interaction matrix element between the rotational 6^+ state and the $f_{7/2}{}^2$ 6^+ state to be the same as between the corresponding 4^+ states, which was determined[5] by fitting the observed energy spectrum. In this way, Gerace obtains an upper limit for the collective amplitude of 0.2. He then calculates $B(E2)$ values for the $6^+\rightarrow4^+$ transition and obtains 2.6, 7.0, and 14.0, all in $e^2\mathrm{F}^4$, for amplitudes of 0, 0.1, and 0.2, respectively. For these results an effective charge of $0.5e$ is assumed for the neutrons. It is seen that the experimental value of $(6.40\pm0.22)e^2\mathrm{F}^4$ is explained exactly with a collective amplitude of about 0.1. Thus the present result is entirely consistent with the earlier picture of levels in the calcium isotopes which contain sizable collective admixtures.

In a recent paper by Flowers and Skouras,[10] essentially the same model is considered, the principal difference being that a variational procedure is used to calculate the deformed orbitals and the unperturbed energies of the deformed states. In this work a collective amplitude of 0.19 is obtained for the 6^+ state at 3.19 MeV which, as seen from Gerace's work above, gives a $B(E2,6^+\rightarrow4_1{}^+)$ somewhat too large, but still in reasonably good agreement with experiment.

ACKNOWLEDGMENTS

The authors are grateful to Geoffrey Hartnell who designed and constructed the beam-pulsing and electronic gating equipment, and to Dr. W. J. Gerace who provided several illuminating communications and kindly gave permission for inclusion of his results here. A helpful discussion with Dr. Pedro Federman is also gratefully acknowledged.

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Lifetime and Angular Distribution Measurements from the ^{31}P(p, γ)^{32}S Reaction*

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The ^{31}P(p, γ)^{32}S reaction has been studied at several resonances with a 20-cm^3 Ge(Li) detector. A level at 6.664 MeV has been populated which may be the same as one found at 6.671 MeV from (He^3, d) experiments. This state is found to branch almost equally to the states at 3.775 and 2.230 MeV. Measured branching ratios for the first six excited states are in agreement with earlier work except for a ground-state transition for the 5.006-MeV level which supports an odd-parity assignment for this state. New branching ratios are assigned for the levels at 5.544 and 6.673 MeV. Doppler-shift lifetime measurements for the states at 3.775, 4.280, 4.694, 5.006, 5.410, 5.544, 6.226, and 6.623 MeV are consistent with other measurements. The lifetime of the level at 6.664 MeV is found to be $0.054_{-0.009}^{+0.013}$ psec. An upper limit of 0.01 psec is set for the mean life of the 7.952-MeV level. Angular distributions have been measured with the Ge(Li) detector for the 1.248- and 1.438-MeV resonances. Spins deduced from combining these results with the lifetime measurements confirm previous assignments for levels at 6.623, 6.226, and 5.410 MeV. The spin of the 4.459-MeV level is limited to 3 or 4, consistent with other recent work and with the hypothesis of Goswami *et al.* that this is the 4^+ level of a rotational band based on the 0^+ ground state. Mixing ratios are given for a number of transitions.

I. INTRODUCTION

It has been well established that some nuclei in the s-d shell exhibit collective effects. A region of nuclear deformation is centered about $A=25$, and many of the recent model calculations in this shell have interpreted the level structure of the neighboring nuclei in terms of strong- or weak-coupling collective models.[1] However, attempts to explain the nucleus ^{32}S in terms of rotational models have so far been unsuccessful; the calculations all fail to explain the low-lying 0^+ level at 3.775 MeV and underestimate the spacing between the ground and first excited states. Particle-hole calculations using a spherical basis[2] have also been unsuccessful. Bar-Touv and Goswami[3] have recently proposed a simple model to explain the lower excited states of ^{32}S. According to this model, the excited 0^+ state corresponds to a spherical equilibrium shape, while the ground state corresponds to a state of axially symmetric deformation and is the head of a rotational band. It is assumed that the energy of the deformed ground state is depressed by the mixing of the two primitive 0^+ states. A value for the ratio of the reduced transition rates between the 0^+ states and the deformed 2^+ first excited state can be

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* Work performed in part under the auspices of the U.S. Atomic Energy Commission.

