{38b}$$

$$A(\Xi^0\to\Sigma^+)=\sqrt{2}A(\Xi^-\to\Sigma^0)\,. \tag{38c}$$

In addition, the first-class requirement leads to one more relation, namely,[16]

$$A(\Sigma^-\to\Lambda)=-A(\Sigma^+\to\Lambda)\,. \tag{38d}$$

The first three parts of Eq. (38) have yet to be verified experimentally, but the fourth is consistent with present data.[11] Further predictions depend upon the $SU(3)$ properties of the currents.

In order to study these additional predictions, we consider the sum rules that hold for a pure octet current [i.e., $G=G'=H=H'=0$ in Eq. (36)]. There are two for strangeness-conserving decays, namely,

$$A(n\to p)+A(\Xi^-\to\Xi^0)=\sqrt{2}A(\Sigma^-\to\Sigma^0)\,, \tag{39a}$$

$$A(n\to p)-A(\Xi^-\to\Xi^0)=-(\sqrt{6})A(\Sigma^-\to\Lambda)\,, \tag{39b}$$

and two for strangeness-violating ones[19]:

$$A(\Lambda\to p)-A(\Xi^-\to\Lambda)=(\sqrt{\tfrac{3}{2}})[A(\Xi^0\to\Sigma^+)-A(\Sigma^-\to n)]\,, \tag{40a}$$

$$A(\Lambda\to p)+A(\Xi^-\to\Lambda)=(\sqrt{\tfrac{1}{6}})[A(\Xi^0\to\Sigma^+)+A(\Sigma^-\to n)]\,. \tag{40b}$$

If we now suppose that the axial-vector current is an admixture of the octet and one other representation, then there will be only one sum rule for each type of decay. For a current of the type **8**+**27** [i.e., $G=G'=0$ in Eq. (36)], the surviving sum rules are Eqs. (39a) and (40a); and for a current of the type **8**+(**10**−**10***) [i.e., $H=H'=0$ in Eq. (36)], they are Eqs. (39b) and (40b).[20] When all representations are present in the current, there are no valid sum rules apart from Eq. (38).

If the CVC hypothesis is correct and the vector current is pure octet, then the vector amplitudes will also satisfy Eqs. (39) and (40). It follows that whenever one of these sum rules is predicted to hold for axial-vector amplitudes, it should also hold for the corresponding total decay amplitudes.

We have already argued that, with the possible exception of the F/D ratios, the coupling constants in Eq. (36) must be treated as independent parameters in the present model. This raises a difficult problem when we attempt to determine these parameters empirically. In strangeness-conserving decay, only two pieces of data,[13] namely, $A(n\to p)$ and $A(\Sigma^-\to\Lambda)$, are available to fit four coupling constants. In strangeness-violating decay, three amplitudes, $A(\Lambda\to p)$, $A(\Sigma^-\to n)$, and $A(\Xi^-\to\Lambda)$, have been measured, and we can, in principle, fit a current consisting of the octet plus one other representation. There are, however, too many ambiguities in the signs and magnitudes of these amplitudes for us to obtain a reliable fit at present. While it is likely that the admixture of non-octet components is about the same as in the preceding models (i.e., about 10%), we cannot be certain of this without more accurate information.

In conclusion, we would like to draw attention to some interesting tests for the reducibility of the axial-vector current. Wolfenstein[16,21] has shown that if the $\Delta T=\frac{1}{2}$ rule holds, the sum rule

$$2[|A(\Lambda\to p)|^2-|A(\Xi^-\to\Lambda)|^2]=|A(\Xi^0\to\Sigma^+)|^2-|A(\Sigma^-\to n)|^2 \tag{41}$$

is satisfied when the current is either (a) pure **8**, or (b) pure (**10**−**10***), or (c) pure **27**. It can also be shown that when $\Delta T=1$, the sum rule

$$|A(n\to p)|^2-|A(\Xi^-\to\Xi^0)|^2=-2\sqrt{3}A(\Sigma^-\to\Lambda)A(\Sigma^-\to\Sigma^0) \tag{42}$$

is satisfied in each of these three cases. Therefore, if the axial-vector current is an admixture of two or more representations, neither Eq. (41) nor Eq. (42) will be satisfied. If it should turn out that one of the two sum rules is satisfied and the other is not, then we will be forced to conclude that the $\Delta S=0$ and $\Delta S=1$ currents have different $SU(3)$ structures—a possibility not allowed in either of the U-spin models discussed in Secs. 2 and 3.

