Stripping Cross-Section Calculations; Comparison between a Coupled-Channel Method and the DWBA for ^{40}Ca†

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(Received 21 January 1969)

Deuteron-stripping differential cross sections are calculated by a coupled-channel (C.Ch.) method and compared to distorted-wave Born-approximation (DWBA) results for incident energies of 2, 4, 5, 7, and 11 MeV. The reaction envisaged is ^{40}Ca$(d, p)^{41}$Ca to a $2p$ state in ^{41}Ca. The optical-model deuteron potential employed in the DWBA calculation is such that for each energy the elastic deuteron cross section reproduces the C.Ch. result as closely as possible. It is found that the maxima of the stripping cross sections are within 10% of each other for energies above 2 MeV. This good agreement does not occur for stripping to $1f$ states, the DWBA result being larger than 40% or more. However, in this case the C.Ch. calculation is only approximate, since the coupling of the f-stripping channel to the deuteron channel is not explicitly included in the calculation. Various other quantities are defined and evaluated in order to illustrate the differences between the C.Ch. and DWBA calculations. They are the "trivially equivalent deuteron potential," the stripping emission density, and an average of the absolute value squared of the radial stripping overlap integrals. The integrand of the latter quantity is also plotted for some cases.

I. INTRODUCTION

For some nuclei, the probablity of a direct reaction such as stripping can be quite large. For example, in the ^{40}Ca$(d, p)^{41}$Ca reaction leading to the $2p_{3/2}$ states in ^{41}Ca, the total stripping cross section is about 10% of the deuteron-^{40}Ca reaction cross section. Whenever such direct reactions take place with a sufficiently large probability, then a coupled-channel (C.Ch) equation should be more suitable to describe the reaction than the distorted-wave Born approximation (DWBA), since the former method automatically describes the multiple transitions which take place between the incident channel and several of the strong reaction channels. By contrast, the DWBA describes explicitly only one single transition at a time, but nevertheless the multiple transitions are implicitly taken into account because the optical potential is adjusted so as to give an elastic cross section which agrees with experiment.

The purpose of this paper is to present a comparison between stripping cross sections obtained by solving a C.Ch. equation and a conventional zero-range local-optical DWBA method.

A rough comparison between the two methods of calculation has been carried out previously.[1] The main result was that the wave function which describes the motion of the c.m. of the deuteron relative to the target nucleus is damped considerably more strongly in the nuclear "interior" as compared to the optical-model wave function, even if conventional nonlocality corrections are applied to the optical-model waves. The main effect of the increased interior damping of the deuteron wave was a decrease in the stripping cross section at backward angles. The peak of the stripping cross section remained nearly unaltered, since it is due mainly to contributions from the surface part of the stripping process, where the optical model and C.Ch-deuteron wave functions are nearly the same. It should be kept in mind that the calculations carried out so far refer only to the nucleus of ^{40}Ca, and the applicability of these result to other nuclei is as yet not known.

In the present study the comparison of C.Ch and DWBA stripping cross sections is carried out in finer detail than previously. The optical potentials in the present study employed for the DWBA stripping calculation are adjusted so that the corresponding elastic deuteron cross section fits the C.Ch.-deuteron cross section. These potentials are the same as those obtained in a previous study[2] of the energy dependence of optical-model deuteron potentials. In addition to a comparison between $\Delta l=1$ (p-state) stripping cross sections, $\Delta l=0$ (s-state) and $\Delta l=3$ (f-state) stripping cross sections are also studied in the present paper, so that insight is gained into the relative spectroscopic factors which would be obtained in the C.Ch calculation for transitions to s, p, and f levels in ^{41}Ca as compared to the DWBA results. However, neither the stripping to the s states nor that to the f states is as yet incorporated into the coupled equations. These transitions are approximated instead with the DWBA formalism for which the deuteron wave function is that obtained from the coupled equation. Only the stripping to p states is explicitly included in the couled equations.

Several functions which help to illustrate the difference between the C.Ch and DWBA treatments are also examined. One is the trivially equivalent local potential V_{TEQ}, which, when used in a uncoupled equation describing the incident channel, has the same effect on the incident wave function as the presence of the coupling to the reaction channel. Another function is the proton emission density $P_J(r)$. The integral of this function over the radial distance r represents the number of protons emitted in all directions per unit

† Work supported by Grant No. GP-6215 of the National Science Foundation and by the Research Foundation of the University of Connecticut.
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[1] G. H. Rawitscher, Phys. Rev. **163**, 1223 (1967).
[2] G. H. Rawitscher, Phys. Rev. Letters **20**, 673 (1968).

time by the particular partial waves which correspond to the total angular momentum J. For a given distance r this function describes the number of protons emitted or absorbed per unit time in the radial interval between r and $r+dr$. A function similar to P_J but related only to the emission and absorption of protons due to the coupling of stripping channels to the incident deuterons is also discussed. It is called the stripping emission density.

II. FORMALISM

The numerical study presented here refers to d-^{40}Ca interaction. The assumptions and the final equation are the same as those in Ref. 1. A short summary is given below. The bulk of the experimental (d, p) stripping cross section occurs to various p states in ^{41}Ca. These states are represented in the equations as one p state bound to ^{41}Ca with 6.41 MeV, giving rise to a Q value for the reaction which is taken to have the value of 4 MeV. The outgoing proton wave function is coupled to the incident deuteron wave via the stripping matrix element of conventional DWBA calculations. The theoretical justification and the approximations involved are discussed in Ref. 1. The nonorthogonality difficulties which occur whenever the incident and outgoing channels have different number of nucleons does remain; further, the effect due to the occurrence of the (d, p) reaction to states other than one p state in ^{41}Ca as well as all (d, n) reactions is incorporated phenomenologically in the value of the constant N in Eq. (1b) below (its value is ~ 3), the spins of the deuteron, proton, and neutron are taken equal to zero, antisymmetrization between nucleons in the nucleus and the deuteron is not carried out, and the breakup process of the deuteron is not explicitly included in the equation.

