# Vector Dominance and the $K_{l4}$ Vector Form Factor

L. E. Wood*
*Center for Theoretical Physics, Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742*

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The $K_{l4}$ vector form factor is estimated using Sakurai's vector-dominance model. For the first time, *all* lowest-order Feynman graphs which contribute to the process have been included. The vector form factor is found to be between $-0.96$ and $-1.4$, depending on the methods and experimental numbers used. Comparisons with previous estimates and present experimental data are made.

## INTRODUCTION

THE $K_{l4}$-vector form factor will be estimated according to Sakurai's vector-dominance model.[1] Vector dominance is of course a rather simple model; however, it has been successfully applied to many other processes.[2,3] Hence, it is interesting to see if it can be applied to $K_{l4}$. In order to do this, it is necessary to include all relevant vector resonances which may contribute. Previous calculations[4-6] have *only* considered the $\rho$ resonance [Fig. 1(a)], while here *both* the $\rho$ and $K^*$ contributions are considered [Figs. 1(a) and 1(b)].[7] Also, at the present time, there does not seem to be a successful calculation of the $K_{l4}$-vector form factor using more sophisticated models (as we discuss further below).

The vector form factor is defined by the relation[8]

$$(2K^0 2q_1^0 2q_2^0)^{1/2}\langle\pi^+(q_1)\pi^-(q_2)|V^\mu(0)|K^+(K)\rangle = i\epsilon^{\mu\lambda\sigma\nu}K_\lambda q_{1\sigma}q_{2\nu}d/m_K^3. \quad (1)$$

We assume the $|\Delta I|=\frac{1}{2}$ rule. Vector dominance means here that the Feynman graphs of Fig. 1 are assumed to give a good approximation to the matrix element in Eq. (1). Assuming $SU(3)$ symmetry, the following Lagrangian can be used to calculate these graphs:

$$\mathcal{L}_{\rm int}=g_{VPP}\,{\rm Tr}(V^\mu[P,\partial_\mu P])+g_{VVP}\,{\rm Tr}(P\partial_\alpha V_\beta\partial_\gamma V_\delta)\epsilon_{\alpha\beta\gamma\delta} +(G/\sqrt{2})g_{K^*e\nu}K^{*\lambda}\bar{u}_e\gamma_\lambda(1+\gamma_5)u_\nu, \quad (2)$$

where $P$ is the octet of pseudoscalar mesons, and $V$ is the octet of vector mesons.

The assumption of $SU(3)$, like constant couplings, is a rather simple approach; however, $SU(3)$ has been remarkably successful, and it is interesting to see if it can be applied to $K_{l4}$. The relevant terms of (2) are

$$\mathcal{L}_{\rm int}=g_{VPP}[K_\mu^{0*}(K^-\overleftrightarrow{\partial}_\mu\pi^+)+\sqrt{2}\rho_\mu^0(\pi^+\overleftrightarrow{\partial}_\mu\pi^-)] +g_{VVP}\epsilon_{\alpha\beta\gamma\delta}[\pi^-\partial_\alpha\bar{K}_\beta^{*0}\partial_\gamma K_\delta^{*+} +(1/\sqrt{2})K^-\partial_\alpha K_\beta^{*+}\partial_\gamma\rho_\delta^0]+(G/\sqrt{2})g_{K^*e\nu}K^{*\lambda}\bar{u}_e \times\gamma_\lambda(1+\gamma_5)u_\nu. \quad (3)$$

Then, by straightforward perturbation theory, one finds for the form factor $d$:

$$d_1=\frac{-2g_{VVP}g_{VPP}g_{K^*e\nu}m_K^3}{(K-q_1-q_2)^2-m_{K^*}^2}\frac{1}{(q_1+q_2)^2-m_\rho^2}, \quad (4)$$

$$d_2=\frac{-2g_{VVP}g_{VPP}g_{K^*e\nu}m_K^3}{(K-q_1-q_2)^2-m_{K^*}^2}\frac{1}{(K-q_1)^2-m_{K^*}^2}, \quad (5)$$

$$d=d_1+d_2, \quad (6)$$

where $d_1$ is the contribution from Fig. 1(a), and $d_2$ from Fig. 1(b).[9]