5. SUMMARY AND DISCUSSION

Our general assumption in this paper has been that the octet rule breaks down in the axial-vector current but not in the vector one. Consequently, we expect that pure vector processes, e.g., $K_L{}^0\to\pi e\nu$, will obey the usual isospin and strangeness selection rules[22] (i.e., $\Delta T=\frac{1}{2}$, $\Delta S=\Delta Q=1$; and $\Delta T=1$, $\Delta S=0$), and that axial-vector processes, e.g., $K_L{}^0\to\pi\pi e\nu$, will violate at least one of them. If, in addition, baryon decays with

[19] For comparison with experimental data, these sum rules are more conveniently expressed as $A(\Lambda\to p)+2A(\Xi^-\to\Lambda)=(3/\sqrt{6})A(\Sigma^-\to n)$, $(\sqrt{6})A(\Lambda\to p)+A(\Sigma^-\to n)=2A(\Xi^0\to\Sigma^+)$.

[20] The sum rule (40b) has been obtained by several authors under various assumptions. M. Ademollo and R. Gatto [Phys. Rev. Letters **13**, 264 (1964)] derived it for second-class axial-vector octet current under first-order $SU(3)$-symmetry breaking by λ_8. K. Kawarabayashi and W. W. Wada [Phys. Rev. **137**, B1002 (1965)] and V. I. Zakharov and I. Yu. Kobzarev {Yadern. Fiz. **1**, 1050 (1965) [English transl.: Soviet J. Nucl. Phys. **1**, 749 (1965)]} derived the same sum rule for conserved vector current under second-order $SU(3)$-symmetry breaking by λ_8. For a discussion on this point see E. C. G. Sudarshan and N. Mukunda, Phys. Rev. **158**, 1424 (1967).

[21] See D. Horn, Nuovo Cimento **33**, 64 (1964).

[22] For the experimental situation on $\Delta S=-\Delta Q$ in K_{l3} decays see B. R. Webber, F. T. Solnitz, F. S. Crawford, Jr., and M. Alston-Garnjost, Phys. Rev. Letters **21**, 498 (1968); **21**, 715(E) (1968).

$\Delta S=-\Delta Q$ or $\Delta S=2$ occur (e.g., $\Sigma^+\to ne^+\nu$ or $\Xi^-\to ne^-\nu^-$), they will be pure axial-vector transitions, and the final-state electron will exhibit no asymmetry relative to the spin of the parent baryon.

The occurrence of $\Delta S=-\Delta Q$ and $\Delta S=2$ decays may also help us choose between the models discussed above. It would appear from the existing $\Delta S=0$ and $\Delta S=\Delta Q$ data that in both the $\Delta U=\frac{1}{2}$ model (Sec. 2) and in the conserved-weak-hypercharge one (Sec. 3) the axial-vector current is roughly 90% octet together with a 10% admixture of **27**. The principal difference between the models is that the $\Delta U=\frac{1}{2}$ rule strictly forbids $\Delta S=-\Delta Q$ and $\Delta S=2$ currents, whereas the conserved-weak hypercharge introduces them in a specific way. Therefore, the absence of both types of decay would lend support to the $\Delta U=\frac{1}{2}$ rule, and their occurrence together would favor the conserved weak hypercharge.

The third model we have considered (Sec. 4) requires $\Delta S=0$ and $\Delta S=\Delta Q$ decays to obey the $\Delta T=1$ and $\Delta T=\frac{1}{2}$ rules, respectively, and it differs from the other models in an important way. In both the $\Delta U=\frac{1}{2}$ and conserved-weak-hypercharge cases, there are simple relationships between currents obeying different strangeness selection rules, but in the third model these currents are independent of one another. For this reason we are not able to make a satisfactory estimate of the admixtures of non-octet components in the currents.

As far as $\Delta S=-\Delta Q$ and $\Delta S=2$ decays are concerned, the third model is rather ambiguous. If the non-octet components are perturbations on a "bare" octet current brought about by $SU(3)$-breaking strong interactions, then only $\Delta S=0$ and $\Delta S=\Delta Q=1$ decays are allowed. If, on the other hand, the non-octet components are intrinsic parts of the current, then $\Delta S=-\Delta Q$ and $\Delta S=2$ currents could come into play. From a purely phenomenological point of view the latter possibility means that if the need arises, we can introduce $\Delta S=-\Delta Q$ and $\Delta S=2$ currents on an *ad hoc* basis. In such circumstances, however, it would be difficult to understand why the $\Delta T=\frac{1}{2}$ and 1 rules should remain valid for the common semileptonic decays.

