Dipole Model of the A_2 Meson and Charge-Exchange Scattering and Polarization

T. J. GAJDICAR AND J. W. MOFFAT*
Department of Physics, University of Toronto, Toronto, Canada
(Received 23 December 1968)

The A_2 meson is described as a dipole resonance and a linear Regge-dipole A_2 trajectory is assumed to dominate the charge-exchange process $\pi^- p \to \eta n$. A least-squares fit to the differential cross-section data for four adjustable parameters is performed, and leads to a $\chi^2 = 11$ for 25 experimental points. A nonzero polarization is predicted. The multichannel problem corresponding to A_2 production in the process $\pi p \to A_2 p$ is studied and a phase-shift model incorporating a dipole A_2 which produces different peaked structures in the various decay models is given by way of example.

1. INTRODUCTION

MISSING-MASS experiments[1] have shown that the $A_2(1300)$ meson has a two-peaked structure in events decaying into $\rho\pi$. The $\eta\pi$ decay events also indicate a double-peaked structure, but the question as to whether the $K_1^0K_1^0$ events show a similar structure is not yet conclusive.

A dipole model of the ρ meson has been investigated[2] and applied to the $\pi^- p \to \pi^0 n$ charge-exchange scattering and polarization, and also the electromagnetic form factors with a resulting good fit to the data with few parameters.

In this paper, we study the charge-exchange process $\pi^- p \to \eta n$, using a Regge-dipole model of the A_2 meson of the form developed in Ref. 2. We also study the properties of the dipole A_2 within a multichannel scheme and discuss a possible model of the different partial modes $\rho\pi$, $\eta\pi$, and $K_1^0K_1^0$.

2. REGGE-DIPOLE A_2 AMPLITUDE

A double-pole model of the A_2 has already been considered,[3] but we concern ourselves only with a model of the A_2 as a dipole trajectory in the t plane generated by the coalescing of two single-pole trajectories for all t, such that the dipole amplitude can be obtained from the single-pole amplitude by differentiating with respect to $\alpha(t)$.[2]

The single Regge-pole amplitude for the process $\pi^- p \to \eta n$ is given by

$$A = f + i(\boldsymbol{\sigma}\cdot\mathbf{q}'\times\mathbf{q}/q^2)\tilde{f}, \qquad (1)$$

where the non-spinflip and spinflip amplitudes f and $\tilde{f}$ are given by

$$f = -(\gamma_A(t)/\sqrt{s})\alpha_A(t)(E/\mu)^{\alpha_A(t)} \times[-i+\cot(\tfrac{1}{2}\pi\alpha_A(t))], \qquad (2)$$

$$\tilde{f} = (\alpha_A(t)/4M)[\gamma_A(t) - \alpha_A(t)\tilde{\gamma}_A(t)] \times(E/\mu)^{\alpha_A(t)}[-i+\cot(\tfrac{1}{2}\pi\alpha_A(t))], \qquad (3)$$

where $\gamma_A(t)$ and $\tilde{\gamma}_A(t)$ are the non-spinflip and spinflip residues, respectively. By differentiating Eqs. (2) and (3) with respect to $\alpha_A(t)$, we obtain for the dipole amplitudes

$$f_D = -(\gamma_A/\sqrt{s})(E/\mu)^{\alpha_A(t)}\{[-i+\cot(\tfrac{1}{2}\pi\alpha_A(t))] \times[1+\alpha_A(t)\ln(E/\mu)] - (\tfrac{1}{2}\pi\alpha_A(t))\csc^2(\tfrac{1}{2}\pi\alpha_A(t))\}, \qquad (4)$$

and

$$\tilde{f}_D = (1/4M)(E/\mu)^{\alpha_A(t)}\{[-i+\cot(\tfrac{1}{2}\pi\alpha_A(t))] \times[\gamma_A - 2\alpha_A(t)\tilde{\gamma}_A + \alpha_A(t)\ln(E/\mu)\times(\gamma_A - \alpha_A(t)\tilde{\gamma}_A)] - \tfrac{1}{2}\alpha_A(t)\pi\times\csc^2(\tfrac{1}{2}\pi\alpha_A(t))(\gamma_A - \alpha_A(t)\tilde{\gamma}_A)\}. \qquad (5)$$

The scale factor $s_0 = 2M\mu$ was used to give the energy dependence $(E/\mu)^{\alpha_A(t)}$, where M is the nucleon mass and μ is the pion mass. The polarization parameter P is then given by

$$P = \frac{-2\,\mathrm{Im}(f\tilde{f}^*)\sin\theta}{|f|^2 - (4t/s)|\tilde{f}|^2}, \qquad (6)$$

