spirator, suppresses its pseudoscalar nature, and was eventually rejected as a possibility. Since the pion has a small mass, Reggeization of the pion is not expected to affect these results substantially. It is also the case that an ordinary Ferrari-Selleri[18] over-all form factor does not affect the higher partial waves. Given that the π^+-exchange pole is present in the np charge-exchange amplitude, it would seem that some sort of vertex form factor is necessary to fit the differential cross-section data.

I am extremely grateful to Dr. R. J. N. Phillips for several discussions and communications, and to Mrs. E. Oxborrow and Dr. E. B. Spratt for the use of the computer.

[18] E. Ferrari and F. Selleri, Nuovo Cimento Suppl. **24**, 453 (1962).

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Certain Implications for High-Energy Total Cross Sections from the Multiperipheral Model*

PETER D. TING
Lawrence Radiation Laboratory, University of California, Berkeley, California 94720
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The multiperipheral model is used to relate the experimental fact, $\sigma_{N\bar{N}}^{\text{tot}} > \sigma_{NN}^{\text{tot}}$, to the possibility that the $N\bar{N}$ channel can annihilate into purely meson final states by exchange of N or $\bar{N}$ trajectories, whereas the NN channel cannot do so. Our argument shows that from the multi-Regge bootstrap point of view, the combination of baryon and antibaryon trajectories will generate a pair of exchange-degenerate trajectories P' and ω. The same line of reasoning can easily be generalized to include strange particles, as well as the (K^-N,K^+N) and (π^-p,π^+p) systems.

I. INTRODUCTION

It is a familiar empirical fact that, if a larger number of two-particle channels communicate with channel $a\bar{b}$ than with channel ab, then the total cross section $\sigma_{a\bar{b}}$ tends to be larger than σ_{ab} at nonasymptotic energies. For example, the $p\bar{p}$ channel can communicate with $p\bar{p}$, $\pi^0\pi^0$, $\pi^+\pi^-$, $K^0\bar{K}^0$, K^+K^-, $\Lambda\bar{\Lambda}$, $\Sigma^0\bar{\Sigma}^0$, etc., whereas the pp channel can only communicate with pp. It is well known that $\sigma_{p\bar{p}} > \sigma_{pp}$. A similar correlation exists for the paired channels, (K^-p,K^+p), (π^-p,π^+p). In this paper we propose to relate these phenomena to the generalized multiperipheral model recently developed by Chew, Goldberger, and Low[1] (CGL). The type of reasoning to be applied is illustrated by the following argument: If more two-particle channels communicate with $a\bar{b}$ than with ab, we will get more unitarity box diagrams (two-particle unitarity contributions) in $a\bar{b}$ than ab. The multiperipheral model iterates the box diagrams to produce a series of multiparticle contributions to the cross section (i.e., to the forward-elastic unitarity integral). Since each such contribution is positive definite, it follows that the channel with the larger number of box diagrams has the larger total cross section, *if* we can demonstrate an equal magnitude for those box diagrams shared by the two channels.

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[1] G. Chew, M. Goldberger, and F. Low, Phys. Rev. Letters **22**, 208 (1969).

In Sec. II, the multiperipheral model of CGL has been generalized to exchange several trajectories. In Sec. III, the multiperipheral model is applied to NN and $N\bar{N}$ channels. Some Reggeon bootstrap arguments are given in Sec. IV. In Sec. V some general remarks are made to include the strangeness and the isospin in the model.

II. MULTIPERIPHERAL MODEL

Let us briefly review the multiperipheral model. A function called B_a was introduced in CGL by

$$A_{ab}(p_a,p_b) \equiv A_{ab}(s,0) = \int B_a(p_a,p_b;Q')(G^b(Q'))^2 \times \delta^+[(Q'-p_b)^2-\mu'^2]d^4Q', \quad (1)$$

where $A_{ab}(p_a,p_b)$ is the absorptive part of the $ab\to ab$ elastic amplitude at forward direction. The function B_a satisfies the integral equation

$$B_a(p_a,p_b;Q') = I_a(p_a,p_b;Q') + \int B_a(p_a,Q';Q)K(Q,Q',p_b) \times \delta^+[(Q-Q')^2-\mu^2]d^4Q, \quad (2)$$