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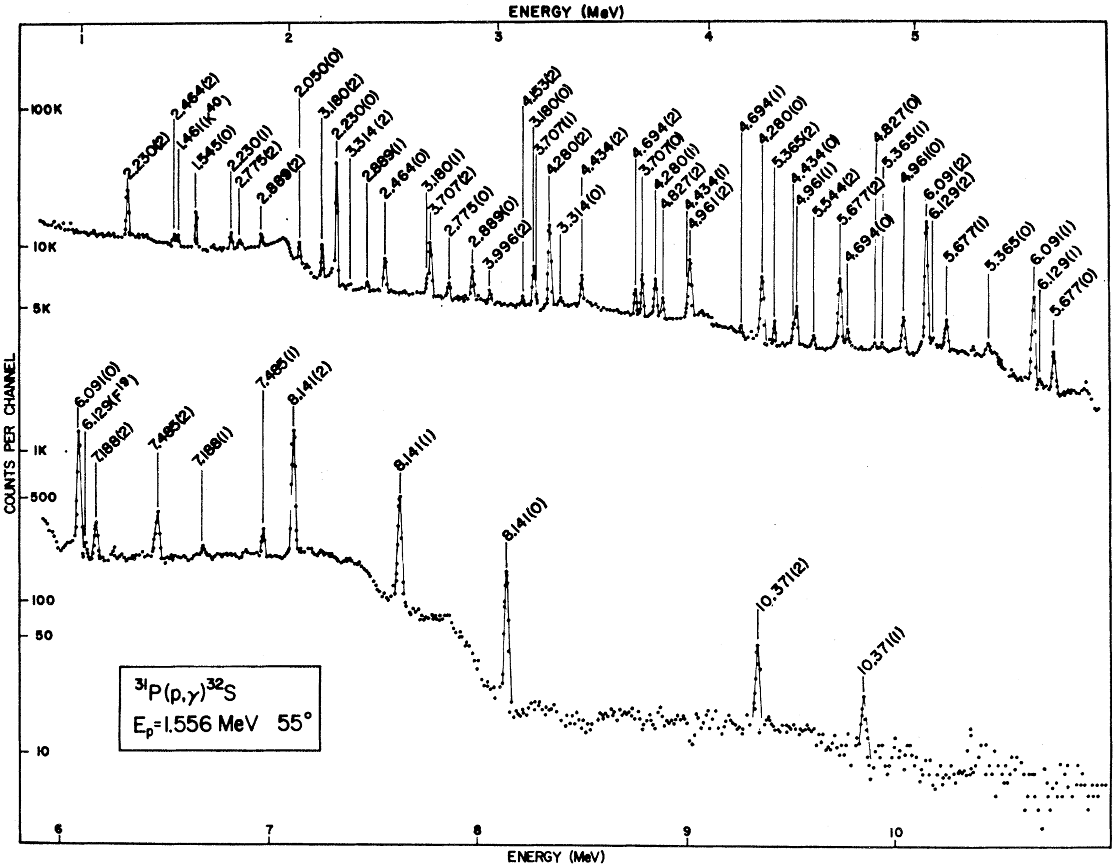

FIG. 1. γ-ray spectrum for the $^{31}P(p, \gamma)^{32}S$ reaction recorded with a 20-cm^3 Ge(Li) detector at 55° to incident beam; proton energy $E_p = 1.556$ MeV.

calculated and checked with a measurement of the lifetimes of the 2.230- and 3.775-MeV states. Also of importance in checking this model is a search for the existence of the 4^+ member of the ground-state rotational band, which is predicted at about 4.5 MeV. The radiative proton capture reaction is particularly useful in providing experimental information about the states in ^{32}S since it produces nuclei in highly excited states, whose subsequent γ decay involves several of the low-lying levels.

The $^{31}P(p, \gamma)^{32}S$ reaction has been studied by many authors[4–10]; the most extensive work in the $E_p = 1.0$–1.6-MeV range is that of Anderson.[10] Previous experimental information on ^{32}S is summarized in the recent review article by Endt and van der Leun.[11] Since that compilation was written, Poletti and Grace[12] have utilized the $(p, p'\gamma)$ reaction to establish the spins of several low-lying levels and to provide mixing-ratio assignments for several of the transitions between these states. We have utilized a 20-cm^3 Ge(Li) detector in a detailed study of four resonances in the $^{31}P(p, \gamma)^{32}S$ reaction to measure accurate level energies, obtain branching ratios, measure lifetimes by the Doppler-shift attenuation method (DSAM), and to investigate γ-ray angular distributions.

II. γ-RAY SPECTRA AND LIFETIME MEASUREMENTS

The proton beam from the University of Oregon 4-MeV Van de Graaff accelerator is focussed onto the target through two $\frac{3}{16}$-in. apertures. The Ge(Li) detector and associated electronics have been described previously.[13] Phosphorus targets were prepared by

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evaporating 99.999% pure Zn_3P_2 onto 0.010-in. gold backings. A fluorine target, used in determining the system asymmetry for the angular distribution measurements, was prepared by evaporating BaF_2 onto a 0.010-in. tantalum backing. Gamma-ray energy, intensity, and Doppler-shift measurements were made with the 20-cm^3 Ge(Li) detector mounted on one of the movable arms of the scattering table. A 3×3-in. NaI counter was mounted on the other movable arm.

Thick targets (8–10 keV) were used for the resonance-spectra measurements. Spectra were recorded with the Ge(Li) detector placed 1.6-in. from the target at 55° and 90° to the incident beam. Accumulation of sufficient statistics required running times of about 12 h with beam currents of 10–12 μA. Spectra were measured at the 1.248-, 1.438-, 1.556-, and 1.583-MeV resonances. Figure 1 shows a γ-ray spectrum obtained at the 1.556-MeV resonance. Location of peak centroids and Ge(Li) detector energy calibration were carried out with computer programs described elsewhere.[13]