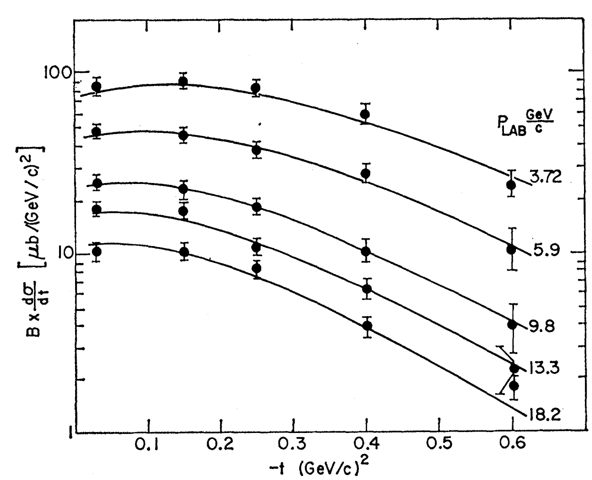

FIG. 1. Fit of $(d\sigma/dt)(\pi^- p \to \eta n)$ reduced by the branching ratio $B(\eta \to 2\gamma)$ which is the only mode observed in the data (Ref. 4).

* Supported in part by the National Research Council of Canada.

[1] D. J. Crennell, Uri Karshon, Kwan Wu Lai, J. Michael Scarr, and Ian O. Skillicorn, Phys. Rev. Letters **20**, 1318 (1968); F. Lefebvres, B. Levrat, H. R. Blieden, L. Dubal, Maria N. Focacci, D. Freytag, J. Geibel, W. Kienzle, B. C. Maglic, M. Mastin, and J. Orear, Phys. Letters **19**, 434 (1965).

[2] R. E. Kreps and J. W. Moffat, Phys. Rev. **175**, 1942 (1968); R. E. Kreps and J. W. Moffat, *ibid.* **175**, 1945 (1968).

[3] D. M. Austin, J. V. Beaupre, and K. E. Lassila, Phys. Rev. **173**, 1573 (1968). The model considered here results from the crossing of two trajectories arising in an eight-parameter problem.

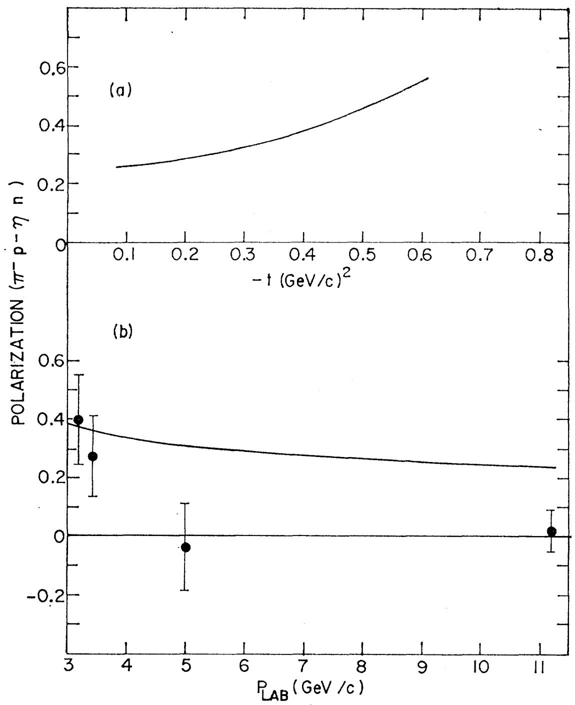

FIG. 2. (a) Neutron polarization predicted by the dipole model for $E_{\text{lab}}=6$ GeV as a function of $-t$. (b) Neutron polarization at $t\sim-0.2$ $(\text{GeV}/c)^2$ in the reaction $\pi^- p\rightarrow\eta n$ as predicted by the dipole model along with the data points from Ref. 5.

and the differential cross section is

$$\frac{d\sigma}{dt}=\frac{\pi}{q^2}\left(|f|^2-\frac{4t}{s}|\tilde{f}|^2\right), \tag{7}$$

where θ is the scattering angle in the s channel.

3. CHARGE-EXCHANGE SCATTERING RESULTS

A least-squares fit was made to 25 differential cross section points[4] at low values of $|t|$. A linear trajectory and constant residue functions were used giving rise to the four adjustable parameters $\alpha_A(0)$, $\alpha_A'(0)$, γ_A, and $\tilde{\gamma}_A$ assuming that they should be adequate to describe the essential features of the dipole model in the t range $0\leqslant|t|\lesssim 0.6$.