To determine $g_{VVP}$, $g_{VPP}$, and $g_{K^*e\nu}$, one can proceed in two different, but theoretically equivalent, ways. In the first method, called method $A$, one can determine $g_{\rho\pi\pi}$ from the experimentally known $\rho$ width, and hence $g_{VPP}$. In fact

$$\Gamma(\rho\to\pi\pi)=(g_{\rho\pi\pi}^2/6\pi m_\rho^2)[(\tfrac{1}{2}m_\rho^2)-m_\pi^2]^{3/2}, \qquad g_{VPP}=g_{\rho\pi\pi}/\sqrt{2}. \quad (7)$$

Using a $\rho$ width of 100 MeV, one finds $g_{\rho\pi\pi}\simeq 4.89$; using a width of 140 MeV, $g_{\rho\pi\pi}\simeq 5.78$.[10] To estimate $g_{VVP}$ and $g_{K^*e\nu}$, the Feynman graph of Fig. 2(a) is used to give

$$eg_{\omega\rho\pi}g_{\rho\gamma}/m_\rho^2=eg_{\omega\pi\gamma}, \quad (8)$$

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[5] J. Ilioupoulos, Nuovo Cimento **38**, 907 (1965); **39**, 413(E) (1965).

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[7] By angular momentum and parity considerations, a $\kappa$ resonance (if it exists) will not contribute to this process.

[8] Our normalization corresponds to that of Ref. 14.

[9] The quantity $d_2$ can be written as $d_2=d_2'+d_2''$, where

$$d_2'=\frac{-2g_{VVP}g_{VPP}g_{K^*e\nu}m_K^3}{(K-q_1-q_2)^2-m_K^{*2}}\times\frac{1}{2}\left[\frac{1}{(K-q_1)^2-m_K^{*2}}-\frac{1}{(K-q_2)^2-m_K^{*2}}\right].$$

$$d_2''=\frac{-2g_{VVP}g_{VPP}g_{K^*e\nu}m_K^3}{(K-q_1-q_2)^2-m_K^{*2}}\times\frac{1}{2}\left[\frac{1}{(K-q_1)^2-m_K^{*2}}+\frac{1}{(K-q_2)^2-m_K^{*2}}\right],$$

Then, $d_2'$ gives the contribution to $d_2$ from the $I=0$, $d$-wave final state of the pions, and $d_2''$ is the $I=1$ $p$-wave contribution. $d_2'$ is small and will be neglected here; we assume that the outgoing pions are in an $I=1$ state, and take $d_2''\simeq d_2$.

[10] See Ref. 13 for a discussion of the $\rho$ width. Note that recent experiments show this width varying from 90 to 150 MeV, but seem to favor a value close to 100 MeV.

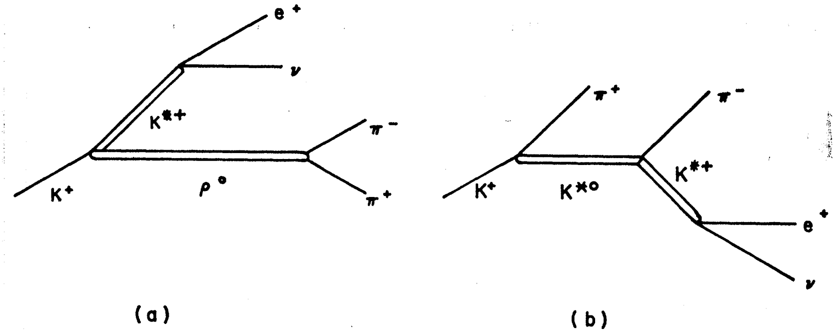


FIG. 1. Feynman graphs used to calculate $d$.

