# Self-Consistent Multiple-Quark-Scattering Analysis of Consistency Relations among $pp$, $p\bar{p}$, $\pi p$, and $\pi\pi$ Cross Sections*

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The self-consistent multiple-quark-scattering (SCMQS) picture that was previously used to analyze and account for the details of the $pp$ differential cross section at high energy is here extended to a comprehensive analysis of high-energy hadron scattering according to a set of consistency relations among $pp$, $p\bar{p}$, $\pi p$, and $\pi\pi$ scattering cross sections and the form factors of $p$ and $\pi$. The consistency relations are a consequence of the SCMQS picture and the composite $SU_3$-quark model which has been suggested on non-group-theoretical grounds by the results of the SCMQS analysis of $pp$ scattering, which also indicate very small pointlike quarks. The essence of the consistency relations is illustrated with the first-order single-quark-scattering analysis, and then they are analyzed according to the generalized higher-order SCMQS treatment. Results of the higher-order SCMQS analysis of $\pi p$ scattering indicate that the pion radius and the inverse diffraction width for $\pi\pi$ diffraction are significantly smaller than indicated by the previously reported results of the single-scattering treatment. We obtain here $r_\pi{}^E \cong 0.36\pm0.12$ F and $\xi_{\pi\pi}\cong 4.8$ (BeV/$c$)$^{-2}$, respectively, for the pion charge radius and the inverse diffraction width for $\pi\pi$ scattering. Our results also indicate that the pion form factor falls off rapidly at high momentum transfer. Multiple-scattering effects and the interferences among them are seen to be very important characteristics of high-energy hadron-scattering phenomena.

## I. INTRODUCTION

On the basis of a self-consistent multiple-quark-scattering (SCMQS) picture the details of the broken-slope structure of the differential cross section for high-energy elastic proton-proton ($pp$) scattering have been successfully analyzed.[1] The main viewpoint of the SCMQS picture is to interpret high-energy collisions of hadrons as if the hadrons behave as though they were comprised effectively of subparticles, quarks, which contribute essentially individually to multiple internal scattering processes within the hadron. The elastic scatterings of hadrons at large momentum transfers are viewed as being due to the cumulative effect of multiple internal (small-angle) elastic scattering processes. Although our discussion refers to a quark picture, nearly every other aspect of our method can also be applied directly in a picture in which the multiple scattering occurs between continuous (or other) matter distributions. Multiple scattering appears to be the important idea, while the quark model and the diffraction-scattering mechanism afford a convenient, simplified operating medium in which to illustrate the basic ideas and methods at first.

The SCMQS picture leads to consistency relations among the differential cross sections for elastic $pp$, $p\bar{p}$, $\pi p$, and $\pi\pi$ scattering and the effective form factors of $p$ and $\pi$. A summary of the first-order scattering treatment and results of these consistency relations has already been reported.[2] Considering only first-order $QQ$ scattering effects, these consistency relations are the same as those derived from the hypothesis of the dominance of exchange of the Pomeranchukon Regge pole with factorizable residues proportional to the charge form factors. However, our present treatment of the SCMQS picture is essentially semiclassical, so the derivation of the consistency relations on this basis is conceptually simpler. Also, the extension of these relations beyond first order is directly understandable in the SCMQS picture, whereas in the Regge picture the corresponding extension involves the Regge cuts. In fact, the multiple-scattering picture may afford one of the most likely ways of studying the Regge cuts by considering the fundamental first-order scattering amplitude as an ordinary simple Regge-pole amplitude (or a sum of simple Regge-pole amplitudes), and considering the Regge-cut effects as generated through the higher-order multiple-scattering series.[3]

It is the purpose of this paper to discuss these consistency relations, not only in the first-order scattering treatment, but also to include the effects of higher-order multiple-scattering contributions as was done for the detailed analysis of $pp$ scattering mentioned above.

In Sec. II, the multiple-scattering formalism, previously developed for the $pp$ analysis, is recollected. In

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Sec. III, the development and results of the self-consistent analysis of the single-channel problem of $pp$ scattering that are used in the present, further analysis are summarized. In Sec. IV, the essence of the consistency relations among several channels in the SCMQS picture is illustrated with the first-order linear quark-scattering analysis. In Sec. V, the consistency relations are analyzed with the generalized higher-order SCMQS treatment with the results that the pion charge radius and inverse diffraction width for $\pi\pi$ scattering are indicated to be somewhat smaller than the results of the first-order treatment.[2] In Sec. VI, the consequences of our results with the SCMQS picture and their more general implications are discussed.

## II. REVIEW OF THE GENERAL FORMALISM

The SCMQS formalism has been previously described with the treatment of $pp$ scattering in Ref. 1. Here we only summarize the formalism developed for the analysis of $pp$ scattering in order to establish notation and terminology in a simple case where, with charge symmetry, all the $Q$'s are similar.

In addition to the basic assumption that large-angle hadron scattering be viewed as the cumulative result of multiple internal scattering within the hadrons, we also assume that the probability distribution for each internal single $QQ$ scattering is so diffractively narrow that longitudinal momentum transfers may be ingored in first approximation. We further assume that we may ignore or average over spin and isospin effects and any effects due to internal motion of $Q$'s within the initial and final states of the hadrons.

The amplitude for scattering of a composite system of $A$ similar $Q$'s from a composite system of $B$ similar $Q$'s may be developed into a multiple-scattering series as

$$f_{AB}(\mathbf{q})=\sum_{n=1}^{AB} f_{AB}{}^{(n)}(\mathbf{q})\,,\quad 1\leqslant n\leqslant AB. \tag{1}$$

The single-scattering contribution is

$$f_{AB}{}^{(1)}(\mathbf{q})=AB\Phi_A(\mathbf{q},0,0,\cdots)\Phi_B(\mathbf{q},0,0,\cdots)f(\mathbf{q})\,. \tag{2}$$

The double-scattering contribution is

$$f_{AB}{}^{(2)}(\mathbf{q})=\frac{i}{2\pi}\int d^2p\; f(\mathbf{p})f(\mathbf{q}-\mathbf{p})\left\{\left[\binom{B}{1}\binom{A}{2}\Phi_B(\mathbf{q},0,\cdots)\Phi_A(\mathbf{q}-\mathbf{p},\mathbf{p},0,\cdots)\right.\right.$$
$$\left.\left.+\binom{B}{2}\binom{A}{2}\Phi_B(\mathbf{q}-\mathbf{p},\mathbf{p},0,\cdots)\Phi_A(\mathbf{q}-\mathbf{p},\mathbf{p},0,\cdots)\right]+[A\rightleftharpoons B]\right\}. \tag{3}$$

The triple-scattering contribution is

$$f_{AB}{}^{(3)}(\mathbf{q})=\left(\frac{i}{2\pi}\right)^2\int d^2pd^2k\; f(\mathbf{p})f(\mathbf{k})f(\mathbf{q}-\mathbf{p}-\mathbf{k})\left\{\left[\binom{B}{1}\binom{A}{3}\Phi_B(\mathbf{q},0,\cdots)\Phi_A(\mathbf{p},\mathbf{k},\mathbf{q}-\mathbf{p}-\mathbf{k},0,\cdots)\right.\right.$$
$$+6\binom{B}{2}\binom{A}{3}\Phi_B(\mathbf{p},\mathbf{q}-\mathbf{p},0,\cdots)\Phi_A(\mathbf{p},\mathbf{k},\mathbf{q}-\mathbf{p}-\mathbf{k},0,\cdots)+3\binom{B}{3}\binom{A}{3}\Phi_B(\mathbf{p},\mathbf{k},\mathbf{q}-\mathbf{p}-\mathbf{k},0,\cdots)$$
$$\left.\left.\times\Phi_A(\mathbf{p},\mathbf{k},\mathbf{q}-\mathbf{p}-\mathbf{k},0,\cdots)+2\binom{B}{2}\binom{A}{2}\Phi_B(\mathbf{p},\mathbf{q}-\mathbf{p},0,\cdots)\Phi_A(\mathbf{k},\mathbf{q}-\mathbf{k},0,\cdots)\right]+[A\rightleftharpoons B]\right\}, \tag{4}$$

and so on. In these amplitudes, $f(\mathbf{q})$ is the effective scattering amplitude for the fundamental $QQ$ scattering process, in which it is assumed that longitudinal momentum transfers may be neglected, and in the interest of simplicity, the effective scattering amplitude between any pair of $Q$'s is, for now, considered to be the same. Only slightly more complicated forms will be required when more than one nonidentical fundamental process, such as $QQ$ and $Q\bar{Q}$ scattering, are involved.

