Chiral Lagrangian Model of Single-Pion Photoproduction*

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(Received 23 October 1968)

We have constructed a chiral-invariant Lagrangian model for single pion photoproduction. We have used this Lagrangian to calculate threshold values for the differential cross sections. These threshold results were found to be in reasonable agreement with recent current-algebra calculations. For the cases where data were available, the comparison with experiment was favorable.

I. INTRODUCTION

RECENTLY, current-algebra techniques have been used to obtain threshold theorems for photoproduction which are in good agreement with experiment.[1,2] The derivation of these threshold results is accompanied by two types of difficulties: one inherent in current-algebra calculations, and the other peculiar to photoproduction. The former concerns the extrapolation of the amplitudes from off-shell to on-shell, and the latter is the question of gauge invariance when the pion is off the mass shell. Although these difficulties are surmountable, we would like to point out that there is another way to obtain these threshold results which avoids the above difficulties altogether. It is the chiral Lagrangian approach.

For soft-pion processes, as is well known,[3,4] one can construct chiral-invariant Lagrangians that are equivalent in their content to the current-algebra hypotheses. Because the pion field is assumed to transform nonlinearly under chiral transformations,[5] these chiral Lagrangians are highly nonlinear. Effectively, however, one always deals with these Lagrangians in the tree approximation.[4]

By using a Lagrangian formalism the gauge-invariance problem is very simple to solve. All that one must do is to either couple the electromagnetic potential A_α to conserved currents or to couple the system directly to the gauge-invariant electromagnetic field tensor $F_{\alpha\beta}$.

A Lagrangian formalism also avoids the problem of extrapolation, since one always obtains physical amplitudes. In this context we should perhaps remark that the Lagrangian method contains a built-in extrapolation *off* the mass shell and that this extrapolation is smooth. In current-algebra calculations one obtains an off-mass-shell amplitude, and then one extrapolates, presumably smoothly, *to* the mass shell. Thus the use of a *particular* chiral Lagrangian is equivalent to an assumption of a *particular* method of extrapolation in the corresponding current-algebra case.

II. CHIRAL-INVARIANT LAGRANGIAN FOR PHOTOPRODUCTION

To construct a chiral-invariant Lagrangian for photoproduction we make use of the usual chiral Lagrangian for pion-nucleon scattering[3,6] and introduce a coupling to the electromagnetic field by a minimal substitution $\partial_\alpha \to \partial_\alpha - ieA_\alpha$. for charged particles. This substitution guarantees that we couple A_α to a conserved current, and therefore preserves guage invariance. We further add terms to the Lagrangian that describe phenomenologically the interaction of the electromagnetic field with the nucleon magnetic moments. Finally, since we also want to take into account the effects of the $N^*(1236)$ at threshold, we add a phenomenological $N^*N\gamma$ interaction. Of course, then we must retain a corresponding $N^*N\pi$ term in the pion-nucleon Lagrangian.[6] Our total Lagrangian for photoproduction is then

$$\mathfrak{L} = \mathfrak{L}_{\text{el}} + \mathfrak{L}_{\pi NN} + \mathfrak{L}_{\pi NN^*},$$

with

$$\begin{aligned}\mathfrak{L}_{\text{el}} = {} & eA^\alpha\{\epsilon_{ij3}\partial_\alpha\pi_i\pi_j + \bar{N}[\tfrac{1}{2}(1+\tau_3)]\gamma_\alpha N + (f\sqrt{2}/m)\bar{N}\gamma_\alpha\gamma_5(\tau_+\pi^- - \tau_-\pi^+)N\} \\ & + eF^{\alpha\beta}\{(\kappa_p/4M)\bar{N}\sigma_{\alpha\beta}[\tfrac{1}{2}(1+\tau_3)]N + (\kappa_n/4M)\bar{N}\sigma_{\alpha\beta}[\tfrac{1}{2}(1-\tau_3)]N + (\kappa^*/4M)[\bar{N}_3{}^{*\lambda}(i\gamma_\alpha g_{\lambda\beta} - i\gamma_\beta g_{\lambda\alpha} \\ & - \tfrac{1}{2}\gamma_\lambda\sigma_{\alpha\beta})\gamma_5 N + \bar{N}(i\gamma_\alpha g_{\lambda\beta} - i\gamma_\beta g_{\lambda\alpha} + \tfrac{1}{2}\sigma_{\alpha\beta}\gamma_\lambda)\gamma_5 N_3{}^{*\lambda}]\},\end{aligned}$$