The coupled radial wave equations are

$$\left[-\frac{\hbar^2}{2\mu_p}\left(\frac{d^2}{dr^2}-\frac{L(L+1)}{r^2}\right)+U_p-E_p\right]F_{(Ll)J}{}^p(r) = -V_l C_{(Ll)J} F_J{}^d(r), \quad (1a)$$

$$\left[-\frac{\hbar^2}{2\mu_d}\left(\frac{d^2}{dr^2}-\frac{J(J+1)}{r^2}\right)+U_d-E_d\right]F_J{}^d = -NV_l \sum_L C_{(Ll)J} F_{(Ll)J}{}^p(r), \quad (1b)$$

where $F_{(Ll)J}{}^p$ and $F_J{}^d$ are the radial functions for the proton and deuteron partial waves, respectively. The asymptotic values of these functions are

$$F_{(Ll)J}{}^p \rightarrow A_{(Ll)J} \exp[i(\kappa_p r+\sigma_L{}^p-\tfrac{1}{2}L\pi)], \quad (1c)$$

$$F_J{}^d \rightarrow \sin(\kappa_d r+\sigma_J{}^d-\tfrac{1}{2}J\pi+K_J{}^d)\exp(iK_J{}^d), \quad (1d)$$

where $\sigma_I{}^{p,d}=\arg\Gamma(I+1+i\eta^{p,d})$ is the usual Coulomb phase shift and κ_p, κ_d are the proton and deuteron wave numbers. The complex coefficient $A_{(Ll)J}$ as well as the complex dueteron phase shifts $K_J{}^d$ are obtained from the solution of the coupled equation. The orbital angular momentum of the neutron bound to ^{41}Ca is given by l and for each value of J, L takes as many values as is compatible with the triangular condition $\Delta(L, l, J)$ and such that $L+l+J$ is an even number. The distorted proton wave obtained from Eqs. (1) which corresponds to a neutron state of magnetic quantum number (m) is

$$^{(m)}\chi^p=[\sqrt{(4\pi)}/\kappa_d r_p]\sum_{L,J} i^J \exp(i\sigma_J{}^d)(2J+1) \times \begin{pmatrix} l & L & J \\ m & -m & 0 \end{pmatrix} F_{(Ll)J}{}^p(r_p)Y_L{}^{-m}(\mathbf{r}_p) \quad (2)$$

and the deuteron distorted wave is

$$\chi^d=[\sqrt{(4\pi)}/\kappa_d r_d]\sum_J (2J+1)^{1/2} i^J \times \exp(i\sigma_J{}^d)F_J{}^d Y_J{}^0(\mathbf{r}_d). \quad (3)$$

The neutron bound-state wave function is

$$\varphi_n=u_l(r_n)Y_l{}^m(\mathbf{r}_n). \quad (4a)$$

The coupling potential V_l is proportional to the radial part of neutron bound-state wave function:

$$V_l=Du_l[(2l+1)/4\pi]^{1/2}, \quad (4b)$$

where D is a constant determined from the zero-range approximation and taken equal to $-(1.68)^{1/2}\times 10^2$ MeV F$^{3/2}$. The coefficients $C_{(Ll)J}$ are given by

$$C_{(Ll)J}=(2L+1)^{1/2}\begin{pmatrix} l & L & J \\ 0 & 0 & 0 \end{pmatrix}. \quad (5)$$

These coefficients have the property that

$$\sum_L C_{(Ll)J}{}^2=1. \quad (6)$$

The real part of the potentials U_p and U_d are of the conventional Woods-Saxon form, while the imaginary parts of the proton and deuteron potentials are respectively given by the derivative of a Woods-Saxon potential and a Woods-Saxon potential; i.e., they are of the surface and volume types, respectively. The values of the parameters for U_d and U_p are given in Table II of Ref. 1 under the headings "coupled channel" and "proton," respectively, and they are reproduced in Table I. The solutions of the homogeneous proton equations (1b), in which the coupling potential V_l is set equal to zero are denoted by $f_L{}^p$. The boundary condition of these functions is such that asymptotically

$$f_L{}^p \rightarrow \sin(\kappa_p r+\sigma_L{}^p-\tfrac{1}{2}L\pi+K_L{}^p)\exp(K_L{}^p). \quad (7)$$

These functions occur in the calculation of stripping overlap integrals:

$$R_{(Ll)J}=\int_0^\infty f_L{}^p V_l F_J{}^d dr, \quad (8)$$

TABLE I. Potential parameters.[a]

		Deuteron optical potential				
		Real			Imaginary[b]	
Energy	V_0	r_0	a	W_0	r_0'	a'
11	115.38	1.0058	0.878	18.766	1.517	0.4823
7	114.0	1.0058	0.903	16.0	1.651	0.4565
5	113.0	1.0058	0.900	14.8	1.713	0.45
4	111.48	1.0058	0.900	14.0	1.780	0.4374
C.Ch.	120.70	0.9060	0.846	60.0[c]	1.40[c]	0.50[c]
		Proton optical potential				
E	$60-0.5E$	1.20	0.65	11.0	1.25	0.47
		Neutron shell potential				
BE[d]						Q Value
-6.41	59.64	1.21	0.65	$2p$ state		4.0
-6.41	40.766	1.21	0.65	$2s$ state		4.0
-8.37	55.50	1.21	0.65	$1f$ state		6.14

[a] All quantities are either in MeV or in Fermi. Unless otherwise indicated, the potentials are of Woods-Saxon type.
[b] Surface derivatives type. The maximum value of the imaginary potential is given by $-W_0$.
[c] Volume type.
[d] Binding energy.

which in turn determine the coefficients in Eq. (1c):

$$A_{(Ll)J}=(2\mu_p/\hbar^2)\kappa_p^{-1}C_{(Ll)J}R_{(Ll)J}. \tag{9}$$

From Eqs. (9), (8), (1c), and (2), the differential stripping cross section can be obtained.