The data points were used at the five laboratory momenta 3.72, 5.9, 9.8, 13.3, and 18.2 GeV/c for $|t|<0.6$ $(\text{GeV}/c)^2$. (See Fig. 1.) The fit to the data resulted in a $\chi^2=11$. The parameter values found were $\alpha_A(0)=0.238$, $\alpha_A'(0)=0.412$, $\gamma_A=0.148$, and $\tilde{\gamma}_A=2.15$. The polarization predicted by these parameters is positive and is shown in Fig. 2 along with the few presently available experimental points.[5] Our dipole model shows quite a substantial polarization decreasing slowly with increasing energy for $t\sim-0.2$ $(\text{GeV}/c)^2$.

4. MULTICHANNEL CONSIDERATIONS

The experiment performed by the BNL group[1] gives some indication that the double-peaked structure in the A_2 mass region occurs only in the $\rho\pi$ and $\eta\pi$ decay modes, while the $K\bar{K}$ events contribute only to the higher-energy peak at about 1315 MeV with a width of about 20 MeV. The BNL group has suggested that the peak at the lower energy $\sim$1270 MeV might be associated with a resonance having $J^P=1^-, 3^-, \cdots$, since this would forbid $K\bar{K}$ decay and only the higher peak at 1315 MeV would be observed. In light of recent experiments,[6] however, a $J^P=2^+$ assignment to both A_2 peaks is favored, and though the single-peaked spectrum for the $K\bar{K}$ decay mode is by no means firmly established,[7] a number of models in which the resonance activity in the A_2 region is 2^+ have been proposed which allow for this possibility.[8]

We examine a multichannel scheme in which the dipole A_2 may interfere with other channels to produce a single $K\bar{K}$ peak in the upper A_2 mass region. We use another $J^P=2^+$ single pole to produce this interference by way of an example, but do not wish to imply necessarily that such a resonance actually exists. In our example, the coupling of this added pole to the $\pi\eta$ channel turns out to be about 40% of the $\pi\eta$ coupling to the dipole A_2, and could be readily made smaller with the same qualitative results. The dipole Regge analysis of the $\pi p\rightarrow\eta n$ process would remain unaltered if it were assumed this coupling were negligible for the region of momentum transfers involved.

Consider the multichannel problem with the sub-S-matrix connecting states with $I^G=1^-$ and $J^P=2^+$. Assuming that the multichannel problem at about 1300 MeV is dominated by $\rho\pi$, $\eta\pi$, and $K\bar{K}$ channels, the diagonalized sub-S matrix can be represented by

$$S=\begin{pmatrix} e^{i2\delta_1} & 0 & 0 \\ 0 & e^{i2\delta_2} & 0 \\ 0 & 0 & e^{i2\delta_3} \end{pmatrix}, \tag{8}$$

where the δ's are *real* eigen phase shifts, since S is unitary. Denoting the $\rho\pi$, $\eta\pi$, and $K\bar{K}$ states by $|1\rangle$, $|2\rangle$, and $|3\rangle$, respectively, they can be expanded in the

[4] O. Guisan, J. Kirz, P. Sonderegger, A. V. Stirling, P. Borgeaud, C. Bruneton, P. Falk-Vairant, B. Amblard, C. Caversasio, J. P. Guillaud, and M. Yvert, Phys. Letters **18**, 200 (1965).
[5] D. D. Drobnis, J. Lales, R. C. Lamb, R. A. Lundy, A. Moretti, R. C. Niemann, T. B. Novey, J. Simanton, A. Yokosawa, and D. D. Yovanovitch, Phys. Rev. Letters **20**, 274 (1968); P. Bonamy, P. Borgeaud, S. Brehin, C. Bruneton, P. Falk-Vairant, O. Guisan, and P. Sonderegger, in *Proceedings of the Heidelberg International Conference on Elementary Particles, Heidelberg, 1967*, edited by H. Filthuth (Wiley-Interscience, Inc., New York, 1968).
[6] G. E. Chikovani, M. N. Focacci, W. Kienzle, U. Kruse, C. Lechanoine, M. Martin, and P. Schubelin, Phys. Letters **28B**, 526 (1969).
[7] B. French, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna 1968* (CERN, Geneva, 1968), p. 91.
[8] J. V. Beaupre, T. P. Coleman, K. E. Lassila, and P. V. Ruuskanen, Phys. Rev. Letters **21**, 1849 (1968); Y. Fujii and M. Kato, Nuovo Cimento **58**, 297 (1968).