$$\mathfrak{L}_{\pi NN} = (f/m)\bar{N}i\gamma_\alpha\gamma_5\tau_i N\partial^\alpha\pi_i,$$

$$\mathfrak{L}_{\pi NN^*} = (ih/m)[\bar{N}_{\alpha i}{}^*(4g^{\alpha\beta} + \gamma^\alpha\gamma^\beta)N - \bar{N}(4g^{\beta\alpha} + \gamma^\beta\gamma^\alpha)N_{\alpha i}{}^*]\partial_\beta\pi_i,$$

* Work supported in part through funds provided by the U. S. Atomic Energy Commission under Contract No. AT (30-1)-2098.
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[2] M. S. Bhatia and P. Narayanaswany, Phys. Rev. **172**, 1742 (1968).
[3] S. Weinberg, Phys. Rev. Letters **18**, 188 (1967).
[4] B. W. Lee and H. T. Nieh, Phys. Rev. **166**, 1507 (1968).
[5] S. Weinberg, Phys. Rev. **166**, 1568 (1968).
[6] R. D. Peccei, Phys. Rev. **176**, 1812 (1968).

where

$$N_1^*=\frac{1}{\sqrt{2}}\begin{pmatrix}N^{++}-N^0/\sqrt{3}\\ N^+/\sqrt{3}-N^-\end{pmatrix},\quad N_2^*=\frac{i}{\sqrt{2}}\begin{pmatrix}N^{++}+N^0/\sqrt{3}\\ N^+/\sqrt{3}+N^-\end{pmatrix},$$

$$N_3^*=-(\sqrt{\tfrac{2}{3}})\begin{pmatrix}N^+\\ N^0\end{pmatrix}.$$

Two comments are in order. First, the contact interaction in $\mathcal{L}_{\mathrm{el}}$ comes about from the minimal replacement of ∂_α in $\mathcal{L}_{\pi NN}$. It has a crucial effect in preserving gauge invariance. Second, the particular structure displayed by the $N^*N\gamma$ and $N^*N\pi$ vertices is dictated by the requirement of coupling the $N\gamma$ and $N\pi$ systems only to the spin-$\frac{3}{2}$ part of the N^* field, both on and off the N^* mass shell.[6] In particular, the terms $\gamma_\lambda\sigma_{\alpha\beta}\gamma_5$ and $\gamma_\alpha\gamma_\beta$ for the $N^*N\gamma$ and $N^*N\pi$ vertices have no effect when the N^* is on the mass shell, but they contribute off the N^* mass shell.

For completeness we record below the numerical values of the various coupling constants and parameters used:

$$f^2/4\pi=0.080\pm0.001,\quad e^2/4\pi=1/137,$$
$$h^2=0.290\pm0.006,$$
$$\kappa_p=1.79,\quad \kappa_n=-1.91,\quad \kappa^*=5.0.$$

The value of κ^* varies depending on the particular analysis of photoproduction considered. Dufner and Tsai[7] give $\kappa^*=5.02$ based on Dalitz and Sutherland's analysis.[8] Matthews[9] gives $\kappa^*=4.90$. Gourdin and Salin[10] obtain $\kappa^*=6.10$. Huang,[11] using a static-model comparison, has $\kappa^*=4.31$. And, finally, the $SU(6)$ prediction[12] is $\kappa^*=3.95$. We shall see later that the threshold results do not depend very much on the N^* contribution, and so the value of κ^* adopted is not particularly critical. Perhaps it is worth mentioning here that the extra term that we have in our $N^*N\gamma$ vertex does not affect the determination of κ^*, since this parameter is evaluated at the resonance. A similar situation occurs in the $N^*N\pi$ vertex, where the value of h^2 is independent of whether we take the vertex with the extra $\gamma_\alpha\gamma_\beta$ term or not.[6]