and the $SU(3)$ relations

$$g_{\omega\rho\pi}=(2/\sqrt{6})g_{VVP}/\sin\lambda\,, \tag{9}$$

$$g_{K^*e\nu}=g_{\rho\gamma}\sin\theta \tag{10}$$

are used, where $\theta$= Cabibbo angle,[11] and $\lambda=\omega-\phi$ mixing angle. ($\sin\lambda\simeq1/\sqrt{3}$).[12] Experimentally, $\Gamma(\omega\rightarrow\pi\gamma)\simeq1.2$ MeV,[13] which gives $g_{\omega\pi\gamma}\simeq2.96\times10^{-3}$ MeV$^{-1}$. Putting this together gives the following:

*method A*

$$\begin{aligned}2m_K{}^3g_{VVP}g_{VPP}g_{K^*e\nu}&=2.24\times10^{11}\text{ MeV}^4\\ &\quad\text{for }\Gamma(\rho\rightarrow\pi\pi)=100\text{ MeV}\\ &=2.65\times10^{11}\text{ MeV}^4\\ &\quad\text{for }\Gamma(\rho\rightarrow\pi\pi)=140\text{ MeV}.\end{aligned} \tag{11}$$

Alternatively, one can proceed by method $B$, in which the Gell-Mann–Sharp–Wagner[2] model is used to estimate $g_{\omega\rho\pi}$. Then the graph of Fig. 2(b) yields[2,5]

$$eg_{\rho\gamma}g_{\rho\pi\pi}/m_\rho{}^2=e. \tag{12}$$

From $\Gamma(\omega\rightarrow3\pi)=11$ MeV,[13] and using[2]

$$\frac{g^2{}_{\omega\rho\pi}}{4\pi}=\frac{\Gamma(\omega\rightarrow3\pi)}{\Gamma(\rho\rightarrow2\pi)}\frac{[(\frac{1}{2}m_\rho)^2-m_\pi{}^2]^{7/2}}{(m_\omega-3m_\pi)^4}\frac{2^7\sqrt{3}}{m_\rho{}^2m_\omega m_\pi{}^2(3.56)}, \tag{13}$$

one finds

$$\begin{aligned}g_{\omega\rho\pi}&=2.03\times10^{-2}\text{ MeV}^{-1}\quad\text{for }\Gamma(\rho\rightarrow\pi\pi)=100\text{ MeV}\\ &=1.73\times10^{-2}\text{ MeV}^{-1}\quad\text{for }\Gamma(\rho\rightarrow\pi\pi)=140\text{ MeV}.\end{aligned}$$

*method B*

$$\begin{aligned}2m_K{}^3g_{VVP}g_{VPP}g_{K^*e\nu}&=2.15\times10^{11}\text{ MeV}^4\\ &\quad\text{for }\Gamma(\rho\rightarrow\pi\pi)=100\text{ MeV}\\ &=2.68\times10^{11}\text{ MeV}^4\\ &\quad\text{for }\Gamma(\rho\rightarrow\pi\pi)=140\text{ MeV}.\end{aligned} \tag{14}$$

The difference between the two sets of numbers in Eqs. (11) and (14) is due to the fact that Eqs. (8) and (12) are not completely consistent with experiment. Combining these two equations, one would predict for $\Gamma(\rho\rightarrow\pi\pi)=100$ MeV,

$$g_{\rho\pi\pi}=g_{\omega\rho\pi}/g_{\omega\pi\gamma}\simeq6.8\,, \tag{15}$$

compared to the value of $g_{\rho\pi\pi}\simeq4.9$ which one would deduce[3] directly from $\Gamma(\rho\rightarrow\pi\pi)=100$ MeV. The situation improves as one increases the $\rho$ width; for $\Gamma(\rho\rightarrow\pi\pi)=140$, inspection of Eqs. (8) and (12) shows that they are consistent. However, one does not necessarily expect much better agreement than Eq. (15) gives for vector-dominance calculations.[3]

