# Effective-Hamiltonian Approach to Hyperon Beta Decay*

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Our approach to hyperon $\beta$ decay is to restrict the theory, from the beginning, to the first order in the velocity of the recoil nucleon. The resulting effective Hamiltonian leads to an approximation which displays the physical content of the theory more explicitly than the exact expressions, through a simple form already familiar from neutron $\beta$ decay. Assuming the current-current form of the weak interaction and a $V-A$ lepton current, the most general hadron current is used, and the weak magnetism and pseudotensor terms are retained. We obtain correlation coefficients for the decay angular distribution of polarized hyperons which are in close agreement with the exact theory.

EXPERIMENTAL results on the process $\Lambda\rightarrow pe\bar{\nu}$ will soon be of sufficient accuracy to be sensitive to the induced form factors, i.e., the terms of order $v/c$ of the recoil proton.[1] There are several treatments of hyperon $\beta$ decays,[2–8] but the resulting complex expressions tend to obscure the physical content of the theory and are not necessary for the analysis of anticipated experiments. Our approach is to restrict the theory, from the beginning, to the first order in the velocity of the recoil nucleon. This procedure clearly displays at all stages the relativistic corrections to ordinary $\beta$ decay. An entirely similar situation obtains in muon capture where Primakoff introduced the effective Hamiltonian,[9,10] i.e., an expansion of the theory to first order in the neutron velocity. The nucleon velocity in hyperon $\beta$ decay is comparable to that of the neutron in muon capture, thereby allowing us to adapt Primakoff's technique to this problem.

We assume the current-current form of the weak interaction and a $V-A$ lepton current. Using the most general matrix element for the hadron current, the interaction Hamiltonian for the process $B\rightarrow be\bar{\nu}$ can be written in the form

$$\mathcal{H}_{\text{int}}=\tfrac{1}{2}\sqrt{2}\bar{\psi}_b(O_\alpha{}^V+O_\alpha{}^A)\psi_B\bar{\psi}_e\gamma_\alpha(1+\gamma_5)\psi_\nu+\text{H.c.}, \tag{1}$$

where

$$\begin{aligned}O_\alpha{}^V&=f_1\gamma_\alpha+(f_2/M_B)\sigma_{\alpha\beta}Q_\beta+i(f_3/M_B)Q_\alpha,\\ O_\alpha{}^A&=g_1\gamma_\alpha\gamma_5+(g_2/M_B)\sigma_{\alpha\beta}\gamma_5Q_\beta+i(g_3/M_B)\gamma_5Q_\alpha,\end{aligned} \tag{2}$$

and

$$Q_\alpha=(P_e+P_\nu)_\alpha=(P_B-P_b)_\alpha.$$

The induced scalar and pseudoscalar form factors, $f_3$ and $g_3$, may be neglected for this process since they are proportional to the electron mass which we shall ignore throughout. All other form factors are retained in keeping with a purely phenomenological analysis. We note that the use of momentum-independent form factors is consistent with the order of approximation we have adopted.

In the usual way, one has for the decay rate

$$d\omega=\frac{|\mathfrak{M}|^2}{(2\pi)^5}\frac{M_b}{M_B}\frac{e^2\nu^3}{e^{\max}-e}ded\Omega_e d\Omega_\nu, \tag{3}$$

where $e$ and $\nu$ are the electron and antineutrino energies, respectively, and $e^{\max}$ is the $\beta$ endpoint.

The matrix element can be written in the form

$$\mathfrak{M}=\langle be|\mathcal{H}_{\text{eff}}|B\nu\rangle, \tag{4}$$

where $\mathcal{H}_{\text{eff}}$ operates on two-component spinors and has the structure

$$\begin{aligned}\tfrac{1}{2}\sqrt{2}\mathcal{H}_{\text{eff}}&=\tfrac{1}{2}(1-\boldsymbol{\sigma}_l\cdot\hat{e})\\ &\times[G_V+G_A\boldsymbol{\sigma}_B\cdot\boldsymbol{\sigma}_l+G_P{}^e\boldsymbol{\sigma}_B\cdot\hat{e}+G_P{}^\nu\boldsymbol{\sigma}_B\cdot\hat{\nu}]\tfrac{1}{2}(1-\boldsymbol{\sigma}_l\cdot\hat{\nu}).\end{aligned} \tag{5}$$