eigenvectors $|e_j\rangle$ of S. Thus,

$$|i\rangle=\sum_{j=1}^{3} a_{ij}|ej\rangle, \qquad (9)$$

where we choose to consider $i=1$ and 3. The two-body scattering events can then be described by the T matrix

$$T=(S-1)/2i. \qquad (10)$$

From the time-reversal invariance of the S matrix, we deduce that the a_{ij} are real coefficients. We have

$$T_{ij}=\sum_{k=1}^{3} a_{ik}a_{jk}e^{i\delta_k}\sin\delta_k, \qquad (11)$$

where

$$\sum_{k=1}^{3} a_{ik}a_{jk}=\delta_{ij}. \qquad (12)$$

For a dipole resonance the unitary S matrix takes the form[9]

$$S=\left(\frac{E-E_0+i(\frac{1}{2}\Gamma_1)}{E-E_0-i(\frac{1}{2}\Gamma_1)}\right)^2, \qquad (13)$$

and if we assume that δ_1 is produced by a dipole resonance then

$$\tan\delta_1=\frac{\Gamma_1(E_0-E)}{(E-E_0)^2-\frac{1}{4}\Gamma_1^2}, \qquad (14)$$

where $\Gamma_1\approx 60$ MeV and $E_0\approx 1300$ MeV. We also assume that δ_2 is produced by a second, single-pole resonance with

$$\tan\delta_2=-\tfrac{1}{2}\Gamma_2/(E-E_1),$$

where $\Gamma_2\approx 30$ MeV and $E_1\approx 1270$ MeV. The third phase shift δ_3 is assumed to be effectively zero. We shall choose the coefficients a_{ij} to be effectively constant in the energy range considered with the values

$$a_{11}=0.9, \quad a_{12}=0.4, \quad a_{31}=0.4, \quad a_{32}=-0.9.$$

Here unitarity demands that

$$a_{11}^2+a_{12}^2\leqslant 1$$

and

$$a_{31}^2\leqslant 1.$$

The amplitudes T_{11} and T_{13} neglecting kinematical factors then describe the processes $\rho\pi\to\rho\pi$ and $\rho\pi\to K\bar{K}$. A calculation of $|T_{11}|^2$ and $|T_{13}|^2$ based on the above assumptions leads to the results shown in Fig. 3. A similar model has been considered by Rosdolsky[10] based on different eigen phase shifts.

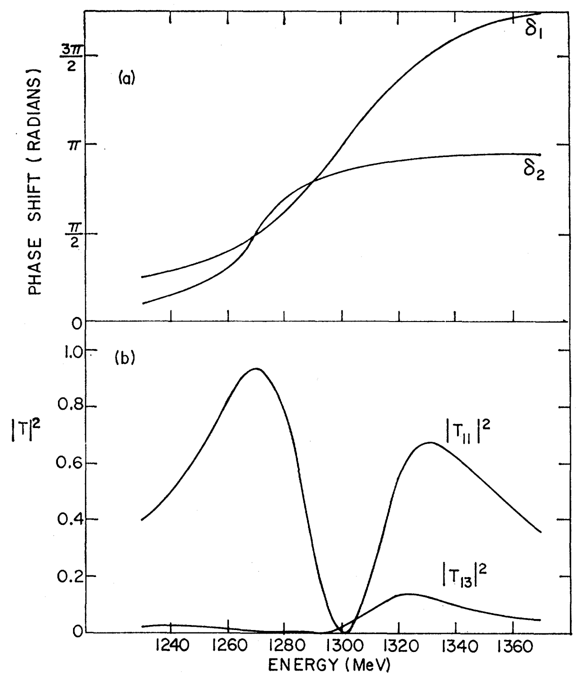

FIG. 3. (a) Eigen phase shifts arising from our multichannel model. δ_1 results from a dipole at 1300 MeV with $\Gamma_1\sim 60$ MeV and δ_2 results from a pole at about 1270 MeV with $\Gamma_2\sim 30$ MeV. (b) $|T_{11}|^2$ and $|T_{13}|^2$ for the respective processes $\rho\pi\to\rho\pi$ and $\rho\pi\to K\bar{K}$ as given by our model of the multichannel problem.

5. CONCLUSIONS

The A_2 as a dipole gives a very good fit to the $\pi^-p\to\eta n$ differential cross sections with a minimum of parameters. Not too much can be said about the predicted polarization in view of the present poor experimental situation. A further investigation of the A_2 dipole trajectory considering KN charge-exchange scattering would be an interesting study. We have also shown that within the multichannel framework an A_2 dipole would still be relevant even if double-peaked structures were not observed in all the decay modes corresponding to $I^G=1^-$ and $J^P=2^+$ resonances in the 1300-MeV region.

---

[9] M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley-Interscience, Inc., New York, 1964), Chap. 8.

[10] H. Rosdolsky, Phys. Rev. **180**, 1403 (1969).