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Proton Polarization near the J-Dependent Cross-Section Minimum in the ^{40}Ca(d, p)^{41}Ca*(3.95 MeV) Reaction*

C. C. Foster,† W. E. Maddox,‡ and D. W. Miller

Department of Physics, Indiana University, Bloomington, Indiana 47401

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Using incident deuterons with energy E_d of 11.0 MeV, the polarization of protons from the ^{40}Ca(d, p) reaction leading to the 3.95-MeV state of ^{41}Ca has been measured at three angles across the J-dependent minimum in the cross-section angular distribution near $\theta_{c.m.}=100°$. A 180° double-focusing magnetic spectrometer focused the proton group under investigation into a carbon polarimeter. The use of three dE/dx and two E solid-state detectors connected in two parallel triple-coincidence configurations kept the background below 4%, even though the cross section was very low (240 μb/sr). The measured polarization passes smoothly through zero near the cross-section minimum as follows: -0.128 ± 0.039, $+0.061\pm0.046$, and $+0.151\pm0.041$ at laboratory angles of 84.2°, 97.1°, and 110.0°, respectively. Cross-section angular distributions for ^{40}Ca(d, p) reactions to the ground, 1.95-, 2.47-, and 3.95-MeV states in ^{41}Ca and for elastic deuteron scattering with $E_d=10.96\pm0.03$ MeV were also measured. Discrepancies of about 25% exist between these distributions and those of Lee *et al.*

I. INTRODUCTION

Dependence of proton differential cross-section angular distributions for (d, p) reactions upon the total neutron angular momentum transfer in some medium-weight target nuclei was reported in 1964 by Lee and Schiffer.[1] Initially, they observed a characteristic difference between the distributions for protons leaving a nucleus in $J=\frac{1}{2}^-$ and $J=\frac{3}{2}^-$ states after (d, p) reactions on even-even target nuclei in which one unit of orbital angular momentum l_n was transferred. Soon after J dependence was seen in these specific reactions, it was also observed in $l_n=2$ and $l_n=3$ transfer reactions.

Lee and Schiffer have summarized the observed J-dependent effects in (d, p) reactions in three rules.[2] The first of these rules, which is the only one of interest in this work, is as follows: "For $l_n=1$ transitions, deuteron energies between 7 and 12 MeV, $40<A<65$, and spectroscopic factors greater than about 0.2, the $J=\frac{1}{2}$ states exhibit a sharp minimum somewhere in the range $90°<\theta<145°$, but $J=\frac{3}{2}$ states do not." No such marked J dependence of differential cross-section angular distributions had been anticipated by direct-reaction theories, and the effect is as yet not fully explained.[3–5]

Proton polarization angular distributions in (d, p) stripping reactions are well known to be J-dependent. In fact, one of the reasons often given for measuring these distributions has been the determination of the total angular momentum of the state in which the residual nucleus is left in the reaction. Unfortunately, earlier rules about the J dependence of proton polarization distributions have been restricted to the angular region near the main stripping peak, and no measurements have previously been made in the region of the J-dependent minimum.

It is also well known that theoretical polarization angular distributions are quite sensitive to the mechanism used to describe the reaction. Because of the lack of understanding of J dependence in (d, p) reactions, and the expected sensitivity of the proton polarization distribution to both the reaction mechanism and to J dependence, it was decided to measure the polarization at several angles across the J-dependent minimum in the cross section for the ^{40}Ca(d, p)^{41}Ca*-(3.95 MeV) reaction ($l_n=1$, $J_n=\frac{1}{2}$) at a bombarding energy of 11.0 MeV. This is very difficult because of the low cross section, and only three angles were attempted. It was also initially intended to make a measurement of the polarization distribution for the ^{40}Ca(d, p)^{41}Ca*(1.95 MeV) reaction ($l_n=1$, $J_n=\frac{3}{2}$) over the same angular region with the same equipment, but time did not permit. The latter measurement has since been carried out by Kelley *et al.*[6] at about the same energy (10.8 MeV), using a scattering chamber and polarimeter assembly designed by Foster and Maddox.[7] It is hoped that these data will eventually be useful in establishing the reaction mechanism responsible for the J dependence of (d, p) cross sections.

At the time that the measurement of the proton polarization angular distribution at back angles for the

* Work supported by the National Science Foundation.

† Present address: Palmer Physical Laboratory, Princeton University, Princeton, N.J.

‡ Present address: Murray State University, Murray, Ky.

[1] L. L. Lee, Jr., and J. P. Schiffer, Phys. Rev. Letters **12**, 108 (1964).

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[4] K. R. Greider, Phys. Rev. **136**, B420 (1964).

[5] R. C. Johnson and F. D. Santos, Phys. Rev. Letters **19**, 364 (1967).

[6] C. T. Kelley, Jr., W. E. Maddox, and D. W. Miller, Bull. Am. Phys. Soc. **13**, 116 (1968).