where I_a corresponds to two-particle unitarity, namely,

$$I_a(p_a,p_b;Q') = [G^a(Q')]^2 |f(p_a,Q',p_b)|^2, \quad (3)$$

while the kernel K is given by

$$K(Q,Q',p_b)=\beta^2(Q,Q')\,|f(Q,Q',p_b)|^2. \qquad (4)$$

In a multi-Regge model, G and β are the external and internal vertex couplings, respectively, and f is the factor $(s_i/s_0)^\alpha$. The kernel can take other forms in other multiperipheral models. Equations (1) and (2) can be written symbolically as

$$A_{ab}=\int B_a(G^b)^2$$

and

$$B_a=I_a+\int B_aK.$$

The CGL equations assumed only one kind of trajectory to be exchanged. If several Reggeons $\alpha_1,\alpha_2,\cdots\alpha_m$ are exchanged, these equations should be written

$$A_{ab}=\sum_x\sum_{i=1}^m\int B_a{}^{\alpha_i}(G_x{}^{\alpha_i b})^2, \qquad (5)$$

$$B_a{}^{\alpha_i}=I_a{}^{\alpha_i}+\sum_y\sum_{j=1}^m\int B_a{}^{\alpha_j}K_y{}^{\alpha_j\alpha_i}, \qquad (6)$$

with [see Fig. 1(b)]

$$I_a{}^{\alpha_i}=\sum_z(G_z{}^{\bar{a}\alpha_i})^2|f^{\alpha_i}|^2, \qquad (7)$$

$$K_y{}^{\alpha_j\alpha_i}=(\beta_y{}^{\alpha_j\alpha_i})^2|f^{\alpha_i}|^2, \qquad (8)$$

where $G_x{}^{\alpha_i b}$ is the external coupling of particles b and x to the Reggeon α_i, in the sense of Fig. 1(a), and similarly for $G_z{}^{\bar{a}\alpha_i}$; $\beta_y{}^{\alpha_j\alpha_i}$ is the internal coupling of the two Reggeons α_i and α_j to the particle y, in the sense of Fig. 1(b); and $\sum_x$ means to sum over all stable particles x which conserve baryon number, strangeness, and isospin[2] at the end vertex coupling b, and similarly for the symbols $\sum_y$ and $\sum_z$.

III. MULTIPERPHERAL MODEL IN NN AND $N\bar{N}$ CHANNELS

For the sake of simplicity and clarity, we begin with the special case of NN and $N\bar{N}$ elastic scattering and include as exchanged trajectories only the nucleon N (or Δ), the antinucleons $\bar{N}$ (or $\bar{\Delta}$), and an averaged effective nonstrange meson M (including both signatures).[3] The justification for using only one kind of meson is that the important nonstrange mesons like P', ρ, ω, and A_2 have roughly the same intercept and slope at $t=0$ on the Chew-Frautschi plot. The Pomeranchon P is different but there are arguments[4,5] against the importance of Pomeranchon repetition in the multiperipheral chain. Of course, in the lowest-order unitarity box diagrams, we have to consider the Pomeranchon as an important trajectory. Hence, in the following discussion the Pomeranchon appears only in unitarity box diagrams. To make this feature explicit, we rewrite Eqs. (5) and (6) as

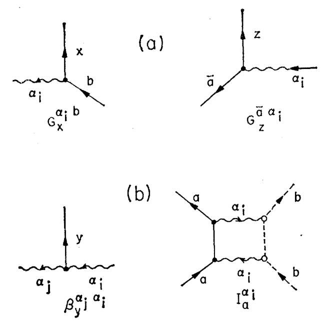

FIG. 1. (a) External couplings $G_x{}^{\alpha_i b}$ and $G_z{}^{\bar{a}\alpha_i}$. (b) Internal coupling $\beta_y{}^{\alpha_j\alpha_i}$, and the two-particle unitarity contribution $I_a{}^{\alpha_i}$.