The total stripping cross section for the angular momentum transfer is the integral over angle of the partial differential cross section and is given by

$$\sigma(d,p)=B\sum_J(2J+1)[\sum_L C_{(Ll)J}{}^2\,|\,R_{(Ll)J}\,|^2], \tag{10a}$$

where

$$B=(16\pi/\hbar^4)(\mu_p\mu_d/\kappa_p\kappa_d{}^3), \tag{10b}$$

and where μ_p and μ_d are the reduced proton and deuteron masses relative to the respective target nuclei. The expression above shows how the various L values contribute to the same total angular momentum J. In view of Eq. (6), it is useful to define R_J as

$$R_J=(2J+1)[\sum_L C_{(Ll)J}{}^2\,|\,R_{(Ll)J}\,|^2]. \tag{11}$$

This quantity is equal to $(2J+1)$ times a physically meaningful average of the absolute value squared of the stripping overlap integrals for each total angular momentum J. Numerical values for R_J are given further on, as well as values for the L averaged overlap integrand of Eq. (8), defined as

$$\mathcal{Y}_J(r)=[\sum_L C_{(Ll)J}{}^2F_{(Ll)J}{}^p(r)]V_l(r)F_J{}^d(r). \tag{12}$$

In conventional DWBA formulations of stripping cross sections a term $(2J_F+1)/(2J_I+1)$ is usually factored out explicitly. In the present case both the deuteron and the target nucleus have spin zero, the spin of the final nucleus J_F is equal to the l, and the term mentioned above reduces to $2l+1$. This factor is contained in the expression for the off-diagonal potential V_l [Eq. (4b)] in the form of $(2l+1)^{1/2}$, and hence it does not appear explicitly in Eq. (10).

The DWBA expression for the stripping cross sections are formally very similar to the ones given above. The only difference is in the procedure for obtaining the deuteron and proton distored waves. The coupled equations (1a) and (1b) are replaced by the optical-model equation

$$\left[-\frac{\hbar^2}{2\mu_d}\left(\frac{d^2}{dr^2}-\frac{J(J+1)}{r^2}\right)+\hat{U}_d-E_d\right]\hat{F}_J{}^d=0, \tag{13}$$

$$\left[-\frac{\hbar^2}{2\mu_p}\left(\frac{d^2}{dr^2}-\frac{L(L+1)}{r^2}\right)+U_p-E_p\right]\hat{F}_{(Ll)J}{}^p=-V_lC_{(Ll)J}\hat{F}_J{}^d, \tag{14}$$

where the caret denotes the optical model or DWBA quantity. In the present approximate treatment, $\hat{U}_p$ is taken as identical to U_p, and hence the solution $\hat{f}_L{}^p$ of the homogeneous part of Eq. (14) is identical to $f_L{}^p$. The goodness of this approximation still remains to be examined. In view of this approximation, the stripping overlap integral

$$\hat{R}_{(Ll)J}=\int_0^\infty f_L{}^pV_l\hat{F}_J{}^d dr \tag{15}$$

differs from the C.Ch. values in Eq. (8) only because $\hat{F}_J{}^d$ and $F_J{}^d$ differ. The deuteron potentials $\hat{U}^d$ are adjusted so that the corresponding elastic deuteron cross section fits the C.Ch. results, and it is found that the deuteron phase shifts $\hat{K}_J$ are very similar to K_J for each value of J. The partial waves $\hat{F}_J{}^d$ and $F_J{}^d$ are therefore very similar in the asymptotic region and in the nuclear "surface" region, and differ progressively in the nuclear interior, as is illustrated in detail in the Sec. IV. The identity of $\hat{f}_L{}^p$ and $f_L{}^p$ does not, of course, imply the identity between $\hat{F}_{(Ll)J}{}^p$ and $F_{(Ll)J}{}^p$, since the right-hand sides of Eqs. (14) and (1a) differ from each other on account of the difference between the radial deuteron waves. In order to discuss the difference between the deuteron waves $\hat{F}_J{}^d$ and $F_J{}^d$, it is useful to introduce the "trivially equivalent potential" (TEQ) in the deuteron channel $V_{\mathrm{TEQ},J}{}^d$ defined by

$$V_{\mathrm{TEQ},J}{}^d(r)=NV_l(r)\Big[\sum_L C_{(Ll)J}F_{(Ll)J}{}^p(r)\Big]/F_J{}^d(r). \tag{16}$$

Of course, $V_{\mathrm{TEQ},J}{}^p$ is known only after the coupled equations (1) have been solved. Equation (1) for the deuteron then reads

$$\left[-\frac{\hbar^2}{2\mu_d}\left(\frac{d^2}{dr^2}-\frac{J(J+1)}{r^2}\right)+U_d+V_{\mathrm{TEQ},J}{}^d-E_d\right]F_J{}^d=0. \tag{17}$$

The potential $V_{\mathrm{TEQ},J}{}^d$ represents the effect of the presence of the C.Ch. If $V_{\mathrm{TEQ},J}{}^d$ were not J-dependent and if it were a smooth function of r, then $V_{\mathrm{TEQ},J}{}^d$ would represent a conventional local complex potential and $U_d+V_{\mathrm{TEQ},J}{}^d$ would be very close to the local optical potential $\hat{U}_d$, and $\hat{F}_J{}^d$ would not be different from $F_J{}^d$. However, the numerical discussion presented in Sec. III shows that $V_{\mathrm{TEQ},J}{}^d$ is strongly J- and r-dependent in the nuclear interior, and that $\hat{F}_J{}^d$ and $F_J{}^d$ are very similar asymptotically but differ considerably at small distances.