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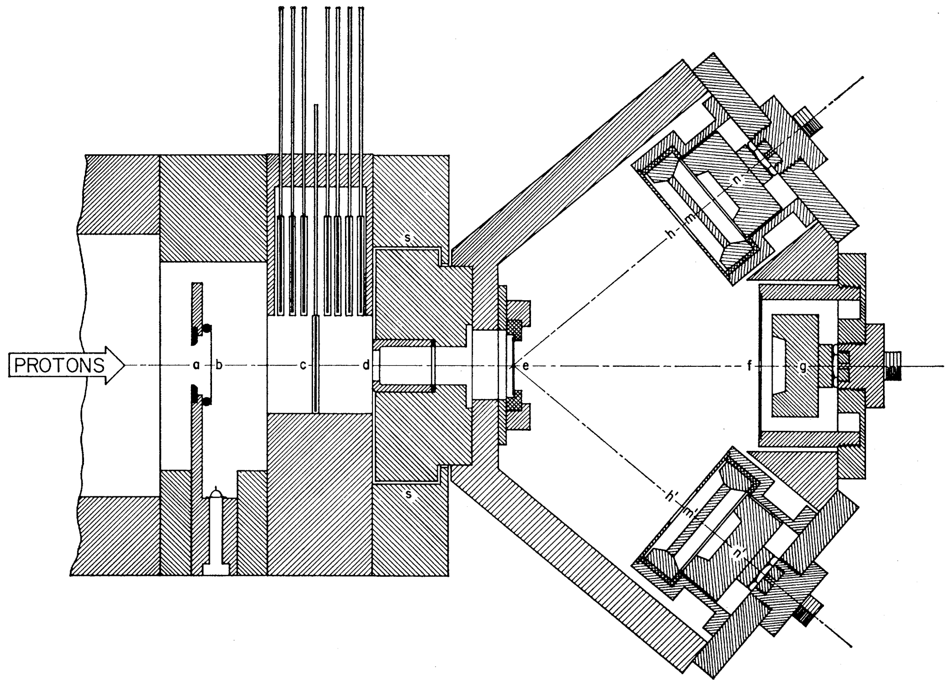

FIG. 1. Carbon polarimeter using solid-state particle detectors. In order to reduce background counts, a triple coincidence was required between counters at b, m, and n and separately between counters at b, m', and n'. To maintain good energy resolution, pulses from the counters at m and n were electronically summed, as were those from the counters at m' and n'.

$^{40}Ca(d, p)^{41}Ca^*$(3.95 MeV) reaction was carried out at 11.0-MeV deuteron energy, measurements of the differential cross-section angular distributions for the same reaction and for elastic deuteron scattering were also performed. The experimental arrangement allowed angular distribution data for reactions leading to the ground, 1.95-, and 2.47-MeV states in $^{41}Ca^*$ to be obtained simultaneously. Soon after this measurement was made, the Argonne group published angular distributions for the same reactions[8] and for elastic deuteron scattering[9] with incident deuteron energies of 7–12 MeV. Certain discrepancies between the present results and the Argonne 11-MeV data will be discussed below.

II. EXPERIMENTAL TECHNIQUES

A. Experimental Apparatus and Procedures

The polarization P_1 of protons produced in the $^{40}Ca(d, p)^{41}Ca^*$(3.95 MeV) reaction was determined by the measurement of the left-right asymmetry P in the elastic scattering of the protons from a carbon target of measured analyzing power P_2, using the standard relation[10]

$$P = P_1 P_2 = (R - L)/(R + L), \qquad (1)$$

where R and L are the number of counts in the right and left counters, respectively. P_2 was determined by measuring the known[11] polarization of recoil protons from the elastic $^1H(\alpha, \alpha)^1H$ process in a separate calibration experiment.

All features of the experimental arrangement were the same as those in a previously published experiment on ^{88}Sr by Ludwig and Miller,[12] except that (a) the incident deuteron energy was 11.0 MeV, (b) the target was a rolled natural-calcium target with a thickness of 12 mg/cm^2, (c) the polarimeter used semiconductor dE/dx-E counter telescopes as side counters, and (d) the electronics were set up to process the pulses from the polarimeter side counters in a sum-coincidence manner to reduce background counts while maintaining energy resolution. All data-accumulation procedures discussed by Ludwig and Miller[12] were also employed in the present measurements, except that (a) data storage was facilitated by the use of a 1024-channel pulse-height analyzer and (b) data analysis was simplified by the clear separation in the side-counter spectra of the elastic proton counts from inelastic and noise counts, and by the low background (less than 4%) which was achieved. Only those aspects of the experiment which were changed significantly from Ref. 12 or are of particular interest will be discussed below.