The $\Phi_A(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3,\cdots,\mathbf{q}_A)$ which occurs in the above amplitude is the effective generalized many-body form factor of the distributions of $Q$'s within the composite $A$. In the particular case when $\mathbf{q}_n=0$ for $n\neq 1$, the $\Phi_A$ reduces to the effective single-$Q$ body form factor of $A$, which is related to the usual effective electromagnetic form factor of $A$, $G_A{}^E(\mathbf{q})$, by

$$G_A{}^E(\mathbf{q})=\Phi_A(\mathbf{q})G_Q{}^E(\mathbf{q})\,, \tag{5}$$

where $G_Q{}^E(\mathbf{q})$ is the effective electromagnetic form factor of $Q$. There seems to be no need to concern ourselves for now with distinctions between effective electric and magnetic form factors. They are very similar for the proton, and the pion has only a single electromagnetic form factor.

## III. SELF-CONSISTENT ANALYSIS OF $pp$ SCATTERING

The self-consistency of the method established in the analysis of $pp$ scattering is essentially threefold: (i)

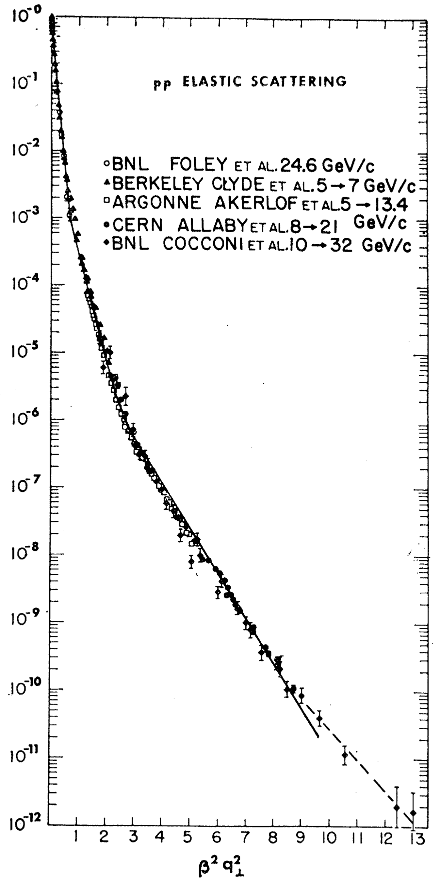


FIG. 1. Results of SCMQS analysis of $pp$ cross section compared with experimental data of $X\equiv(d\sigma^\dagger/dt)/(d\sigma^\dagger/dt)_{t=0}$ versus $\beta^2\mathbf{q}_\perp^2$ (BeV/$c$)$^2$ as described in Sec. III of the present paper and in Ref. 1. The data are from Ref. 11, and we are grateful to Professor A. D. Krisch for permission to reproduce his plot here.

Since the formalism directly involves only the effective $QQ$ scattering amplitudes and the effective generalized many-body form factors of the distributions of quarks within the hadrons, minimal conjectures about these phenomenological functions are introduced directly without having to determine the internal wave functions. (ii) The next aspect of the self-consistent program is the determination of the effective fundamental $QQ$ scattering amplitude by extracting it from the analysis of the first slope of the differential $pp$ cross section at small momentum transfer, where single internal $QQ$ scattering dominates. Then, using this effective fundamental $QQ$ scattering amplitude, the amplitudes for the higher-order multiple-scattering contributions are self-consistently calculated. (iii) After the full amplitude, including contributions of all significant orders of multiple quark scattering, has been constructed, both the effective $QQ$ amplitude and the effective generalized form factors of $p$ are adjusted variationally in order to isolate and identify the physical effects of their various functional parameters and to obtain finally the best self-consistent fit to the experimental data.

Figure 1 shows the results of the SCMQS analysis of $pp$ scattering.[1]

The variation of the effective $QQ$ scattering amplitude consists principally of determining its effective phase behavior and indicates that a rather strong dependence of its phase upon momentum transfer is required.[1] The absolute magnitude of the effective $QQ$ amplitude is already rather definitely determined directly from the $pp$ cross section at small momentum transfer. The effective $QQ$ scattering amplitude determined in the $pp$ analysis is

$$f(\mathbf{q})=\Phi_p(\mathbf{q})^{-2}(\sigma_{QQ}/4\pi)(i-0.413)e^{-\frac{1}{2}\xi_{QQ}\mathbf{q}^2}, \tag{6}$$

in which the effective inverse diffraction width is

$$\xi_{QQ}=9.8-i13\ (\mathrm{BeV}/c)^{-2}, \tag{7}$$

the effective $QQ$ total cross section is

$$\sigma_{QQ}=35.2/9\ \mathrm{mb} \tag{8}$$

(based on the asymptotic $pp$ total cross section $\sigma_{pp}\cong 36.0$ mb), and the effective one-body form factor of the quark distribution within $p$ is

$$\Phi_p(\mathbf{q})=(1+\mathbf{q}^2/\mu^2)^{-2}, \tag{9}$$

with

$$\mu^2=0.71\ (\mathrm{BeV}/c)^2.$$

The self-consistently determined form of $f$ given in (6), except for the large-momentum-transfer dependence of the phase, is clearly only slightly refined in most aspects beyond the first trial form determined from the single-scattering approximation for the forward peak.

The variations of the effective body form factors of $p$ have been performed with single- and double-pole functions of the single variable $x$ which is the function of the many momentum-transfer variables

$$x\equiv(A-1)^{-1}\sum_{i>j}^{A}(\mathbf{q}_i-\mathbf{q}_j)^2/\mu^2.$$

The double pole is the most rapid rate of falloff at large momentum transfers that can be allowed for the body form factors of $p$ because, owing to the relation (5), any more rapid rate of decrease of the single-$Q$ body form factor of $p$ would imply that the charge form factor of $Q$ increases with increasing large momentum transfer. The variable $x$ is selected because it arises in several specific models and would allow continuity with the case of independent Gaussian single-particle distributions constrained by a c.m. correlation. Variation over discrete values of $A$ for reasonably small $Q$'s rather easily selects $A=3$; no remotely satisfactory fit to the $pp$ cross section could be obtained with $A=2$, and with $A=4$ approaches to the data region would require $Q$'s to be comparable in spatial extent with the $p$'s. Then for $A=3$ functional variation of the effective form factors with respect to the effective size (spatial distribution) of the $Q$ within $p$ confirms that the $Q$'s are rather small compared to the size of $p$—perhaps pointlike.[1] For $A=3$, the best fit to the data is obtained with

$$\Phi_p(\mathbf{q}_1,\mathbf{q}_2)=[1+(A-1)^{-1}\sum_{i>j}^{A}(\mathbf{q}_i-\mathbf{q}_j)^2/\mu^2]^{-2}, \quad (10)$$

with

$$\mu^2=0.71\ (\mathrm{BeV}/c)^2,$$

which, because of relation (5), implies that the effective electromagnetic radius of the quarks is much smaller than that of the proton.

For all subsequent analyses, we will take as definitive these results of the $pp$ analysis for $f$ as given in (6), for $\Phi_p$ as given in (10), for $A=3$ and the implication that our subparticles are $SU_3$ quarks, and we shall assume the rudimentary structure of hadrons according to the composite $SU_3$-quark model.