In the expression above $\hat{e}$ and $\hat{\nu}$ are unit vectors along the electron and antineutrino directions, and $\boldsymbol{\sigma}_l$ and $\boldsymbol{\sigma}_B$ operate solely on lepton and baryon states, respectively. The effective coupling constants in Eq. (5) are simple functions of the form factors

$$\begin{aligned}G_V&=f_1-\frac{M_B-M_b}{M_B}f_2+\frac{e+\nu}{2M_b}\left(f_1+\frac{M_B+M_b}{M_B}f_2\right),\\ G_A&=-g_1+\frac{M_B-M_b}{M_B}g_2-\frac{e-\nu}{2M_b}\left(f_1+\frac{M_B+M_b}{M_B}f_2\right),\\ G_P{}^e&=\frac{e}{2M_b}\left(-f_1+g_1-\frac{M_B+M_b}{M_B}(f_2-g_2)\right),\\ G_P{}^\nu&=\frac{\nu}{2M_b}\left(f_1+g_1+\frac{M_B+M_b}{M_B}(f_2+g_2)\right).\end{aligned} \tag{6}$$

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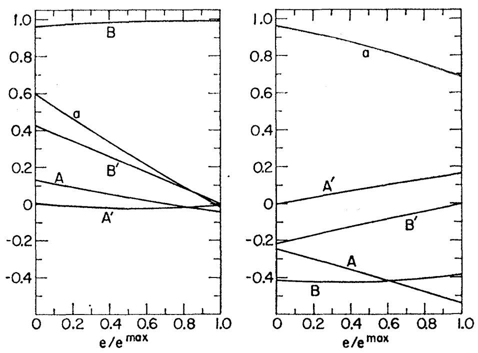


(a) (b)

FIG. 1. (a) $\Lambda$ $\beta$ correlations; (b) $\Sigma$ $\beta$ correlations.

Without the first-order momentum terms in the coupling constants, $G_P{}^e = G_P{}^\nu = 0$ and Eq. (5) reduces to the familiar neutron $\beta$ decay Hamiltonian. The assumption of a $V-A$ lepton current leads to the explicit appearance of left-handed projection operators which bracket the effective Hamiltonian.

Since polarized hyperons are readily available, an experimental analysis should include nucleon, electron, and antineutrino correlations with the hyperon spin direction. However, their spins are not usually observed, so we sum over these in computing the distribution:

$$\sum_{e \text{ spins},\, b \text{ spins}} |\langle be|\mathcal{H}_{\text{eff}}|B\nu\rangle|^2 = \langle B\nu|\mathcal{H}_{\text{eff}}{}^\dagger \mathcal{H}_{\text{eff}}|B\nu\rangle \tag{7}$$

and

$$\sum_{\nu \text{ spins}} \langle B\nu|\mathcal{H}_{\text{eff}}{}^\dagger \mathcal{H}_{\text{eff}}|B\nu\rangle = \langle B|\operatorname{Tr}(\mathcal{H}_{\text{eff}}{}^\dagger \mathcal{H}_{\text{eff}})|B\rangle. \tag{8}$$

The trace calculation is over the antineutrino spin. Keeping only the first-order momentum terms:

$$\begin{aligned}
|\mathfrak{M}|^2 = \xi[&1 + a\hat{e}\cdot\hat{\nu} + A\langle\boldsymbol{\sigma}_B\rangle\cdot\hat{e} + B\langle\boldsymbol{\sigma}_B\rangle\cdot\hat{\nu} + D\langle\boldsymbol{\sigma}_B\rangle\cdot\hat{e}\times\hat{\nu} \\
&+ A'\langle\boldsymbol{\sigma}_B\rangle\cdot\hat{e}(\hat{e}\cdot\hat{\nu}) + B'\langle\boldsymbol{\sigma}_B\rangle\cdot\hat{\nu}(\hat{e}\cdot\hat{\nu})], \\
\xi &= |G_V|^2 + 3|G_A|^2 - 2\operatorname{Re}[G_A{}^*(G_P{}^e + G_P{}^\nu)], \\
\xi a &= |G_V|^2 - |G_A|^2 - 2\operatorname{Re}[G_A{}^*(G_P{}^e + G_P{}^\nu)], \\
\xi A &= -2\operatorname{Re}[G_V{}^* G_A] - 2|G_A|^2 \\
&\quad + 2\operatorname{Re}[G_V{}^* G_P{}^e + G_A{}^* G_P{}^\nu], \\
\xi B &= -2\operatorname{Re}[G_V{}^* G_A] + 2|G_A|^2 \\
&\quad + 2\operatorname{Re}[G_V{}^* G_P{}^\nu - G_A{}^* G_P{}^e], \\
\xi D &= 2\operatorname{Im}[G_V{}^* G_A] + 2\operatorname{Im}[G_A{}^*(G_P{}^\nu - G_P{}^e)], \\
\xi A' &= 2\operatorname{Re}[G_P{}^{e*}(G_V + G_A)], \\
\xi B' &= 2\operatorname{Re}[G_P{}^{\nu*}(G_V - G_A)].
\end{aligned} \tag{9}$$

TABLE I Form factors and no-recoil correlation coefficients.

| | $a$ | $A$ | $B$ | $A'$ | $B'$ | $g_1/f_1$ | $f_2/f_1$ | $g_2/f_1$ |
|---|---|---|---|---|---|---|---|---|
| $\Lambda \to pe\bar{\nu}$ | 0.18 | 0.15 | 0.97 | 0 | 0 | 0.73[a] | 1.0[b] | 0 |
| $\Sigma^- \to ne\bar{\nu}$ | 0.80 | −0.51 | −0.31 | 0 | 0 | −0.24[a] | −1.1[b] | 0 |

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Note that in the no-recoil approximation $A' = B' = 0$ and the other terms become those familiar from neutron $\beta$ decay.[11] We have maintained complex coupling constants throughout, since time-reversal invariance is still an open question in hyperon decays.

To illustrate the effects of the first-order momentum terms, Figs. 1(a) and 1(b) show the correlation coefficients introduced in Eq. (9) as a function of $e/e^{\max}$ for the decays $\Lambda \to pe\bar{\nu}$ and $\Sigma^- \to ne\bar{\nu}$. These are to be compared with the no-recoil values shown in Table I. Also included are the form factors that we have used.

We have compared our results with computations based on the exact expression of Ref. 5 and find an over-all agreement which is typically better than 2% and never worse than 3% of the leading term.[12] If we work with the exact angular distribution resulting from $\mathcal{H}_{\text{eff}}$ of Eq. (5), the agreement is even better.[13] Such accuracy is adequate for the analysis of all hyperon $\beta$-decay experiments in the near future. In addition, our approximation displays the physical content of the theory more explicitly than the exact expressions through a simple form already familiar from neutron $\beta$ decay.

*Note added in proof.* After submitting this paper, the authors learned of a pertinent paper by P. S. Desai, Phys. Rev. **179**, 1327 (1969), which treats decays for which the only observed polarization is that of the final baryon. The expression for the lepton correlations with respect to $\langle\sigma_b\rangle$ is obtained from Eq. (9) by interchanging $e$ and $\nu$ throughout and reversing the sign of $D$. Eq. (6) is unchanged.

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[12] Not so for Ref. 2, with which we disagree when the weak magnetism term $f_2$ is included.

[13] The exact angular distribution has $|G_P{}^e|^2 + |G_P{}^\nu|^2$ added to $\xi$, $|G_P{}^e + G_P{}^\nu|^2 + 2\operatorname{Re}[G_P{}^{e*} G_P{}^\nu]\hat{e}\cdot\hat{\nu}$ added to $\xi a$, and $2\operatorname{Im}[G_P{}^e G_P{}^{\nu *}] \times (1 + \hat{e}\cdot\hat{\nu})$ added to $\xi D$.