III. THRESHOLD RESULTS

To proceed we decompose the photoproduction amplitude in the usual CGLN way.[13]

$$T=\bar{u}(p_2)[A\gamma\cdot\epsilon\gamma\cdot k+B(P\cdot kq\cdot\epsilon-P\cdot\epsilon q\cdot k)$$
$$+C(q\cdot\epsilon\gamma\cdot k-q\cdot k\gamma\cdot\epsilon)$$
$$+2D(P\cdot k\gamma\cdot\epsilon-P\cdot\epsilon\gamma\cdot k+M\gamma\cdot\epsilon\gamma\cdot k)]\gamma_5 u(p_1),$$

where p_1 and p_2 are the incoming and outgoing nucleon momenta, k is the photon momentum, q is the meson momentum, and $P=\frac{1}{2}(p_1+p_2)$.

The isospin content of the amplitude A is given by

$$A=A^+\delta_{\beta 3}+A^-\times\tfrac{1}{2}[\tau_\beta,\tau_3]+A^0\tau_\beta,$$

with similar relations holding for B, C, and D. Here β is the isospin of the outgoing nucleon.

The amplitudes for physical processes are then given by

$$T(\gamma+p\to p+\pi^0)=T^++T^0,$$
$$T(\gamma+n\to n+\pi^0)=T^+-T^0,$$
$$T(\gamma+p\to n+\pi^+)=\sqrt{2}(T^0+T^-),$$
$$T(\gamma+n\to p+\pi^-)=\sqrt{2}(T^0-T^-).$$

At threshold the differential cross section in the center-of-mass system is given by

$$\frac{w}{|\mathbf{q}|}\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{c.m.}}=\frac{1}{16\pi^2}\frac{M^3}{(M+m)^3}\left(\frac{2Mm+m^2}{2M}\right)^2$$
$$\times|mD-A-w(D+C)|^2,$$

where $w=|\mathbf{k}|=m(2M+m)/(2M+2m)$ at threshold.

To obtain the desired threshold results we need only compute the amplitudes A, C, and D in the appropriate isospin combinations and make use of the above formula for the differential cross section. These amplitudes, along with B, are given in the Appendix and can be calculated in a straightforward manner from our chiral Lagrangian. We find, in this way,

$$(w/|\mathbf{q}|)(d\sigma/d\Omega)(\gamma+p\to n+\pi^+)$$
$$=\lambda[1+r^2(\kappa_p+\kappa_n+\xi\kappa^*)]^2,$$

$$(w/|\mathbf{q}|)(d\sigma/d\Omega)(\gamma+n\to p+\pi^-)$$
$$=\lambda[1+2r-r^2(\kappa_p+\kappa_n-\xi\kappa^*)]^2,$$

$$(w/|\mathbf{q}|)(d\sigma/d\Omega)(\gamma+p\to p+\pi^0)$$
$$=2\lambda r^2[1-r(\kappa_p+\xi'\kappa^*)]^2,$$

$$(w/|\mathbf{q}|)(d\sigma/d\Omega)(\gamma+n\to n+\pi^0)=2\lambda r^4[\kappa_n-\xi'\kappa^*]^2,$$

where

$$\lambda=\frac{e^2}{4\pi}\frac{f^2}{4\pi}\frac{2M^3}{(M+m)^3}\frac{1}{m},\quad r=\frac{m}{2M},$$

$$\xi=\frac{h}{f}\frac{(2M+m)^2}{3(M+m)M^{*2}}$$
$$\times\left(2M^*+M-\frac{2Mm[M^*(M+m)+M^2]}{3[(M^{*2}+Mm)(M+m)-M^3]}\right),$$