## IV. CONSISTENCY RELATIONS FOR $pp$, $p\bar{p}$, $\pi p$, AND $\pi\pi$ SCATTERING IN FIRST ORDER

Before adapting the formulas of Sec. II to the general consistency relations among $pp$, $p\bar{p}$, $\pi p$, and $\pi\pi$ diffraction-scattering cross sections and the form factors of $p$ and $\pi$, we will first illustrate the essentials of the self-consistent method with a treatment of only first-order quark-scattering effects. This will bring into focus the adjustments that are required in order to extend the consistency relations to a general higher-order SCMQS treatment.

Considering only first-order quark-scattering effects, consistency relations among elastic $pp$, $p\bar{p}$, $\pi p$, and $\pi\pi$ diffraction cross sections and effective form factors of $p$ and $\pi$ can be obtained from which the $\pi\pi$ diffraction cross section and the pion charge radius can be solved.[2] Considering only the low-momentum-transfer region, where single $QQ$ scattering dominates, formula (2) gives for the leading diffraction-scattering amplitude for $pp$ scattering

$$f_{pp}^{(1)}(\mathbf{q})\cong[3\Phi_p(\mathbf{q})]^2 f(\mathbf{q}), \quad (11)$$

where

$$d\sigma_{pp}^{(1)}/d\mathbf{q}^2\cong\pi|f_{pp}^{(1)}(\mathbf{q})|^2 \quad (12)$$

is the $pp$ cross section and $\mathbf{q}_\perp^2\simeq\mathbf{q}^2\simeq -t$ in the region of the first diffraction slope. Similarly, for $p\bar{p}$ scattering,

$$f_{p\bar{p}}^{(1)}(\mathbf{q})\cong[3\Phi_{\bar{p}}(\mathbf{q})]^2\bar{f}(\mathbf{q}), \quad (13)$$

where, since the $\bar{p}$ is composed of $\bar{Q}$'s in the same way as $p$ is composed of $Q$'s the $\bar{p}$ form factors are all the same functions as the $p$ form factors,[1,2] $\Phi_{\bar{p}}=\Phi_p$, and $\bar{f}$ is the effective amplitude for the fundamental $Q\bar{Q}$ scattering process. For $\pi p$ scattering,

$$f_{\pi p}^{(1)}(\mathbf{q})\cong 3\Phi_p(\mathbf{q})\Phi_\pi(\mathbf{q})[f(\mathbf{q})+\bar{f}(\mathbf{q})], \quad (14)$$

where the effective $\bar{Q}$ form factor of $\pi$ is the same as the effective $Q$ form factor of $\pi$. For $\pi\pi$ scattering,

$$f_{\pi\pi}^{(1)}(\mathbf{q})\cong\Phi_\pi(\mathbf{q})^2[f(\mathbf{q})+\bar{f}^-(\mathbf{q})+2\bar{f}(\mathbf{q})], \quad (15)$$

where $\bar{f}^-$ is the effective amplitude for the fundamental $\bar{Q}\bar{Q}$ scattering process.

Charge symmetry gives $\bar{f}^-=f$, which allows (14) and (15) to be combined as

$$f_{\pi\pi}^{(1)}(\mathbf{q})\cong\tfrac{2}{3}[\Phi_\pi(\mathbf{q})/\Phi_p(\mathbf{q})]f_{\pi p}^{(1)}(\mathbf{q}), \quad (16)$$

or, squaring both sides of (16) and using (5) to relate electromagnetic and $Q$ body form factors, we obtain the consistency relation

$$\frac{d\sigma_{\pi\pi}^{(1)}}{d\mathbf{q}^2}\cong\left(\frac{2}{3}\frac{G_\pi^E(\mathbf{q})}{G_p^E(\mathbf{q})}\right)^2\frac{d\sigma_{\pi p}^{(1)}}{d\mathbf{q}^2}, \quad (17)$$

where $G_\pi^E$ and $G_p^E$ are the usual electromagnetic form factors of $\pi$ and $p$, respectively.

Using (11) and (12) to determine $|f|$ in terms of the $pp$ cross section at small momentum transfer, and similarly using (13) and (12) to determine $|\bar{f}|$ from the $p\bar{p}$ cross section, then inserting these into (14), we can solve for $\Phi_\pi$ in terms of $pp$, $\pi p$, and $p\bar{p}$ cross sections and $\Phi_p$. Then multiplying both sides by $G_Q^E$, the effective electromagnetic form factor of $Q$, and using (5), we obtain a consistency relation for the effective electromagnetic form factor of the $\pi$ in terms of the effective electromagnetic form factor of $p$, $G_p^E$, and the $pp$, $p\bar{p}$, and $\pi p$ cross sections:

$$G_\pi^E(\mathbf{q})\cong 3G_p^E(\mathbf{q})\left(\frac{d\sigma_{\pi p}^{(1)}}{d\mathbf{q}^2}\right)^{1/2} \times\left[\left(\frac{d\sigma_{pp}^{(1)}}{d\mathbf{q}^2}\right)^{1/2}+\left(\frac{d\sigma_{p\bar{p}}^{(1)}}{d\mathbf{q}^2}\right)^{1/2}\right]^{-1}. \quad (18)$$

We assume all these scattering processes to be so diffractive that the leading diffraction amplitudes for all these processes have roughly the same phases over the region of the first diffraction slopes, starting from nearly purely imaginary at zero momentum transfer. This will be made more definite in the more complete treatment to follow.

Another consistency relation for the $\pi\pi$ cross section in terms of the $pp$, $\pi p$, and $p\bar{p}$ cross sections and $G_p^E$ may be obtained by eliminating $G_\pi^E$ from (17) by substituting (18) into (17):

$$\frac{d\sigma_{\pi\pi}^{(1)}}{d\mathbf{q}^2}\cong 4\left[\frac{d\sigma_{\pi p}^{(1)}}{d\mathbf{q}^2}\right]^2\left[\left(\frac{d\sigma_{pp}^{(1)}}{d\mathbf{q}^2}\right)^{1/2}+\left(\frac{d\sigma_{p\bar{p}}^{(1)}}{d\mathbf{q}^2}\right)^{1/2}\right]^{-2}. \quad (19)$$

Assuming leading diffraction peaks of the (energy-independent) form

$$\frac{d\sigma_{pp}^{(1)}}{d\mathbf{q}^2}\cong\left.\frac{d\sigma_{pp}^{(1)}}{d\mathbf{q}^2}\right|_{\mathbf{q}=0}e^{-\xi_{pp}\mathbf{q}^2}, \quad (20)$$

where $\xi_{pp}\cong 9$–$10$, $\xi_{\pi p}\cong 8$–$9$, and $\xi_{p\bar{p}}\cong 11$–$12$ $(\mathrm{BeV}/c)^{-2}$, and, assuming that the magnitudes of the amplitudes at the forward angle are given roughly by the optical theorem as proportional to the total cross sections $\sigma_{pp}\cong 38$ mb, $\sigma_{p\bar{p}}\cong 48$ mb, and $\sigma_{\pi p}\cong 25$ mb, formula (18) becomes

$$G_\pi{}^E(\mathbf{q})\cong 3G_p{}^E(\mathbf{q})\left(\frac{\sigma_{pp}}{\sigma_{\pi p}}e^{-\frac{1}{2}(\xi_{pp}-\xi_{\pi p})\mathbf{q}^2}+\frac{\sigma_{p\bar{p}}}{\sigma_{\pi p}}e^{-\frac{1}{2}(\xi_{p\bar{p}}-\xi_{\pi p})\mathbf{q}^2}\right)^{-1} \quad (21)$$

for momentum transfer squared less than about 1 $(\mathrm{BeV}/c)^2$. This same relation between $G_\pi{}^E$ and $G_p{}^E$ also holds between the single-quark body form factors $\Phi_\pi$ and $\Phi_p$. In the same way the formula (19) for the $\pi\pi$ cross section becomes

$$\frac{d\sigma_{\pi\pi}{}^{(1)}}{d\mathbf{q}^2}\cong 4\frac{d\sigma_{\pi p}{}^{(1)}}{d\mathbf{q}^2}\left(\frac{\sigma_{pp}}{\sigma_{\pi p}}e^{-\frac{1}{2}(\xi_{pp}-\xi_{\pi p})\mathbf{q}^2}+\frac{\sigma_{p\bar{p}}}{\sigma_{\pi p}}e^{-\frac{1}{2}(\xi_{p\bar{p}}-\xi_{\pi p})\mathbf{q}^2}\right)^{-2}. \quad (22)$$

