Level Order in ^{22}Ne, ^{24}Mg, and ^{26}Mg: Inference on the Effective Interaction

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It has been found that the experimental order of the low levels in ^{22}Ne, ^{24}Mg, and ^{26}Mg can be reproduced by a residual interaction which has the "correct" amount of Majorana exchange. The dependence of the level order on the range appears to be different for the three nuclei. A change of range is more effective on the spectra for a Gaussian than for a Yukawa interaction.

IN the first half of the *sd* shell the only even-even nuclei which are expected to exhibit two or more low SU_3 bands (i.e., two or more bands which belong to the lowest SU_3 representation) are ^{22}Ne, ^{22}Mg, ^{24}Mg, and ^{26}Mg.[1] The latest experimental results[2–4] on the low spectra of ^{22}Ne, ^{24}Mg, and ^{26}Mg are shown in Fig. 1, columns b, d, and f (^{22}Mg has not been included owing to the scantiness of experimental data[3]).

From inspection of Fig. 1 one sees that ^{22}Ne and ^{24}Mg exhibit what we have called[5] "anomalous level ordering," while ^{26}Mg does not; namely, the sequence of the two lowest excited states is 2^+, 4^+, in ^{22}Ne and ^{24}Mg and 2^+, 2^+ in ^{26}Mg.

In this work we seek a residual interaction which can reproduce these features.

We have shown[6] that in ^{22}Ne both the increase of the amount M of Majorana exchange and the increase of the ratio b/a (between the harmonic oscillator length parameter and the range of the residual interaction) produce a shift up of the $K=2$ band relative to the $K=0$ band. As a consequence, by carefully choosing the parameters of the residual interaction, we were able to reproduce the experimental order of the ^{22}Ne levels.

We now have extended the scope of our investigation to ^{24}Mg and ^{26}Mg, with the further aim of finding out whether the dependence of the level order on b/a and on M which we found in ^{22}Ne is peculiar to this nucleus or represents a general trend.

The lowest SU_3 representations occurring in ^{22}Ne, ^{24}Mg, and ^{26}Mg are $(\lambda\mu)=(82)$, $(\lambda\mu)=(84)$, and $(\lambda\mu)=(10, 2)$, respectively.[1] Therefore in the lowest SU_3 representation two bands ($K=0, 2$) are included for ^{22}Ne and ^{26}Mg and three ($K=0$, 2, 4) for ^{24}Mg. We have computed the energy levels belonging to these bands with the following effective interactions:

$$V_1(r)=V_{01}(b/a)[\exp(-r/a)/(r/a)](W+MP^x), \quad (1)$$

$$V_2(r)=V_{02}(b/a)[\exp(-r^2/a^2)](W+MP^x), \quad (2)$$

where $W+M=1$, b/a varies from 0.6 to 1.7, and M varies from 0.5 to 1.0. Since we had expected[1,5–7] that the spacing between levels would be too small for every reasonable value of V_{01} and V_{02}, and anyhow we were principally interested in the order of levels (and not in their spacing), V_{01} and V_{02} were not considered relevant parameters at this stage of the calculation.

Our results show that the dependence of the relative order of the second 2^+ and the first 4^+ level on the exchange factor M is analogous in all three nuclei for both effective interactions. In all six cases, for nearly every b/a value in the range considered, if M is sufficiently large, the "inversion" occurs, i.e., the first 4^+ level appears below the second 2^+ level. The dependence on M of the excitation energies of the three lowest excited states is qualitatively described for all of the three nuclei and for both interactions by Fig. 2 of our previous work on ^{22}Ne.[6] On the other hand, the dependence of the level order on b/a is different for the three nuclei: the amount of Majorana exchange M_{inv} necessary to produce the inversion grows with b/a in ^{24}Mg and in ^{26}Mg, while in ^{22}Ne the opposite feature appears. This feature, too, is qualitatively present for both the Yukawa and Gaussian radial interactions, but M_{inv} varies more slowly with b/a in the Yukawa case than in the Gaussian one. This is easily explained if one remembers that the Hamiltonian can be written as[6]

$$H=\sum_{\rho=1}^{10} M_\rho H_\rho,$$

where only the M_ρ depend on the shape of the two-body interaction, and only M_8, M_9, and M_{10} are affected by a change of M; therefore, the ratio

$$R_M=(\sum_{\rho=8}^{10} M_\rho)/(\sum_{i=1}^{7} M_i)$$

(where the M_ρ are computed for a Wigner force) is a measure of the effectiveness of a change of M. Now, when b/a ranges from 0.6 to 1.7, R_M varies from 0.50 to 0.15 for a Gaussian interaction, and only from 0.40 to 0.23 for a Yukawa interaction. In other words, in the language recently introduced by Vincent[8] the relative amount of the basic interactions $(00)_3$, $(22)_4$, and $(22)_5$ is a more rapidly varying function of b/a for a Gaussian interaction than for a Yukawa one. [$(00)_3$,