A function which provides some insight into the calculation of stripping reactions is the proton emission density $P_J(r)$, which will now be defined. If ${}^{(m)}\mathbf{j}^p(r)$ denotes the proton current which corresponds to the proton wave function defined in Eq. (2), then its divergence integrated over the whole space can be written as a sum over the contributions from each value of J as follows:

$$\sum_m \int \mathrm{div}^{(m)}\mathbf{j}^p(r)\,d^3r=\sum_J \int P_J(r)\,dr. \tag{18}$$

By definition, $P_J(r)$ is the proton emission density for each angular momentum J. By manipulating Eqs. (1) and (2) it can be shown that

$$P_J(r)=\hbar^{-1}(8\pi/\kappa_d{}^2)(2J+1)\sum_L\{V_lC_{(Ll)J}\,\mathrm{Im}(F_{(Ll)J}{}^{p\,*}F_J{}^d)+\mathrm{Im}(U_p)\,|\,F_{(Ll)J}{}^p\,|^2\}. \tag{19}$$

The second term in curly brackets is negative. It represents the protons removed from the stripping channel into other inelastic channels or else into the compound nucleus. The first term is on the average positive. It represents the protons which originate from the incident deuterons. This term is the negative of the imaginary part of $V_{\mathrm{TEQ},J}{}^d\,|\,F_J{}^d\,|^2$, which represents deuterons absorbed from the indicent beam on account of the presence of stripping channels. Because of its intimate relation with the C.Ch. calculation, the first term in the curly brackets in Eq. (19) is called the stripping emission density $S_J(r)$:

$$S_J(r)=\sum_L V_lC_{(Ll)J}\,\mathrm{Im}[F_{(Ll)J}{}^p(r)F_J{}^d(r)], \tag{19$'$}$$

and will be illustrated by Fig. 4 in Sec. III.

The total number of protons emitted in all direction per second, given by Eq. (18), is directly related to the (d,p) cross section integrated over angle. Comparison with Eq. (10) shows that

$$\int_0^\infty P_J(r)\,dr=\frac{2\mu_p}{\hbar^2}\kappa_p{}^{-1}\frac{R_J}{2J+1}, \tag{20}$$

with R_J given by Eqs. (11) and (8). Equation (20) provides a useful check on the numerical calculation. For the DWBA calculation the proton emission density can be defined in an analogous fashion by the same relations as above, after the careted quantities are substituted for the C.Ch. quantities. However, contrary to what is the case for the integrand of the stripping overlap integrals $R_{(Ll)J}$, the DWBA and C.Ch. proton-emission densities will, in general, be different from each other for large radial distances. The reason is that $\hat{F}_{(Ll)J}{}^p$ and $F_{(Ll)J}{}^p$ which occur in Eq. (19) are not close to each other at large distances. Nevertheless, for large radial distances, the difference between P_J and $\hat{P}_J$ is determined by the difference between the C.Ch. and DWBA values of the complex quantity $\sum_L C_{(Ll)J}{}^2R_{(Ll)J}$. This follows from Eqs. (1c) and (9), which relate the asymptotic value of $F_{(Ll)J}{}^p$ to $C_{(Ll)J}R_{(Ll)J}$. Hence most of the insight for the difference between the C.Ch. and DWBA calculations of stripping cross sections can be gained by comparison of $R_{(Ll)J}$ and $\hat{R}_{(Ll)J}$.

The reasoning which led to Eqs. (18)–(20) for the stripping channel can also be carried out for the deuteron channel. The divergence of the deuteron current integrated over-all space is given by

$$\int \mathrm{div}\mathbf{j}^d d^3r=\frac{8\pi}{\hbar\kappa_d{}^2}\sum_J(2J+1)\times\int_0^\infty \mathrm{Im}[U_d(r)+V_{\mathrm{TEQ},J}{}^d(r)]\,|\,F_J{}^d(r)\,|^2dr. \tag{21}$$

The first term in the integral represents the absorption due to all processes other than stripping, such as absorption due to compound nucleus formation. The second portion represents the absorption due to the stripping process which is treated explicitly by the coupled equations.

The total reaction cross section in the deuteron channel is given by

$$-\frac{1}{v_d}\int \mathbf{j}^d\cdot d\mathbf{S} = -\frac{\pi}{\kappa_d^2}\sum_J (2J+1)T_J, \qquad (22)$$

where v_d is the velocity of the incident deuteron and T_J is the deuteron absorption coefficient in the angular momentum channel J:

$$T_J = 1 - |\exp(2iK_J{}^d)|^2.$$

From the expressions given above, it is now possible to estimate the portion $T_J{}^S$ of the deuteron absorption coefficient which is due to the stripping reaction, and which therefore should be subtracted from the total absorption coefficient T_J in order to calculate the amount of compound nucleus formation induced by the deuteron. Combining Eqs. (21) and (22) one finds that

$$T_J = -\frac{8\mu_d}{\hbar^2\kappa_d}\int_0^\infty (\mathrm{Im}U_d + \mathrm{Im}V_{\mathrm{TEQ},J}{}^d)\,|F_J{}^d|^2 dr. \qquad (23)$$

It is the second term in the integrand which gives rise to $T_J{}^S$. The first term in curly brackets in Eq. (19) is equal to the negative of the second term in the integrand in Eq. (23). If one neglects the second term in curly brackets in Eq. (19) one obtains for $T_J{}^S$ the expression

$$T_J{}^S = -\frac{8\mu_d}{\hbar^2\kappa_d}\int_0^\infty \mathrm{Im}V_{\mathrm{TEQ},J}{}^d\,|F_J{}^d|^2 dr$$
$$\cong \frac{N}{\kappa_d\kappa_p}\frac{16\mu_p\mu_d}{\hbar^4}\frac{R_J}{2J+1}. \qquad (24)$$

Equation (24) relates $T_J{}^S$ to the stripping overlap integral R_J defined in Eq. (11). This equation should be useful for Hauser-Feshbach calculations of compound elastic or compound stripping cross sections, as has recently been performed by Hodgson and collaborators.[3] As an illustration, Eq. (24) was utilized in order to calculate the reduction factor X which, when multiplied into the total absorption coefficient T_J, obtained from the elastic deuteron cross section calculation, gives the portion which leads to compound nucleus formation. For J ranging between 0 and 6, X decreased steadily from 0.9 to 0.5.