$$A_{NN}=A_{NN}{}^P+\sum_{x,i=M,N,\bar{N}}\int B_N{}^i(G_x{}^{iN})^2, \qquad (9a)$$

$$A_{N\bar{N}}=A_{N\bar{N}}{}^P+\sum_{x,i=M,N,\bar{N}}\int B_N{}^i(G_x{}^{i\bar{N}})^2, \qquad (9b)$$

$$B_N{}^i=I_N{}^i+\sum_{y,j=M,N,\bar{N}}\int B_N{}^jK_y{}^{ji},\quad i=M,N,\bar{N}, \qquad (10)$$

where

$$A_{ab}{}^P=\int[|T_{ab}{}^P|^2+(T_{ab}{}^P)^*(T_{ab}{}^M)+(T_{ab}{}^M)^*(T_{ab}{}^P)] \qquad (11)$$

and

$$T_{ab}{}^\alpha=\sum_x I_a{}^\alpha(G_x{}^{\alpha b})^2=\sum_{z,x}(G_z{}^{\bar{a}\alpha})^2|f^\alpha|^2(G_x{}^{\alpha b})^2, \qquad (12)$$

where $T_{ab}{}^\alpha$ is the part of the elastic amplitude where only the α trajectory is exchanged, and P denotes "Pomeranchon" and M our average effective meson.

Now we propose to argue that $A_{NN}{}^P\approx A_{N\bar{N}}{}^P$. From the experimental data, the total elastic cross sections[6] for NN and $N\bar{N}$ are roughly equal[7] for $p_L>2$ GeV/c,

[2] We treat all particles as spinless. This is equivalent to summing and averaging over spin orientations.

[3] For simplicity we neglect isospin in the calculation, because the isospin dependence of NN and $N\bar{N}$ elastic amplitude is very weak, but we should consider isospin in calculating A_{π^+p} and A_{π^-p}.

[4] G. Chew and A. Pignotti, Phys. Rev. **176**, 2112 (1968).

[5] Chew and Pignotti (Ref. 4) have argued that the internal coupling of the Pomeranchon is small compared with internal couplings which involve P', ρ, ω, and A_2. Some physicists even think that, because the Pomeranchon trajectory is associated with diffractive phenomena, multiple Pomeranchon exchange is meaningless.

[6] Yoshio Sumi and Toshihiro Yoshida, Progr. Theoret. Phys. (Kyoto) Suppl. **41**, 53 (1967); **42**, 53 (1967).

[7] The combination of $\sigma_{N\bar{N}}>\sigma_{NN}$ with $\sigma_{N\bar{N}}{}^{\rm el}\approx\sigma_{NN}{}^{\rm el}$ implies the crossover in the elastic scattering angular distribution. We wish to thank Dr. G. Fox for pointing out this fact.

i.e., $\sigma_{NN}{}^{\text{el}} \approx \sigma_{N\bar{N}}{}^{\text{el}}$, but by definition

$$64\pi^2 s\sigma_{NN}{}^{\text{el}} \equiv A_{NN}{}^{P} + \int |T_{NN}{}^{M}|^2 ,$$

$$64\pi^2 s\sigma_{N\bar{N}}{}^{\text{el}} \equiv A_{N\bar{N}}{}^{P} + \int |T_{N\bar{N}}{}^{M}|^2 .$$

Thus our first task is to show that

$$\int |T_{NN}{}^{M}|^2 = \int |T_{N\bar{N}}{}^{M}|^2 .$$

Rewrite

$$T_{ab}{}^{M} = T_{ab}{}^{M(+)} + T_{ab}{}^{M(-)} ,$$

where $M(+)$ denote the effective meson trajectory with signature $+$, and $M(-)$ denote the effective meson trajectory with signature $-$.

Now $M(+)$ and $M(-)$ have the same intercept and slope at $t=0$, so the usual Regge signature factor $1 \pm e^{-i\pi\alpha}$ leads to a 90-deg phase difference between $T_{ab}{}^{M(+)}$ and $T_{ab}{}^{M(-)}$. Thus $|T_{ab}{}^{M}|^2 = |T_{ab}{}^{M(+)}|^2 + |T_{ab}{}^{M(-)}|^2$. Then, since the CPT theorem implies $|T_{ab}{}^{M(\pm)}|^2 = |T_{a\bar{b}}{}^{M(\pm)}|^2$, it follows that $A_{NN}{}^{P} = A_{N\bar{N}}{}^{P}$.