1. *Polarimeter*

Protons produced at the desired laboratory angle from the $^{40}Ca(d, p)$ reaction leaving ^{41}Ca in its 3.95-MeV

[8] L. L. Lee, Jr., J. P. Schiffer, B. Zeidman, G. R. Satchler, R. M. Drisko, and R. H. Bassel, Phys. Rev. **136**, B971 (1964); **138**, AB6(E) (1965).
[9] R. H. Bassel, R. M. Drisko, G. R. Satchler, L. L. Lee, Jr., J. P. Schiffer, and B. Zeidman, Phys. Rev. **136**, B960 (1964).
[10] L. Wolfenstein, Ann. Rev. Nucl. Sci. **6**, 43 (1956).
[11] A. C. L. Barnard, C. M. Jones, and J. L. Weil, Bull. Am. Phys. Soc. **8**, 124 (1963).
[12] E. J. Ludwig and D. W. Miller, Phys. Rev. **138**, B365 (1964).

excited state were momentum-selected by the 180° double-focusing magnetic spectrometer and directed into the carbon polarimeter, as indicated by the arrow in Fig. 1. A typical momentum spectrum of these protons is shown in Fig. 2.

After collimation at a in Fig. 1, the protons passed through a 0.005-in.-thick diffused junction dE/dx semiconductor detector[13] at b, one or more aluminum degrading foils at c, and an additional collimator at d. By the use of these foils the energy of the protons incident on the 44-mg/cm^2 amorphous carbon second-scatterer at e was adjusted to be 7.57±0.05 MeV for each laboratory angle. This uncertainty in the incident proton energy introduces an uncertainty of less than 3% in the analyzing power P_2 of the polarimeter.

Protons scattered right and left from the carbon second scatterer at a mean laboratory angle of 40° were detected by semiconductor dE/dx-E counter pairs at m, n and m', n'. Collimators at h and h' restricted the scattering angles accepted by the counters to ±5°, assuming a point second scatterer located on the polarimeter axis at e. A $\frac{5}{16}$-in.-diam collimator at d defined the finite size of the proton flux incident on the second scatterer such that the actual maximum angular acceptance of the side counters was ±9°. Protons passing directly through the carbon target were detected in the center counter at g after collimation by a $\frac{1}{8}$-in.-diam hole at f.

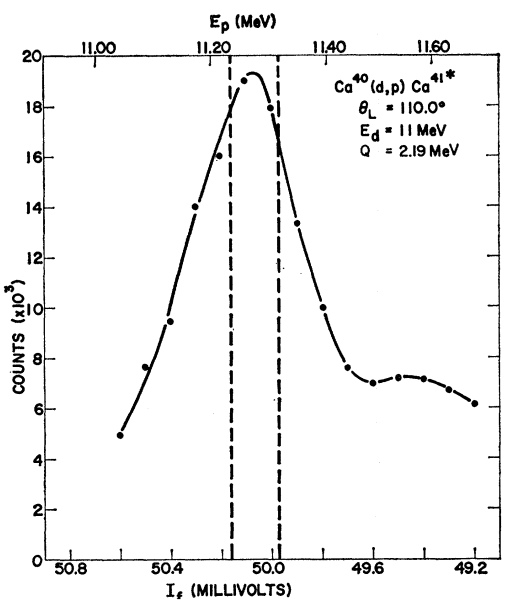

FIG. 2. Typical momentum spectrum of the proton group corresponding to the 3.95-MeV excited state of ^{41}Ca. Such spectra are plots of the polarimeter center counter yield versus magnetic field as determined by a fluxmeter current I_f. The dashed vertical lines represent the maximum momentum interval accepted by the polarimeter apertures during the measurements.

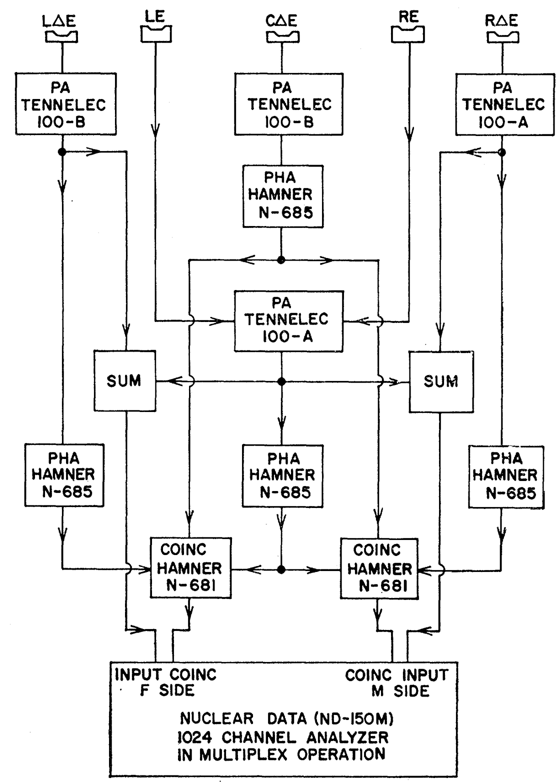

FIG. 3. Block diagram of the electronics used for the dual-channel–triple-coincidence–double-sum technique described in the text.