If the Pomeranchuk theorem were exact for $pp$ and $p\bar{p}$, and also for the $QQ$ and $Q\bar{Q}$ cross sections such that $\sigma_{\pi p}/\sigma_{pp}=\sigma_{\pi p}/\sigma_{p\bar{p}}=\frac{2}{3}$, $\xi_{pp}=\xi_{p\bar{p}}$, and we take $\xi_{\pi p}\cong\xi_{pp}$, then

$$\Phi_\pi(\mathbf{q})\cong\Phi_p(\mathbf{q}),\quad G_\pi{}^E(\mathbf{q})\cong G_p{}^E(\mathbf{q}),\quad \text{(Pomeranchuk limit)} \quad (23)$$

and

$$\frac{d\sigma_{\pi\pi}{}^{(1)}}{d\mathbf{q}^2}\cong\frac{4}{9}\frac{d\sigma_{\pi p}{}^{(1)}}{d\mathbf{q}^2}, \quad \text{(Pomeranchuk limit)}. \quad (24)$$

On the other hand, ignoring the Pomeranchuk theorem and treating the present (approximately energy-independent) high-energy cross sections as the asymptotic values, we obtain

$$G_\pi{}^E(\mathbf{q})\cong 0.87/(1+\mathbf{q}^2/M^2) \quad (25)$$

for $\mathbf{q}^2\lesssim M^2=0.57$ $(\mathrm{BeV}/c)^2$. The normalization is off here, since $G_\pi{}^E(0)\cong 3\sigma_{\pi p}(\sigma_{pp}+\sigma_{p\bar{p}})^{-1}\cong 0.87$, which is perhaps indicative of the order of accuracy of this first-order treatment. However, the mean-square charge radius of the pion does not depend on this normalization and is given directly from (18) as

$$\langle r_\pi{}^2\rangle\cong\frac{-6dG_\pi{}^E/d\mathbf{q}^2|_{\mathbf{q}=0}}{G_\pi{}^E(0)}, \quad (26)$$

or

$$r_\pi\cong 0.65\ \mathrm{F}.$$

This value is in good agreement with the best experimental determinations of $r_\pi=0.8\pm0.1$ F from pion electroproduction[4] and $r_\pi\cong 0.9$ F from $\pi$-He$^4$ Coulomb scattering,[5] and with the value $r_\pi\leq 0.6$ F calculated from current algebra and pole dominance.[6]

Similarly, the inverse diffraction width in $\pi\pi$ scattering can be determined directly from (22) as

$$\xi_{\pi\pi}\cong 2\xi_{\pi p}-\left(\xi_{pp}+\xi_{p\bar{p}}\frac{\sigma_{p\bar{p}}}{\sigma_{pp}}\right)\times\left(1+\frac{\sigma_{p\bar{p}}}{\sigma_{pp}}\right)^{-1}\cong 5.8\ (\mathrm{BeV}/c)^{-2}, \quad (27)$$

and the total cross section as

$$\sigma_{\pi\pi}\cong 2(\sigma_{\pi p})^2(\sigma_{pp}+\sigma_{p\bar{p}})^{-1}\cong 14\ \mathrm{mb}. \quad (28)$$

These values agree closely with those obtained from Regge theory: $\xi_{\pi\pi}\cong 2\xi_{\pi p}-\xi_{pp}\cong 7$ $(\mathrm{BeV}/c)^{-2}$ and $\sigma_{\pi\pi}\cong(\sigma_{\pi p})^2/\sigma_{pp}\cong 16$ mb.[7]

We have succeeded in obtaining the effective electromagnetic form factor of the pion $G_\pi{}^E$ and the effective $\pi\pi$ diffraction cross section as phenomenological solutions of a first-order treatment of the consistency relations. But expressions (21) for $G_\pi{}^E$ and (22) for $d\sigma_{\pi\pi}{}^{(1)}/d\mathbf{q}^2$ so obtained do not fall to zero as $\mathbf{q}^2$ becomes very large if, instead of imposing the Pomeranchuk theorem, the slope $\xi_{pp}$ of $d\sigma_{pp}{}^{(1)}/d\mathbf{q}^2$ is allowed to be greater than the slope $\xi_{\pi p}$ of $d\sigma_{\pi p}{}^{(1)}/d\mathbf{q}^2$, as is observed at realistic energies. Resolution of this difficulty is apparently beyond the scope of our simple first-order SCMQS treatment but we will see that it can be accomplished with the higher-order SCMQS treatment of these consistency relations.

## V. CONSISTENCY RELATIONS IN HIGHER-ORDER SCMQS

It is thus desirable to investigate these consistency relations in more detail by considering higher-order multiple-scattering effects and using form factors that do not increase at large momentum transfers. Simple algebraic formulas such as are explicitly solved with the first-order treatment can no longer be obtained. Rather, the whole amplitudes, including the higher-order contributions, must now be variationally adjusted to determine the pion form factor self-consistently as was done in the higher-order analysis of $pp$ scattering.

Although the formulas of Sec. II are suitable as they stand for direct application to $pp$ scattering, their applications to higher-order analysis of $p\bar{p}$, $\pi p$, and $\pi\pi$ scattering require that they be modified somewhat because all the subparticles involved in each of these scattering processes are no longer similar as in the $pp$ case. Thus, $Q\bar{Q}$ scattering is involved in $p\bar{p}$ and $\pi p$ scattering, and in $\pi\pi$ scattering $\bar{Q}\bar{Q}$ scattering is also involved. Although the required modifications in the formulas reduce to trivial formalities under the two

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approaches concerning $Q\bar{Q}$ scattering that we consider, this matter must be explained more fully.

It is quite reasonable to expect $\bar{Q}\bar{Q}$ scattering to be the same as $QQ$ scattering on the basis of charge symmetry. There is, however, something different about the effective $Q\bar{Q}$ interaction, as can be seen by comparison of the $p\bar{p}$ cross section with the $pp$ cross section. This difference might be expected to be largely due to the access, or lack of access, to intermediate annihilation channels. We will consider two different approaches to this issue. The first approach is based on the ansatz that the annihilation contributions are to be lumped in with the rest of the effective $Q\bar{Q}$ scattering amplitude and will be referred to as the "lumped approach." The second approach is based on the ansatz that the extra annihilation contributions in $p\bar{p}$ scattering which cannot contribute in $pp$ scattering are restricted by $SU_3$-triality conservation.[8] We refer to this as the "Kokkedee approach."[9]

Under the lumped approach to the annihilation contributions in the effective $Q\bar{Q}$ scattering amplitude, it may be determined from the $p\bar{p}$ cross section by the same SCMQS analysis as was used to determine the effective $QQ$ scattering amplitude from the $pp$ cross section. The form factors of the $\bar{Q}$ distributions within $\bar{p}$ are taken by charge symmetry to be the same as the form factors of the $Q$ distributions within $p$, so the effective $Q\bar{Q}$ scattering amplitude is the only function to be variationally adjusted in this analysis. Figure 2 shows the results of such an analysis of $p\bar{p}$ scattering. The effective $Q\bar{Q}$ scattering amplitude determined this way is[10]

$$\bar{f}(\mathbf{q})\cong\Phi_p(\mathbf{q})^{-2}(\sigma_{Q\bar{Q}}/4\pi)(i-0.109)e^{-\frac{1}{2}\xi_{Q\bar{Q}}\mathbf{q}^2}, \qquad (29)$$

with

$$\xi_{Q\bar{Q}}=12.5-i13\ (\text{BeV}/c)^{-2},$$
$$\sigma_{Q\bar{Q}}=49.9/9\ \text{mb}$$