Gamma rays emitted by the recoil nucleus in the capture γ process are Doppler shifted. The time dependence induced in the recoil velocity by the slowing of the excited nucleus in the target material gives rise to an attenuation of the Doppler shift,[14] so that the energy of the γ ray observed at an angle θ is

$$E(\theta)=E_0\{1+[\beta(0)\cos\theta]F(\tau)\}. \tag{1}$$

The attenuation factor $F(\tau)$ is given by[15]

$$F(\tau)=\frac{1}{\tau}\int_0^\infty\left(\frac{\beta(\tau)}{\beta(0)}\right)e^{-t/\tau}\langle\cos\phi\rangle dt, \tag{2}$$

where $\langle\cos\phi\rangle$ is the mean recoil multiple-scattering angle, E_0 is the nuclear transition energy, $\beta(0)$ is the initial recoil velocity of the compound system, and τ is the mean lifetime of the decaying state. The ratio between the observed shift ΔE and the fully Doppler-shifted energy difference ΔE_{full} at two angles yields an experimental measurement of $F(\tau)$. The lifetime of the γ-emitting state is determined by comparing this measured value of $F(\tau)$ with the numerical solutions of Eq. (2). Since the lifetimes of the resonance levels generally are immeasurably short, this technique can be used to determine the lifetimes of states populated directly from the resonant level. A somewhat more complicated situation arises when the level of interest is populated from a state with a measurable lifetime. The attenuated shift expression must then be generalized to

$$\Delta E_2=\Delta E_{2,\mathrm{full}}\{\tau_1F(\tau_1)-\tau_2F(\tau_2)\}/(\tau_1-\tau_2), \tag{3}$$

where the subscripts 1 and 2 refer to the parent and daughter levels, respectively. The ratio of observed-to-full Doppler shift can be used in this case to determine the lifetime of the daughter level, provided that the mean life of the parent level is known.

The 1.556-MeV resonance level populates many of the lower states in ^{32}S, and consequently is a convenient resonance for the simultaneous measurement of the lifetimes of several levels. One measurement was made at this resonance with a dispersion of approximately 1 keV/channel, over the γ-ray energy range 1–5 MeV. A second measurement was made at the 1.438-MeV resonance with special emphasis on determining the lifetime of the 6.623-MeV level, which is not populated at the 1.556-MeV resonance. Typical Doppler-shift data from the 1.556-MeV resonance measurement are illustrated in Fig. 2.

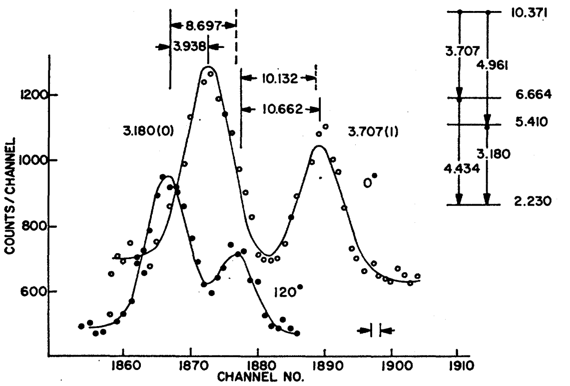

Fig. 2. Typical Doppler-shift data at the 1.556-MeV resonance. The dashed lines denote the centroid positions which would have resulted if the γ ray were fully Doppler-shifted. The magnitude of the zero shift at this energy is indicated in the lower right corner of the graph. The relevant portion of the decay scheme at this resonance is shown at the upper right. The attenuated and full Doppler-shift differences are given in keV.

III. ANGULAR DISTRIBUTION MEASUREMENTS

Experimental Procedure

Since it is advantageous to have many degrees of freedom in the analysis by χ^2 methods, the present measurements were made at seven angles: 90°, 75°, 60°, 52°, 41°, 30°, and 0°. The Ge(Li) detector was placed 2.7 in. from the target to facilitate corrections for the solid angle attenuation factors.[16] At this distance, the counting times necessary for the accumulation of sufficient statistics were 12 h per angle with a beam current of 10 μA. The NaI monitor detector was placed 6 in. from the target at an angle of 130°. Angular distributions were measured at the 1.248- and 1.438-MeV resonances.

Figure 3 is a block diagram of the electronics used in the angular distribution measurements. The single-channel analyzer (SCA) in the NaI channel was biased to select a convenient high-energy portion of the

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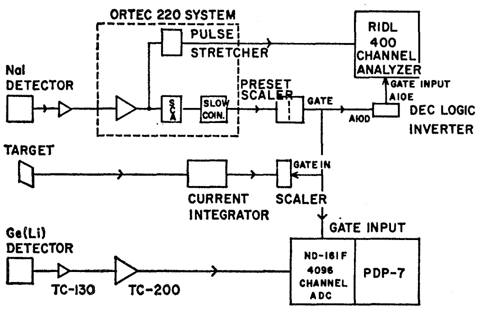

FIG. 3. Block diagram of electronics used in the angular distribution measurements.

resonance spectrum for use in normalizing the spectra measured with the Ge(Li) detector at different angles. The SCA output pulses were counted by a preset scaler which was used to gate the Ge(Li) spectrum in the 4096-channel analyzer, a scaler counting the accumulated target charge, and a 400-channel RIDL analyzer recording the entire NaI monitor spectrum.

Several sources of systematic errors arise in the measurement of angular distributions, the most serious being the possible deviation of the target area from the center of rotation of the detector. Accurate target centering was checked by measuring the isotropic[17] angular distribution of the 6.129-MeV γ ray from the $^{19}F(p, \alpha\gamma)^{16}O$ reaction at the 0.936-MeV resonance. The magnitude of the system asymmetry is indicated in Fig. 4(a). Another systematic error arises from gain shifts in the NaI channel, which alter the normalization of the Ge-detector spectra. In order to separate this effect from that due to the system asymmetry, the Ge-detector spectra were measured in the order 90°, 60°, 30°, 0°, 41°, 75°, and 52°. Figure 4(b) shows the integrated monitor spectrum from the 1.248-MeV data as a function of Ge-detector position, the variation clearly indicating the effect of gain shifts.