Ortec[14] surface-barrier detectors were used for the side counters and the center counter. The dE/dx counters were about 70 μ thick (corresponding to an energy loss of about 1.0 MeV for the 4.9-MeV protons incident upon them), and had a surface area of 150 mm.^{2} Up to 6.2-MeV protons could be stopped in the side E counter, which had the same area.

2. *Electronics*

Analysis and collimation of the deuteron beam from the cyclotron resulted in a $\frac{3}{16}\times\frac{3}{16}$-in. rectangular spot centered on the calcium target. The beam current was maintained between 4 and 5 μA on the Faraday cup. Due to this high beam intensity and the proximity of the beam stop and slits, the side counters were subjected to high fluxes of fast neutrons and γ rays, which initially caused a background that tended to obscure the proton peaks. This background was greatly reduced by requiring a triple coincidence (resolving time of 150 nsec) between the dE/dx counters at b and the side-counter pair at m and n for one side, or the counter pair at m' and n' on the other side. An energy loss of 3.1 MeV occurred in the carbon second scatterer. The additional energy loss in the dE/dx counters on each

[13] C. N. Inskeep, W. W. Eidson, and R. A. Lasalle, IRE Trans. Nucl. Sci. 9, 167 (1962). Counter constructed locally by Dr. Inskeep.

[14] Oak Ridge Technical Enterprises Corporation, P.O. Box C, Oak Ridge, Tenn. 37831.

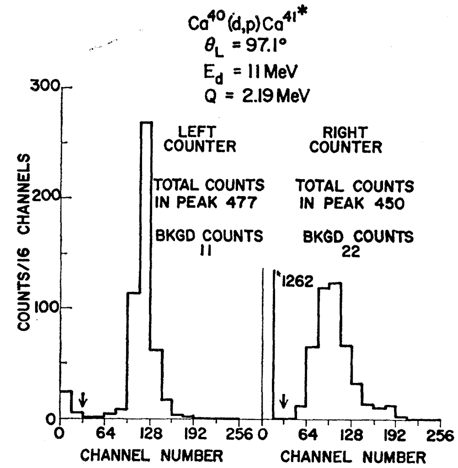

FIG. 4. Spectra of proton pulses from the left and right counters at the J-dependent minimum in the ^{40}Ca$(d,p)^{41}$Ca*(3.95 MeV) reaction, prior to background subtraction. The arrows indicate the cutoff in channel 32. The background is less than 4% of the signal, and the peaks are cleanly separated from noise and inelastic peaks.

side resulted in poorly resolved energy spectra in the E counters alone. In order to regain adequate resolution, the E and dE/dx counter pulses were electronically summed on each side.

A block diagram of the electronics employed is shown in Fig. 3. Standard circuitry was used throughout. It was possible to use a common preamplifier and pulse-height analyzer for the left and right E counters because counting rates were low (about 150 counts/h).

B. Data Storage

In order to obtain data for corrections for background and inherent asymmetry (due to possible imperfections in the electronics or mechanical systems), data were taken in the following sequence: signal run and background run (polarimeter normally oriented); signal run and background run (polarimeter rotated about the axis e-f by 180°). Each signal or background run was stored in a 512-channel memory block of the 1024-channel pulse-height analyzer. Summed pulses from one side-counter telescope which satisfied the coincidence requirements were routed into the first 256 channels of this block, while those from the other side were simultaneously routed into the second 256 channels.

A background run was taken by placing a duplicate 44-mg/cm^2 carbon foil into the proton beam on one of the foil holders at c, removing the carbon target from the beam, and recording counts to the left and right as for a signal run. The background foil could not be "seen" directly by the side counters, but simulated the energy conditions of a signal run. The number of particles detected in the center counter was recorded for each run and used to determine background normalization factors.

The ratio of background to signal counting times was chosen to yield the minimum statistical error in the true product polarization P. As a result of the low observed background (less than 4%), only 10% of the total run time was spent taking background data. Signal runs were about 1h in duration, and the data-taking sequence was repeated at each laboratory angle until the sum of the counts on the left and right sides was approximately 1000 counts for each polarimeter orientation.

C. Data Analysis and Corrections

At each laboratory angle in the calibration and reaction measurements, the data consisted of several signal and background spectra from the right and left counter assemblies for each of the two (0° and 180°) polarimeter orientations. Figure 4 shows typical spectra in which the raw signal counts in each channel have been summed for all runs with the same polarimeter orientation. For each of the two polarimeter orientations, the data were combined by summing the counts for all signal runs, and separately the counts for all background runs, before making the appropriate normalized subtraction. The normalization factors were determined experimentally in order to account for effects of target-thickness variation with exposure to the beam and differences in counting rates in the center counter due to multiple scattering in the carbon target and background foils.