[1] J. P. Elliott and M. Harvey, Proc. Roy. Soc. (London) **A272**, 557 (1963).
[2] S. Buhl, D. Pelte, and B. Povh, Nucl. Phys. **A91**, 319 (1967).
[3] P. M. Endt and C. Van Der Leun, Nucl. Phys. **A105**, 1 (1967).
[4] P. Spilling, H. Gruppelaar, and A. M. F. Opden Kamp, Nucl. Phys. **A102**, 209 (1967).
[5] C. Abulaffio, Phys. Letters **11**, 156 (1964).
[6] C. Abulaffio, Phys. Rev. **161**, 925 (1967).
[7] M. K. Banerjee, C. A. Levinson, and S. Meshkov, Phys. Rev. **130**, 1064 (1963).
[8] C. M. Vincent, Nucl. Phys. **A106**, 35 (1968).

TABLE I. Amount of Majorana exchange sufficient to create the first 4^+ level lower than the second 2^+ level, tabulated versus b/a.

b/a	^{22}Ne Yukawa	^{22}Ne Gaussian	^{24}Mg Yukawa	^{24}Mg Gaussian	^{26}Mg Yukawa	^{26}Mg Gaussian
0.6	0.85	...	0.90	0.60	0.95	0.75
0.7	0.85	1.00	0.90	0.65	0.95	0.75
0.8	0.85	0.95	0.90	0.70	0.95	0.80
0.9	0.80	0.90	0.90	0.75	0.95	0.80
1.0	0.80	0.85	0.90	0.80	0.95	0.85
1.1	0.80	0.75	0.95	0.80	0.95	0.85
1.2	0.75	0.70	0.95	0.85	1.00	0.90
1.3	0.75	0.70	0.95	0.90	1.00	0.90
1.4	0.75	0.65	0.95	0.95	1.00	0.95
1.5	0.70	0.60	1.00	0.95	1.00	1.00
1.6	0.70	0.55	1.00	1.00	1.00	1.00
1.7	0.70	0.50	1.00	...	...	...

$(22)_4$ and $(22)_5$ are the only basic interactions that have nonzero matrix elements in an orbitally antisymmetric two-particle state.]

All the above-mentioned features are summarized in Table I, where M_{inv} is given as function of b/a for the three nuclei, and for both a Yukawa and a Gaussian residual interaction. From Table I one can immediately pick up the parameters of those residual interactions which satisfy our requirements.

The following are the allowed parameters for (a) a Yukawa interaction, and (b) a Gaussian interaction:

(a) $M=0.90$ $b/a=0.6, 0.7, 0.8, 0.9, 1.0,$
$M=0.95$ $b/a=1.2, 1.3, 1.4,$

(b) $M=0.80$ $b/a=1.1,$
$M=0.85$ $b/a=1.2,$
$M=0.95$ $b/a=1.5.$

In Fig. 1 we report the spectra calculated from ^{22}Ne, ^{24}Mg, and ^{26}Mg with that interaction which meets our requirements and has the lowest M. The spectra for the other eleven allowed cases are very similar. We choose $V_{02}=68$ MeV; this is the strength required to reproduce the excitation energy of the first 2^+ ^{20}Ne level with $b/a=1.1$. It is apparent from Fig. 1 that for ^{24}Mg the order of the five lowest-excited levels is correctly reproduced, while for ^{22}Ne and ^{24}Mg the order is correct only for the three lowest and the two lowest-excited states, respectively. The experimental ^{26}Mg 0^+ level at 3.58 MeV obviously does not belong to the lowest SU_3 representation. The corresponding level in ^{24}Mg is much higher (6.44 MeV); this fact is a further indication that the SU_3 basis is better for ^{24}Mg than for ^{26}Mg.

Let us now check our assumptions. To confine the basis to the lowest SU_3 representation is too restrictive an assumption, as has been pointed out repeatedly.[1,6,7,9]

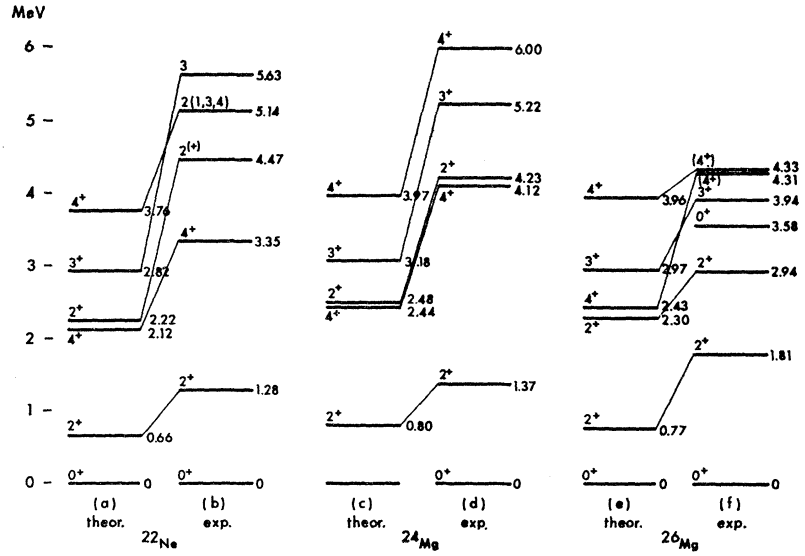

FIG. 1. The two lowest bands in ^{22}Ne, ^{24}Mg, and ^{26}Mg. (a), (c), and (e)—Gaussian interaction with $b/a=1.1$, $M=0.8$, and $V_{02}=68$ MeV, respectively.

