Muon Polarization in $K_{\mu 3}{}^0$ Decay*

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An experiment to measure the polarization of muons from $K_{\mu 3}{}^0$ decay was carried out at the Bevatron with counter techniques. All three components of the polarization were measured. The time-reversal-violating component transverse to the decay plane was found to be $\langle P_T{}^{\text{c.m.}}\rangle = -0.005\pm0.019$, which is consistent with T invariance. The result for the form-factor ratio $\xi \equiv f_-/f_+$ was $\text{Re}\xi = -1.81_{-0.26}{}^{+0.50}$, $\text{Im}\xi = -0.02\pm0.08$, $\text{Arg}(\xi)-\pi = (0.6\pm2.6)°$, if ξ is assumed to be independent of q^2. If ξ is not too strongly dependent on q^2, this result still holds true for ξ at $q^2 \cong 2.65 m_\pi{}^2$, the average q^2 selected by our apparatus.

I. INTRODUCTION

In the past few years K-meson decays have been intensively studied to learn more about the properties of strangeness-nonconserving weak interactions. The leptonic and semileptonic decay modes are particularly interesting in this respect because it is possible to make reliable calculations to compare with experimental data. The unexpected discovery of a violation of CP invariance in K^0 decays has greatly increased interest in K-meson decays. The present experimental and theoretical situation is summarized in review articles by Willis,[1] Lee and Wu,[2] and Camerini and Murphy.[3]

The experiment described here was designed to study the polarization of the muons from $K_{\mu 3}{}^0$ decay with significantly better accuracy than previous experiments. The specific decay mode studied was $K_L{}^0 \to \pi^- + \mu^+ + \nu$. This mode constitutes $\approx 14\%$ of the total for long-lived neutral K mesons.[2] The polarization of the muons was measured by the standard technique of stopping the muons in graphite and observing the asymmetry of the decay positrons.[4] In the course of the experiment, all three mutually perpendicular components of the average muon polarization were measured. Brief reports of the results of the experiment have been published previously.[5]

The goal of the experiment was twofold. The primary aim was to test time-reversal (T) invariance in K^0 decays by searching for a component of polarization transverse to the decay plane, a component which is forbidden by T invariance. This test of T invariance was first proposed by Sakurai[6] soon after the discovery that the weak interaction violates C and P invariance. With the discovery that the combined symmetry operation CP is also violated in K^0 decays,[7] there is the expectation that T should also be violated, since symmetry under CPT is thought to be valid. A violation of CP therefore implies a complementary violation of T. However, no direct evidence for such a violation has been found. The second goal of the experiment was to measure the other two components of polarization in order to determine the ratio ξ of the two form factors which appear in the usual theory describing K decays. (See Sec. II A.)

II. THEORY

A. Form Factors

In the usual $V-A$ theory[8] of $K_{\mu 3}{}^0$ decays,[2] the matrix element governing the decay is proportional to $\langle\pi|J_\lambda|K\rangle$, where J_λ is the strangeness-changing hadronic current. This can be written

$$\langle\pi|J_\lambda|K\rangle = \tfrac{1}{2}f_+(q_K+q_\pi)_\lambda + \tfrac{1}{2}f_-(q_K-q_\pi)_\lambda\,, \qquad (1)$$

where q_K and q_π are the four-momenta of the kaon and pion, respectively, and f_+ and f_- are form factors. These can depend only on q^2, the absolute value of the invariant four-momentum transfer to the lepton pair,

$$q^2 = |q_K - q_\pi|^2 = M_K{}^2 - 2M_K E_\pi + M_\pi{}^2\,. \qquad (2)$$

In writing the matrix element in this form, we assume

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[1] W. J. Willis, in *Proceedings of the Heidelberg International Conference on Elementary Particles*, edited by H. Filthuth (North-Holland Publishing Co., Amsterdam, 1968).

[2] (a) T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. **15**, 381 (1965); (b) **16**, 471 (1966); (c) **16**, 511 (1966).

[3] U. Camerini and C. T. Murphy, in *Proceedings of the Thirteenth Annual International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, Calif., 1967).

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[6] J. J. Sakurai, Phys. Rev. **109**, 980 (1958).

[7] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964).

[8] In what follows, we refer to long-lived K^0's unless specifically stated otherwise.

that the interaction between the muon and neutrino is local; i.e., they are produced at the same vertex, and the coupling does not involve derivatives of the lepton fields. We also assume that no scalar or tensor terms appear in the matrix element[9] and that the $\Delta S=\Delta Q$ rule holds.[10] While the approximate validity of these conditions seems likely on the basis of present experimental results, the reader should keep in mind that this form for the matrix element may be only approximately true.[11]

In this formalism all experimental quantities except absolute rates can be written in terms of a single complex parameter $\xi(q^2)$, the ratio of f_- to f_+. Since the simplest models predict a slow variation of f_- and f_+ with q^2, a linear dependence with q^2 is generally assumed.[1] the usual way of parametrizing this dependence is the form

$$f_-(q^2)=f_-(0)[1+\lambda_-(q^2/m_\pi^2)] \tag{3a}$$

and

$$f_+(q^2)=f_+(0)[1+\lambda_+(q^2/m_\pi^2)], \tag{3b}$$

so that

$$\xi(q^2)=\xi(0)[1+\lambda_-(q^2/m_\pi^2)]/[1+\lambda_+(q^2/m_\pi^2)]. \tag{3c}$$

If we assume the validity of the $|\Delta I|=\frac{1}{2}$ rule for K decays, then the form factors for charged $K_{\mu 3}$ decays are the same as for neutral $K_{\mu 3}$ decays (except for a multiplicative factor of $\sqrt{2}$).[1,2] If we further assume μ-e universality, then f_+ and f_- are also the same as the form factors that appear in $K_{e3}{}^0$ and $K_{e3}{}^{\pm}$ decay.[12] However, the latter are not sensitive to f_- because the f_- term leads to factors which are proportional to the lepton mass and are therefore negligible in K_{e3} decays.

The experimental situation regarding the parameters ξ, λ_+, and λ_- is at present confused.[1,3] Measurements of ξ based on the $K_{e3}/K_{\mu 3}$ branching ratio generally give $\xi\approx+0.5$, while those based on a study of the muon polarization give $\xi\approx-1.2$.[1] (See Sec. V.) Most recent experiments give $\lambda_+\approx0.02$.[1] Little is known about λ_-.

If T invariance is not assumed, f_+ and f_- can be out of phase and their ratio ξ is then a complex quantity. $\mathrm{Im}\xi$ is therefore a measure of the violation of time-reversal invariance.

B. Expression for the Muon Polarization

Expressions for the polarization of the muons from $K_{\mu 3}$ decays have been given by several authors.[13] Because the neutrinos from the decay have unit helicity, all muons coming off at a given angle and momentum relative to the neutrino have a unique spin orientation. In the K rest system the polarization is therefore a unit vector whose orientation is specified when the decay kinematics are specified. The expression given by Cabibbo and Maksymowicz[13] for the muon polarization in the K rest system is $\mathbf{P}=\mathbf{A}/|\mathbf{A}|$, where

$$\mathbf{A}=a_1(\xi)\mathbf{p}_\mu-a_2(\xi)\{(\mathbf{p}_\mu/m_\mu)[(M_K-E_\pi)+(\mathbf{p}_\pi\cdot\mathbf{p}_\mu)\times((E_\mu-m_\mu)/|\mathbf{p}_\mu|^2)]+\mathbf{p}_\pi\}+M_K\,\mathrm{Im}\xi(q^2)(\mathbf{p}_\pi\times\mathbf{p}_\mu) \tag{4}$$

with

$$a_1(\xi)=2(M_K{}^2/m_\mu)[E_\nu+\mathrm{Re}b(q^2)(E_\pi{}^*-E_\pi)],$$
$$a_2(\xi)=M_K{}^2+2\,\mathrm{Re}b(q^2)M_KE_\mu+|b(q^2)|^2m_\mu{}^2,$$
$$b(q^2)=\tfrac{1}{2}[\xi(q^2)-1],$$

and

$$E_\pi{}^*=(M_K{}^2+m_\pi{}^2-m_\mu{}^2)/2M_K.$$

Here $\mathbf{p}_\pi$ and $\mathbf{p}_\mu$ are the three-momenta of the pion and muon, respectively, in the K rest system. The last term in Eq. (4) violates time-reversal invariance and vanishes when $\mathrm{Im}\xi=0$.

The muon polarization vector is generally rotated relative to its momentum vector in the transformation from the K^0 rest system to the lab system.[2,14] Any transformation to another system whose relative velocity is parallel to the muon momentum vector leaves the polarization vector unchanged. (The muon polarization is measured with the muons at rest in the laboratory system; the relevant polarization is that in the rest system of the muon.) In this experiment events were selected so that the average angle between the muon momentum and the K^0 momentum was small; therefore, the polarization direction in the lab system did not differ greatly from that in the K^0 rest system.

An expression for the polarization in the laboratory system has been derived by Cabibbo and Maksymowicz.[13] If the momentum of the K^0 in the laboratory system is $\mathbf{p}_K$, then the polarization vector is $\mathbf{P}=\mathbf{B}/|\mathbf{B}|$, where

$$\mathbf{B}=b_1(\xi)[(\mathbf{p}_\mu/m_\mu)(\mathbf{p}_\nu\cdot\mathbf{p}_\mu/(E_\mu+m_\mu)-E_\nu)+\mathbf{p}_\nu]+b_2(\xi)[(\mathbf{p}_\mu/m_\mu)(\mathbf{p}_K\cdot\mathbf{p}_\mu/(E_\mu+m_\mu)-E_K)+\mathbf{p}_K]-(\mathrm{Im}\xi)\mathbf{d}, \tag{5}$$

$$b_1(\xi)=M_K{}^2+m_\mu{}^2|b(q^2)|^2+2[\mathrm{Re}b(q^2)](q_\mu q_K),$$
$$b_2(\xi)=-\{2(q_\nu q_K)+[\mathrm{Re}b(q^2)](q^2-m_\mu{}^2)\},$$
$$\mathbf{d}=E_K(\mathbf{p}_\mu\times\mathbf{p}_\pi)+E_\mu(\mathbf{p}_\pi\times\mathbf{p}_K)+E_\pi(\mathbf{p}_K\times\mathbf{p}_\mu)+[\mathbf{p}_\mu\cdot(\mathbf{p}_K\times\mathbf{p}_\pi)/(E_\mu+m_\mu)]\mathbf{p}_\mu,$$

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[9] Experimental limits for the ratio of the scalar or tensor coupling constants to the vector coupling constant in K_{e3} decays are $\lesssim0.1$. [See, for example, P. T. Eschstruth *et al.*, Phys. Rev. **165**, 1487 (1968).]

[10] Results of experimental tests of the $\Delta S=+\Delta Q$ rule are discussed in Ref. 1. The situation is confused, but Willis concludes that $0\lesssim|\mathrm{Re}x|\lesssim0.2$ and $0\lesssim|\mathrm{Im}x|\lesssim0.4$, where x is the complex ratio of the $\Delta S=-\Delta Q$ to the $\Delta S=+\Delta Q$ amplitude.