The total left and right counts obtained for each polarimeter orientation were then combined by the use of the following equations to get the inherent polarimeter asymmetry K and the product polarization P (corrected for K):

$$K=(R_0/L_0)^{1/2}/(L_{180°}/R_{180°})^{1/2}, \quad (2)$$

$$\epsilon=(1-K)/(1+K), \quad (3)$$

$$A=(R_0-L_0)/(R_0+L_0), \quad (4)$$

$$P=P_1P_2=(\epsilon+A)/(1+\epsilon A), \quad (5)$$

where R_0, L_0 and $R_{180°}$, $L_{180°}$ are the counts in the right and the left counters in the 0° and 180° polarimeter orientations, after background subtractions.

Corrections were made for false asymmetries due to the shift of the "mean proton" from the polarimeter axis as a result of the asymmetrical shape of the spectrometer aperture and the variation of the differential cross section with angle. Since the spectrometer acceptance aperture was 12.2° below the beam plane, the spin direction of the incident protons was not, in general, parallel to the magnetic field, and subsequent precession about the magnetic field produced a longitudinal (and unmeasurable) component of the polarization at the polarimeter. Corrections for this effect[12] are included in the quoted results.

D. (d, p) and Elastic Angular Distribution Measurements

An evaporated natural-calcium target of thickness 0.54 ± 0.03 mg/cm^2 and a doubly collimated Ortec[14] surface-barrier detector capable of stopping up to 18-MeV protons were used in the measurement of the (d, p) and elastic cross-section angular distributions at an incident deuteron energy of 10.96 ± 0.03 MeV. Counting rates and discriminator levels were adjusted so that the elastic deuteron group and the proton groups associated with the ^{40}Ca(d, p) reactions leading to the ground, 1.95-, 2.47-, and 3.95-MeV states in ^{41}Ca were clearly seen in each pulse-height spectrum, with adequate statistics for all peaks at each angle. A typical pulse-height spectrum is shown in Fig. 5.

III. EXPERIMENTAL RESULTS

A. Calibration and Polarization Results

The polarization data of Fig. 4 were taken at $\theta_{\rm lab}=97.1°$, which is near the low point of the J-dependent cross-section minimum with a measured cross section of 240 μb/sr. No radiation shielding was required around the polarimeter to obtain the low background rates shown. The difference in the left and right peak shapes is attributed to differences in energy resolution and discriminator levels in the left and right counter systems. It is reflected in a somewhat large average measured inherent polarimeter asymmetry factor K of 1.064.

Polarimeter calibration was accomplished by degrading the energy of the recoil protons from α-p elastic scattering at $\theta_{\rm lab}=12.2°$ to an energy of 7.57 MeV and measuring the product polarization. The polarimeter analyzing power P_2 was then obtained from the value $P_1=-0.542\pm0.035$ for the polarization of the recoil protons as obtained from the p-α scattering

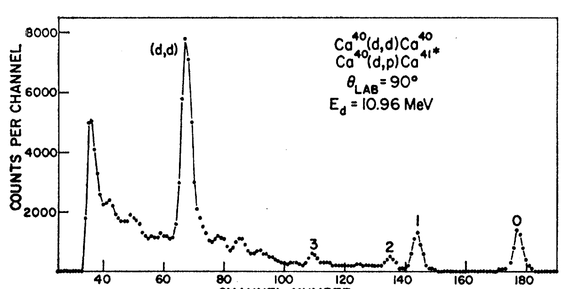

FIG. 5. Typical pulse-height spectrum for the cross-section measurements. Proton groups leaving ^{41}Ca in the ground, 1.95-, 2.47-, and 3.95-MeV states are labeled 0, 1, 2, and 3, respectively. The elastic deuteron peak appears at channel 67. The energy resolution is 1.2% full width at half-maximum.

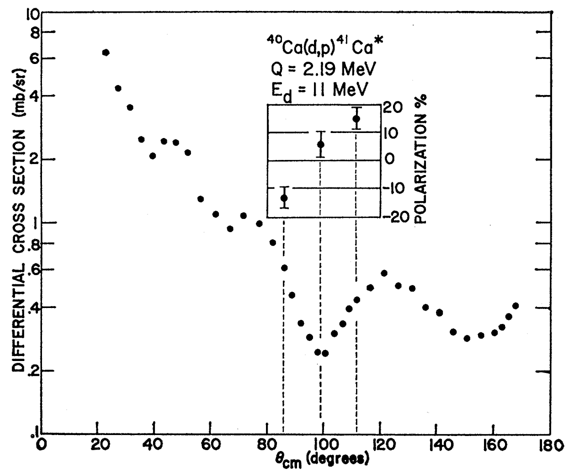

FIG. 6. Differential cross-section and polarization results for the ^{40}Ca(d, p)^{41}Ca*(3.95 MeV) reaction. The measured polarization appears to pass smoothly through zero at the J-dependent minimum.

results of Barnard *et al.*[11] The only corrections applied which were not negligible were the correction due to the cross-section angular variation (5%) and the correction due to K (1.084 ± 0.036 [15]). At $\theta_{\rm lab}=12.2°$ the reaction plane is coincident with the spectrometer median plane, so that no depolarization correction is required. The analyzing power P_2 obtained in this calibration can be compared to that of Ludwig and Miller,[12] whose polarimeter employed the same carbon second scatterers at the same polarimeter angle of 40°. The earlier value of -0.680 ± 0.051 agrees to within statistics with the result for the new polarimeter of -0.719 ± 0.057.