The greatest uncertainty in the analysis of the angular distributions arose in the determination of the areas under the peaks at different angles. Uncertainties in the measured areas were taken as the errors in the calculated parameters for the peaks which could be fitted by the computer program, and were estimated from the graphical fits for weak peaks which had to be plotted by hand.

Analysis

When the capture reaction proceeds through the formation of a compound state of sharp spin and parity J^π the angular distribution of the subsequently emitted γ rays can be written in the form of a series of Legendre polynomials:

$$W(\theta)=\sum_{K=0}^{K_{\max}} A_{2K}P_{2K}(\cos\theta)\,. \qquad (4)$$

The Legendre polynomial summation is terminated by the angular-momentum selection rules. Theoretical expressions for the coefficients A_{2K} can be found in the literature.[18–21] In addition to the spins involved in the capture and decay, the coefficients A_{2K} depend on continuous parameters involving the details of formation and decay of the resonant state. In the case of protons incident on ^{31}P with a spin of $\frac{1}{2}$, only one continuous formation parameter is needed. In this case, the formation is characterized by either x, the ratio of reduced matrix elements for higher to lower angular momenta in channel spin 1, or by τ_1, the fraction of formation through channel spin 0. Explicitly, the formation parameter becomes τ_1 for resonances with natural parity $(1^-, 2^+, 3^-, \cdots)$ and x for those with unnatural parity $(1^+, 2^-, 3^+, \cdots)$. The subsequent γ decay of the resonance level is characterized by the multipole mixing ratio δ defined as the ratio of reduced matrix elements for decay through higher and lower multipoles. It is also possible to analyze the angular distributions of the secondary γ rays, corresponding to transitions from states populated only from the resonant level. The analysis and interpretation of the secondary distributions are similar to those of the primary transitions, except that the theoretical expressions are somewhat more complicated in that they also depend on the mixing ratio in the primary transition.[21]

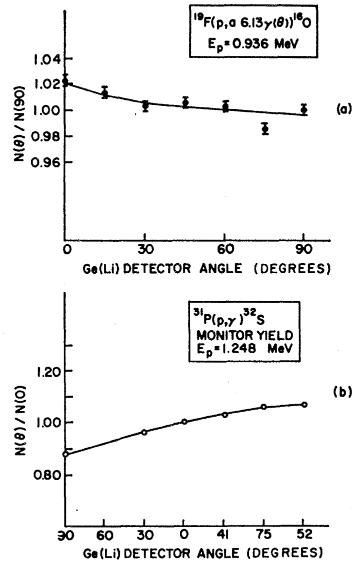

FIG. 4. Correction curves for the angular distribution measurements at 1.248 MeV. (a) Isotropic $^{19}F(p, \alpha\ 6.13\text{-MeV}\ \gamma)^{16}O$ angular distribution showing magnitude of system asymmetry. (b) Monitor yield as a function of detector position, showing effects of gain shifts in the NaI channel.

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TABLE I. Summary of branching ratios in percent of total γ decay from resonant state.

Final state (MeV)	1.248 MeV ($E^*=10.073$)		1.438 MeV ($E^*=10.257$)		1.556 MeV ($E^*=10.371$)		1.583 MeV ($E^*=10.398$)	
	a	b	a	b	a	b	a	b
0	2	2	<0.1	···	<0.5	1	<0.5	···
2.230	29	32	1	<1	11	10	<1	<1
4.280	2	4			40	35	2	3
4.459			9	10			1	8
4.694	2	3			11	5	···	3
5.006	14	12	5	10	2	5	7	10
5.410	4	···			12	···		
5.544	···	3			4	10		
6.226	47	42	···	4	3	4	···	3
6.623			76	60	···	15	85	58
6.664					13	···		
7.11	···	2	···	1	···	8	···	5
7.188					1	···		
7.43			···	4	···	8	···	5
7.485					2	···		
7.952			9	···			4	···
8.30			···	4			···	5

[a] Present work.

[b] Reference 10.

The method employed to determine spins and mixing ratios from the observed angular distributions is based on minimizing the function

$$Q^2 = \sum_{i=1}^{N} [W_{\text{expt}}(\theta_i) - W_{\text{theoret}}(\theta_i)]^2 [\sigma^2(\theta_i)]^{-1}, \qquad (5)$$

where $W_{\text{theoret}}(\theta_i)$ is given by Eq. (4), $\sigma^2(\theta_i)$ is the error in the ith experimental point, and the parameters to be determined, for a given set of spins J, J_f, and f and δ, where f is either x or τ_1. The function $Q^2(f, \delta)$ is evaluated for every set of mixing ratios and examined at each point (f, δ) to determine its absolute minimum. The minimum value of Q^2 is χ^2 distributed with $N-K$ degrees of freedom,[21] where N is the number of data points and K is the number of adjustable parameters. The goodness of fit of the theoretical function to the experimental data can be evaluated by comparing the values of $Q^2(f, \delta)$ with the confidence limits of the appropriate χ^2 distribution. Evaluation of the function Q^2 at each point (f, δ) requires a large number of calculations over a closely spaced mesh, since the mixing ratios are continuously variable functions. A FORTRAN IV program GRAD has been written for the IBM 360/50 computer for this analysis. Input to the program consists of the raw experimental data, the finite solid-angle corrections for the detector, and any number of spin sequences.[22]