[11] To the extent that the neglected amplitudes are small our conclusions regarding T invariance will not be affected significantly. However, our result for $\mathrm{Re}\xi$ may need to be substantially revised if, for example, significant contributions from tensor amplitudes were found to exist.

[12] Muon-electron universality is not well tested in strangeness-changing weak decays. See Sec. V for further discussion of this point.

[13] (a) N. Cabibbo and A. Maksymowicz, Phys. Letters **9**, 352 (1964); **11**, 360 (1964); **14**, 72 (1965); (b) Ref. 2; (c) N. Brene, L. Egardt, and B. Quist, Nucl. Phys. **22**, 553 (1961).

[14] G. Ascoli, Z. Physik **150**, 407 (1958).

with

$$(AB)\equiv A^0B^0-\mathbf{A}\cdot\mathbf{B}\,,\quad q^2=(q_K-q_\pi)^2\,,\quad b(q^2)=\tfrac{1}{2}[\xi(q^2)-1].$$

The polarization measured experimentally is an average over the sample of events selected by the apparatus. Since it is an average, it need not be a unit vector, and its value depends on the geometry of the experiment.

C. Time-Reversal Invariance and the Transverse Polarization of the Muon

The prediction of T invariance for decays of real (in contrast to virtual) particles is that if there is no final-state interaction between the decay products, then terms which are odd under T do not contribute to observable quantities.[15] A transverse polarization of the muon corresponds to a term $\langle \boldsymbol{\sigma}_\mu\cdot\mathbf{p}_\pi\times\mathbf{p}_\mu\rangle\neq 0$. Since spins and momenta are odd under T, this term also is odd and therefore forbidden to the extent that final-state interactions can be neglected. The electromagnetic final-state interaction between the $\pi^\pm$ and $\mu^\mp$ in $K_{\mu3}{}^0$ decay yields a transverse polarization and Imξ which are negligible for our purposes. Byers, MacDowell, and Yang[16] have calculated this effect and find that typically $(\mathrm{Im}\xi)_{\mathrm{e.m.}}$ is $\approx 1/137$, well below present experimental upper limits.

III. EXPERIMENTAL PROCEDURE

A. General

This experiment utilized a neutral beam taken off at an angle of approximately 4° from a copper target in the external proton beam of the Bevatron. The estimated $K_L{}^0$ flux of $\approx 5\times 10^5$ per pulse was considerably greater than that available in previous experiments with either neutral or charged kaons. This enabled us to design a pure counter experiment with a very high ratio of good events to background and obtain a high counting rate with a comparatively modest setup. The high rate and quick availability of the data allowed many checks on the results which would not have been possible otherwise. All three mutually perpendicular components of the muon polarization were measured with essentially the same apparatus. Of these, the T-violating component was very small ($\lesssim 0.015$), and the T-conserving components were quite large (up to ≈ 0.9). This provided important internal checks on possible systematic effects that might give false asymmetries on the one hand or effects that would tend to decrease the polarization on the other (e.g., depolarization of the muons or contamination of the sample by muons from pion decay).

B. Neutral Beam

In the design of the beam, several considerations were decisive. The design arrived at was a compromise between maximizing the K^0 flux and maintaining the neutron background at a permissible level. Compatibility with other experiments and limitations on available space were also major factors.

A plan view of the beam layout is shown in Fig. 1. An 8.9-cm-long copper target was used in the external proton beam. We also tried a 10.2-cm-long aluminum target and a 5.1-cm tungsten target and found little difference in event rate or event/background ratio. The neutral beam was taken off at an angle of $\approx 4°$ relative to the proton beam. The first sweeping magnet B_1 bent the proton beam away from the mouth of the collimator into a uranium beam stopper. Lead and steel shielding was used in the upstream part of the shielding wall to make most efficient use of the limited space available for shielding.

The defining aperture in the neutral beam was an 8.25-cm-wide by 12.0-cm-high brass slit. This was followed by a second sweeping magnet to remove charged particles formed in the defining collimator. The decay region for the K^0's started just after the second sweeping magnet B_2. The defining aperture was about 6.8 m from the production target which gave a solid angle of 2.3×10^{-4} sr. The estimated K^0 flux through the decay region was $\approx 5\times 10^5$ per beam pulse of 5×10^{11} protons in the external beam. The neutron/kaon ratio was estimated at 500/1.

The beam was carefully designed so that after it passed the defining aperture it stayed well clear of any counters or other objects that might generate false events or increase the background in the vicinity of the apparatus. Beyond the defining aperture the beam traveled in helium until well past the apparatus. It was found that interactions in the helium produced a negligible fraction of the "events" studied (see Sec. III H). The main effect of the neutron interactions in the helium was to increase the singles rates in the counters near the beam.

The intensity profile of the beam was studied by placing x-ray film in the beam just upstream of the pion counters (Fig. 1). A 1.2-cm-thick piece of aluminum was placed in front of the film to convert the neutrons. After the film was developed it was scanned with a densitometer. A correspondence between density and beam intensity was established by exposing a piece of the same type of film to a β source for varying lengths of time and scanning it with a densitometer. Beam profiles arrived at with this technique are shown in Fig. 2. The edges of the pion counters, which were the closest objects to the beam, are indicated.

[15] J. J. Sakurai, *Invariance Pricniples and Elementary Particles* (Princeton University Press, Princeton, N. J., 1964).

[16] N. Byers, S. W. MacDowell, and C. N. Yang, in *Proceedings of the International Seminar on High-Energy Physics and Elementary Particles Trieste, 1965* (International Atomic Energy Agency, Vienna, 1965), p. 953.

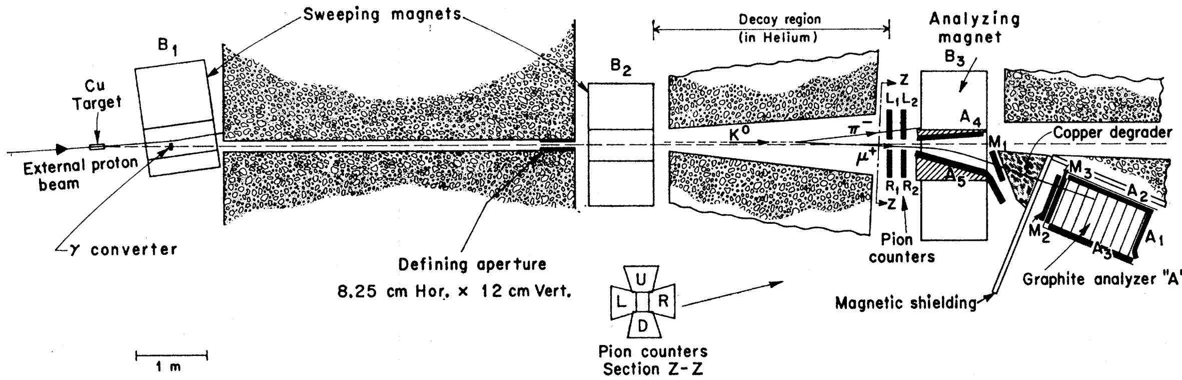


FIG. 1. Plan view of experiment. Analyzer B is not shown.

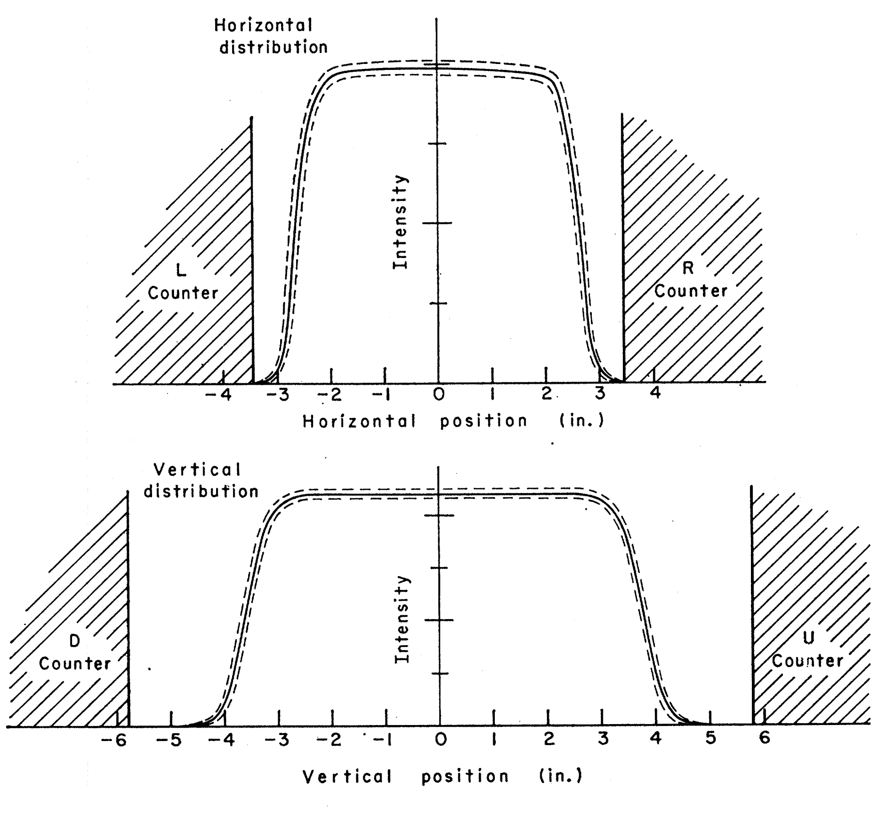
C. Counter Geometry

The major consideration in the design of the counter geometry was to minimize the statistical error in the T-violating component of the muon polarization. This procedure was decided upon because it was felt that by various checks and design precautions instrumental asymmetries in the measurement of the T-violating component could be reduced to the point where the only limitation would be statistics. For the other two components, which are large, systematic errors were expected to be the limiting factor. The checks and precautions against instrumental asymmetries affecting the T-violating component are discussed below.

No attempt was made to completely determine the kinematics for each decay. Since the momentum of the incident K^0 was unknown, this would require measuring the momentum and angle of both the pion and muon which would lead to a prohibitive reduction of solid angle and event rate. The polarizations that were measured, therefore, were average values for the sample of events selected by our apparatus. This is not a great disadvantage, since for a particular choice of form factors it is possible to make a unique prediction of the average polarization measured with our geometry.

The counter geometry was optimized by means of a Monte Carlo program which simulated the experi-

FIG. 2. Intensity profiles of the neutral beam. The inside edges of the pion counters are indicated.