(based on the asymptotic $p\bar{p}$ total cross section $\sigma_{p\bar{p}}\cong47.0$ mb), and $\Phi_p(\mathbf{q})=(1+\mathbf{q}^2/\mu^2)^{-2}$, with $\mu^2=0.71$ $(\text{BeV}/c)^2$, as previously determined. The self-consistent fit was made to the $p\bar{p}$ differential cross-section data plotted as a function of $\beta^2\mathbf{q}_\perp{}^2$ as was done for the $pp$ analysis, although the removal of the energy dependence by this trick is not so complete in the $p\bar{p}$ case. In the

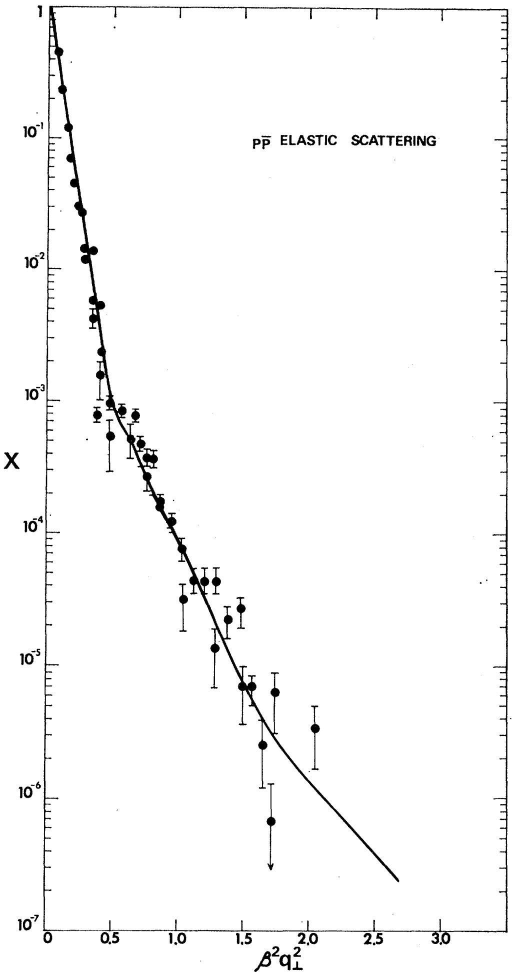


FIG. 2. Results of SCMQS analysis of $p\bar{p}$ cross section calculated under our lumped approach to the question of direct $Q\bar{Q}$ annihilation contribution to $p\bar{p}$ elastic scattering compared with experimental data of $X\equiv(d\sigma/dt)/(d\sigma/dt)_{t=0}$ versus $\beta^2\mathbf{q}_\perp{}^2$ $(\text{BeV}/c)^2$. The data shown are from J. Orear, D. P. Owen, F. C. Peterson, A. L. Read, D. G. Ryan, D. H. White, A. Ashmore, C. J. S. Damerell, W. R. Frisken, and R. Rubenstein, Brookhaven National Laboratory Report No. BNL-12767 (unpublished); K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters **11**, 503 (1963); K. J. Foley, R. S. Gilmore, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. H. Willen, R. Yamada, and L. C. L. Yuan, *ibid.* **15**, 45 (1965); A. Ashmore, C. J. S. Damerell, W. R. Frisken, R. Rubinstein, J. Orear, D. P. Owen, F. C. Peterson, A. L. Read, D. G. Ryan, and D. H. White, *ibid.* **21**, 387 (1968).

$pp$ analysis we fitted to Krisch's[11] plot of $d\sigma_{pp}{}^\dagger/dt$ versus $\beta^2\mathbf{q}_\perp{}^2$, which includes some corrections near 90° for identical-particle effects that are clearly irrelevant for the case of $p\bar{p}$ scattering.

The Kokkedee approach is predicated on the idea that, since free quarks have never been observed as physical states and the imaginary forward scattering amplitude may be given through the unitarity relation as a sum over asymptotic physical states, we should not allow intermediate states that correspond to single or

[8] Triality is defined and discussed in several texts on unitary symmetry; see, for instance, P. Carruthers, *Introduction to Unitary Symmetry* (Wiley-Interscience, Inc., New York, 1966).

[9] J. J. J. Kokkedee [in Proceedings of the Sixth Internationale Universitätwochen für Kernphysik, 1967 (unpublished)] gave one of the first references to these considerations of triality conservation within the context of quark-scattering models.

[10] K. J. Foley, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. D. Platner, C. A. Quarles, and E. H. Willen, Phys. Rev. Letters **19**, 857 (1967). Although the phase of the forward $p\bar{p}$ scattering amplitude has not been experimentally well measured so far, some theoretical arguments have been advanced for this phase to be the negative of the corresponding $pp$ phase. See, for instance, R. J. Eden, *High-Energy Collisions of Elementary Particles* (Cambridge University Press, New York, 1967).

[11] A. D. Krisch, Phys. Rev. Letters **19**, 1149 (1967).

double $Q\bar{Q}$ annihilation to contribute to $p\bar{p}$ scattering in our multiple-quark-scattering formalism. This would suggest that only simultaneous threefold $Q\bar{Q}$ annihilation be allowed in $p\bar{p}$ scattering, and that $Q\bar{Q}$ annihilation processes contributing to $p\bar{p}$ scattering be considered third-order as multiple $Q\bar{Q}$ scattering processes.[9] Thus, in this approach the effective $Q\bar{Q}$ interaction includes a part that contributes to multiple-internal-scattering effects exactly the same as the effective $QQ$ interaction, but also included in the effective $Q\bar{Q}$ interaction is a part which contributes as a triple $Q\bar{Q}$ annihilation process. The amplitudes for single and double $Q\bar{Q}$ scattering contributions to $p\bar{p}$ scattering in this approach are, respectively, the same as the single and double $QQ$ scattering contributions to $pp$ scattering. But in third order, the $p\bar{p}$ amplitude includes an additional part $f_{p\bar{p}a}{}^{(3)}$ which describes the pure annihilation contribution

$$f_{p\bar{p}}=f_{pp}+f_{p\bar{p}a}{}^{(3)}+(4°)\,, \tag{30}$$

where (4°) indicates the contributions of higher-than-third-order multiple-scattering effects. Although the annihilation contribution in $p\bar{p}$ scattering is considered under this approach to be a third-order multiple $Q\bar{Q}$ scattering process, its magnitude is more nearly commensurate with the ordinary double-quark-scattering contribution at small momentum transfers, so that its effect shows up even in the first slope of the observed $p\bar{p}$ cross section. Thus, in this approach $p\bar{p}$ scattering gives no additional information that is useful for the consistency relations, because the annihilation amplitude $f_{p\bar{p}a}{}^{(3)}$ occurs only in $p\bar{p}$ scattering and it is, by definition, that amplitude which must be added to the previously determined $pp$ amplitude so that the $p\bar{p}$ cross section is fitted by the combined amplitude (30). Although, in principle, this is simple, we found it difficult to determine actually such an amplitude $f_{p\bar{p}a}{}^{(3)}$.

The effective $Q\bar{Q}$ scattering amplitude extracted after allowance for effects of simultaneous triple $Q\bar{Q}$ annihilation for the analysis of $p\bar{p}$ scattering is used directly in calculating the SCMQS amplitudes for $\pi p$ and $\pi\pi$ scattering in this approach. Direct $Q\bar{Q}$ annihilation processes contributing specifically in $\pi p$ and $\pi\pi$ scattering could also be considered in this approach, but we stop short of this here. It will be suggested later that such contributions would be small. Thus, in our second approach the formulas of Sec. II apply already as they are to $\pi p$ and $\pi\pi$, as well as $pp$ scattering, with the effective $Q\bar{Q}$ scattering amplitude equal to the $QQ$ amplitude in them.

Formulas for the SCMQS amplitudes for $\pi p$ and $\pi\pi$ scattering with both these two approaches can be written formally the same, with a numerical distinction between the $\bar{f}$ and the $f$ that occur in them when they are applied under our lumped approach to the $Q\bar{Q}$ annihilation question and numerical equivalence of $\bar{f}=f$ in them when applied under our Kokkedee approach.