$$\xi=1.41,$$

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and

$$\xi' = -\frac{h}{f}\frac{(2M+m)m}{3(M+m)M^{*2}} \times\left(-M+2M^* - \frac{2M(2M+m)[M^*(M+m)+M^2]}{3[(M^{*2}+Mm)(M+m)-M^3]}\right),$$

$$\xi' = -0.040.$$

We remark that Bhatia and Narayanaswany[2] obtain the same results as ours by current-algebra methods, except that their N^* contribution at threshold differs from ours because of the different $N^*N\gamma$ and $N^*N\pi$ vertices adopted in our work. Nevertheless, in both cases the contribution is small. We find that the N^* contributes about 8% for the charged-pion photoproduction, 3% for π^0 photoproduction by protons, and about 25% for π^0 photoproduction by neutrons.

In conclusion, we record in Table I our final results at threshold obtained by the chiral Lagrangian model, and we compare them to the current-algebra work and, when possible, to experiment.[14]

TABLE I. Reduced cross section $(|\mathbf{k}|/|\mathbf{q}|)d\sigma/d\Omega$ for threshold pion photoproduction, in μb/sr.

Process	Chiral Lagrangian	Current algebra	Experiment[a]
$\gamma+p\rightarrow n+\pi^+$	16.4	16.79	15.6±0.5
$\gamma+n\rightarrow p+\pi^-$	21.7	21.96	b
$\gamma+p\rightarrow p+\pi^0$	0.13	0.116	...
$\gamma+n\rightarrow n+\pi^0$	0.0027	0.0018	...

[a] Reference 14.
[b] What is measured is the Panofsky ratio.

$$R=\frac{d\sigma}{d\Omega}(\gamma+n\rightarrow p+\pi^-)\bigg/\frac{d\sigma}{d\Omega}(\gamma+p\rightarrow n+\pi^+)=1.265\pm0.075.$$

We obtain $R=1.32$.

ACKNOWLEDGMENT

We would like to thank Professor Kenneth Johnson for his constant guidance and encouragement.

APPENDIX

We give below the chiral Lagrangian contributions to the twelve CGLN amplitudes:

$$A^+=\frac{ef}{2m}\left\{2M\left(\frac{1}{M^2-s}+\frac{1}{M^2-u}\right)-\frac{1}{M}(\kappa_p-\kappa_n)-\frac{h}{f}\frac{\kappa^*}{3M}\left[\frac{1}{M^{*2}-s}\left(4t+\frac{4M}{3M^*}(s-M^2+2m^2)+\frac{4M^2}{3M^{*2}}(s-M^2+m^2)\right)\right.\right.$$
$$\left.\left.+(s\rightarrow u,\, t\rightarrow t)\right]\right\},$$

$$B^+=\frac{ef}{2m}\left[\frac{-8M}{(M^2-s)(M^2-u)}+\frac{h}{f}\frac{\kappa^*}{3M}\left(\frac{16}{M^{*2}-s}+\frac{16}{M^{*2}-u}\right)\right],$$

$$C^+=\frac{ef}{2m}\left\{(\kappa_p-\kappa_n)\left(\frac{1}{M^2-u}-\frac{1}{M^2-s}\right)+\frac{h}{f}\frac{\kappa^*}{3M}\left[\frac{1}{M^{*2}-s}\left(2M+4M^*-\frac{4M^2}{3M^*}-\frac{2M}{3M^{*2}}(s-2M^2+2m^2)\right)-(s\rightarrow u,\, t\rightarrow t)\right]\right\},$$

$$D^+=\frac{ef}{2m}\left\{(\kappa_p-\kappa_n)\left(\frac{1}{M^2-u}+\frac{1}{M^2-s}\right)+\frac{h}{f}\frac{\kappa^*}{3M}\left[\frac{1}{M^{*2}-s}\left(6M+4M^*+\frac{4M^2}{3M^*}+\frac{2M}{3M^{*2}}(s-2M^2+2m^2)\right)+(s\rightarrow u,\, t\rightarrow t)\right]\right\},$$