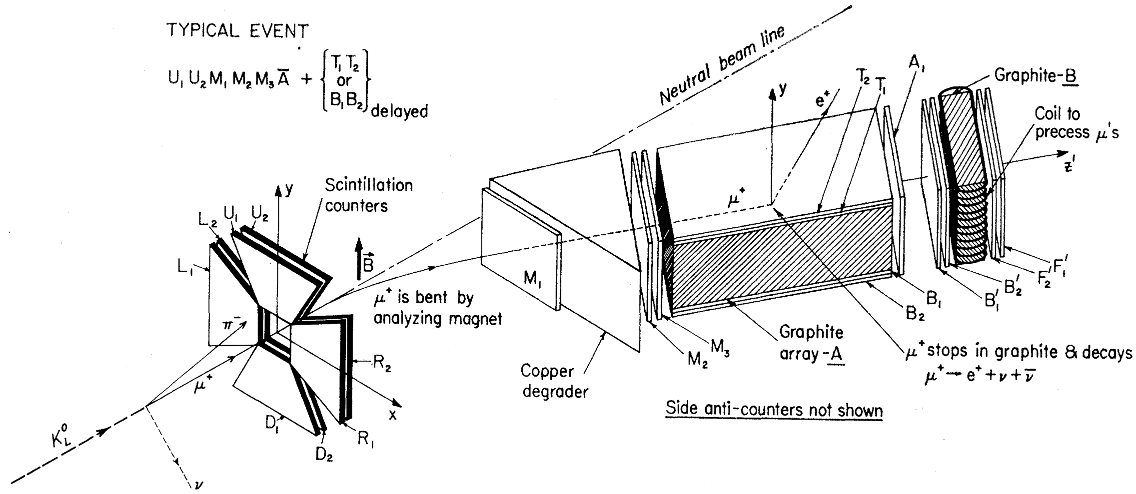


FIG. 3. Isometric view of experiment.

mental arrangement and calculated average polarizations for the sample of events "collected." Many geometries were tried until a nearly optimum one was established. The Monte Carlo program is discussed more thoroughly in Sec. III F.

The experimental arrangement decided upon is shown in Figs. 1 and 3. The useful decay region for the K_L^0's extended from the exit of the second sweeping magnet B_2 to the pion counters (see Fig. 1). For the typical K_L^0 momentum of 2 BeV/c, approximately 5% decayed in this region. Of these, $\approx 14\%$ decayed via the desired mode $K_L^0 \rightarrow \pi^- + \mu^+ + \nu$. This is the only decay mode that yields positive muons directly, and this fact was used as a basis for selecting the desired mode.

Muons that came off at nearly 0° in the laboratory system passed between the pion counters, were bent out of the neutral beam by the analyzing magnet B_3 (Figs. 1 and 3), and were detected by counter M_1. Anticounters A_4 and A_5 together with lead shielding blocks behind the pion counters prevented muons from passing through the pion counters and reaching M_1. The muons were then slowed down by a copper degrader which was wedge shaped to allow a first-order correction for the momentum dispersion of magnet B_3. The muons were then detected by counters M_2 and M_3, and some of them stopped in either of the two analyzers which were used to measure the muon polarization. The first analyzer (graphite array "A" in Fig. 3) was arranged to measure the vertical asymmetry in the angular distribution of the positrons from the muon decay. It was thus sensitive to a vertical component of polarization in the laboratory system. The second analyzer, "B", was arranged to measure the asymmetry along the direction of the incoming muons and was thus sensitive to a longitudinal component of polarization. The analyzers are discussed in detail in Sec. III D.

Before reaching counter M_3 the muons had to pass through a thickness of copper and other material ranging from ≈ 600 to 850 g/cm^2. This was sufficient to ensure that all but a very small fraction of the pions and electrons from K_L^0 decays or other sources were stopped before reaching the analyzers. A small fraction of the pions were able to fake muons from K^0 decays by decaying in flight with the decay muon then passing through the degrader and entering the analyzer. The correction for the contamination from this effect is discussed in Sec. III H.

The pions from $K_{\mu3}^0$ decays were detected by means of scintillation counters at the end of the decay region. A beam's-eye view of the pion counters is shown in section Z-Z in Fig. 1. The neutral beam and the muons from the $K_{\mu3}^0$ decays passed through the rectangular aperature between these counters. The detection of a pion in one of the pion counter pairs in fast coincidence with a muon defined the orientation of the decay plane relative to the vertical. Lead shielding behind the pion counters prevented the pions from reaching the anticounters A_4 and A_5. Figure 3 shows a "typical" event with the pion going into L_1L_2 so that the normal to the decay plane is approximately downward along the $-y$ axis.[17] On the average all decays with pions going into L_1L_2 have the normal to the decay plane downward (though in some cases the normal is at a rather large angle to the vertical). Thus for pions going into L_1L_2, the vertical asymmetry in the positron distribution measured in analyzer "A" is proportional to the average component of polarization normal to the decay plane. For pions going into R_1R_2, the situation is the same except that the asymmetry has the opposite sign.

Similarly, for events with pions detected in U_1U_2 the vertical asymmetry measured in analyzer "A" is proportional to the component of polarization in the decay plane and perpendicular to the momentum vector of the muon. Again for pions going into D_1D_2, the asymmetry has the opposite sign.

[17] We take the positive sense of the normal to be along $\mathbf{p}_\pi \times \mathbf{p}_\mu$.

The relation between the observed asymmetries and the polarization in the rest system of the K^0 depends on many factors including the counter geometry, the $K_L{}^0$ spectrum, the magnetic field in B_3, the geometry of the copper degrader, and the configuration of the analyzer. This relation was established by using the Monte Carlo program described in Sec. III F. The apparatus was most efficient for $K_L{}^0$ decays from which the muon went off approximately forward (i.e., along the beam direction) in the $K_L{}^0$ rest frame with the pion going off near 90°. It therefore tended to select decays for which the transverse polarization was largest.[18] Another noteworthy feature of the design is that magnetic field in B_3 and the thickness of the copper degrader were chosen to allow only the higher-energy muons to reach the analyzer. This tended to strongly reject decays with the muons going backward in the $K_L{}^0$ rest frame. (See also Sec. III F.)

Analyzer "A", which measured the vertical component of polarization, could not be used to measure the longitudinal component. Analyzer "B" was used for this purpose. In the case of the longitudinal component there was no need to restrict the pion to come off in a horizontal or vertical plane. However, only decays with the pion going into the U or D telescopes were used for reasons discussed in Sec. IV C.

D. Analyzers

The technique used to measure the polarization of the muons was to bring the muons to rest in graphite and then observe the asymmetry in the angular distribution of the positrons from the muon decay, $\mu^+ \to e^+ + \nu + \bar{\nu}$. The positrons come off preferentially parallel to the muon polarization vector. This technique is a standard one that has been used for many years.[4] Graphite is a favorite material for stopping the muons since it is known not to depolarize the muons,[19] it is fairly dense, and its Z is relatively low which makes it easier to get the positrons out. This technique is not practical for negative muons because they are captured in atomic orbits and almost completely depolarized.

The basic geometry of the analyzer is shown schematically in Fig. 4. We define the asymmetry ϵ in the positron distribution as

$$\epsilon = (T-B)/(T+B),$$

where T and B refer to positron counts in the top and bottom counters, respectively (in delayed coincidence with appropriate pion and muon counters). The component of polarization P along the $+Y$ direction in Fig. 4 is related to the asymmetry by

$$\epsilon = \mathcal{A} P, \tag{6}$$

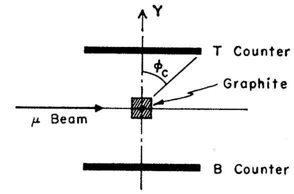


FIG. 4. Geometry of analyzer used to measure the muon polarization (schematic).

where the "analyzing power" $\mathcal{A}$ is determined by the geometry of the apparatus. The analyzing power is equal to the asymmetry obtained when muons completely polarized along the $+Y$ axis decay in the graphite.

The analyzing power for a particular geometry can be readily calculated using well-known expressions for the positron distribution.[4] If φ is the angle between the muon spin and the positron momentum and x the momentum in units of $\frac{1}{2}m_\mu c$, then for completely polarized muons the distribution is[20]

$$d^2N/dx d\Omega \propto x^2[(\tfrac{3}{2}-x)+(x-\tfrac{1}{2})\cos\varphi]. \tag{7}$$

If we consider an idealized geometry such that the graphite can be approximated as a point source of positrons and the T and B counters are circular so the cutoff angle φ_c (Fig. 4) is well defined, then the numbers of counts in the T and B counters are given by

$$T \propto \int_{x_c}^{1}\int_{0}^{\varphi_c} x^2[(\tfrac{3}{2}-x)+(x-\tfrac{1}{2})\cos\varphi]\,dx\,\sin\varphi\,d\varphi,$$
$$B \propto \int_{x_c}^{1}\int_{\pi-\varphi_c}^{\pi} x^2[(\tfrac{3}{2}-x)+(x-\tfrac{1}{2})\cos\varphi]\,dx\,\sin\varphi\,d\varphi, \tag{8}$$

where x_c is the cutoff momentum; i.e., the minimum positron momentum required to get out of the graphite and reach the counters.

Recalling that the analyzing power is equal to the asymmetry with completely polarized incoming muons, we obtain for $\mathcal{A}$

$$\mathcal{A} = \frac{T-B}{T+B} = [\tfrac{1}{2}(1+\cos\varphi_c)]\left[\frac{1+2x_c{}^3-3x_c{}^4}{3(1-2x_c{}^3+x_c{}^4)}\right]. \tag{9}$$

On a purely statistical basis, the optimum geometry for the analyzer occurs when the quantity $Q \equiv \mathcal{A}^2(T+B)$ is maximized, since the statistical error squared is inversely proportional to the product of $\mathcal{A}^2$ and the rate $(T+B)$. Q is maximized when $\varphi_c \approx 70°$ and $x_c \approx 0.67$. These values yield $\mathcal{A} \approx 0.36$ with $\approx 41\%$ of the positrons collected. $x_c = 0.67$ corresponds to a positron of momentum 35 MeV/c and a range of 17 g/cm^2 in graphite.

The geometry of the analyzer actually used is greatly different from the idealized situation considered

[18] From Eq. (4) the transverse polarization is proportional to $\mathbf{p}_\pi \times \mathbf{p}_\mu \propto \sin\theta_{\pi\mu}$ when $|\mathrm{Im}\xi| \ll |\mathrm{Re}\xi|$.

[19] Experiments indicate that positive muons retain $100 \pm 2\%$ of their original polarization when brought to rest in graphite. (See Ref. 27.)

[20] There are three parameters which appear in the general expression for the positron distribution. Of these, two (ρ,δ) are determined in the two-component neutrino theory to be 0.75, in agreement with experiment. The third parameter η is found experimentally to be 0.96 ± 0.05 and is taken as unity in Eq. (7). [See. Refs. 2(a) and 4.]