III. RESULTS

The optical-model deuteron potentials which fit the C.Ch. cross sections for elastic deuteron scattering from the nucleus of ^{40}Ca at deuteron incident energies of 4, 7 and 11 MeV are listed in Table I. These parameters are taken from Ref. 2. The values at 5 MeV are obtained by interpolation of the values at the other energies, and

[3] P. E. Hodgson and D. Wilmore, Proc. Phys. Soc. (London) **90**, 361 (1967); O. Dietzsch *et al.*, Nucl. Phys. **A114**, 330 (1968).

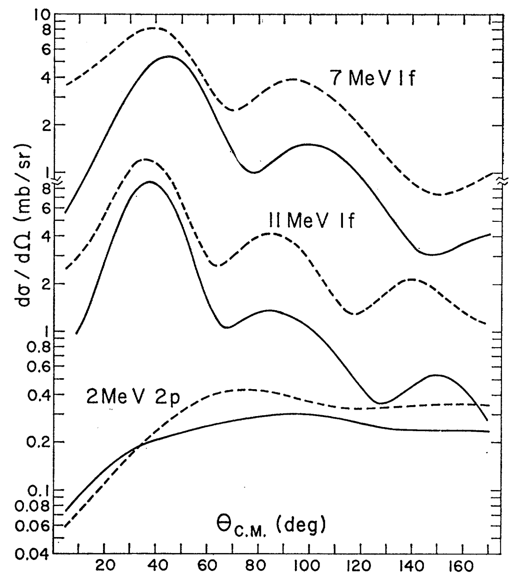

FIG. 1. Differential (d, p) stripping cross section to the $1f$ and $2p$ states in ^{41}Ca. The dashed and solid curves represent the DWBA and C.Ch. results, respectively. The incident deuteron energies are indicated near the curves. In this, as well as in all other figures, the optical deuteron potential is adjusted at each energy to fit the C.Ch. elastic scattering cross section. The parameters are listed in Table I.

are also listed in the table. The proton potentials used in both the C.Ch. and DWBA calculations, as well as the deuteron potentials used in the coupled equations, are those given in Table II of Ref. 1 and are repeated in Table I. The neutron bound-state potentials are also listed in this table. The $2p$-state neutron function gives rise to the coupling potential given by Eq. (4b). The same bound state is used in the DWBA calculations of the stripping reaction. The bound s state is a hypothetical state, introduced for the purpose of this numerical study. Neither the s nor the f stripping channel is included in the set of coupled equations, Eq. (1). Instead, the corresponding C.Ch stripping cross section is obtained by the approximate procedure of using the C.Ch. deuteron radial waves in the DWBA-like expression represented by Eq. (14). This procedure is correct if the coupling to the s and f states has a negligible effect on the deuteron channel. In Eq. (14) the value of l is set equal to 0 and 3 in the expression of $C_{(Ll)J}$ and of V_l, and the appropriate Clebsch-Gordan coefficients are used in the final expression for the differential stripping cross sections to the s and f states, respectively. The DWBA value for the stripping cross section is obtained by employing in Eq. (14) the optical-model values for the distorted deuteron radial waves $F_J{}^d$, and proceeding in the same way as described above. The outgoing proton energies are obtained from the values of Q listed in Table I.

The angular dependence of the $\Delta l = 1$ stripping cross sections are of the type already illustrated in Fig. 5 of Ref. 1. For the larger deuteron incident energies the

C.Ch. values decrease more steeply with angle than the DWBA counterparts, the minima are more pronounced, and are variously shifted to larger angles. On the other hand, for the hypotherical 2s transition the large angle minima of the C.Ch. stripping cross sections are shifted to smaller angles relative to the DWBA result. The inverse is the case for the 1f transitions, as is illustrated in Fig. 1. The $\Delta l=1$ stripping angular distributions for incident deuterons of 2 MeV are also illustrated in Fig. 1, since for this energy the differences between C.Ch. and DWBA results are particularly pronounced for this transition.

With the exception of the 2-MeV case illustrated in Fig. 1, the maxima of the p, s, and f stripping cross sections occur in the forward hemisphere. The values of these cross sections at the maxima are usually fitted to the experimental values and hence determine the values of the spectroscopic factors. The ratio of the corresponding maxima for the C.Ch. and DWBA cases are plotted in Fig. 2. Figure 2 shows that for the stripping cross section to the p states in ^{41}Ca the C.Ch method would yield a spectroscopic factor which is larger than the DWBA one by 40% at 2 MeV, smaller by about 10% at 7 MeV, and about the same at 11 MeV. The spectroscopic factor for the hypothetical 2s state determined by the C.Ch. method would be smaller by 20% at 7 MeV and again nearly the same as 11 MeV as compared to the DWBA result. For the $\Delta l=3$ transition, the differences are more pronounced. The source of all these differences is due to the difference between the optical-model and C.Ch. deuteron wave function, which occurs at small cistances, as is discussed further below.

The maxima of the stripping cross sections for the transitions to the p states, as calculated with the coupled equations, are in reasonable agreement with the experimental results of Lee *et al.*[4] between 7 and 11 MeV, and of Leighton *et al.*[5] at 5 MeV, as is shown in Table II. The results for the f stripping are in disagreement with experiment,[4,5,6,7] increasingly so the lower the incident deuteron energy. Furthermore, the elastic deuteron cross section at 5 MeV is in disagreement with the results of Leighton *et al.* For angles larger than 100° the theoretical result is approximately 50% larger than the experimental value, while for angles less than 60° theory and experiment agree.