With presently existing data it is not feasible to carry the analysis beyond this point. It is therefore of considerable interest not only to obtain more accurate information on processes that have already been studied, but also to learn the empirical properties of hitherto unexamined Ξ decay modes. Another reason for caution is that in the matrix element of Eq. (6) we have neglected both the q^2 dependence of the form factor V and A, and also various induced terms (e.g., the induced pseudoscalar interaction). Since these effects are expected to be of order 10%, and since the present analysis indicates that non-octet terms may form about 10% of the current, a careful resolution between induced effects and non-octet components would require much more data than is available at present time.

We conclude this discussion by observing that the presence of non-octet components in the axial-vector current would raise general questions about the PCAC hypothesis, the algebra of currents, and the current $\times$current interaction for nonleptonic decay. In order for the PCAC hypothesis[6] not to break down, one of two conditions would have to be fulfilled: either the four-divergence of the non-octet components must vanish; or there must exist no low-lying multiplets of pseudoscalar mesons other than the (π,K,η) octet. The latter condition is consistent with the known meson spectrum[13] and it ensures that at low-momentum transfer, the pion pole is the only one available to dominate the matrix element $\langle n|\partial_\mu A_\mu{}^0|p\rangle$.

The present version of current algebra[23] would not remain valid because, in the presence of non-octet components, the axial-vector "charges" could no longer be identified as generators of chiral $SU(3)\times SU(3)$. Consequently, the commutation rule[24]

$$[A^0,(A^0)^\dagger]=2V^{(3)} \tag{43}$$

would not hold, and the basis for the Adler-Weisberger sum rule[7] and other important results may be lost. It might be possible, however, to overcome this difficulty by embedding the vector and axial-vector charges in a larger chiral algebra [for example, $R(8)\times R(8)$] in such a way that Eq. (43) is replaced by

$$[A^0,(A^0)^\dagger]=2V^{(3)}+V'. \tag{44}$$

If the matrix element of V' between nucleon states were zero, then the Adler-Weisberger sum rule could be derived from Eq. (44) in much the same way as it was obtained from Eq. (43). Whether other consequences of Eq. (43) could be retained would depend upon the other matrix elements of V'.

As far as nonleptonic decay is concerned, the existence of non-octet axial-vector terms in a current$\times$current interaction would affect the properties of both parity-conserving and parity-violating amplitudes. The interaction would no longer possess the simple chiral $SU(3)\times SU(3)$ properties which hold in the usual case, and consequently the attractive results derived from PCAC and current algebra would no longer hold.[25] It may, however, be possible to retain some of the results that depend only upon the unitary symmetry properties of the interaction, for example the forbiddenness of $K\to 2\pi$.[25]

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One of us (S.P.R.) would like to thank Professor M. Sugawara and Professor T. K. Kuo for useful conversations.

[23] M. Gell-Mann, Physics **1**, 63 (1964); Phys. Rev. **125**, 1067 (1962). See also S. L. Adler and R. F. Dashen, *Current Algebras and Applications to Particle Physics* (W. A. Benjamin, Inc., New York, 1968).

[24] A^0 is the axial-vector "charge" derived from the strangeness-conserving β-decay current, and $V^{(3)}$ is the third component of the isospin operator.

[25] S. P. Rosen and S. Pakvasa, in *Advances in Particle Physics*, edited by R. L. Cool and R. E. Marshak (John Wiley & Sons, Inc., New York, 1968), Vol. 2.

APPENDIX

The tensors used in the body of the paper are defined in terms of the baryon and antibaryon octets as follows:

$$F_\nu{}^\mu=\bar{B}_\lambda{}^\mu B_\nu{}^\lambda-\bar{B}_\nu{}^\lambda B_\lambda{}^\mu,$$

$$D_\nu{}^\mu=\bar{B}_\lambda{}^\mu B_\nu{}^\lambda+\bar{B}_\nu{}^\lambda B_\lambda{}^\mu-\tfrac{2}{3}\delta_\nu{}^\mu(\bar{B}\cdot B),$$

$$[10]_{\nu\beta}{}^{\mu\alpha}=3\sqrt{2}(1+P_{\nu\beta})(1-P^{\mu\alpha})\times[\bar{B}_\nu{}^\mu B_\beta{}^\alpha+\tfrac{1}{3}\delta_\nu{}^\mu F_\beta{}^\alpha], \quad \text{(A1)}$$