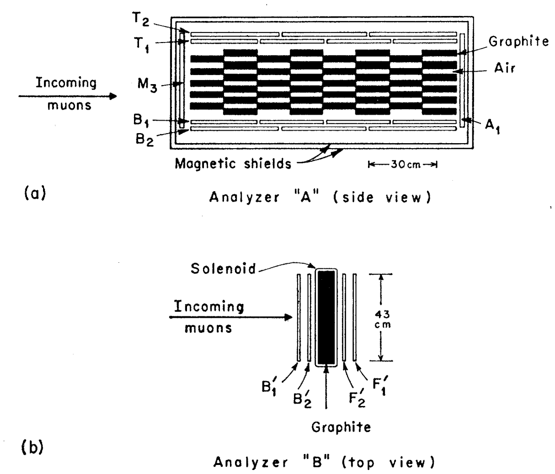


FIG. 5. Actual configuration of analyzers used in this experiment.

here. However, the maximum of Q is quite broad, and the values obtained above serve for the purpose of orientation.

The actual geometry of analyzer A is shown in Fig. 5(a) and analyzer B in Fig. 5(b). In order to reduce accidental rates in the T and B counters, a fast coincidence bwteen T_1 and T_2 or B_1 and B_2 was required The analyzing power for each of the analyzers was calculated from the distribution function given in Eq. (7) by means of a Monte Carlo program. This program took into account the details of the counter geometry and used experimental information on range straggling for electrons in graphite.[21] In the course of the experiment the vertical distribution of muons in analyzer A was measured, and this was also used in the calculation. The result of the calculation for the analyzing power was

$$\mathcal{A}_A = 0.39 \pm 0.01 \quad \text{for analyzer } A\,,$$

$$\mathcal{A}_B = 0.34 \pm 0.01 \quad \text{for analyzer } B\,.$$

The quoted errors are based on estimates of the uncertainties in the muon distribution and effective positron ranges in graphite. For reasons to be discussed in Sec. IV D the uncertainties in $\mathcal{A}$ have little effect on our value for Reξ. The rather large value of $\mathcal{A}_A$ reflects the fact that in the design of the analyzer the range straggling and multiple Coulomb scattering of the positrons were not taken into account. This led to a somewhat higher average cutoff momentum than expected and a larger $\mathcal{A}_A$ at the expense of a reduced positron collection efficiency.

The graphite in analyzer A was spaced to provide a mean density of 0.8 g/cm^3 for the graphite. This allowed the graphite in the analyzer to be 38-cm high, thereby increasing the solid angle for collecting muons while still maintaining a reasonably high efficiency for collecting the decay positrons.

The graphite in analyzer A was carefully shielded from stray magnetic fields that might precess the muon spin in such a way that a false transverse polarization would arise. The field due to the magnet B_3 was greatly reduced by a 5-cm-thick steel "wall" just in front of counter M_2 (Fig. 1). The graphite was enclosed in a double magnetic shield consisting of an outer layer of 3.2-mm mild steel and an inner layer of 1.6-mm Permalloy. [See Fig. 5(a).] The magnetic field within the shield was measured by inserting a probe through small holes in the shield. The field was found to be $\lesssim 0.07$ G. The frequency of precession of the muons is 10.7°/(lifetime G). The precession resulting from the residual field is estimated to cause an apparent transverse polarization <0.007, compared with a statistical error in the polarization of ± 0.013.

No special magnetic shielding of analyzer B was required. The stray field measured at its location was ≈ 3 G. This is estimated to cause a change in the longitudinal polarization of 0.04 (compared with the statistical error of ± 0.25), and a correction was made. The graphite in analyzer B was also considerably thinner to minimize false asymmetries due to the gradient in the stopping muon distribution along the direction of the incoming muons [Fig. 5(b)]. A solenoid surrounding the graphite, when powered, provided an average field of 75 G in the vertical direction. This was sufficient to effectively depolarize the muons completely since their spin vectors processed rapidly about a vertical

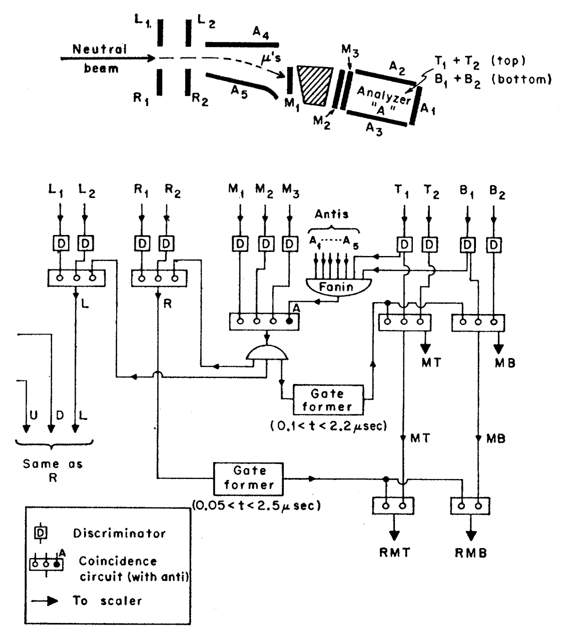


FIG. 6. Simplified diagram of electronics associated with analyzer A. The various concidences are defined as $L = L_1L_2$, $R = R_1R_2, \cdots$, $T = T_1T_2$, $B = B_1B_2$, $M = M_1M_2M_3\bar{A}$.

[21] A. T. Helms, Natl. Bur. Std. (U. S.) Circ. Suppl. 577, 1958; J. E. Leiss, S. Penner, and C. S. Robinson, Tables of Electron Range Straggling in Carbon, University of Illinois Technical Report, 1956 (unpublished).

axis as they decayed.[22] By alternating "solenoid-on" and "solenoid-off" runs we were able to subtract out false asymmetries (such as the one mentioned above) to a high degree of accuracy.

E. Electronics

A simplified diagram of the electronics associated with analyzer A is shown in Fig. 6. The electronics associated with analyzer B was quite similar. The circuits used for the fast electronics were standard, commercially available units with minimum resolving times $\sim$1 nsec and maximum counting rates up to 100 MHz.[23] The "slow" electronics (in the μsec range) was mostly Standard Lawrence Radiation Laboratory design.[24] All photomultiplier tubes were RCA 6810A's.

As shown in Fig. 6, a master gate was formed by a fast coincidence of M_1, M_2, and M_3 with no anticoincidences. This signified that a muon had entered the analyzer and stopped. The master gate defined the time interval within which a decay positron could be detected This period began 0.1 μsec after the muon entered in order to discriminate against prompt events. The inner T and B counters (T_1 and B_1) were part of the anticoincidence shield around the analyzer; this also prevented prompt T and B coincidences from simulating positrons. The master gate ended 2.2 μsec after the muon entered. This time was set arbitrarily as a compromise between positron collection efficiency and accidental rates. Coincidences between the various pairs of pion counters (e.g., L_1L_2, R_1R_2) started gates which were longer than the master gate, so that the master gate was always the defining one. These "pion gates" thus sorted out which of the T and B pulses passed by the master gate corresponded to muons in fast coincidence with one of the pion counter telescopes.

Various important accidental rates were also monitored continuously. (The associated electronics is omitted from Fig. 6 for clarity.) Accidentals involving a muon in $M_1M_2M_3$ (with no anti) in chance coincidence with an unrelated event in one of the pion counters were typically 1.8%. Accidentals of this type correspond to events in which the decay plane is undefined and therefore only dilute the observed polarizations (Sec. III H).

Accidentals involving the T and B counters were also monitored continuously. On the average throughout the run $\approx$4% of the "positrons" were chance coincidences from unrelated background in the T and B counters. No appreciable difference between the accidental rates in the T and B counters was observed. Because of the manner in which the data from the various pion counters were averaged (see Sec. III G), it is extremely unlikely that an asymmetry in the accidental rates, even if present, would lead to a false asymmetry. Therefore the effect of this type of accidental is also to dilute any real polarization.

The outputs of the various coincidence circuits were scaled with conventional scalers, and the data were recorded by hand. A typical run lasted 2 h. Typical rates were 10/min in each of MT and MB, and 1/min in each of LMT, LMB, RMT, RMB, $\cdots$. The rela-

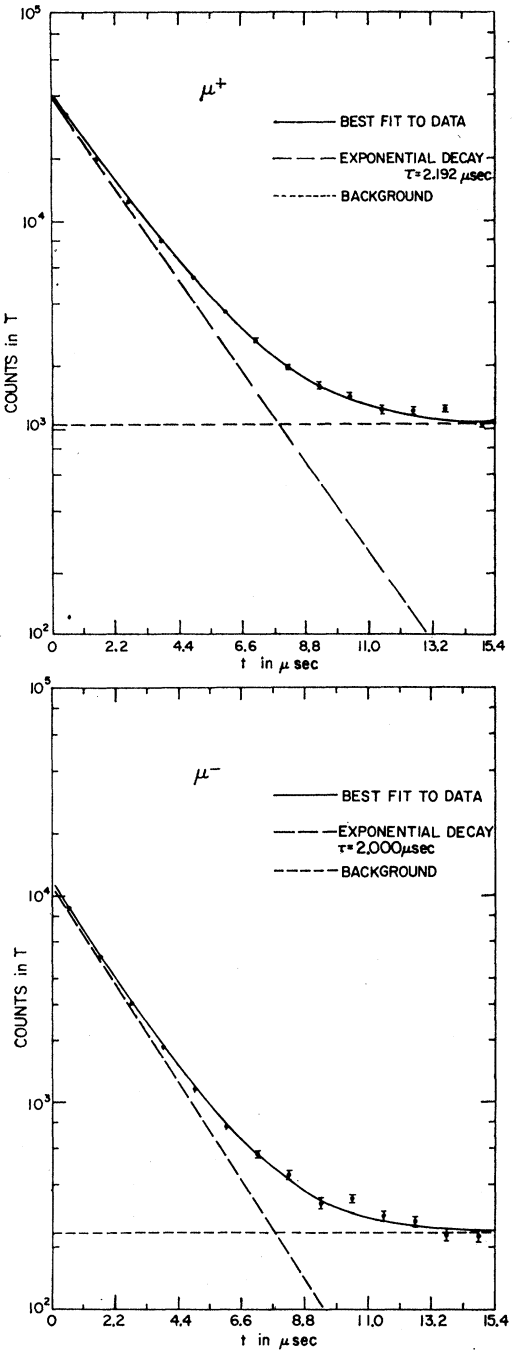


FIG. 7. Time distribution of positrons (or electrons) from muon decays as measured with a time-to-pulse-height converter.

[22] The time-averaged residual polarization was $\lesssim 1\%$.

[23] Chronetics, Inc., Mt. Vernon, N. Y.

[24] Lawrence Radiation Laboratory Counting Handbook, UCRL-3307, revised ed. (unpublished).