Table I presents the results of the polarization measurements.[15] In this table, $\theta_{\rm c.m.}$ is the c.m. angle, P is the corrected product polarization, K is the inherent polarimeter asymmetry factor determined separately at each angle, and P_1 is the polarization of the protons from the ^{40}Ca(d, p) reaction.

Figure 6 shows a plot of the measured cross-section angular distribution results for the ^{40}Ca(d, p) reaction leaving ^{41}Ca in the 3.95-MeV state. Results of the polarization measurements are presented for comparison on the same angular scale in the small insert in this figure. The J-dependent cross-section minimum is clearly seen near $\theta_{\rm c.m.}=100°$, while the polarization distribution is observed to pass smoothly through zero near this minimum with a positive slope.

The dashed lines on the momentum spectrum presented earlier in Fig. 2 show the maximum momentum acceptance of the spectrometer for protons from the ^{40}Ca(d, p)^{41}Ca*(3.95 MeV) reaction at 110°. The in-

[15] All errors quoted are standard statistical errors unless otherwise noted.

TABLE I. Results of proton polarization measurements across the J-dependent cross-section minimum near $\theta_{c.m.}=100°$ for the $^{40}Ca(d,p)^{41}Ca^*$(3.95 MeV) $(l_n=1, J_n=\frac{1}{2})$ reaction at an incident deuteron energy of 11.0 MeV.

θ_{lab} (deg)	$\theta_{c.m.}$ (deg)	P (corrected)	e_P	K	e_K	P_1 (transverse)	e_{P_1} (transverse)	P_1	e_{P_1}
84.2	86.0	+0.079	±0.023	1.062	±0.047	−0.110	±0.033	−0.128	±0.039
97.1	98.9	−0.038	±0.028	0.990	±0.045	+0.052	±0.039	+0.061	±0.046
110.0	111.7	−0.094	±0.024	1.111	±0.051	+0.130	±0.036	+0.151	±0.041

completely resolved peak on the high-energy side is made up of protons from (d, p) reactions exciting neighboring states. It is estimated that at this angle (the worst case), less than 10% of the accepted protons do not result from the excitation of the 3.95-MeV state. Further, since several states add to give this small contribution, it is reasonable to assume that their net effect on the polarization is probably within the quoted statistical errors. Nevertheless, calculations were made at each angle in which limiting values of ±1 for the polarizations of these protons were assumed. The results of these calculations do not alter the conclusion that the polarization angular distribution for protons from the $^{40}Ca(d,p)^{41}Ca^*$(3.95 MeV) reaction passes smoothly through zero near the minimum of the cross-section angular distribution.

B. Angular Distribution Measurements

Plots of the measured deuteron elastic scattering cross-section angular distribution and the calculated Rutherford distribution are shown in Fig. 7. Figure 8 is a composite plot of the measured cross-section angular distributions for the $^{40}Ca(d,p)$ reactions exciting the ground, 1.95-, 2.47-, and 3.95-MeV states of ^{41}Ca. Angular measurements have a relative error of ±0.1°, and the zero angle is in error by no more than ±1°.

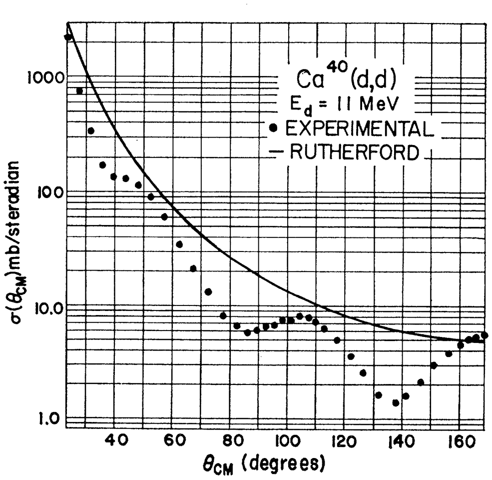

FIG. 7. Measured angular distribution of deuterons elastically scattered from natural calcium. Statistical errors are less than the size of the points.