[9] J. C. Parikh and K. H. Bhatt, Nucl. Phys. **A103**, 496 (1967).

Our underlying assumption has been that configuration mixing would improve the spacing, but not change the order of levels. This has now been conclusively proved by Wathne and Engeland in the ^{24}Mg case.[10] They found that by gradually expanding the basis, the same order (the wrong one) consistently appears in the ^{24}Mg low levels, while the spacing is almost doubled. (The calculation was made with a Gaussian Rosenfeld potential.) As for the effective interaction, the main unorthodox feature in our assumption is the dropping of both the spin and the isotopic spin exchange. This assumption is however not so unrealistic as it may seem, since it is based on recent theoretical results of Parikh and Bhatt.[9] Their analysis of the effective residual interaction of Kuo and Brown[11] showed that the latter's matrix elements have in fact a Majorana exchange character. Finally the strength of the effective interaction has been derived from the excitation energy of the first excited ^{20}Ne state because detailed calculations show[12,13] that this value is hardly affected by configuration mixing.

[10] K. Wathne and T. Engeland, Nucl. Phys. **A94**, 129 (1967).

[11] T. T. S. Kuo and G. E. Brown, Nucl. Phys. **85**, 40 (1966).
[12] C. Abulaffio, Nucl. Phys. **81**, 71 (1966).
[13] J. Flores and R. Perez, Phys. Letters **26B**, 55 (1967).

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Statistical Theory of Nuclei. II. Medium and Heavy Nuclei*

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The total energies of finite nuclei are expressed as a functional $E[\rho_p, \rho_n]$ of the local proton and neutron densities ρ_p and ρ_n. The binding energies and densities of any nucleus are found by minimizing $E[\rho_p, \rho_n]$ with respect to ρ_p and ρ_n separately. The potential-energy functional is initially derived from a nuclear matter calculation but later adjusted to reproduce experimental binding energies. Results for light nuclei have already been reported, and here the calculation is extended to medium and heavy nuclei. Binding energies and mean radii are well reproduced but surface thicknesses are too large and rms radii too small.

I. INTRODUCTION

THE idea of deriving bulk properties of nuclei by means of statistical methods goes back to the earlier days of nuclear physics. The main task consists of reproducing the binding energies, the sizes, and the shapes of nuclei without appealing to a microscopic description. Remarkable results have already been recorded within the framework of the Thomas-Fermi approximation using simple nuclear forces. Improvements in our understanding of the nucleon-nucleon interactions, as well as in the description of many-fermion systems, allow us to accomplish the program with greater accuracy and more confidence in the theoretical background. The Thomas-Fermi theory, for instance, has been recently reviewed in detail by Bethe,[1] assuming realistic nuclear forces with a repulsive core, which ensured its validity in the nuclear case.

The presence of a shell structure may well require the use of Hartree-Fock-like theories rather than statistical approaches. This is obviously the case for observables strongly depending on single-particle states. As an example, the shell-model potential experienced by a particle inside the nucleus is hardly given by a statistical expression. Nevertheless, solving Hartree-Fock-type equations results in a tedious computation, especially if use is made of a realistic two-body force, for which the Brueckner-Goldstone formalism has to be introduced. Thus, it remains worthwhile to derive general properties by means of more practical methods.

Another reason for dealing with statistical theories has been pointed out by Myers and Swiatecki.[2] It is essential to separate liquid-drop and shell effects in the semiempirical mass formula, in order to extrapolate the mass formula and to investigate heavy and superheavy nuclei. Higher-order corrections to the usual Bethe-Weizsäcker expression, arising from the surface symmetry energy, curvature effects, or exchange Coulomb energy, are difficult to extract from the actual

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[1] H. A. Bethe, Phys. Rev. **167**, 879 (1968), in which references to earlier works can be found as well as in Ref. 5; see also *Proceedings of the International Conference on Nuclear Physics, Gatlingsburg, Tenn., 1966* (Academic Press Inc., New York, 1967).

[2] W. D. Myers and W. J. Swiatecki, Nucl. Phys. **81**, 1 (1966); see also W. D. Myers and W. J. Swiatecki, Arkiv Fysik **36**, 343 (1967); W. D. Myers, University of California Radiation Laboratory Report No. UCRL-17725, 1967 (unpublished).