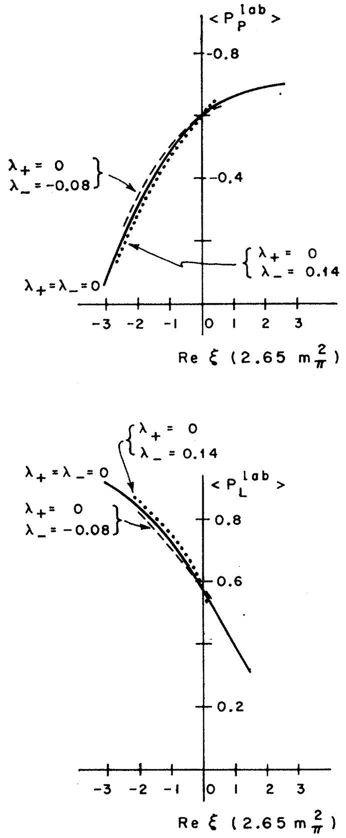


FIG. 8. $\langle P_L^{\text{lab}} \rangle$ and $\langle P_P^{\text{lab}} \rangle$ versus Reξ for Im$\xi=0$ and λ_-, λ_+ as indicated. The abscissa is Reξ evaluated at $q^2=2.65m_\pi^2$, the average q^2 for the sample of events selected by our apparatus.

tively high event rate and quick accessibility of the data allowed essentially continuous monitoring of the operation of the system. This together with the many kinds of coincidences scaled also allowed many checks on the consistency and quality of the data. As a precaution against false asymmetries due to differences in the electronics in the T and B channels, the circuits for the two sets of scintillation counters were periodically interchanged. Other checks will be discussed in Sec. III G.

As a check on the purity of the sample of muons collected, the decay of the muons with time was also studied by means of two time-to-pulse-height converters (not shown in Fig. 6). These were started by a pulse corresponding to the entry of a muon into analyzer A. One was stopped by the next coincidence from the T counters and the other by the B counters. The time distributions obtained for the T and B counters were consistent to a high degree of accuracy. The distributions obtained with both μ^+ and μ^- entering the analyzer are shown in Fig. 7. The most probable mean life was found by a maximum-likelihood method. The mean life we obtained for μ^+ was 2.192 ± 0.012 μsec compared with the accepted value of 2.200 μsec, and for μ^- we obtained 2.000 ± 0.032 μsec, compared with the accepted value of 2.026 μsec for μ^- decaying in graphite.[4] (The latter differs from the mean life in vacuum because $\approx 10\%$ of the negative muons undergo nuclear capture before decaying.) The consistency between the lifetimes of muons from pion decay with those for muons from K^0 decay also is evidence against the existence of any quantum number which distinguishes muons from the decay of strange particles from "ordinary" muons.[5(c)]

F. Monte Carlo Simulation

The Monte Carlo program used to simulate the experimental arrangement served a dual purpose. In the planning stage, it was used to optimize the geometry. This was done on the basis of a trial-and-error procedure. As was mentioned previously, the geometry was optimized for the time-reversal-violating component of polarization; however, this procedure also tended to give a nearly optimum arrangement for the other components as well. The over-all philosophy was to maximize the product of the rate and the asymmetry squared, which gives the smallest statistical error. However, this philosophy often had to be modified because of other practical considerations.

The same program also serves as a means of relating the experimentally observed polarizations with $\xi(q^2)$. The procedure essentially was to generate a sample of $K_L{}^0$ decays for various choices of $\xi(q^2)$. The pions and the muons from each "decay" were then traced through the simulated counter system and subjected to the same geometrical constraints the real particles had to satisfy. Any decay that did not satisfy these requirements was discarded. The average polarization of the muons from decays that satisfy the requirements could then easily be calculated and compared with the experimental results.

The Monte Carlo simulation can be considered in two parts. The vertical distribution of the muons entering analyzer A was measured experimentally. This is effectively the only information (other than the geometry of the analyzer) needed to calculate the analyzing power. This phase of the Monte Carlo simulation was discussed in Sec. III D. In the following we discuss only that part of the program which simulates the muon *before* it reaches the analyzer.

The steps in the Monte Carlo routine are as follows:

(i) The momentum for the decaying $K_L{}^0$ was generated according to an estimated $K_L{}^0$ spectrum weighted with the appropriate probability of decay. Very little experimental information was available on K^0 spectra at Bevatron energies. The spectrum chosen was based on a statistical-model calculation.[25] The results were

[25] D. Morgan, Atomic Energy Research Establishment (Harwell) Report R3242, 1960 (unpublished). This spectrum agrees generally with the $K_L{}^0$ spectrum measured by W. Galbraith *et al.* (private communication).

found to be nearly independent of the choice of the spectrum. This is discussed further in Secs. IV D and V.

(ii) A decay point within the decay volume was selected according to the experimentally observed intensity distribution of the neutral beam (Fig. 2). The direction of the incident $K_L{}^0$ was then determined assuming the $K_L{}^0$ originated in the copper target in the external proton beam.

(iii) A direction for the muon momentum vector and the orientation of the decay plane in the $K_L{}^0$ rest system were then chosen according to a uniform angular distribution in this system.

(iv) The angle of the pion relative to the muon and the muon momentum were then selected according to the distribution functions given by Brene *et al.*[13c] With the assumptions given in Sec. II A, the distribution is specified completely in terms of the four parameters Re$\xi(0)$, Im$\xi(0)$, λ_+, and λ_-.

The kinematics of the decay are now completely determined, and the laboratory momentum and angles of the pion and muon can be calculated. The polarization of the muon was then calculated in the $K_L{}^0$ rest system from Eq. (4) and in the lab system from Eq. (5).

(v) The pion trajectory was then traced to determine if it passed through one of the pion telescopes. Events were binned according to which telescope was involved.

(vi) The muon was traced through the magnet and the copper degrader and subjected to the same geometric constraints required of the actual muons. Only muons which stopped in one of the analyzers were considered as part of the final sample.

(vii) The appropriate component of polarization was then calculated for the survivors and the average value determined.

Many tests were made to verify that the Monte Carlo program faithfully simulated the experimental arrangement and generated events according to the proper distributions. Some of these included the following:

(i) The c.m.-system distributions of momenta and polarization for various values of ξ were checked with the calculated distributions of Brene *et al.*[13c]

(ii) The spatial distribution of muons stopping in analyzer A was found to be consistent with the experimentally observed distribution.

(iii) The event rate as a function of current in magnet B_3 determined from the program was found to be consistent with the experimentally observed dependence.

(iv) Events generated without requiring a pion counter were found to give zero average polarization as expected. The average polarization for events associated with the U pion counters was equal and opposite in sign to that for the D counters. Similarly, for the L and R counters the average polarizations were nearly equal but opposite in sign as expected.

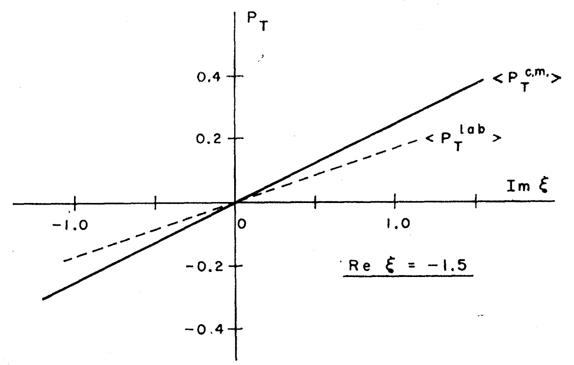


FIG. 9. Comparison of $\langle P_T{}^{\rm lab}\rangle$, the transverse polarization measured in our apparatus (as calculated from the Monte Carlo program) with $\langle P_T{}^{\rm c.m.}\rangle$, the average of the transverse component in the K^0 rest system if all decays are accepted.

(v) For Im$\xi=0$ the transverse polarization was zero.

Figure 8 shows the expected variation of the two T-allowed components of polarization with Reξ (as determined from the Monte Carlo program). We define $\langle P_L{}^{\rm lab}\rangle$ as the average polarization measured for the sample of muons stopping in analyzer B. $\langle P_L{}^{\rm lab}\rangle$ can be roughly identified with the average component of polarization along the muon direction in the laboratory system. (Note that $\langle P_L{}^{\rm lab}\rangle$ is somewhat different from the longitudinal polarization in the $K_L{}^0$ rest system because of the effect of the transformation from the $K_L{}^0$ system to the lab system on the polarization vector.) We define $\langle P_P{}^{\rm lab}\rangle$ as the average polarization measured for the sample of muons stopping in analyzer A with the associated pion going into either the U or D telescope. $\langle P_P{}^{\rm lab}\rangle$ can be roughly identified with the average component of polarization perpendicular to the muon momentum vector and in the decay plane. From Fig. 8 it can be seen that the relation between the experimentally observed polarizations and Reξ is not sensitive to λ_+ and λ_- if Reξ is evaluated at $q^2=2.65\ m_\pi{}^2$, the average q^2 for the sample of events selected by our apparatus. (See also Secs. IV D and IV E.)

For future use we also define $\langle P_T{}^{\rm lab}\rangle$ as the average polarization measured in analyzer A with the pion going into the R or L telescope. $\langle P_T{}^{\rm lab}\rangle$ is closely related to the average value of the T-violating component of polarization normal to the decay plane. This component is hardly affected by the transformation from the $K_L{}^0$ rest system to the lab system. In Fig. 9 we compare the variation of $\langle P_T{}^{\rm lab}\rangle$ versus Imξ with that of $\langle P_T{}^{\rm c.m.}\rangle$. The latter is defined as the average value of the component of polarization (in the $K_L{}^0$ rest system) normal to the decay plane, where the average is taken over *all* decays. It can be seen that $\langle P_T{}^{\rm lab}\rangle$ is $\approx 0.7\langle P_T{}^{\rm c.m.}\rangle$. This relationship is determined primarily by two factors. $\langle P_T{}^{\rm lab}\rangle$ is somewhat enhanced relative to $\langle P_T{}^{\rm c.m.}\rangle$ because the apparatus has been designed to select decays with larger-than-average polarization. (See also Sec. III C.) The factor that tends to reduce

$\langle P_T^{\text{lab}}\rangle$ relative to $\langle P_T^{\text{c.m.}}\rangle$ is the tipping of the decay plane relative to the horizontal. For a typical choice of $\text{Re}\xi(q^2)$ this effect reduces $\langle P_T^{\text{lab}}\rangle$ by $\approx 30\%$.

G. Discussion of Instrumental Asymmetries

As mentioned previously, the principal goal of the experiment was a sensitive test of T invariance. This involved a search for a component of polarization likely to be very small. A major consideration in the design of the apparatus and during the run was to avoid false asymmetries arising from instrumental effects. The question of possible false asymmetries is most crucial in the measurement of $\langle P_T\rangle$ since the experimental error for $\langle P_T\rangle$ is considerably smaller than those for $\langle P_P\rangle$ and $\langle P_L\rangle$. Section III G 1 is principally concerned with possible instrumental asymmetries in the measurement of $\langle P_T\rangle$, but most of the considerations also apply to $\langle P_P\rangle$ and, to a lesser extent, to $\langle P_L\rangle$.