It is possible that the discrepancy with the elastic deuteron cross section at 5 MeV is due to the neglect of the inclusion of the stripping to the f state in the coupled equations. At the lower energy the f-stripping

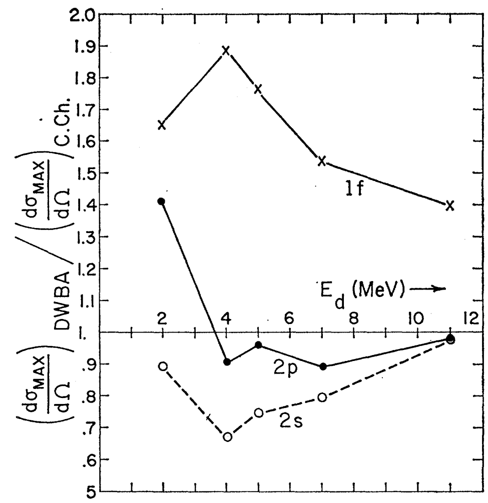

FIG. 2. Ratio of DWBA to C.Ch. stripping cross-section maxima as a function of incident deuteron energy for transitions to 1f, 2p, and 2s states in ^{41}Ca. The latter is a hypothetical state.

cross section becomes comparable to the p-stripping cross section, while at 11 MeV it is quite negligible. It is unlikely that the effect of the deuteron breakup, also not included in the coupled equations, is responsible for the discrepancies with experiment. The reason can be seen from the comparison of the deuteron optical potential obtained by Leighton *et al.*[5] to fit experiment at 5 MeV, and the optical potential obtained here which is equivalent to the coupled equation. The real parts of the two potentials are nearly identical. The values of V_0, r_0, and a are 110 MeV, 1.03 F, and 0.92 F and 113 MeV, 1.006 F, and 0.90 F, respectively. However, the imaginary part of the experimental potential is smaller than the theoretical one. The values of W_d, r_0' and a' are 9.8 MeV, 1.64 F, 0.53 F and 14.8 MeV, 1.78 F, 0.437 F, respectively. If breakup had a large effect, then one would expect an increased value of the imaginary potential in the surface region, where the break up takes place, contrary to what is the case experimentally. The experimental f-stripping cross section is larger than the coupled-channel result. The reason could be due in part to contributions from nuclear compound effects, since it is known[8] that the f-stripping fluctuates rather strongly with energy at lower energies. The p-stripping cross section, on the other hand, is a much smoother function of the incident deuteron energy.[8] As a result of the above discussion it is concluded that comparison of theory with experiment for f-stripping cross sections is still premature. In performing the C.Ch. calculations it was noticed that the effective coupling between the stripping channel and the deuteron channel is particularly large near 4 and 5 MeV. This is seen from the fact that the iterations carried out in solving the equation converge poorly, and can be attributed to the fact that

[4] L. L. Lee, Jr., J. P. Schiffer, B. Zeidman, G. R. Satchler, R. M. Drisco, and R. H. Bassel, Phys. Rev. **136**, B971 (1964); **138**, AB 6 (E) (1964).

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TABLE II. Maxima of the stripping cross sections.

Deuteron energy (MeV)		Peak value of stripping cross section (mb/sr) $p_{3/2}$	$f_{7/2}$	Ratio Expt/Theory $p_{3/2}$	$f_{7/2}$
5	Theory[a]	14.5	1.65	1.0	1.9
	Expt[b]	14.7[c]	3.15[c]		
7	Theory[a]	31.5	3.08	0.96	1.4
	Expt[b]	33.0[d]	4.2[d]		1.4
	Expt	...	4.2[e]		
10	Theory[a]	48.7	4.52	1.1	1.2
	Expt[b]	41.0[d]	5.4[d]		1.0
	Expt	...	4.48[f]		

[a] Coupled channel calculation. The 10-MeV values are obtained by linear extrapolation between the 11- and 7-MeV results. The p- and f-stripping cross sections are scaled down by the statistical spin factors 3/2 and 7/4, respectively.
[b] The $p_{3/2}$ cross section is the sum of the two $p_{3/2}$ transitions with Q values of 4.19 and 3.67 MeV.
[c] Reference 5.
[d] Reference 4.
[e] Reference 9.
[f] Reference 10.

the overlap between the wave functions in the two channels is particularly large at these energies.