IV. RESULTS

γ-Ray Spectra

γ-ray spectra were measured at the 1.248-, 1.438-, 1.556-, and 1.583-MeV resonances. Estimated errors in the γ-ray energies are approximately 1 keV for the peaks <3 MeV and ±3 keV for those of higher energy. Errors in the branching ratios are estimated to be less than 10% for the stronger branches, but may be as high as 50% for the decays involving weakly populated levels. Transitions from the resonance levels are listed in Table I, along with the results of a previous NaI-detector study by Anderson.[10] Definite evidence is found for population of a level at 6.664 MeV, which is quite likely the level observed at 6.671±0.008 MeV in the $^{31}P(He^3, d)^{32}S$ reaction.[11]

Measured branching ratios for the bound states are summarized in Fig. 5. Results for the first six levels are generally in agreement with Anderson's,[10] with the exception of the 5.006-MeV level, where he reports no ground-state transition. Existence of the ground-state branch has also recently been reported elsewhere,[23,24] supporting an odd-parity assignment for this $J=3$ level. Decay of the 5.410-MeV level is found to be in agreement with the results of Poletti and Grace,[12] but with

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TABLE II. Summary of lifetime measurements.

Level (MeV)	Present results $F(\tau)$	Present results τ (psec)	Other τ (psec)
3.775	0.052±0.015	$0.52_{-0.10}^{+0.30}$	1.00[a]
4.280	0.726±0.008	0.029±0.002	0.074[a]
			$0.035_{-0.008}^{+0.003}$ [b]
4.694	0.103±0.009	0.53±0.05	0.49[a]
5.006	0.022±0.010	$1.50_{-0.70}^{+2.50}$	0.75[a]
			$0.66_{-0.13}^{+0.20}$ [b]
5.410	0.415±0.011	0.097±0.005	0.19[a]
5.544	0.640±0.174	0.043±0.035	0.064±0.001[b]
6.226	0.380±0.092	$0.11_{-0.03}^{+0.05}$	0.064±0.007[a]
6.623	0.145±0.025	0.37±0.08	$0.57_{-0.17}^{+0.29}$ [b]
6.664	0.57±0.05	$0.054_{-0.009}^{+0.013}$	
7.952	>0.94	<0.01	

[a] Reference 23.

[b] Reference 24.

an upper limit of 1% placed on the absent ground-state branch. The 5.544-MeV level branching to the ground and first excited state is found to be 22 and 78%, respectively, in disagreement with both Anderson[10] and Poletti and Grace,[12] who report 45 and 55% for these branches. The previously measured branching of the 6.623-MeV level of 90 and 10% for the branches to the 5.006- and 2.230-MeV levels[10] disagrees with the present results of 74 and 2%, plus a 24% branch to the 4.459-MeV level. The 6.664-MeV level, which has not previously been studied, shows approximately equal branching to the 3.775- and 2.230-MeV levels. Decay modes for the 7.188-, 7.485-, and 7.952-MeV levels have not previously been reported; the present results are only tentative, since these states are so weakly populated that precise intensity measurements could not be performed.

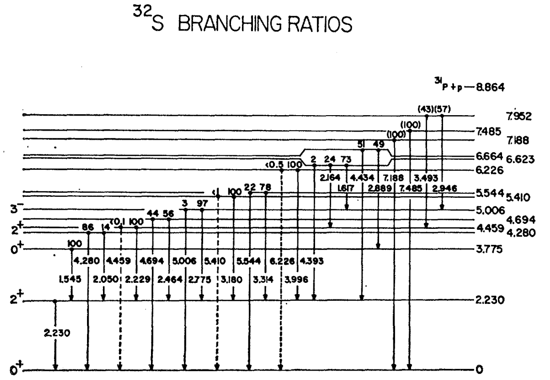

FIG. 5. Summary of the measured branching ratios for the low-lying states in ^{32}S.

Lifetime Measurements

Doppler-shift measurements were made at both the 1.438- and 1.556-MeV resonances. Results for the lifetimes of the 3.775-, 4.280-, 4.694-, 5.006-, 5.410-, 5.544-, 6.226-, 6.623-, and 6.664-MeV levels are summarized in Table II. Lifetimes of the levels at 3.775 MeV, populated entirely from the 6.664-MeV state at the 1.556-MeV resonance, and at 5.006 MeV, fed mostly from the 6.623-MeV level at the same resonance, were determined using Eq. (3). The error for the latter measurement was large due to the fact that only 80% of the feeding of this level comes from the 6.623-MeV state. Presence of the unresolved line depopulating the 4.459-MeV state to the state at 2.230 MeV rendered determination of the lifetime of the latter state impossible. The 7.188- and 7.485-MeV levels are too

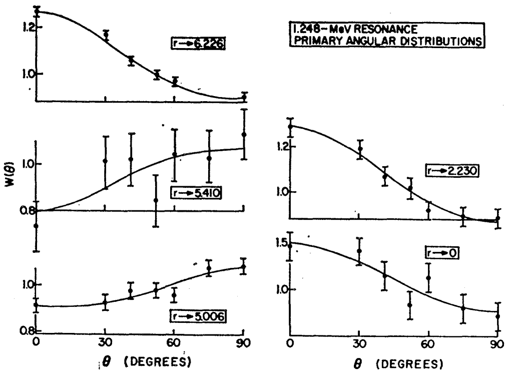

FIG. 6. Primary angular distributions measured at the 1.248-MeV resonance. Solid curves are least-squares fits to Eq. (4).