From Eqs. (4) and (5), it is clear that $d$ is a function of $(q_1+q_2)^2$, $(K-q_1-q_2)^2$, and $(K-q_1)^2$. The lepton momentum squared $(K-q_1-q_2)^2$ varies from $m_e{}^2\simeq0$ up to $(m_K-2m_\pi)^2$. This is small compared to $m_{K^*}{}^2$, and furthermore, phase space is largest at the lower end of this range; hence we can reasonably set $(K-q_1-q_2)^2=0$. The quantity $(K-q_1)^2$ varies from $m_\pi{}^2$ to $(m_K-m_\pi)^2$; using Eqs. (5) and (11) (i.e., method $A$), one finds that $d_2$ varies from $-0.35$ to $-0.41$.[9] [In this discussion we use $\Gamma(\rho\rightarrow\pi\pi)=100$ MeV.[13]] Hence, an adequate estimate is $d_2\simeq-0.38$. From

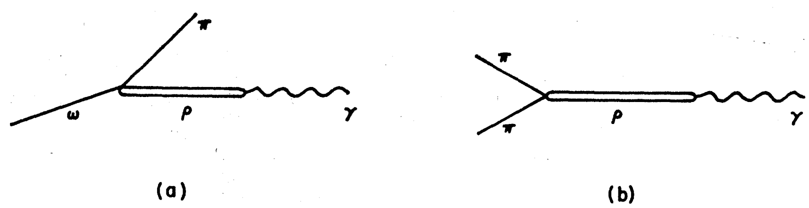


FIG. 2. Feynman graphs used to estimate strong-coupling constants.

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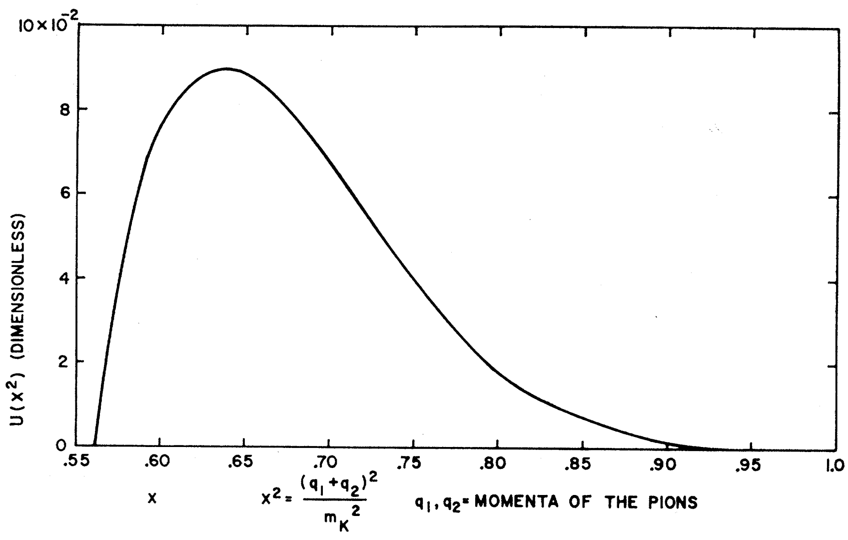


FIG. 3. Plot of Cabibbo and Maksymowicz's function $U(x^2)$.