$$[10^*]_{\nu\beta}{}^{\mu\alpha}=3\sqrt{2}(1-P_{\nu\beta})(1+P^{\mu\alpha})[\bar{B}_\nu{}^\mu B_\beta{}^\alpha-\tfrac{1}{3}\delta_\nu{}^\mu F_\beta{}^\alpha],$$

$$[27]_{\nu\beta}{}^{\mu\alpha}=5\sqrt{2}(1+P_{\nu\beta})(1+P^{\mu\alpha})\times[\bar{B}_\nu{}^\mu B_\beta{}^\alpha-\tfrac{1}{5}\delta_\nu{}^\mu D_\beta{}^\alpha-\tfrac{1}{12}\delta_\nu{}^\mu\delta_\beta{}^\alpha(\bar{B}\cdot B)],$$

where $P_{\nu\beta}$ and $P^{\mu\alpha}$ are permutation operators. The normalization of the various tensor components used in Eqs. (8), (28), and (36) are

$$\|F\|=\sqrt{6}, \quad \|D\|=\sqrt{(10/3)}, \quad \|[10]\|=\|[10^*]\|=2\sqrt{3}, \quad \|[27]\|=2\sqrt{15}. \quad \text{(A2)}$$

The identification between components of $B_\nu{}^\mu$ and hyperon states is given in Ref. 26.

[26] S. P. Rosen, Phys. Rev. **137**, B431 (1965).

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Anomalies in Ward Identities for Three-Point Functions*

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We study the Ward identities for three-point functions of vector, axial-vector, scalar, and pseudoscalar densities constructed in a free-quark model. Divergences in the integral representations for the two- and three-point functions have the effect that not all of the formal Ward identities are satisfied in the model. After making full use of the ambiguities inherent in the definition of linearly divergent objects, as well as adding known polynomials to certain of the three-point functions, we find that only the Ward identities for three axial-vector densities and for one axial-vector and two vector densities remain unsatisfied. Arguments are presented that for n-point functions with $n>3$, the Ward identities are satisfied.

I. INTRODUCTION

It has been argued on the basis of the partial conservation of axial-vector current (PCAC), that the invariant coupling constant for the two-photon decay of the neutral pion vanishes.[1,2] Recently it has been recognized[1,3] that this argument must be modified whenever the axial-vector current, whose divergence is proportional to the pion field, contains bilinear products of fermion fields.[4] This anomalous behavior can be traced to the presence of a three-current triangle graph which is not well defined and which modifies the Heisenberg equation for the axial-vector current.

In the interaction picture, we may use the triangle graph to compute the Ward identity satisfied by the three-point function it represents. The anomaly now is apparent in the fact that one finds that the "naive" Ward identity expected from the commutation relations satisfied by the currents, together with PCAC, does not hold. The reason for this can be traced to an ambiguity in the two-point functions appearing in the Ward identity.

The breakdown of the Ward identities for the neutral-axial-vector current two-photon three-point function throws doubt on the validity of hard-pion calculations[5]

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[1] J. S. Bell and R. Jackiw, Nuovo Cimento **60**, 47 (1969).
[2] D. G. Sutherland, Nucl. Phys. **B2**, 433 (1967).
[3] These anomalies were first discussed by J. Schwinger, Phys. Rev. **82**, 664 (1951). Contemporary examination of this problem is found in Ref. (1) and S. Adler, Phys. Rev. **177**, 2426 (1969); C. R. Hagen, **177**, 2622 (1969); R. Jackiw and K. Johnson, *ibid.* (to be published); K. G. Wilson, *ibid.* **179**, 1499 (1969).
[4] Similar anomalies will arise in the algebra of fields [T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 1029 (1967)] because the divergence of the field current variable is proportional to the Noether current, which is bilinear in the fermion fields.
[5] The bibliography of hard-pion calculations is quite extensive by now. For work on three-point functions, see H. J. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967); I. S. Gerstein, H. J. Schnitzer and S. Weinberg, *ibid.* **175**, 1873 (1968); I. S. Gerstein and H. J. Schnitzer, *ibid.* **175**, 1876 (1968); for an equivalent phenomenological Lagrangian approach, see J. Wess and B. Zumino, *ibid.* **163**, 1727 (1967); B. W. Lee and H. T. Nieh, *ibid.* **166**, 1507 (1968); R. Arnowitt, M. Friedman, and P. Nath, *ibid.* **174**, 1999 (1968). This list is not meant to be exhaustive and we apologize to authors whose work we did not mention.