The complete amplitude for $\pi p$ scattering is composed of a first-order contribution which is formally the same as formula (14) plus a double-scattering contribution

$$\begin{aligned}f_{\pi p}{}^{(2)}(\mathbf{q})=3\frac{i}{2\pi}\int d^2\mathbf{p}\,\{&\Phi_p(\mathbf{q})\Phi_\pi(\mathbf{q}-2\mathbf{p})f(\mathbf{q})\bar{f}(\mathbf{q}-\mathbf{p})\\&+[\Phi_\pi(\mathbf{q})(f(\mathbf{p})f(\mathbf{q}-\mathbf{p})+\bar{f}(\mathbf{p})\bar{f}(\mathbf{q}-\mathbf{p}))\\&+2\Phi_\pi(\mathbf{q}-2\mathbf{p})f(\mathbf{p})\bar{f}(\mathbf{q}-\mathbf{p})]\Phi_p(\mathbf{p},\mathbf{q}-\mathbf{p})\}\,,\end{aligned} \tag{31}$$

plus a triple-scattering contribution

$$\begin{aligned}f_{\pi p}{}^{(3)}(\mathbf{q})=&\left(\frac{i}{2\pi}\right)^2\int d^2\mathbf{p}d^2\mathbf{k}\,\{6\Phi_p(\mathbf{p},\mathbf{q}-\mathbf{p})\Phi_\pi(\mathbf{q}-2\mathbf{k})\\&\times[f(\mathbf{p})\bar{f}(\mathbf{k})f(\mathbf{q}-\mathbf{p}-\mathbf{k})+\bar{f}(\mathbf{p})f(\mathbf{k})\bar{f}(\mathbf{q}-\mathbf{p}-\mathbf{k})]\\&+\Phi_p(\mathbf{p},\mathbf{k},\mathbf{q}-\mathbf{p}-\mathbf{k})\Phi_\pi(\mathbf{q})[f(\mathbf{p})f(\mathbf{k})f(\mathbf{q}-\mathbf{p}-\mathbf{k})\\&+\bar{f}(\mathbf{p})\bar{f}(\mathbf{k})\bar{f}(\mathbf{q}-\mathbf{p}-\mathbf{k})]+3\Phi_p(\mathbf{p},\mathbf{k},\mathbf{q}-\mathbf{p}-\mathbf{k})\\&\times\Phi_\pi(\mathbf{q}-2\mathbf{k})[f(\mathbf{p})\bar{f}(\mathbf{k})f(\mathbf{q}-\mathbf{p}-\mathbf{k})\\&+\bar{f}(\mathbf{p})f(\mathbf{k})f(\mathbf{q}-\mathbf{p}-\mathbf{k})]\}\,,\end{aligned} \tag{32}$$

where we have used the fact that there is only one pion form factor and $\Phi_\pi(\mathbf{q}_1,\mathbf{q}_2)=\Phi_\pi(\mathbf{q}_1-\mathbf{q}_2)$.

The amplitude for $\pi\pi$ scattering is likewise composed of a first-order contribution which is formally the same as formula (15) plus a double-scattering contribution

$$\begin{aligned}f_{\pi\pi}{}^{(2)}(\mathbf{q})=\frac{i}{2\pi}\int d^2\mathbf{p}\,\Phi_\pi(\mathbf{q}-2\mathbf{p})\{&4\Phi_\pi(\mathbf{q})f(\mathbf{p})\bar{f}(\mathbf{q}-\mathbf{p})\\&+\Phi_\pi(\mathbf{q}-2\mathbf{p})[f(\mathbf{p})f(\mathbf{q}-\mathbf{p})+\bar{f}(\mathbf{p})\bar{f}(\mathbf{q}-\mathbf{p})]\}\,,\end{aligned} \tag{33}$$

plus a triple-scattering contribution

$$\begin{aligned}f_{\pi\pi}{}^{(3)}(\mathbf{q})=&2\left(\frac{i}{2\pi}\right)^2\int d^2\mathbf{p}d^2\mathbf{k}\,\Phi_\pi(\mathbf{q}-2\mathbf{p})\Phi_\pi(\mathbf{q}-2\mathbf{k})\\&\times\{f(\mathbf{p})f(\mathbf{k})\bar{f}(\mathbf{q}-\mathbf{p}-\mathbf{k})+f(\mathbf{p})\bar{f}(\mathbf{k})\bar{f}(\mathbf{q}-\mathbf{p}-\mathbf{k})\}\,,\end{aligned} \tag{34}$$

where we have imposed charge symmetry so that $\bar{f}^-=f$.

While with the higher-order SCMQS treatment the consistency relations are complicated and are to be solved by functional variation, still the pion form factor $\Phi_\pi$ is the only unknown to be determined from analysis of $\pi p$ scattering after once having selected either our lumped or our Kokkedee criterion and determined $\bar{f}$. In this sense the qualitative content of the consistency relations remains the same as in the first-order treatment, while their quantitative precision should be much improved. We have seen in the first-order treatment that the effective pion radius is an important objective of the analysis of $\pi p$ scattering, and we expect that it can be determined by analysis of the $\pi p$ cross section at low momentum transfers. Therefore, we adopt the following program for our SCMQS analysis of $\pi p$ scattering:

First, the $\pi p$ cross section is fitted at low momentum transfers in order to determine the effective pion radius. Then the functional form and large-momentum-transfer dependence of the pion form factor with this radius is determined from the fit to the $\pi p$ cross section over the high-momentum-transfer range. [Clearly, the dichotomic form of this rule is merely an operational convenience, since there is only a single function of a single (momentum-transfer) variable to be variationally determined.]

In the region of momentum transfers below $\mathbf{q}_\perp^2 \cong 0.5$ $(\mathrm{BeV}/c)^2$, the data in the plot of the $\pi p$ cross section $d\sigma_{\pi p}/dt$ versus $\mathbf{q}_\perp^2$ form a very narrow straight line. Fits to the data in this region were made with first- and second-order amplitudes $f_{\pi p}^{(1)}+f_{\pi p}^{(2)}$ calculated from formulas (14) and (31) for four cases. Fits were made for both our lumped and our Kokkedee approaches to the treatment of direct $Q\bar{Q}$ annihilation processes and for each approach fits were made with pion form factors of both simple pole form, $\Phi_\pi = (1+\mathbf{q}^2/\mu^2)^{-1}$, and Gaussian form, $\Phi_\pi = e^{-\mathbf{q}^2/\mu^2}$. For our lumped approach, $\bar{f}$ was taken as in formula (29) determined from the $p\bar{p}$ analysis, and the resulting pion radius was $r_\pi \cong 0.25$ F for both functional forms of the pion form factor. For our Kokkedee approach $\bar{f}$ was taken the same as $f$ as determined from the $pp$ analysis and given in formula (6), and the resulting pion radius was $r_\pi \cong 0.40$ F. For both our lumped and our Kokkedee approaches there was almost no distinction between the results for the $\pi p$ cross section at low momentum transfers calculated with Gaussian or simple pole functions for $\Phi_\pi$ for the given values of $r_\pi$. This indicates that the high-momentum-transfer dependence of $\Phi_\pi$ is not crucial for the cross section at small angles. For both approaches the resulting pion radius is significantly smaller than indicated by the results of the first-order treatment. Two factors contribute to this discrepancy: The first-order treatment in Sec. IV treated $-t \cong \mathbf{q}_\perp^2$ in the region of small-angle scattering, but if even the first-order fits are made to $d\sigma_{\pi p}/d\mathbf{q}^2$ versus $\mathbf{q}_\perp^2$ instead of $d\sigma_{\pi p}/dt$ versus $t$, then the resulting value of the pion radius obtained is $r_\pi \cong 0.55$ F instead of $r_\pi = 0.65$ F. This effect was overlooked in the previous report of the results of the first-order treatment.[2] The second factor contributing to this discrepancy is, of course, simply the contribution of the double-scattering effects, which are about 20% even at the forward angle. The double-scattering contribution at the forward angle tends to cancel the single-scattering effects so that their combination gives a narrower-peaked cross section than the single scattering alone would with all other parameters held fixed. Thus, to maintain the fit to the observed cross section the smaller pion radius serves to compensate for the narrowing effect of higher-order scattering.