Next if we subtract Eq. (9a) from (9b), we get

$$A_{N\bar{N}} - A_{NN} = \sum_{i=M,N,\bar{N}} \int B_N{}^{i} \Big[\sum_{x} (G_x{}^{i\bar{N}})^2 - \sum_{x'} (G_{x'}{}^{iN})^2 \Big] . \tag{13}$$

From the dearth of stable $B=2$, $s=0$ baryons produced in high energy collisions and conversely the abundance of nonstrange meson ($B=0, s=0$) production, we conclude that $G_x{}^{N\bar{N}} \approx 0$, $K_y{}^{N\bar{N}} \approx 0$, and $I_N{}^{N} \approx 0$. By the charge conjugation we also know

$$(G_x{}^{NN})^2 = (G_x{}^{\bar{N}\bar{N}})^2 , \quad (G_x{}^{\bar{N}N})^2 = (G_x{}^{N\bar{N}})^2 , \quad \alpha_N = \alpha_{\bar{N}} ,$$
$$K_x{}^{NN} = K_x{}^{\bar{N}\bar{N}} , \quad \text{and } K_y{}^{N\bar{N}} = K_y{}^{\bar{N}N} .$$

Equations (10) and (13) can then be reduced to

$$A_{N\bar{N}} - A_{NN} \approx \int B_N{}^{M} (G_{\bar{N}}{}^{M\bar{N}})^2 - (G_N{}^{MN})^2 + \sum_{x} \int (B_N{}^{\bar{N}} - B_N{}^{N})(G_x{}^{\bar{N}\bar{N}})^2 , \tag{14}$$

$$B_N{}^{\bar{N}} - B_N{}^{N} \approx I_N{}^{\bar{N}} + \int B_N{}^{M} (K_N{}^{M\bar{N}} - K_{\bar{N}}{}^{MN}) + \sum_{y} \int (B_N{}^{\bar{N}} - B_N{}^{N}) K_y{}^{\bar{N}\bar{N}} . \tag{15}$$

Because the symbol M in our model contains both positive- and negative-signature mesons, in general, the CPT theorem *will not* give $(G_N{}^{MN})^2 = (G_{\bar{N}}{}^{M\bar{N}})^2$, $K_N{}^{MN} = K_{\bar{N}}{}^{M\bar{N}}$, and $K_{\bar{N}}{}^{NM} = K_N{}^{\bar{N}M}$. However, from the rough equality $\sigma_{NN}{}^{\text{el}} \approx \sigma_{N\bar{N}}{}^{\text{el}}$, exchange degeneracy among mesons, and other experimental results,[8–10] we believe it would be a reasonable zeroth-order approximation to take $\int B_N{}^{M} (G^{MN})^2 \approx \int B_N{}^{M} (G^{M\bar{N}})^2$ and $\int B_N{}^{M} K_N{}^{MN} \approx \int B_N{}^{M} K_{\bar{N}}{}^{M\bar{N}}$ as our input. Then Eqs. (14) and (15) will become

$$A_{N\bar{N}} - A_{NN} = \sum_{x} \int (B_N{}^{\bar{N}} - B_N{}^{N})(G_x{}^{\bar{N}\bar{N}})^2 , \tag{16}$$

$$B_N{}^{\bar{N}} - B_N{}^{N} = I_N{}^{\bar{N}} + \sum_{y} \int (B_N{}^{\bar{N}} - B_N{}^{N}) K_y{}^{\bar{N}\bar{N}} . \tag{17}$$

Since all quantities in Eqs. (16) and (17) are evaluated at forward direction, both $I_N{}^{\bar{N}}$ and $K_y{}^{N\bar{N}}$ are positive definite; Eq. (17) can be solved by iteration, so both $(B_N{}^{\bar{N}} - B_N{}^{N})$ and $(A_{N\bar{N}} - A_{NN})$ must be positive. From the optical theorem we have thus shown that $\sigma_{N\bar{N}} > \sigma_{NN}$ at nonasymptotic energies.