1. Instrumental Asymmetries in $\langle P_T\rangle$

Instrumental asymmetries leading to a spurious transverse component of polarization could arise from a variety of causes. These are associated with some kind of asymmetry of the apparatus with respect to the horizontal median plane. Some of the precautions and checks against such effects were as follows:

(i) The apparatus was designed to be as nearly symmetric about the horizontal median plane as possible. The counters were then carefully aligned relative to the neutral beam. This ensured that there were no geometrical factors leading to a false up-down asymmetry for the positrons.

(ii) The T and B counters were carefully plateaued with cosmic rays to yield equal counting rates in each set of positron counters.

(iii) An important test of the success of these efforts to ensure the symmetry of the apparatus about the median plane was made by measuring the asymmetry for muons without requiring a coincidence with one of the pion telescopes. In this case there can be no preferred orientation of the decay plane, and there should be no asymmetry in the positron distribution. The measured asymmetry for this type of event averaged over the entire run was 0.0011 ± 0.0018. This showed that instrumental asymmetries not correlated with the pion counters were very small.

(iv) A further check was possible by reversing the field in magnet B_3, thus bending μ^- instead of μ^+ into the analyzers. The μ^- are almost completely depolarized in the graphite, and any real asymmetry should almost vanish. The average asymmetry for all runs with negative muons for pions in the R or L telescopes was $\bar{\epsilon}=-0.0034\pm0.0083$ where we define $\bar{\epsilon}$ as $\frac{1}{2}(\epsilon_R-\epsilon_L)$. Since the distribution of stopping μ^- is the same as that for μ^+ this check is sensitive to any kind of false asymmetry affecting the measured polarizations.

(v) For decays from which the pion goes into counters L_1L_2, the normal to the decay plane has the opposite sense compared with those from which the pion goes into R_1R_2. Any real asymmetry should then be equal and opposite in sign for the two sets of data. When the two sets of data are averaged, instrumental asymmetries should cancel out in $\bar{\epsilon}$ *if* the vertical distribution of stopping muons is independent of which pion counter is involved. In the measurement of $\langle P_T^{\text{lab}}\rangle$ the R and L counters are symmetrically placed with respect to the horizontal median plane. The vertical distribution of stopping muons is therefore expected to be the same for events associated with the R and the L counters. This was confirmed by a direct measurement which showed the two distributions agreed to a high degree of accuracy. The data for ϵ_R and ϵ_L are presented in Sec. IV A. The results are consistent with *no* residual instrumental asymmetry so this last precaution was an additional guarantee against instrumental asymmetries.

2. Instrumental Asymmetries in $\langle P_P\rangle$

In the measurement of $\langle P_P^{\text{lab}}\rangle$, the perpendicular component in the decay plane, the U and D pion telescopes were used so that the decay plane was roughly a vertical plane. In this case there is a significant vertical asymmetry inherent in the apparatus. For pions going into the D counters, the muons tend to stop in the upper part of the analyzer, thus causing a significant bias in favor of the T counters. Furthermore, in the case of the U counters this effect has the opposite sense and so does not cancel when the data from the two sets of pion counters are averaged.

This source of instrumental asymmetry could be readily corrected by using the asymmetry measured with μ^- in the analyzer (by reversing the field in B_3). The distribution functions for μ^- are the same, but they are mostly depolarized in the graphite. It has been found experimentally that μ^- retain $\approx 16\%$ of their polarization in graphite.[26] We define the average polarization for $\mu^\pm$ as $\bar{\epsilon}^\pm=\frac{1}{2}(\epsilon_D^\pm-\epsilon_U^\pm)$. If the real and instrumental asymmetries are small, the measured asymmetries are

$$\bar{\epsilon}^+=\epsilon_0+\epsilon_I \quad \text{and} \quad \bar{\epsilon}^-=0.16\epsilon_0+\epsilon_I\,, \tag{10}$$

where ϵ_0 is the true asymmetry and ϵ_I the instrumental asymmetry. This gives

$$\epsilon_0=[\bar{\epsilon}^+-\bar{\epsilon}^-]/0.84 \quad \text{and} \quad \epsilon_I=\bar{\epsilon}^+-\epsilon_0\,. \tag{11}$$

The measured values for the asymmetries were

$$\bar{\epsilon}^+=+0.2053\pm0.0055\,,\quad \bar{\epsilon}^-=+0.1034\pm0.0108\,.$$

[26] A. Astbury, P. M. Hattersley, M. Hussain, M. A. R. Kemps, H. Muirhead, and T. Woodhead, Proc. Phys. Soc. (London) **78**, 1144 (1961), A. E. Ignatenko, L. B. Egorov, S. Khalnpa, and D. Chultem, Zh. Eksperim. i Teor. Fiz. **35**, 1131 (1958) [English transl.: Soviet Phys.—JETP **8**, 792 (1958)].

This gives

$$\epsilon_0 = +0.1213 \pm 0.0145,$$
$$\epsilon_I = +0.084 \pm 0.0130.$$

As a check on this procedure, ϵ_I was determined independently directly from the measured distributions of muons associated with the U or D counters by means of the Monte Carlo program for the analyzer. This gave a value for ϵ_I of 0.08, in good agreement with the value obtained above. Checks were also made to verify that Eq. (10) was an adequate approximation to obtain ϵ_0.

3. *Instrumental Asymmetries in* $\langle P_L \rangle$

Due to the different orientation of analyzer B, which was used in the measurement of $\langle P_L{}^{\text{lab}} \rangle$, the factors which might cause instrumental asymmetries were somewhat different than those affecting $\langle P_T{}^{\text{lab}} \rangle$ and $\langle P_P{}^{\text{lab}} \rangle$. Basically, any asymmetry in analyzer B, or in the distribution of muons stopping in it, relative to its median plane would lead to a false asymmetry in the measurement.

Because of the gradient in the density of stopping muons along the direction of the incident muons in analyzer B [Fig. 5(b)], there was a significant instrumental asymmetry in the data for $\langle P_L \rangle$. This was corrected by alternating runs with the solenoid off and solenoid on as described in Sec. III D. The solenoid produced a magnetic field of ≈ 75 G, which effectively depolarized the muons. The asymmetry with the solenoid on is purely instrumental so that the corrected asymmetry is

$$\epsilon_0 = \epsilon_{\text{off}} - \epsilon_{\text{on}}.$$

We find

$$\epsilon_{\text{off}} = 0.0781 \pm 0.0509$$

and

$$\epsilon_{\text{on}} = -0.1697 \pm 0.0504,$$

where

$$\epsilon \equiv (F' - B')/(F' + B').$$

This gives

$$\epsilon_0 = +0.248 \pm 0.071.$$

As a check, we found that the asymmetry for μ^- decay was consistent with a residual polarization of $\approx 16\%$ as expected.

H. Dilution Factors

In addition to possible instrumental asymmetries there are also effects that tend to reduce the measured polarization from the true value.

One possibility is that for some reason the muons might depolarize partially in flight or in the graphite. The vertical component of the magnetic field in B_3 has no effect on the vertical component of polarization. Its effect on the longitudinal component is essentially to precess it along with the momentum vector. The other components of the field are very small and are symmetric about the median plane of the magnet. They have no significant effect on the average polarization. Extensive theoretical and experimental work on the depolarization of positive muons indicates that no significant depolarization takes place in flight, in the degrader, or when stopped in conductors such as graphite.[27]

In addition to depolarization in flight or in graphite there are other effects which dilute the polarization of the sample of events studied. We first discuss those affecting the measurements of $\langle P_T{}^{\text{lab}} \rangle$ and $\langle P_P{}^{\text{lab}} \rangle$.

One such factor is the contamination of the sample of events with muons originating from pion decays rather than directly from K decays. The muons from pion decays can only be polarized longitudinally (by symmetry considerations). The pions can arise from either K decays or neutron interactions in the helium. The latter source was studied experimentally by replacing the helium in the helium bag with denser gases (N_2 and CO_2). This caused no significant increase in the event rate, and we estimate the dilution of the polarization by pions from this source to be $< 4\%$. The biggest source of contamination was muons from pions (originating from $K_L{}^0$ decays) that decayed in flight with the muon then going on and stopping in the graphite. This effect is easy to calculate and causes a dilution of $\approx 7\%$. The chance of a positive pion getting through the copper degrader and decaying in analyzer A is estimated to be $\approx 0.5\%$. When the branching ratios are taken into account this leads to a dilution of $\approx 2\%$.

Other potential sources of dilution are as follows:

(i) Negative muons. Because of the large bend in magnet B_3, negative muons could not reach the analyzer and satisfy the coincidence requirements.

(ii) Muons which stop in depolarizing media rather than graphite. The graphite was completely surrounded by an anticoincidence shield except on the upstream end. This prevented any muon which passed out of the graphite from counting. Muons stopping in the latter part of scintillator M_3 or the early part of one of the anticounters are the only ones which can stop in a depolarizing medium and satisfy the fast logic criteria. This causes a dilution of $\approx 4\%$.

(iii) The effect of accidentals on the T and B counters or in the pion telescopes is essentially to dilute the sample of events studied (see Sec. III E). The accidental rate in the T and B counters was $\approx 4\%$ and in the pion counters $\approx 2\%$, giving a total of $\approx 6\%$.

The combination of these factors gives an over-all dilution factor of 0.82 for $\langle P_T{}^{\text{lab}} \rangle$ and $\langle P_P{}^{\text{lab}} \rangle$. A direct test for possible depolarization of the muons and/or unexpected dilution of the sample by unpolarized

[27] The best experimental limit on μ^+ depolarization in graphite with no magnetic field is that of A. Buhler *et al.* [Nuovo Cimento **39**, 824 (1965)]. They obtain a residual polarization of $100 \pm 2\%$ of the expected value. They also find no depolarization when muons are slowed down, even in media which strongly depolarize positive muons at rest.

muons was possible because all three components of polarization were measured. This test is discussed in Sec. IV D.

The corresponding factor for $\langle P_L{}^{\rm lab}\rangle$ is somewhat different because of the following reasons:

(i) Analyzer B was not magnetically shielded. The measured field was 3 G in an approximately vertical direction. The average precession angle θ due to this field is given by

$$\langle\cos\theta\rangle=\left(\int_{0.1\,\mu\rm sec}^{2.2\,\mu\rm sec} e^{-t/\tau}\cos\omega t\,dt\right)\Big/\int_{0.1\,\mu\rm sec}^{2.2\,\mu\rm sec} e^{-t/\tau}dt\,,$$

where τ is the muon mean life, ω is the angular velocity of precession$=10°/(\text{G lifetime})=30°/\tau$. This causes a 4% reduction in $\langle P_L{}^{\rm lab}\rangle$; i.e., $\langle\cos\theta\rangle\cong0.96$.

(ii) The muons from pion decay were partially longitudinally polarized with an estimated polarization of $\approx16\%$ for our geometry in a direction opposite to the polarization of the muons from K^0 decay. Since the contamination of muons from this source was $\approx7\%$, the net reduction in $\langle P_L{}^{\rm lab}\rangle$ was $\approx8\%$.

(iii) Accidentals in the pion counters would not affect $\langle P_L{}^{\rm lab}\rangle$ significantly. Accidentals in the F' and B' counters were $\approx4\%$.