The results including triple-scattering effects as given by the sum of amplitudes (14), (31), and (32) were calculated for $\pi p$ scattering under both our approaches to the treatment of $Q\bar{Q}$ annihilation effects.

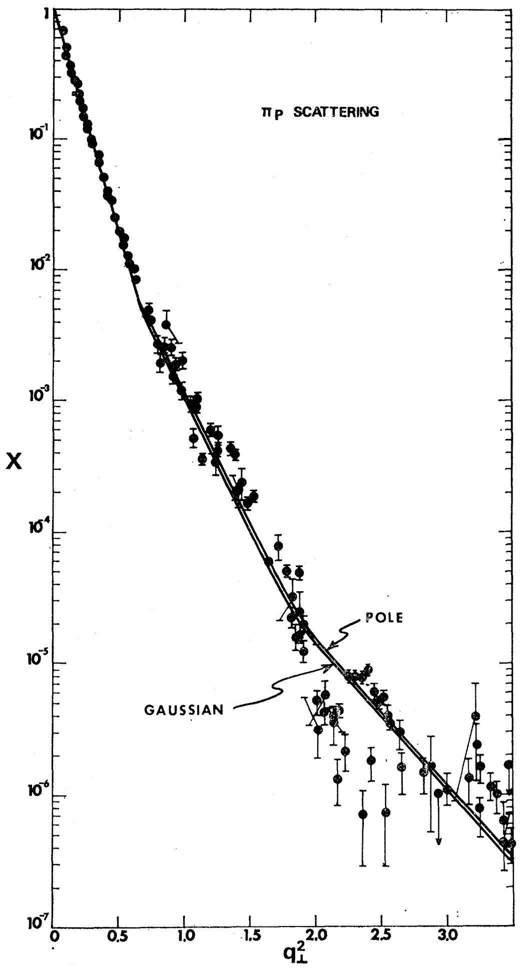


FIG. 3. Results for $X \equiv (d\sigma/dt)/(d\sigma/dt)_{t=0}$ versus $\mathbf{q}_\perp^2$ $(\mathrm{BeV}/c)^2$ of SCMQS analysis of $\pi p$ cross section calculated from the consistency relations under our lumped approach to the direct $Q\bar{Q}$ annihilation contribution. Results shown are calculated with $r_\pi = 0.25$ F for both Gaussian and simple pole functions for the pion form factor. The high-energy elastic $\pi^- p$ scattering data shown are from A. Ashmore *et al.*, Phys. Rev. Letters **32**, 387 (1968); D. Harting, P. Blackall, B. Elsner, A. C. Helmholz, W. C. Middelkoop, B. Powell, B. Zacharov, P. Zanella, P. Dalpiaz, M. N. Focacci, S. Focardi, G. Giacomelli, L. Monari, J. A. Beaney, R. A. Donald, P. Mason, L. W. Jones, and D. O. Caldwell, Nuovo Cimento **38**, 60 (1965); J. Orear, R. Rubenstein, D. B. Scarl, D. H. White, A. D. Krisch, W. R. Frisken, A. L. Read, and H. Ruderman, Phys. Rev. **152**, 1162 (1966); J. Orear *et al.*, Brookhaven National Laboratory Report No. BNL-12767 (unpublished).

Figure 3 shows the results for $\pi p$ scattering calculated under our lumped approach with $\bar{f}$ as from the $p\bar{p}$ analysis in formula (29) and with both Gaussian and simple pole functions for the pion form factor, both with pion radius $r_\pi \cong 0.25$ F. Figure 4 shows the results for the Kokkedee approach with $\bar{f}=f$ as from the $pp$ analysis in formula (6) and with both Gaussian and simple pole functions for the pion form factor, both with pion radius $r_\pi \cong 0.40$ F.

Results with both our lumped approach shown in Fig. 3 and our Kokkedee approach shown in Fig. 4,

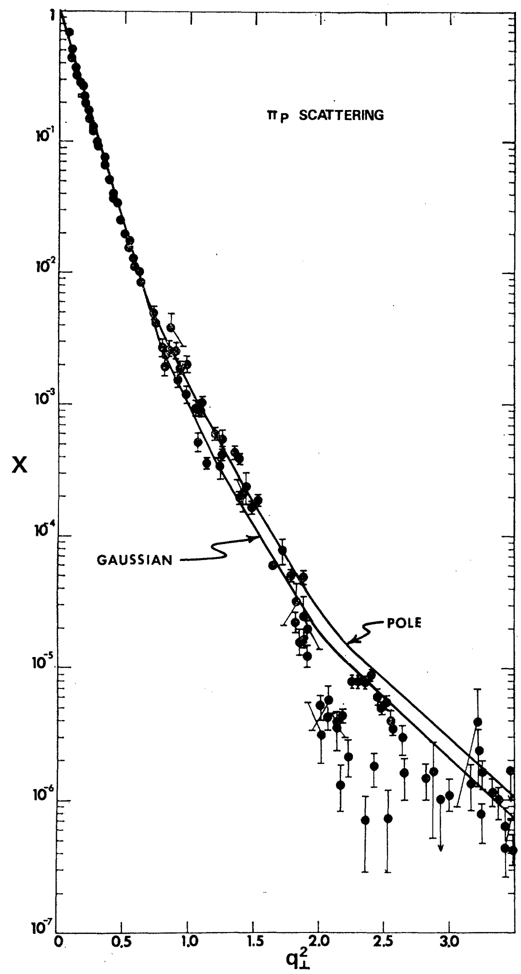


FIG. 4. Results for $X\equiv(d\sigma/dt)/(d\sigma/dt)_{t=0}$ versus $\mathbf{q}_\perp^2$ $(\text{BeV}/c)^2$ of SCMQS analysis of $\pi p$ cross section calculated from the consistency relations under our Kokkedee approach to the direct $Q\bar{Q}$ annhilation contribution. Results shown are calculated with $r_\pi=0.40$ F for both Gaussian and simple pole functions for the pion form factor. The high-energy elastic $\pi^-p$ scattering data shown are from A. Ashmore *et al.*, Phys. Rev. Letters **21**, 387 (1968); D. Harting *et al.*, Nuovo Cimento **38**, 60 (1968); J. Orear *et al.*, Phys. Rev. **152**, 1162 (1966); J. Orear *et al.*, Brookhaven National Laboratory Report No. BNL-12767 (unpublished).

but especially the former, indicate that the high-momentum-transfer dependence of $\Phi_\pi$ is not very crucial. This is to be expected in our multiple-diffraction-scattering picture, and tends to confirm the assumptions upon which our present formalism is based. There is, however, in Fig. 4 some indication that the more rapid falloff at large momentum transfer results in $\pi p$ cross-section curves lying nearer the lower envelope of the data toward which the higher-energy data may tend to accumulate similarly as in the $pp$ case. In all, our fits to the $\pi p$ cross-section data may be said to be consistent with a pion radius

$$r_\pi=0.36\pm0.12\ \text{F}\,, \tag{35}$$

and a Gaussian behavior of the pion form factor at large momentum transfers,

$$\Phi_\pi(\mathbf{q})=e^{-\mathbf{q}^2/\mu^2}\,, \tag{36}$$

with $\mu^2=1.85\ (\text{BeV}/c)^2$.

Having determined $\Phi_\pi$ from the analysis of $\pi p$ scattering, the $\pi\pi$ scattering amplitudes (15), (33), and (34) are completely determined, except possibly for a second-order $Q\bar{Q}$ annihilation contribution $f_{\pi\pi a}{}^{(2)}$ which could contribute in the same way $f_{p\bar{p}a}{}^{(3)}$ does for $p\bar{p}$ in the Kokkedee approach. We expect that this second-order $Q\bar{Q}$ annihilation contribution to $\pi\pi$, $f_{\pi\pi a}{}^{(2)}$, is not important since results calculated ignoring this $f_{\pi\pi a}{}^{(2)}$ in our Kokkedee approach agree closely already with the results calculated for $\pi\pi$ scattering in our lumped approach. Figure 5 shows the results for the $\pi\pi$ cross section calculated with $f_{\pi\pi}{}^{(1)}+f_{\pi\pi}{}^{(2)}$ (without third-order multiple-scattering contribution) for our lumped approach with $r_\pi\cong0.25$ F, and for our Kokkedee approach with $r_\pi\cong0.40$ F and ignoring possible second-order $Q\bar{Q}$ annihilations. Gaussian form factors of the pion are used in both calculations. Results calculated for $\pi\pi$ scattering with both approaches look very similar.