$$A^-=\frac{ef}{2m}\left\{2M\left(\frac{1}{M^2-s}-\frac{1}{M^2-u}\right)+\frac{h}{f}\frac{\kappa^*}{3M}\left[\frac{1}{M^{*2}-s}\left(2t+\frac{2M}{3M^*}(s-M^2+2m^2)+\frac{2M^2}{3M^{*2}}(s-M^2+m^2)\right)-(s\rightarrow u,\, t\rightarrow t)\right]\right\},$$

$$B^-=\frac{ef}{2m}\left[\frac{8M}{m^2-t}\left(\frac{1}{M^2-s}-\frac{1}{M^2-u}\right)-\frac{h}{f}\frac{\kappa^*}{3M}\left(\frac{8}{M^{*2}-s}-\frac{8}{M^{*2}-u}\right)\right],$$

$$C^-=\frac{ef}{2m}\left\{(\kappa_p-\kappa_n)\left(-\frac{1}{M^2-u}-\frac{1}{M^2-s}\right)-\frac{h}{f}\frac{\kappa^*}{3M}\left[\frac{1}{M^{*2}-s}\left(M+2M^*-\frac{2M^2}{3M^*}-\frac{M}{3M^{*2}}(s-2M^2+2m^2)\right)\right.\right.$$
$$\left.\left.+(s\rightarrow u,\, t\rightarrow t)\right]\right\},$$

$$D^-=\frac{ef}{2m}\left\{(\kappa_p-\kappa_n)\left(\frac{1}{M^2-s}-\frac{1}{M^2-u}\right)-\frac{h}{f}\frac{\kappa^*}{3M}\left[\frac{1}{M^{*2}-s}\left(3M+2M^*+\frac{2M^2}{3M^*}+\frac{M}{3M^{*2}}(s-2M+2m^2)^2\right)\right.\right.$$
$$\left.\left.-(s\rightarrow u,\, t\rightarrow t)\right]\right\},$$

[14] J. P. Burq, Ann. Phys. (Paris) **10**, 363 (1965).

$$A^0=\frac{ef}{2m}\left[2M\left(\frac{1}{M^2-s}+\frac{1}{M^2-u}\right)-\frac{1}{M}(\kappa_p+\kappa_n)\right],\quad B^0=\frac{ef}{2m}\left[\frac{-8M}{(M^2-s)(M^2-u)}\right],$$

$$C^0=\frac{ef}{2m}\left[(\kappa_p+\kappa_n)\left(\frac{1}{M^2-u}-\frac{1}{M^2-s}\right)\right],\quad D^0=\frac{ef}{2m}\left[(\kappa_p+\kappa_n)\left(\frac{1}{M^2-u}+\frac{1}{M^2-s}\right)\right].$$

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An $I=1$ $\pi\pi$ Sum Rule and the Derivatives of the S-Wave $\pi\pi$ Amplitudes*

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A sum rule for the $I=1$ $\pi\pi$ amplitude is written at the symmetry point, $s=t=u=\frac{4}{3}m_\pi^2$, under the assumption that $\partial A^1(t,V)/\partial V$ satisfies an unsubtracted fixed-t dispersion relation, where t and V are the energy and momentum-transfer variables. From the symmetry-point conditions, $\partial A^0/\partial t$ and $\partial A^2/\partial t$ are obtained and compared with the current-algebra estimates.

HERE we present a sum rule for the $I=1$ $\pi\pi$ amplitude from which one can obtain sum rules for the derivative of the $I=0$ and $I=2$ $\pi\pi$ amplitudes at the symmetry point, $s=t=u=\frac{4}{3}m_\pi^2$. The sum rule has the property that the contributions of high-mass resonances drop off very rapidly.