In order to provide some insight into the effect due to the coupling of stripping channels upon the deuteron waves, the trivially equivalent potential defined in Eqs. (16) and (17) will now be discussed. Such a potential was first considered by Perey[9] in a study of a nonlocal potential introduced by Perey and Buck[7] to describe nucleon-nucleus scattering, and was later discussed further by Austern[10] in connection with the damping of the nonlocal wave function in the nuclear interior. A plot of $U_d+V_{\mathrm{TEQ},J}{}^d$ versus radial distance is shown in Fig. 3 for incident deuterons of 7 MeV. This potential, when used in an uncoupled optical-model deuteron potential, will give rise to the same radial deuteron wave function as the coupled equations, for each angular momentum value J. Only the behavior for distances beyond 4 F is shown in the figure. The heavy line marked $J=\infty$ illustrates the value of U_d, and the deviations from this line for each value of J represents $V_{\mathrm{TEQ},J}{}^d$ and is due to the effect of the proton channel. Figure 3 shows that the real part of $V_{\mathrm{TEQ},}{}^d$ is predominantly negative (i.e., attractive) between 5 and 6 F, strongly J-dependent and of the same order of magnitude as U_d. Beyond 6 F $V_{\mathrm{TEQ},J}{}^d$ can be either repulsive or attractive, and it becomes negligible only beyond 7.5 F. At distances less than 4 F, $V_{\mathrm{TEQ},J}{}^d$ can be as large as 100 MeV or more, and it changes sign repeatedly. The imaginary part of $V_{\mathrm{TEQ},J}{}^d$ is systematically negative for distances beyond 5.5 F, which is consistent with the fact that deuterons are being used up in order to feed the stripping channel. Beyond 6.5 F the imaginary part of $V_{\mathrm{TEQ},J}{}^d$ is remarkably J-independent. For distances in the vicinity of 4 F the imaginary part of $V_{\mathrm{TEQ},J}{}^d$ is seen to be strongly positive for $J=0$ and 2. Despite the fact that $\mathrm{Im}U_d$ is a Woods-Saxon potential of the volume type, the sum $\mathrm{Im}(U_d+V_{\mathrm{TEQ},Jd})$ is surface peaked between 4 and 6 F. For $J=1$ the effect is also seen but is less pronounced. The positive value of $\mathrm{Im}V_{\mathrm{TEQ},J}{}^d$ indicates that deuterons are being reemitted into the deuteron channel due to a recombination of the protons and neutrons. The optical potential description of course has no such reemission mechanism. Possibly the optical potential allows for the reemission by not absorbing the deuterons in the interior of the potential. The reflections of deuterons at the interior turning point may thus simulate the reemission mechanism.

The emission of protons into the stripping channel is closely connected to the absorption of deuterons from the incident channel. After multiplying the imaginary part of the deuteron trivially equivalent potential with the absolute value squared of the deuteron radial function, one obtains the number of deuterons absorbed per unit volume per unit time due to the presence of the stripping channel. According to Eqs. (19) and (19′), this quantity, to within the factor $8\pi(2J+1)/\hbar\kappa_d{}^2$,

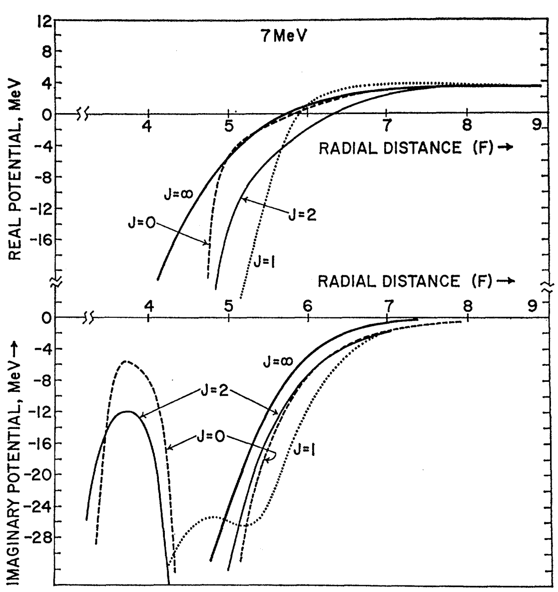

FIG. 3. Deuteron potentials which when used in an uncoupled Schrödinger equation give rise to the same deuteron partial waves as the C.Ch. equations. The curves $J=\infty$ illustrate the potential in the absence of coupling to the stripping channel, and the deviation from these curves represent the potential $V_{\mathrm{TEQ},J}{}^d$ defined in Eq. (16). The imaginary part of $V_{\mathrm{TEQ},J}{}^d$ is seen to be attractive beyond 5.5 F and strongly repulsive in the vicinity of 4 F.

[9] F. G. Perey, in *Direct Interactions and Nuclear Reactions Mechanisms*, edited by E. Clementel and C. Villi (Gordon and Breach Science Publishers, Inc., New York, 1963), p. 125; F. G. Perey and B. Buck, Nucl. Phys. **32**, 353 (1962).
[10] N. Austern, Phys. Rev **137**, B752 (1965).

is equal to the number of protons emitted per unit time per unit volume, and is called the stripping emission density. This function is illustrated in Fig. 4 for three values of the deuteron angular momentum J, calculated by the coupled-channel method, for the stripping reaction to the $2p$ state in ^{41}Ca. It can be seen that the major peak occurs between 6 and 8 F, which lies in the outer region of the nuclear surface, as is to be expected. The negative values between 2 and 5 F are related to the emission of deuterons in this region, as was discussed above.

The purpose of the remainder of this section is to explore the reasons for the difference between the DWBA and C.Ch. stripping cross sections illustrated in Figs. 1 and 2. The difference between the stripping overlap integrals $R_{(Ll)J}$ defined in Eqs. (8) and (15) must of course fully determine the difference between the resulting stripping cross sections. However, the $R_{(Ll)J}$'s are complex quantites and for each value of L, several values of J must be considered. A quantity which is easier to examine is R_J, defined in Eq. (11). Figure 5 illustrates the comparison of the DWBA and C.Ch. values of R_J for three incident deuteron energies. The top half of the figure describes transitions in which the neutron is caputred in a $2p$ state, the middle part illustrates the hypothetical $2s$ transitions, and the bottom part shows the $1f$ transitions. The most prominent feature for the $2p$ transitions is that the DWBA points are alternatively higher (even-J values) and lower (odd-J values) than the corresponding C.Ch. results. However, the sum over J of R_J, which according