TABLE III. Angular distributions—least-squares fits.

Resonance (MeV)	Transition	A_2	A_4
1.248	10.073→6.226	0.238±0.015	0.044±0.014
	10.073→5.410	−0.172±0.090	−0.046±0.100
	10.073→5.006	−0.130±0.024	0.042±0.027
	10.073→2.230	0.278±0.032	0.015±0.034
	10.073→0	0.516±0.129	0.004±0.130
	6.226→2.230	0.100±0.004	0.001±0.004
	5.410→2.230	0.066±0.040	0.077±0.041
	5.006→2.230	−0.279±0.027	−0.063±0.030
1.438	10.257→6.623	0.655±0.028	0.028±0.031
	10.257→5.006	−0.305±0.038	−0.223±0.040
	10.257→4.459	0.609±0.055	0.010±0.056
	6.623→5.006	0.491±0.037	0.224±0.041
	6.623→4.459	0.559±0.020	−0.031±0.022

weakly populated at these resonances to permit accurate DSAM measurements; for the same reason only an upper limit of 0.01 psec could be placed on the mean life of the level at 7.952 MeV.

Angular Distributions

The angular distributions measured at the 1.248-MeV resonance are shown in Figs. 6 and 7. The solid curves are least-squares fits to Eq. (4). The coefficients A_2 and A_4 are listed in Table III for both the 1.248- and 1.438-MeV resonance distributions. The allowed range in the Legendre-polynomial expansion of these distributions immediately excludes certain spin combinations. For example, the 10.073-, 6.226-, and 5.410-MeV levels cannot have spin 0. Also, the nonzero A_4 coefficient in the 10.073→6.226 distribution excludes spin 1 for the 1.248-MeV resonance level.

The spins and parities of the ground, 2.230- and 5.006-MeV levels have been established as 0^+, 2^+, and 3^-, respectively.[11] In analyzing the distributions at the 1.248-MeV resonance a resonant level spin and parity of 2^-, taken from proton elastic scattering,[10] was assumed. Figure 8 shows a typical plot of the two-dimensional Q^2 surface of Eq. (5) for one of the primary angular distributions illustrating a characteristic feature of the analysis: Most of the distributions are nearly independent of the formation parameter. Consequently, the actual multipole mixing ratios will not differ significantly from those found at the value of x or τ_1 at which a minimum of Q^2 occurs. Hence the first step in the analysis was to determine the best value x_0 of the formation parameter for the 1.248-MeV resonance. This is illustrated in Fig. 9 for three primary transitions for which the final-state spin was known. The r→0 transition is particularly useful since it depends only on the formation parameter.

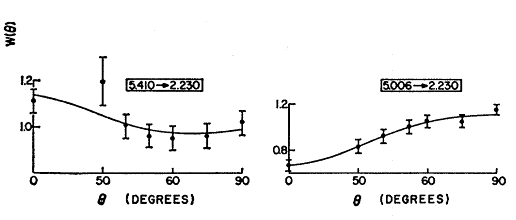

FIG. 7. Secondary angular distributions measured at the 1.248-MeV resonance. Solid curves are least-squares fits to Eq. (4).

Once x_0 was determined the remainder of the distributions were analyzed as a function of δ only. A typical distribution analyzed in this manner is shown in Fig. 10 for the case of the 5.001–2.230-MeV transition. For the most part, the lack of sensitivity of the angular distributions to the choice of spins and mixing ratios permitted only a few possibilities to be ruled out at the 1% confidence level. All the measured distributions have been examined in this manner and the results are summarized in Table IV.

The values and errors of the multipole mixing ratios have been determined by least-squares fitting to a parabola the region around each of the minima in the $Q^2(x_0, \delta)$ curves. The range of allowed mixing ratios was defined by the values of δ for which the 10% confidence limit was exceeded on either side of the point of best fit.

Analysis of the 1.438-MeV resonance was complicated

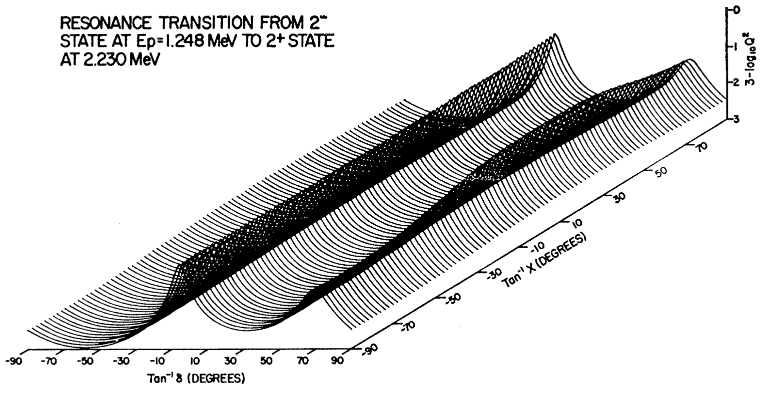

FIG. 8. An isometric plot of Q^2 versus the multipole mixing ratio δ and the orbital mixing ratio x at the $E_p=1.248$-MeV resonance for the resonant to first excited state transition.

by the lack of a significant ground-state transition, so that the formation parameter τ_1 for this 4^- resonance[10] was poorly determined. Because of this, no results will be quoted except that the analysis is consistent with the spin of the interesting 4.459-MeV level being either 3 or 4, and the spin of the state at 6.623 MeV is most consistent with a 3^- assignment.