The physical meaning behind Eqs. (16) and (17) is very simple. Within our model the difference $\sigma_{NN} - \sigma_{N\bar{N}}$ arises because the $N\bar{N}$ channel can annihilate into pure meson final states[11] by exchange of N (or $\bar{N}$) trajectories while the NN channel cannot.

IV. REGGEON BOOTSTRAP

In order to calculate $A_{N\bar{N}} - A_{NN}$ explicitly, we must assume explicit forms for the kernel $K_y{}^{\bar{N}\bar{N}}$ as well as for $I_N{}^{\bar{N}}$ and $G^{N\bar{N}}$, a step we do not propose to take here. However, we wish to discuss this problem from the bootstrap point of view. In the usual Regge theory, if we neglect isospin dependence in $N\bar{N}$ and NN, Regge fitting says

$$A_{N\bar{N}} \approx \beta_P (s/s_0)^{\alpha_P(0)} + \beta_{P'} (s/s_0)^{\alpha_{P'}(0)} + \beta_\omega (s/s_0)^{\alpha_\omega(0)} , \tag{18}$$

$$A_{NN} \approx \beta_P (s/s_0)^{\alpha_P(0)} + \beta_{P'} (s/s_0)^{\alpha_{P'}(0)} - \beta_\omega (s/s_0)^{\alpha_\omega(0)} , \tag{19}$$

at $t=0$, where s_0 is a constant. Then

$$A_{N\bar{N}} - A_{NN} \approx 2\beta_\omega (s/s_0)^{\alpha_\omega(0)} . \tag{20}$$

[8] Atsuko Iwaki, Progr. Theoret. Phys. (Kyoto) Suppl. **41**, 316 (1967); **42**, 316 (1967).

[9] G. Alexander, O. Benary, and U. Maor, Nucl. Phys. **B5**, 1 (1968).

[10] From Refs. 8 and 9, we get

$$\sum_{m=1}^{\infty} \sigma_{p\bar{p}\to N\bar{N}+m\pi} \approx \sum_{m=1}^{\infty} \sigma_{pp\to NN+m\pi} \quad \text{for } 2\ \text{GeV}/c < p_L ,$$
$$\sigma_{p\bar{p}\to p\bar{p}\pi^+\pi^-} \approx \sigma_{pp\to pp\pi^+\pi^-} \quad \text{for } 2\ \text{GeV}/c < p_L ,$$
$$\sigma_{p\bar{p}\to N\bar{N}+3\pi} \approx \sigma_{pp\to NN+3\pi} \quad \text{for } 2\ \text{GeV}/c < p_L ,$$
$$\sum_{m=2}^{\infty} \sigma_{p\bar{p}\to m\pi} \approx (\sigma_{p\bar{p}} - \sigma_{pp}) \quad \text{for } 3\ \text{GeV}/c < p_L < 7\ \text{GeV}/c ,$$
$$\sum_{m=2}^{\infty} \sigma_{p\bar{p}\to m\pi} \gg \sum_{m=1}^{\infty} (\sigma_{p\bar{p}\to N\bar{N}+m\pi} - \sigma_{pp\to NN+m\pi}) \quad \text{for } 3\ \text{GeV}/c < p_L < 7\ \text{GeV}/c .$$

[11] Because we have not considered K^* and Y^* (or Λ, Σ) exchange in this calculation, we neglect those final states which involve strange particles. See the concluding paragraph of this paper.

Comparing Eq. (20) with Eqs. (16) and (17), we get a result of interest to bootstrap arguments, since it can be proved that equations of the latter form lead to Regge poles. According to Eqs. (16) and (17) our model forms a combination of baryon and antibaryon trajectories. The meson-meson contribution, we suggest, is small.