(iv) The chance of a pion reaching analyzer B and decaying there is considerably less than for analyzer A because of the attenuation of the additional graphite. The dilution from this source is estimated to be $\approx0.6\%$.

(v) The dilution due to muons stopping in a depolarizing medium, in this case the latter part of counter B_2' or the first part of F_2' (Fig. 3), is $\approx8\%$.

The net effect was to give a dilution factor of 0.77 for $\langle P_L{}^{\rm lab}\rangle$. Our result for Re$\xi$ is insensitive to uncertainties in the dilution because of the technique used in extracting Reξ from the data. (See Sec. IV D.)

IV. RESULTS

A. T-Violating Component $\langle P_T{}^{\rm lab}\rangle$

Precautions against possible instrumental asymmetries in the measurement of $\langle P_T\rangle$ were discussed in Sec. III G. One of the most important is that instrumental asymmetries should cancel out when data for the R and L pion telescopes are averaged, since any real asymmetry should be equal and opposite in sign for the two sets of data (Sec. III G).

The measured asymmetries were

$$\epsilon_R=-0.0007\pm0.0066\,,\quad \epsilon_L=0.0014\pm0.0056\,.$$

Note that each of these is consistent with zero asymmetry. The average of these is

$$\bar\epsilon\equiv\tfrac{1}{2}(\epsilon_R-\epsilon_L)=-0.0011\pm0.0043\,,$$

where a positive $\bar\epsilon$ corresponds to a preferred direction of the positrons along $\mathbf{p}_\pi\times\mathbf{p}_\mu$. To get $\langle P_T{}^{\rm lab}\rangle$ from this we have to divide by the analyzing power (0.39 from Sec. III D) and by the dilution factor (0.82 from Sec. III H). This gives

$$\langle P_T{}^{\rm lab}\rangle=-0.0034\pm0.0134\,.$$

Our result is therefore consistent with no time-reversal violation. The limit on Imξ is discussed in Sec. IV E. An internal check on the values of the analyzing power and dilution factors used was possible because we measured all three components of polarization. This is discussed in Sec. IV D.

B. Perpendicular Component in the Decay Plane $\langle P_P{}^{\rm lab}\rangle$

The corrected asymmetry associated with this component was found in Sec. III G to be $\epsilon_0=+0.1213\pm0.0145$. The analyzing power and dilution factor are the same as for $\langle P_T{}^{\rm lab}\rangle$. This gives

$$\langle P_P{}^{\rm lab}\rangle=-0.379\pm0.045\,,$$

if the positive direction of polarization is along $+\mathbf{p}_\mu\times(\mathbf{p}_\pi\times\mathbf{p}_\mu)$. (The positrons are found to come off preferentially in the opposite direction from the pions.)

C. Longitudinal Component of Polarization $\langle P_L{}^{\rm lab}\rangle$

The apparatus for measuring the longitudinal polarization was added half-way through the run. The rate for collecting data was considerably lower than for the other two components, mostly because analyzer B had to be considerably thinner than A to reduce systematic asymmetries. Of the muons that passed through counter M_3, $\approx35\%$ stopped in analyzer A, $\approx3\%$ in analyzer B, and the remainder passed out the sides of A or went on through B. Because of these factors the statistical error in $\langle P_L{}^{\rm lab}\rangle$ is considerably larger than for the other two components.

Only decays with the pion going into the U or D telescope were used in the measurement of $\langle P_L\rangle$. This

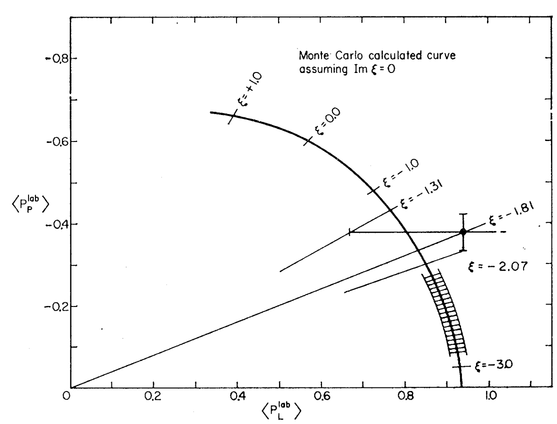


FIG. 10. Curve showing relation of $\langle P_P{}^{\rm lab}\rangle$ and $\langle P_L{}^{\rm lab}\rangle$ (calculated from the Monte Carlo program) for our geometry when Reξ is varied. The experimental point with error flags shows our measured values.

ensured that the samples of decays studied were essentially the same for $\langle P_L \rangle$ and $\langle P_P \rangle$ and allowed us to determine Reξ in a way that is insensitive to effects of depolarization or dilution. (See Sec. IV D.)

From Sec. III G the corrected asymmetry associated with $\langle P_L{}^{\text{lab}} \rangle$ was 0.248 ± 0.071. The analyzing power for analyzer B from Sec. III D is 0.34, and the dilution factor from Sec. III H is 0.77. The result for $\langle P_L{}^{\text{lab}} \rangle$ is

$$\langle P_L{}^{\text{lab}} \rangle = 0.94 \pm 0.27 .$$

The direction of $\langle P_L{}^{\text{lab}} \rangle$ is parallel to the muon momentum vector. (The positrons are preferentially emitted along the muon direction.)

D. Determination of Reξ

Since we found that $\langle P_T{}^{\text{lab}} \rangle$ was consistent with no T violation, we can neglect Imξ in the determination of Reξ. Accordingly, we write Eq. (3) as

$$\xi(q^2) \cong \text{Re}\xi(q^2) = \xi(0)[1+\lambda_-(q^2/m_\pi{}^2)]/[1+\lambda_+(q^2/m_\pi{}^2)] .$$

For the moment we shall assume that both λ_- and λ_+ are zero so that $\xi \equiv \xi(0)$. By means of the Monte Carlo program (Sec. III F), the measured polarizations can be related to $\xi(q^2)$. The curve in Fig. 10, determined from this program, shows the relationship between $\langle P_L{}^{\text{lab}} \rangle$ and $\langle P_P{}^{\text{lab}} \rangle$ calculated for our apparatus as Reξ is varied. The shaded portion of the curve indicates the uncertainty due to the fact that the momentum spectrum of the $K_L{}^0$ beam is not well known. The resulting uncertainty in ξ is $\lesssim 0.05$. The data point with errors shown in Fig. 10 gives our measured values of $\langle P_L{}^{\text{lab}} \rangle$ and $\langle P_P{}^{\text{lab}} \rangle$. ξ can be determined from either $\langle P_L{}^{\text{lab}} \rangle$ or $\langle P_P{}^{\text{lab}} \rangle$. However, because of the possibility of unknown dilution factors or depolarization effects we prefer to use the procedure outlined below.

If the dilution in our sample were larger than expected, or if our values for the analyzing power were systematically high, the experimental point would tend to fall closer to the origin than the Monte Carlo curve. The fact that the experimental point is consistent with the curve is strong evidence that such effects cannot be large. We can determine ξ in a manner that is almost independent of possible dilution or depolarization effects by drawing rays from the origin to the Monte Carlo curve as illustrated in Fig. 10. The experimental arrangements used in the measurements of $\langle P_P{}^{\text{lab}} \rangle$ and $\langle P_L{}^{\text{lab}} \rangle$ were very similar so that the dilution factors and analyzing powers are closely related. Therefore the experimental point moves approximately along a radius if the dilution is varied, and the value of ξ is nearly independent of dilution.

TABLE I. Variation of our result for Re$\xi(0)$ and Re$\xi(2.65m_\pi{}^2)$ with λ_- and λ_+.

λ_-	λ_+	Re$\xi(0)$	Re$\xi(2.65m_\pi{}^2)$
0	0	−1.81	−1.81
0	+0.1	−2.43	−1.92
−0.08	0	−2.43	−1.92
0	−0.1	−1.24	−1.69
0.136	0	−1.24	−1.69

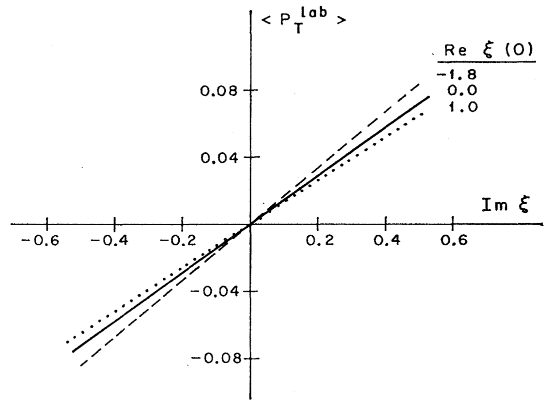


FIG. 11. Variation of $\langle P_T{}^{\text{lab}} \rangle$ with Imξ for several choices of Reξ with $\lambda_- = \lambda_+ = 0$.

The result for ξ is

$$\xi = -1.81_{-0.26}^{+0.50} .$$

This assumes that ξ is independent of q^2. Experiments are consistent with $\lambda_+ \lesssim 0.02$; however, almost nothing is known about λ_-.[1] The mean value of q^2 selected by our apparatus was $\approx 2.65m_\pi{}^2$. Table I gives $\xi(0)$ and $\xi(2.65m_\pi{}^2)$ for various values of λ_- and λ_+. Note that $\xi(2.65m_\pi{}^2)$ is nearly independent of λ_- and λ_+ over the range given.

E. Upper Limits on Imξ and Argξ

Figure 11 shows the variation of $\langle P_T{}^{\text{lab}} \rangle$ with Imξ for various values of Reξ. It can be seen that for a given $\langle P_T{}^{\text{lab}} \rangle$ the value of Imξ is almost independent of Reξ. From Fig. 11, using $\langle P_T{}^{\text{lab}} \rangle$ from Sec. IV A and Reξ ≈ -1.8 as found in Sec. IV E, we obtain

$$\text{Im}\xi = -0.02 \pm 0.08 .$$

The q^2 dependence of Imξ is, of course, unknown, but the above value can be considered to be Imξ at $q^2 \cong 2.65m_\pi{}^2$. The distribution in q^2 of the sample of decays accepted by our apparatus is shown in Fig. 12. For comparison, the distribution that would be obtained with an unbiased selection of decays is also shown. This result for Imξ is insensitive to variations in the momentum spectrum assumed for the incident $K_L{}^0$ beam.