V. DISCUSSION

The set of spins and mixing ratios determined from the angular distribution measurements can be further truncated by utilizing the transition strengths deduced from the known lifetimes (Table II). For example, if the spin of the 6.226-MeV level is 1 or 3 in the $r\rightarrow 6.226$ transition, the measured mixing ratios listed in Table IV require transition strengths which are substantially greater than the typical strengths found in *s-d* shell nuclei.[25] This possibility is therefore highly unlikely. Furthermore, even with a mixing ratio as small as 0.16, a spin of 4 for this level would require unreasonably high octupole strength. One is then left with the mixing ratio $\delta_1=-0.09\pm0.06$ and a spin and parity of $2^\pm$ for the 6.226-MeV state. The even-parity solution would require an $M2$ strength of 6.5 Wu in the $r\rightarrow 6.226$-MeV transition, which is quite large.[25] If the parity is odd, the

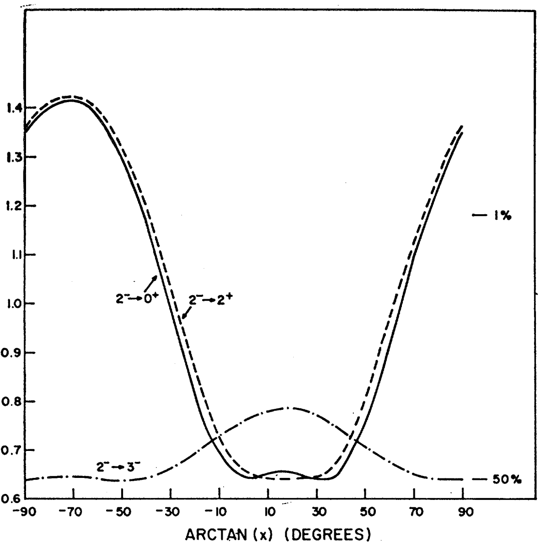

FIG. 9. Plots of $\log_{10}Q^2(x)$ versus $\arctan x$ for the three primary decays from the 1.248-MeV resonance used to determine the best value of the formation parameter x_0. The solid curve is for the ground-state transition. The dashed curve is for the transition to the first excited state (2.230 MeV) with $\delta_1=2.75$ and the dash-dot curve is for the transition to the 5.006-MeV level with $\delta_1=-7.12$. A value of $x_0=0.29$ corresponding to $\arctan x=16°$ was used in the remainder of the angular distribution analysis.

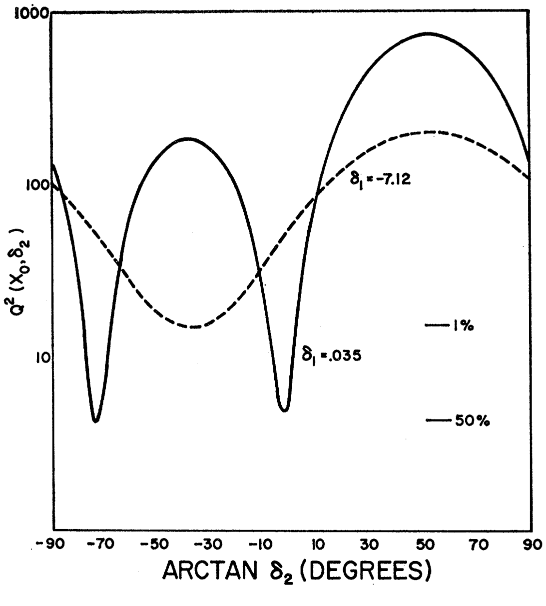

FIG. 10. Plots of $Q^2(x_0, \delta_2)$ versus $\arctan\delta_2$ for the decay of the 5.001-MeV level populated at the 1.248-MeV resonance. The solid curve is for the decay to the 2.230-MeV level with $\delta_1=-7.12$ and the dashed curve is for the same transition with $\delta_1=0.035$.

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[25] C. van der Leun, in *Proceedings of the Symposium on the Structure of Low-Medium Mass Nuclei* (University of Kansas, Lawrence, 1964), p. 109.

TABLE IV. Summary of angular distribution results at 1.248-MeV resonance.

Transition	J_i^π	Intermediate spin (parity)	J_F^π	Primary mixing ratio	Secondary mixing ratio
$r\rightarrow 0$	2^-	...	0^+	...	...
$r\rightarrow 2.230$	2^-	...	2^+	-0.07 ± 0.05	...
				2.7 ± 0.5	
$r\rightarrow 5.006\rightarrow 2.230$	2^-	3^-	2^+	0.03 ± 0.05	-3.5 ± 0.8
$r\rightarrow 5.410\rightarrow 2.230$	2^-	1	2^+	$-3.8_{-0.6}^{+1.2}$	0.14 ± 0.23
					$-7.1_{-\infty}^{+4.4}$
	2^-	1	2^+	0.10 ± 0.01	-0.36 ± 0.28
					-1.4 ± 1.1
	2^-	2	2^+	-14	$-2.1_{-\infty}^{+1.8}$
	2^-	2	2^+	-0.6 ± 0.4	2.5 ± 1.4
					-0.03 ± 0.30
	2^-	3	2^+	-9.5	$4.7_{-2.0}^{+\infty}$
					0.53 ± 0.34
	2^-	3	2^+	0.1 ± 0.1	$+\infty$
					0.25 ± 0.07
	2^-	4	2^+	-4.0	1.3 ± 2.5
	2^-	4	2^+	0.4 ± 0.3	-3.1 ± 2.0
					-0.29 ± 0.20
$r\rightarrow 6.226\rightarrow 2.230$	2^-	1	2^+	-14	-7.1 ± 0.2
					0.14 ± 0.01
	2^-	1	2^+	0.4 ± 0.2	-0.73 ± 0.23
	2^-	2	2^+	-0.07 ± 0.03	-0.14 ± 0.02
	2^-	3	2^+	-1.9 ± 0.8	0.40 ± 0.01
	2^-	4	2^+	-0.16 ± 0.07	-3.08 ± 0.03
	2^-	4	2^+	4.7 ± 0.4	-14.3 ± 3.0
					-0.03 ± 0.02