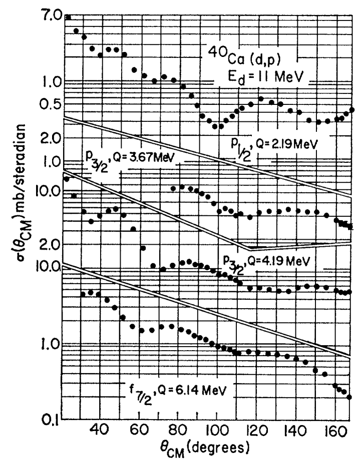

FIG. 8. $^{40}Ca(d,p)$ cross-section angular distributions corresponding to reactions exciting the ground, 1.95-, 2.47-, and 3.95-MeV states of ^{41}Ca.

The total angular width accepted by the detectors was 1.1°, and the energy resolution (full width at half-maximum) was 1.2%. Tables of all cross-section data are on file with the American Society for Information Science.[16]

Table II gives the relative and absolute errors assigned to the cross-section measurements. The major contribution to the relative errors arose from estimates of the background and contributions from incompletely resolved peaks. The absolute errors result from un-

TABLE II. Estimated relative and absolute errors for the present elastic deuteron and (d, p) reaction cross-section measurements at an incident deuteron energy of 10.96 MeV.

		(d, p)			
	(d, d) Elastic	Ground	1.95 MeV	2.47 MeV	3.95 MeV
Relative errors (%)	6	5	4	17	15
Absolute errors (%)	35	29	30	57	52

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TABLE III. Parameters employed by Satchler[a] for the DWBA predictions in Fig. 9. The deuteron potentials are those of Yule and Haeberli,[b] while the proton and neutron potentials are those of Lee *et al.*[c] and Becchetti and Greenlees.[d]

	V (MeV)	r_0 (fm)	a (fm)	W (MeV)	W_d (MeV)	r_0' (fm)	a' (fm)	V_s (MeV)	r_s (fm)	a_s (fm)	r_{charge} (fm)
Deuteron[b]	118.6	1.0	0.81	0	10	1.55	0.6	10	0.65	0.5	1.3
Proton, neutron[c]	54.5	1.20	0.65	0	11	1.25	0.47	8	1.20	0.65	1.25
Proton, neutron[d]	51.2	1.17	0.75	0.8	6.15	1.32	0.51	5	1.01	0.5	1.3

[a] Reference 17.
[b] T. J. Yule and W. Haeberli, Nucl. Phys. **A117**, 1 (1968).
[c] Reference 8.
[d] Becchetti, M.S. thesis, University of Minnesota, 1968 (unpublished).

certainties in target thickness, beam integration, solid angle, and background subtraction, with the latter being the major contribution.

IV. DISCUSSION

A. (*d*, *p*) Polarization Measurements

The primary purposes of the present work were to obtain experimental data for the polarization near the J-dependent cross-section minimum in the $^{40}Ca(d, p)$ $^{41}Ca^*$(3.95 MeV) reaction, and to measure elastic scattering and (d, p) cross sections at the same energy, in order to assist in future theoretical attempts to understand J-dependent effects. For completeness, however, comparisons between the present experimental results at 11-MeV bombarding energy and existing distorted-wave Born-approximation (DWBA) calculations performed by Satchler[17] for 12 MeV are shown in Fig. 9. The theoretical curves in the upper part of this figure should be considered simply as predictions for the polarization; no attempt to fit the present polarization results has been made. These polarization predictions do appear to pass through zero at angles not far from the cross-section minimum, but may exhibit larger predicted oscillations than the few experimental results might seem to indicate. It is also apparent that the position of the predicted cross-section minimum agrees reasonably well with that of the observed minimum in our angular distribution (for graphical clarity the theoretical curves in the lower half of Fig. 9 are plotted assuming a spectroscopic factor of 1). Information on the specific calculations and the parameters employed by Satchler are given in the caption to Fig. 9 and in Table III.

The discovery of J dependence has given rise to other attempts to obtain theoretical understanding of the cross-section behavior in (d, p) reactions, but with only partial success.[4,5] With the exception of DWBA predictions, to our knowledge no quantitative theoretical predictions for the polarization near the characteristic cross-section minimum have been made, however. Predictions of the polarization expected by the "sudden approximation" theory[18–21] or the "weakly bound projectile" model[22] for the particular reaction studied are also not yet available.