method $B$, one finds $d_2 \simeq -0.55$. The quantity $(q_1+q_2)^2$ varies from $4m_\pi^2$ to $m_K^2$, which results in $d_1$ varying from $-0.53$ to $-0.79$. However, $d$ has been measured experimentally by determining the factor of $\cos\phi$ in the observed $\phi$ angular distribution,[6] determined from the angular correlation scheme of Cabibbo and Maksymowicz.[14] This term is multiplied by the phase-space factor $U(x^2)$, (where $x^2=(q_1+q_2)^2/m_K^2$), as defined by Cabibbo and Maksymowicz. Then, from Fig. 3 one can see that the most important contribution is around $x=0.65$. For this value of $x^2$, one finds $d_1 \simeq -0.58$ using method $A$, and $d_1 \simeq -0.82$ using method $B$. Finally, our best estimates are[15]: For $\Gamma(\rho \to \pi\pi)=100$ MeV,

$$d_1=-0.58, \quad d_2=-0.38, \quad d=-0.96 \text{ (method } A);$$

$$d_1=-0.82, \quad d_2=-0.55, \quad d=-1.37 \text{ (method } B).$$

For $\Gamma(\rho \to \pi\pi)=140$ MeV,

$$d_1=-0.68, \quad d_2=-0.45, \quad d=-1.13 \text{ (method } A);$$

$$d_1=-0.69, \quad d_2=-0.47, \quad d=-1.16 \text{ (method } B).$$

There have been numerous previous estimates of $d$. Mohanti and Marshak[4] have estimated $d$ by current-algebra and soft-pion techniques and found $d \simeq 1.5$.[16] Sarker,[17] using a Ward-indentity approach combined with a soft-pion approximation (i.e., setting $\partial_\lambda A^\lambda=0$) of some matrix elements (including the neglect of all "$\sigma$-type" terms), has found $d \simeq 5.07$. The only justification for his neglect of the $\sigma$ terms is that the results are not inconsistent with experiment.

Previous vector-dominance calculations of $d$ have only considered $d_1$. Mohanti and Marshak[4] have estimated $d_1 \simeq 0.7$. Berends, Donnachie, and Oades,[6] using method $A$, obtained a value of $d_1 \simeq 1.24$. However, they relate $g_{\omega\pi\gamma}$ directly to a $\rho K e\nu$ vertex by the relation

$$g_{\rho K e\nu}=(\sqrt{\tfrac{3}{2}})g_{\omega\pi\gamma}\sin\theta\sin\lambda\,,$$

while we have found

$$g_{\rho K K^*}=g_{VVP}/\sqrt{2}$$
$$=\tfrac{1}{2}\sqrt{3}g_{\omega\pi\rho}\sin\lambda\,,$$

and hence

$$g_{K^*e\nu}g_{\rho K K^*}=\tfrac{1}{2}\sqrt{3}g_{\omega\pi\rho}g_{\rho\gamma}\sin\theta\sin\lambda\,.$$

Earlier estimates, such as that of Ilioupoulos,[5] have changed because of newer experimental data.

The experimental situation of $d$ is uncertain. A preliminary analysis of 310 events gives a value of $d \simeq 10 \pm 4$.[6] However, this analysis assumes that all $K_{l4}$ form factors depend only on $(q_1+q_2)^2$. Also, the value of $d$ depends to some extent upon what assumptions are made concerning the axial form factors and the $\pi$-$\pi$ interaction. Thus, although our estimate is smaller than the current (preliminary) experimental estimate, the experimental situation seems uncertain, and hence the status of the vector-dominance model in $K_{l4}$ seems also to be still open.

*Note added in proof.* A recent experimental analysis[18] finds that the values $|d| \simeq 1.38$ or $|d| \simeq 0.68$ fit the present experimental data.


## ACKNOWLEDGMENT

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[15] Our sign of $d$ is determined from Condon-Shortley phase conventions and standard perturbation theory. Most previous estimates appear to be concerned only with the absolute value of $d_1$. Hence, it is the *absolute value* of our result that is to be compared with the results of others.

[16] In obtaining this estimate, Mohanti and Marshak neglect what they call a "higher-order" term which is quadratic in pion momenta; this seems to be a most dubious procedure since the matrix element is itself quadratic in pion momenta. See, for example, S. L. Adler and R. F. Dashen, *Current Algebras and Applications to Particle Physics* (W. A. Benjamin, Inc., New York, 1968), p. 121.

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