With the optical-theorem relations at forward angle the results of our calculations shown in Fig. 5 correspond to a total $\pi\pi$ cross section and inverse $\pi\pi$ diffraction width

$$\sigma_{\pi\pi}\cong19.3\ \text{mb} \quad\text{and}\quad \xi_{\pi\pi}\cong4.6\ (\text{BeV}/c)^{-2} \tag{37}$$

based on our lumped approach, or

$$\sigma_{\pi\pi}\cong16.1\ \text{mb} \quad\text{and}\quad \xi_{\pi\pi}\cong5.0\ (\text{BeV}/c)^{-2} \tag{38}$$

based on our Kokkedee approach. These compare with

$$\sigma_{\pi\pi}\cong14.0\ \text{mb} \quad\text{and}\quad \xi_{\pi\pi}\cong5.8\ (\text{BeV}/c)^{-2} \tag{39}$$

based on the results of the first-order treatment of Sec. IV, and

$$\sigma_{\pi\pi}\cong16.0\ \text{mb} \quad\text{and}\quad \xi_{\pi\pi}\cong7.0\ (\text{BeV}/c)^{-2} \tag{40}$$

based on the results of the "Regge pole with factorizable residues" treatment.[7]

The higher-order SCMQS treatment results in a $\pi\pi$ differential cross section that *cannot* be said to be about 4/9 of the $\pi p$ cross section, while the results of the first-order treatment of Sec. IV indicated a $\pi\pi$ cross section that was compatible with 4/9 of the $\pi p$ cross section. The multiple-scattering effects are clearly quite important for determination of the pion characteristics $r_\pi$, $\Phi_\pi$, $\xi_{\pi\pi}$, and $d\sigma_{\pi\pi}$ from these consistency relations.

## VI. CONCLUSION

We have suggested and applied a rather comprehensive analysis of high-energy hadron scattering according to the SCMQS picture. The SCMQS analysis of $pp$ scattering has led to interesting results[1]: (i) The differential cross section for high-energy $pp$ scattering has been calculated accounting for all its principal details. (ii) The effective body form factor of the distribution of quarks within $p$ has been self-consistently determined with the result that the single-quark body form factor of $p$ has been determined to be very similar to the electromagnetic form factor of $p$, $\Phi_p(\mathbf{q}) \cong G_p{}^E(\mathbf{q})$. (iii) This means that the effective electromagnetic form factor of the quark is much more slowly varying than that of $p$, $G_Q{}^E(\mathbf{q}) \cong 1$, or that the effective electromagnetic radius of the quark is much smaller than that of $p$, $r_Q{}^E \ll r_p{}^E$, and is consistent for purposes of our analysis with a pointlike spatial structure of the quarks. (iv) By variation over discrete values the effective number of quarks per $p$ has been suggested to be $A=3$, and with this confirmation the composite $SU_3$-quark model has been assumed which leads to consistency relations among $pp$, $p\bar{p}$, $\pi p$, and $\pi\pi$ scattering amplitudes and cross sections and the effective form factors of $p$ and $\pi$.

The consistency relations comprise first the determination of the effective $QQ$ scattering amplitude from the analysis of $pp$ the scattering and effective $Q\bar{Q}$ amplitude from the analysis of $p\bar{p}$ scattering with some choice of ansatz about treatment of direct $Q\bar{Q}$ annihilation effects. Then, using these basic $QQ$ and $Q\bar{Q}$ scattering amplitudes, the full SCMQS amplitudes for $pp$, $p\bar{p}$, $\pi p$, and $\pi\pi$ scattering have been calculated. Analysis of $\pi p$ scattering has led to determination of the effective pion form factor as a Gaussian function with effective pion radius $r_\pi \cong r_\pi{}^E = 0.36 \pm 0.12$ F; with point quarks the hadronic, electromagnetic, and quark body form factors of the pion are the same, as are also the corresponding radii. The rapid falloff of the Gaussian $\Phi_\pi$ at large momentum transfers seems to be indicated, although it is not really well determined in our diffraction-scattering picture. This tentative observation is in agreement with a similar observation of Chou and Yang.[12,13] The smallness of the effective pion radius seems to be an important consequence of the consideration of multiple-scattering effects. The results of the single-scattering treatment, as reported earlier, indicated nearly twice this large an effective pion radius, similar to the effective electromagnetic radius of $p$. Results of consideration of higher-order multiple-scattering effects also indicate a considerably broader diffraction peak for $\pi\pi$ scattering than is indicated by the results of the first-order treatment.

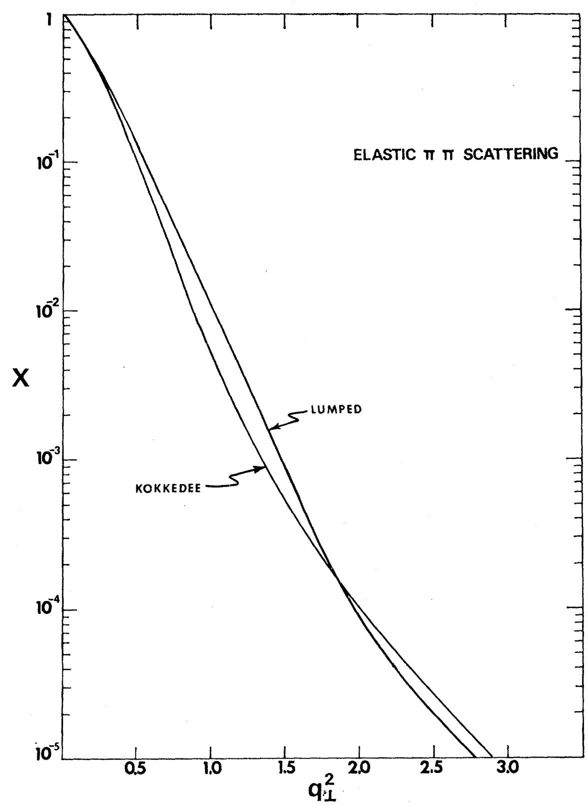


FIG. 5. Results for $X \equiv (d\sigma/dt)/(d\sigma/dt)_{t=0}$ versus $\mathbf{q}_\perp{}^2$ (BeV/$c$)$^2$ of SCMQS analysis of $\pi\pi$ cross section calculated from the consistency relations. Results are shown that are calculated with Gaussian pion form factor and $r_\pi = 0.25$ F for our lumped approach to the direct $Q\bar{Q}$ annihilation contribution and with Gaussian form factor and $r_\pi = 0.40$ F for our Kokkedee approach.

In general, it seems apparent that multiple-scattering effects and the interferences among them are important characteristics of high-energy hadron-scattering phenomena. It is perhaps well to keep in mind that this multiple-scattering viewpoint is the important thing, and that attempts to extract too much detail by analysis within this viewpoint according to specific model constructions may be premature at this time. That is, probably some information about things like the effective pion interaction radius or the width of the effective $\pi\pi$ diffraction peak is the most specific detail that can be sought at present. As to details about the basic scattering interaction process, we will probably have to be satisfied with showing that these processes seem to iterate giving multiple-scattering effects, their attendant interferences, and the much-sought cut structures of analytic amplitudes.[3]

[12] T. T. Chou and C. N. Yang, Phys. Rev. **170**, 1591 (1968).

[13] T. T. Chou and C. N. Yang, in Proceedings of the Second International Conference on High-Energy Physics and Nuclear Structure, held at the Weizman Institute of Science (unpublished).