Furthermore, we notice that in high-energy production process, mainly mesons are produced; $(N\bar{N})$ pairs are rare.[12] This implies $\beta_N{}^{\bar{N}M}, \beta_{\bar{N}}{}^{NM} \ll \beta_x{}^{MM}$, so $K_N{}^{JM}, K_{\bar{N}}{}^{NM} \ll K_{y'}{}^{MM}$, and $K_N{}^{MN}, K_{\bar{N}}{}^{M\bar{N}} \ll K_y{}^{\bar{N}\bar{N}}$. If we accept this experimental fact, the meson and nucleon channels are decoupled. That is, from Eqs. (9a), (9b), and (10), we get

$$A_{N\bar{N}}+A_{NN}\approx 2A_{N\bar{N}}{}^P+2\int B_N{}^M(G_{\bar{N}}{}^{M\bar{N}})^2 +\sum_x \int (B_N{}^{\bar{N}}+B_N{}^N)(G_x{}^{\bar{N}\bar{N}})^2, \quad (21)$$

$$B_N{}^M\approx I_N{}^M+\sum_{y'}\int B_N{}^M K_{y'}{}^{MM} +\int (B_N{}^{\bar{N}}K_N{}^{\bar{N}M}+B_N{}^N K_{\bar{N}}{}^{NM}) \approx I_N{}^M+\sum_{y'}\int B_N{}^M K_{y'}{}^{MM}, \quad (22)$$

$$(B_N{}^{\bar{N}}+B_N{}^N)\approx I_N{}^{\bar{N}}+\int B_N{}^M(K_N{}^{MN}+K_{\bar{N}}{}^{M\bar{N}}) +\sum_y \int (B_N{}^{\bar{N}}+B_N{}^N)K_y{}^{\bar{N}\bar{N}} \approx I_N{}^{\bar{N}}+\sum_y \int (B_N{}^{\bar{N}}+B_N{}^N)K_y{}^{\bar{N}\bar{N}}. \quad (23)$$

By adding Eqs. (18) and (19), we get

$$A_{N\bar{N}}+A_{NN}\approx 2\beta_P(s/s_0)^{\alpha_P(0)}+2\beta_{P'}(s/s_0)^{\alpha_{P'}(0)}. \quad (24)$$

Because the kernels of Eqs. (23) and (17) are identical, it follows that besides the ω trajectory, the combination of N and $\bar{N}$ trajectories will generate another meson trajectory $\alpha_{P'}$ which has the same intercept and slope as ω. The $\alpha_{P'}$ contributes only to $(A_{N\bar{N}}+A_{NN})$, not to $(A_{N\bar{N}}-A_{NN})$, whereas for ω the converse is true; i.e., $\alpha_{P'}$ has positive signature, α_ω has negative signature. Thus our model gives an explanation of the exchange degeneracy between $\alpha_{P'}$ and α_ω.

The effective meson trajectory M may be expected to generate an even signature Regge pole, which Chew and Pignotti[4] (CP) have identified with Pomeranchon. For more discussion of bootstrap questions, see CGL[1] and CP.[4]

V. SUMMARY AND DISCUSSION

The above line of reasoning is easily generalized to include strange particles. The key points in our previous argument, leading to partial approximate diagonalization of the multiperipheral kernel, were the low rates of production of $B=2$ particles and $B\bar{B}$ pairs. These points remain true in general. Furthermore, the low rate of production of mesons with $B=0$, $|S|=2$; $B=0$, $S=0$, $I=2$, together with the fact that the nucleon and kaon have $I=\frac{1}{2}$, allows a partial diagonalization with respect to $A_{\bar{K}N}\pm A_{KN}$ and with respect to $A_{\bar{\pi}N}\pm A_{\pi N}$, by arguments which completely parallel the preceding. We then can conclude that $\sigma_{K^-p}>\sigma_{K^+p}$, $\sigma_{\pi^-p}>\sigma_{\pi^+p}$. We do not attempt here to exhaust all applications of our general argument. The principle should be clear from the above examples.

ACKNOWLEDGMENTS

It is a pleasure to thank Professor Geoffrey Chew for suggesting this problem and for his unfailing advice and encouragement. I would also like to thank Dr. A. Pignotti, Dr. G. Fox, and C. K. Chen for helpful discussions.

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[12] V. Barashenkov, M. Valtsev, I. Patera, and V. Toneev, Fortschr. Physik **14**, 357 (1966).