B. (*d*, *p*) and Elastic-Deuteron Angular Distribution Measurements

The cross-section angular distributions obtained in this work have been compared graphically with those of Lee *et al.*[8,9] and Kato *et al.*[23] at 11-MeV incident deuteron energy. Since special efforts were made by Bassel *et al.*[9] to keep the uncertainties in their absolute cross sections for elastic deuteron scattering small (±5%), using procedures not feasible in the present work, the present elastic deuteron data have been normalized to that of Bassel *et al.*[9] at forward angles

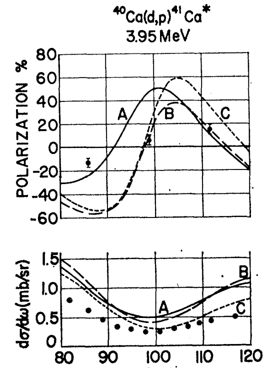

FIG. 9. Excerpts from DWBA predictions for the $^{40}Ca(d, p)^{41}Ca^*$-(3.95 MeV) reaction made by Satchler for 12-MeV bombarding energy (Ref. 17). Polarization and cross-section results of the present experiment carried out at 11 MeV are plotted for comparison, but no attempt has been made to fit these results. The predicted curves all use the Yule-Haeberli deuteron potentials (Ref. b in Table III). Nonlocality and finite-range corrections are included in the local-energy approximation: range 1.25 F, nonlocalities $\beta_d = 0.54$ F, $\beta_p = 0.85$ F (not included for the neutron). The solid curve (A) uses the proton and neutron potentials of Lee *et al.* (Table III and Ref. 8), while those of Becchetti and Greenlees (Ref. d in Table III). are used for the dashed curve (B). The dotted curve (C) is similar to curve B except that extra damping of interior contributions was tried by increasing β_d to 1 and β_p to 2 F.

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(<84°). A normalization factor of 1.56 was required. This normalization was used for both the (d, d) and (d, p) distributions and is included in Figs. 7 and 8. When this absolute normalization factor is used, the present elastic results at back angles (>90°) are higher than those of Bassel *et al.* by about 25%, although the distributions agree well relatively at forward and backward angles separately. The present work used ten overlapping angles to determine the forward-to-backward normalization factor, which was found to be 1.018.

The results of the comparison of the ground-state distributions show good *relative* agreement over the entire angular range between the present data and the results of Kato *etal.* and Lee *et al.* However, while the present absolute cross sections and those of Kato *et al.* agree well (to within 10%), the absolute cross section of Lee *et al.* is about 25% larger than the present results. Further, the $E_d=11$ MeV absolute cross section of Lee *et al.* is about 25% higher than it might reasonably be expected to be with respect to their own $E_d=10$ MeV and $E_d=12$ MeV distributions. This discrepancy has been independently noted by Schwandt and Haeberli.[24]

Comparisons of the remaining (d, p) angular distributions at $E_d=11$ MeV are as follows: (a) The relative agreement of the 1.95-MeV distributions is good, but the absolute cross section of Lee *et al.* is about 25% higher than in this work, as was the case of the ground state. (b) The relative agreement of the 2.47-MeV distributions is good, but the absolute cross section of Lee *et al.* is higher than the present results by about 50%. (c) The relative agreement of the 3.95-MeV distributions is not as good as for the other distributions, and the absolute cross section of Lee *et al.* still seems to be 50% higher than obtained here. The larger difference between the absolute cross sections for the 2.47- and 3.95-MeV distributions as well as the poorer relative agreement for the 3.95-MeV distribution may be due to the presence of several lower-intensity proton groups associated with (d, p) reactions to neighboring states which were incompletely resolved.

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Nuclear Energy Levels of V^{51}, Mn^{53}, and Co^{55} by the Quasiparticle Method

RAM RAJ, M. L. RUSTGI, AND R. P. SINGH
Physics Department, State University of New York, Buffalo, New York 14214
(Received 26 November 1968)

The modified Tamm-Dancoff approximation has been applied to the calculation of the nuclear energy levels of V^{51}, Mn^{53}, and Co^{55}. The shell-model reaction matrix elements of Kuo and Brown, calculated with the Hamada-Johnston nucleon-nucleon potential and renormalized for core polarization, are used with the aim of ascertaining the accuracy of these matrix elements. The effects of the extra term in the BCS equations and of the ground-state correlation are studied. Only a qualitative agreement between theoretical and experimental spectra is found for all three nuclei investigated.

1. INTRODUCTION

IN large measure, the degree of success in a shell-model calculation of the structure of a given nucleus depends on the degree to which the assumed closed core is really closed and on the residual interaction employed. Some of the difficulties found in shell-model calculations in which O^{16} and Ca^{40} are assumed to be closed cores come about because these nuclei are not really good closed cores.[1] There is some evidence that Ca^{48} forms a good closed core.[2,3] The conventional approach of assuming some simple but reasonable forms for the residual interaction has been questionable. An alternative approach is to treat the shell-model matrix elements themselves as adjustable parameters,[4] without specifying the algebraic forms of the interactions. This approach becomes futile because one has to decide beforehand which configurations should be included and which experimental levels are to be fitted. Yet there is a third approach, pursued by Kuo and Brown,[5] in which the effective interactions are deduced from the free nucleon-nucleon potential determined by the scattering data below the meson threshold. Two such potentials, which are known to be numerically similar, have been obtained by Breit and collaborators[6] and by Hamada and Johnston.[7]

Recently Kuo and Brown[5] have calculated the shell-

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