The derivative relations at the symmetry point for the amplitudes $A^I(t,V)$, where t is the energy variable and $V=\frac{1}{2}(s-u)$ are (we take $\hbar=c=m_\pi=1$)[1]

$$\partial A^0/\partial t(\tfrac{4}{3},0)=-2(\partial A^2/\partial t)(\tfrac{4}{3},0) \tag{1}$$

$$=\partial A^1/\partial V(\tfrac{4}{3},0)\,. \tag{2}$$

Now if $A^I(t,V)\to\beta(t)V^{\alpha_I(t)}$ as $V\to\infty$, then since $\alpha_I(t)<1$ for $I=1$ and $t<m_\rho^2$ we can write an unsubtracted, fixed-t, dispersion relation in the neighborhood of $t=\frac{4}{3}$,

$$\frac{\partial A^1}{\partial V}(t,V)=\frac{1}{\pi}\int_{V_0}^{\infty}dV'A_V{}^1(t,V')\times\left(\frac{1}{(V'-V)^2}+\frac{1}{(V'+V)^2}\right). \tag{3}$$

Using crossing symmetry for the absorptive part $A_V{}^1$ we get at $t=\frac{4}{3}$ and $V=0$,

$$\frac{\partial A^1}{\partial V}(\tfrac{4}{3},0)=\frac{2}{\pi}\left\{\int_{V_0}^{N}\frac{dV'}{V'^2}\left[\tfrac{1}{3}\operatorname{Im}A^0(V',t)+\tfrac{1}{2}\operatorname{Im}A^1(V',t)-\tfrac{5}{6}\operatorname{Im}A^2(V',t)\right]+c(t)\frac{N^{\alpha_1(t)-1}}{1-\alpha_1(t)}\right\}, \tag{4}$$

where $\operatorname{Im}A^I(V,t)$ are the imagninary parts in the s channel, and the second term on the right-hand side of (4) is the Regge term.

An important point to notice in relation (4) above is that the $I=0$ and $I=1$ contributions, as well as the Regge contribution, are all *positive.* Since no resonance in $I=2$ has been observed, the derivative $\partial A^1/\partial V$ is positive and, therefore, from (2) $\partial A^0/\partial t$ is positive at the symmetry point.

In Table I we have given the contributions of different $\pi\pi$ resonances to the right-hand side of the sum rule (4). In addition to ρ and f_0, we have also included higher resonances such as g (the recurrence of ρ) and h (a possible recurrence of f^0). The experimental value of the $\pi\pi$-partial width of the g meson is not known precisely. It varies between 30 and 75 MeV.[2] The values in Table I correspond to $\Gamma_g=100$ MeV, $\Gamma_g=30$ MeV, and $m_g{}^2=2.6$ BeV^2. Experimentally very little is known about h, but we expect the f^0 trajectory to be roughly parallel to the ρ trajectory, so that a recurrence of f^0 should exist around $m_h{}^2=3.6$ BeV^2. The values given in Table I correspond to $\Gamma_h=100$ MeV and $\Gamma_h=30$ MeV. The main point we want to emphasize about the sum rule (4) is that the contributions of higher resonances fall off very rapidly and, therefore, the value of $\partial A^0/\partial t$ obtained through (2), is *not* sensitive to the contributions of g, h, and other higher resonances.

* Work supported in part by the Atomic Energy Commission Contract No. AEC AT(11-1)34 P107A and in part by Contract No. W-7405-Eng-48.

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[2] For the upper limit of the g partial width we have used 50% of the value of g total width [A. H. Rosenfeld *et al.*, University of California Radiation Laboratory Report No. UCRL-8030 (rev.) 1968 (unpublished)] where it is indicated that the 2π decay mode is dominant. For the lower limit we have taken the partial widths given by T. F. Johnston *et al.*, Phys. Rev. Letters **20**, 1414 (1968); D. J. Crennell *et al.*, *ibid.* **18**, 323 (1967); and N. N. Biswas *et al.*, *ibid.*. **21**, 50 (1968).