The phase of ξ is given by

$$\text{Arg}(\xi) - \pi = \tan^{-1}\left(\frac{\text{Im}\xi}{\text{Re}\xi}\right) = \tan^{-1}\left[\frac{-0.02 \pm 0.08}{-1.8}\right]$$
$$= +0.6° \pm 2.6° \text{ for } q^2 \cong 2.65m_\pi{}^2 .$$

The present situation regarding Reξ from other experi-

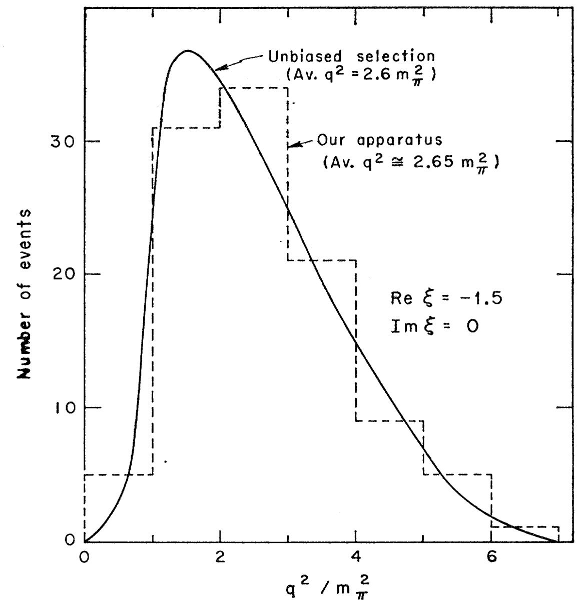


FIG. 12. Distribution of q^2 values for the events selected by our apparatus (histogram) and the distribution that would be obtained with an unbiased selection if all decays are collected (solid curve).

ments is unclear (see Sec. V). Using the value of Reξ from this experiment to obtain Argξ bypasses many of the questions raised by the uncertainty in Reξ (for example, the possibility of a failure of muon-electron universality).[28] Our result for Reξ is consistent with other measurements based on the studies of the muon polarization.

V. DISCUSSION AND SUMMARY

The measured polarizations $\langle P_T^{\text{lab}}\rangle$, $\langle P_L^{\text{lab}}\rangle$, and $\langle P_P^{\text{lab}}\rangle$ can be related to the average polarization components in the K_L^0 rest system, $\langle P_T^{\text{c.m.}}\rangle$, $\langle P_L^{\text{c.m.}}\rangle$, and $\langle P_P^{\text{c.m.}}\rangle$ through the Monte Carlo program. (See Sec. III F for definitions of these quantities.) Since the measured polarizations depend to a large extent on the geometry of the experiment, the components of polarization in the K_L^0 rest system are useful to compare with theory and the results of other experiments.

In Fig. 9 we gave the relation between $\langle P_T^{\text{lab}}\rangle$ and $\langle P_T^{\text{c.m.}}\rangle$. Using our experiment result for $\langle P_T^{\text{lab}}\rangle$, we obtain from Fig. 9

$$\langle P_T^{\text{c.m.}}\rangle = -0.005\pm0.019.$$

This result for $\langle P_T^{\text{c.m.}}\rangle$ is relatively insensitive to Reξ (see Fig. 11).

We obtain $\langle P_L^{\text{c.m.}}\rangle$ and $\langle P_P^{\text{c.m.}}\rangle$ in a similar fashion from our result for Reξ. Figure 13 shows the relationship between these quantities. For Re$\xi = -1.81_{-0.26}^{+0.50}$ we obtain

$$\langle P_L^{\text{c.m.}}\rangle = 0.92_{-0.04}^{+0.01}$$

[28] B. Kenny, Phys. Rev. Letters **20**, 1217 (1968).

TABLE II. Summary of results for polarization and ξ.

$\langle P_T^{\text{lab}}\rangle = -0.003\pm0.0134$	$\langle P_T^{\text{c.m.}}\rangle = -0.005\pm0.019$
$\langle P_P^{\text{lab}}\rangle = -0.38\ \pm0.045$	$\langle P_P^{\text{c.m.}}\rangle = -0.14_{-0.14}^{+0.06}$
$\langle P_L^{\text{lab}}\rangle = 0.94\ \pm0.27$	$\langle P_L^{\text{c.m.}}\rangle = 0.92_{-0.04}^{+0.01}$
$\text{Im}\xi = -0.02\ \pm0.08$	$\text{Re}\xi = -1.81_{-0.26}^{+0.50}$
$\text{Arg}\xi - \pi = +0.6\ \pm2.6^0$	(for $q_{\text{av}}^2 \approx 2.65 m_\pi^2$)

and

$$\langle P_P^{\text{c.m.}}\rangle = -0.14_{-0.14}^{+0.06}.$$

The small errors for $\langle P_L^{\text{c.m.}}\rangle$ reflect the fact that $\langle P_L^{\text{c.m.}}\rangle$ is insensitive to Reξ for Re$\xi \sim -1.5$. (See Fig. 13.)

Our results for the polarization and ξ are tabulated in Table II. For comparison the best previous limits in Imξ were 0.11 ± 0.35 obtained by Bartlett *et al.*[29] for K_L^0 decays, and -0.1 ± 0.4 obtained by the $X2$ bubble chamber collaboration for K^+ decays.[30]

The situation regarding Reξ at present is confused.

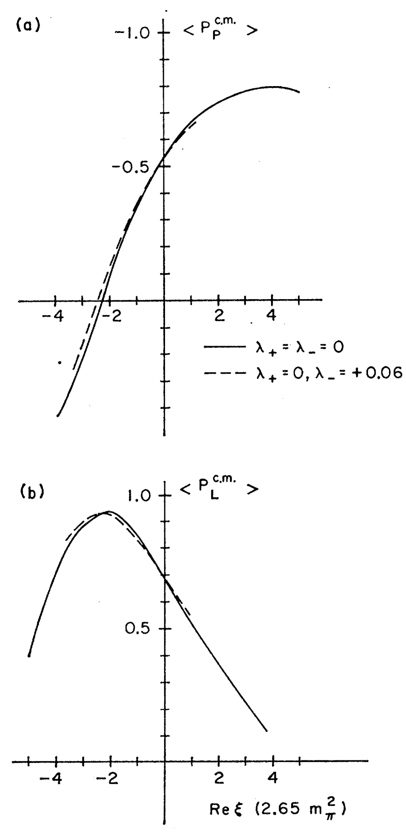


FIG. 13. (a) Relation between $\langle P_P^{\text{c.m.}}\rangle$, the component of polarization normal to the muon momentum and in the decay plane as averaged over the entire Dalitz plot, and Re$\xi(2.65m_\pi^2)$ for two choices of λ_-. (b) Relation between $\langle P_L^{\text{c.m.}}\rangle$, the longitudinal component of polarization averaged over the entire Dalitz plot, and Re$\xi(2.65m_\pi^2)$ for two choices of λ_-.

[29] D. Bartlett, C. Friedberg, K. Goulianos, and D. Hutchinson, Phys. Rev. Letters **16**, 282 (1966).
[30] Bettels *et al.*, Nuovo Cimento **56A**, 1106 (1968).

Willis has given an excellent summary of the existing data.[1] He concludes that the measurements of Reξ based on the $K_{\mu3}/K_{e3}$ branching ratio are generally consistent with each other. The world averages are

$$\mathrm{Re}\xi(K_{\mu3}/K_{e3})=+0.3\pm0.4 \quad \text{for } K_{l3}{}^+ \text{ decays}$$

and

$$\mathrm{Re}\xi(K_{\mu3}/K_{e3})=+0.7\pm0.3 \quad \text{for } K_{l3}{}^0 \text{ decays}.$$

On the other hand, world averages for measurements of Reξ based on studies of the muon polarization are

$$\mathrm{Re}\xi(\mu \text{ polarization})=-1.25\pm0.32 \quad \text{for } K_{\mu3}{}^+ \text{ decays}$$

and

$$\mathrm{Re}\xi(\mu \text{ polarization})=-1.15\pm0.35 \quad \text{for } K_{\mu3}{}^0 \text{ decays}.$$

This gives a combined average for the polarization measurements of -1.2 ± 0.3. Both the branching ratio and polarization measurements are separately consistent with the $|\Delta I|=\frac{1}{2}$ rule.

Our result ($-1.81_{-0.26}^{+0.50}$) is in fairly good agreement with the world average of the polarization measurements. It is interesting to note that our results for both $\langle P_P{}^{\mathrm{lab}}\rangle$ and $\langle P_L{}^{\mathrm{lab}}\rangle$ are inconsistent with the average value of Reξ from the branching ratio measurements; $\langle P_P{}^{\mathrm{lab}}\rangle$ is too small and $\langle P_L{}^{\mathrm{lab}}\rangle$ is too large (Fig. 10). These two results are statistically independent, and systematic errors would probably tend to cause both to be too high or too low. It therefore seems unlikely that systematic errors in the muon polarization experiments are the cause of the discrepancy.

A recent result based on the branching ratio gives Re$\xi=-0.5\pm0.3$.[31] This may indicate the possibility of unsuspected systematic errors in previous branching ratio measurements. Explanations which postulate large variations of Reξ with q^2 (Ref. 32) seem unable to explain the discrepancy since recent results show that Reξ is approximately independent of q^2.[30-33]

[31] Aachen-Bari-CERN-Padua-Valencia-Madrid Heavy Liquid Collaboration (quoted in Ref. 1).
[32] L. Auerbach, A. Mann. W. McFarlane, and F. Sciulli, Phys. Rev. Letters **19**, 464 (1967).
[33] D. Cutts, R. Stiening, C. Wiegand, and M. Deutsch, Phys. Rev. Letters **20**, 955 (1968).

Possible theoretical explanations of the discrepancy have been discussed by Kenny,[28] who finds that the assumption of a sizable contribution from a scalar term in the matrix element does not help. A breakdown of muon-electron universality in the decays of strange particles could cause the discrepancy.

Recent theoretical estimates of Reξ have ranged from (at least) -1.54 (Ref. 34) to $+0.43$ (Ref. 35) and are not especially helpful in resolving the discrepancy between the experimental results.

Our limits on Imξ and Argξ are particularly interesting in relation to possible theories of CP violation. Theories which postulate that the violation occurs in some part of the weak interaction can lead to large T-violating effects in $K_{\mu3}{}^0$ decay. Such theories have been proposed by Sachs[36] and Cabibbo.[37] Sachs attributes the CP violation in K^0 decay to an interference between the $\Delta S=\Delta Q$ and $\Delta S=-\Delta Q$ amplitudes, which are 90° out of phase. This theory predicts transverse polarizations $\langle P_T{}^{\mathrm{c.m.}}\rangle$ as large as $\approx20\%$ compared to our experimental limit of $\approx2\%$. Cabibbo has proposed that the CP violation is due to an interference between "regular" and "irregular" parts of the weak-interaction Lagrangian. In the limit of exact SU_3 invariance $|\mathrm{Arg}\xi|$ should be 90° in his theory, compared to our limit of $\approx2.6°$.

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[34] J. P. Hsu, Nuovo Cimento **58A**, 775 (1968).
[35] S. N. Biswas and J. Smith, Nuovo Cimento **51A**, 214 (1967).
[36] R. G. Sachs, Phys. Rev. Letters **13**, 286 (1964); B. G. Kenny and R. G. Sachs, Phys. Rev. **138**, B943 (1965).
[37] N. Cabibbo, Phys. Letters **12**, 137 (1964); **14**, 965 (1965).