secondary mixing ratio $\delta_2=3.5\pm 0.2$ requires the large $M2$ strength of 40 Wu for the 6.226→2.230 transition which tends to rule out this solution. We conclude that the most likely spin and parity for the 6.226-MeV state is 2^-, in agreement with Ref. 12, and that the most probable values of the mixing ratios are $\delta_1=-0.09\pm 0.06$ and $\delta_2=-0.14\pm 0.02$. Similar arguments for the other transitions have assisted in obtaining the information on spin assignments and mixing ratios which is summarized in Table V. Good agreement with previous results is found except for the mixing ratios of the 6.226→2.230 and 5.006→2.230 transitions.

Bar-Touv and Goswami[3] have recently proposed a simple collective model explanation of the lower levels of ^{32}S. This model is termed "inverted nuclear coexistence," since it is the inverse of the model suggested[26] to explain the occurrence of a low-lying 0^+ state in ^{16}O. According to this model, the 0^+ ground state and 0^+ 3.775-MeV state are the Hartree-Fock solutions corresponding to deformed and spherical equilibrium shapes, respectively. Using the calculated moment of inertia[3] and the experimental energies of the first excited 2^+ state and of the excited 0^+ state, the admix-

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[26] P. Goldhammer and F. Prosser, Phys. Rev. **163**, 950 (1967); J. Lowe, A. R. Poletti, and D. H. Wilkinson, *ibid.* **148**, 1045 (1966).

TABLE V. Summary of spin assignments and mixing ratios.

Level (MeV)	Spin Present	Spin Other	Transition	Mixing ratio Present	Mixing ratio Other
6.623	$3^{(-)}$	(3^-)[a]	10.073→2.230	0.06 ± 0.02	0.05 ± 0.02[a]
6.226	$2^{(-)}$	$2^{(+)}$[a]	10.073→5.006	0.02 ± 0.03	0.05 ± 0.07[a]
		2^-[b]	10.073→5.410($1^\pm$)	0.10 ± 0.01	
5.410	1, 3	3[b]	($3^\pm$)	0.1 ± 0.1	
4.459	3, 4	2, 3, 4[b]	10.073→6.226	-0.07 ± 0.03	-0.04 ± 0.04[a]
		4[c]	6.226→2.230	-0.14 ± 0.02	0.75 ± 0.15[a]
					-1.9 ± 0.1
					-0.1 ± 0.1[b]
			5.410($1^\pm$)→2.230	-0.36 ± 0.28	
				-1.9 ± 1.1	
			5.410($3^\pm$)→2.230	∞	12_{-5}^{+36}[b]
				-0.25 ± 0.07	
			5.006→2.230	-3.5 ± 0.8	-0.09 ± 0.06[a]

[a] Reference 10.
[b] Reference 12.
[c] Reference 27.

ture of the two primitive 0^+ states in the observed 0^+ levels can be calculated. The ratio R of the reduced transition probabilities between the 2^+ state and the two 0^+ states,

$$R=\frac{B(E2;\, 2_{\mathrm{def}}{}^+\rightarrow 0_{\mathrm{sph}}{}^+)}{B(E2;\, 2_{\mathrm{def}}{}^+\rightarrow 0_{\mathrm{def}}{}^+)}\,,$$

can be calculated directly and is $R_{\mathrm{theoret}}=0.51$. This ratio is also obtainable from the measured lifetime of the 3.775- and 2.230-MeV states. The measured lifetime of the 2.230-MeV state is 0.33 psec,[17] while our measurement for the lifetime of the 3.775-MeV state (Table II) is $0.52_{-0.10}^{+0.30}$ psec. These values lead to an experimental value for this ratio of $R_{\mathrm{expt}}=0.79_{-0.30}^{+0.19}$. With the moment of inertia used in this calculation, the resulting energy of the primitive deformed 0^+ state, the 4^+ member of the ground-state rotational band is predicted to be at 4.50 MeV. The best candidate for this 4^+ state is the level at 4.459 MeV. Earlier work had limited the spin of the level to 2, 3, or 4. The present angular distributions are consistent with either 3 or 4. Recent studies from elastic deuteron scattering[27] and inelastic proton scattering[28] strongly favor a spin and parity of 4^+.

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[27] M. Mermaz, C. Whitten, and D. A. Bromley, Bull. Am. Phys. Soc. **13**, 675 (1968).

[28] G. M. Crawley and G. T. Garvey, Phys. Rev. **160**, 981 (1967).