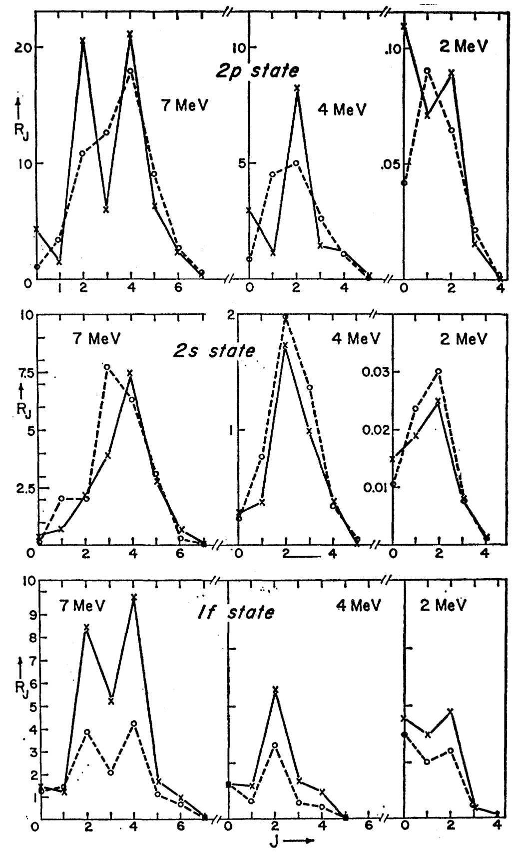

FIG. 5. Absolute value squared of stripping overlap integrals R_J, defined in Eq. (11) as a function of the deuteron angular momentum J. The crosses connected by solid lines represent the DWBA values, the open circles connected by dashed lines are the C.Ch. results. Transitions to $2p$, $2s$, and $1f$ states in ^{41}Ca for the same incident deuteron energy are ordered vertically. Plots which represent transitions to the same state for various deuteron energies are ordered horizontally.

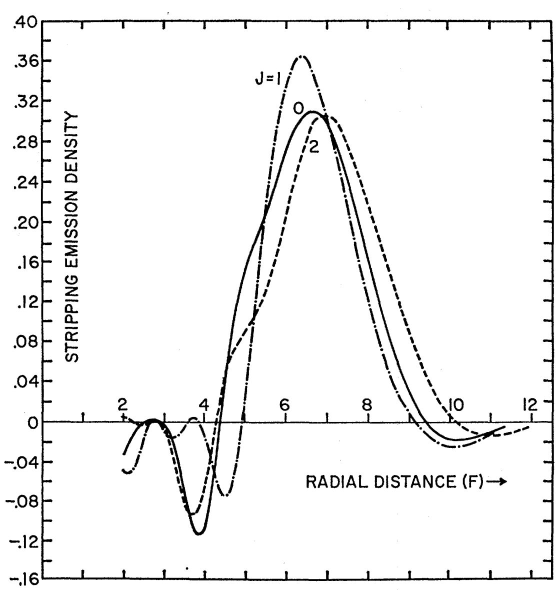

FIG. 4. Stripping emission density, defined in Eq. (19′). This function is proportional to the number of protons emitted by the stripping process per radial interval per second. Negative values indicate absorption of protons and emission of deuterons, which occurs as a result of the recombination of neutrons and protons into deuterons.

to Eq. (10a) is proportional to the total stripping cross section, is nearly the same for both methods of calculation at 7 and 4 MeV; this can be seen from Table III, showing that the difference between the C.Ch. and DWBA nearly averages to zero when the contributions from all J values are considered. At 2 MeV the discrepancy between C.Ch. and DWBA $2p$ cross section is more pronounced for several reasons. The number of J values for which contributions are significant is much smaller than at 7 MeV, and therefore the averaging process is less complete, the difference between the C.Ch. and DWBA values of R_J is now much more pronounced, and the DWBA values are, on the average, systematically higher. By contrast, the behavior of the values for the hypothetical $2s$ transition is quite different. Both the DWBA and C.Ch. values of R_J are now reasonably smooth functions of J, and at both 4 and 2 Mev the C.Ch. values lie systematically higher.

The difference between the DWBA and C.Ch, value of R_J is due to differences in the contributions to the stripping overlap integral which arise in the interior of

TABLE III. Total cross section.[a]

E_d (MeV)		2	4	5	7	11
σ_{2p}[b]	DWBA	4.04	68.49	89.97	103.1	91.96
	C.Ch.	2.78	63.98	77.71	95.07	84.16
σ_{2s}[b]	DWBA	0.966	16.96	24.27	30.23	27.77
	C.Ch.	1.077	21.31	30.71	37.42	30.85
σ_{1f}[b]	DWBA	0.675	18.40	29.36	42.69	46.87
	C.Ch.	0.463	10.94	16.92	22.19	23.69
σ_d[c]	DWBA	14.42	443.60	675.3	979.3	1201
	C.Ch.	18.46	442.5	698.75	1020.5	1253

[a] In units of mb.
[b] (d, p) cross section integrated over angle. The corresponding values listed in Table III of Ref. 1 are in error.
[c] The total deuteron reaction cross section.

the interaction region. For example, Fig. 6 illustrates the comparison between the DWBA and C.Ch. values of the overlap integrand $\mathcal{Y}_J(r)$ [Eq. (12)] for transitions to $2p$ states. This quantity occurs in the expression for $R_{(Ll)J}$, averaged over the various proton L values which are coupled to the same deuteron J value. It is seen from this figure that the DWBA and C.Ch. integrand are very similar at the large distances, but at distances less than 6 F large differences occur. These differences are particularly systematic for the real part of the overlap integrand. For $L=0$ the DWBA excess is negative and tends to add to the main stripping peak, which is also negative and which occurs beyond 6 F. For $J=1$ the DWBA excess is positive and tends to subtract from the main stripping peak beyond 6 F, thus giving rise to a decreased value of R_J. The pattern repeats itself for other J values, and the net result is a systematic enhancement of the DWBA value of R_J for the even values of J, and a decrease for odd J. The top part of Fig. 7 shows the stripping integrand for the case $J=2$, where the discrepancy between C.Ch. and DWBA values of R_J, as seen in Fig. 5, is particularly pronounced. Figure 7 is intended to illustrate the radial distances involved. The top part of the figure shows that the pronounced differences between the C.Ch, and DWBA integrand occur for distances less than 5 F. In this region the optical-model deuteron partial wave functions are, for small J values and to within a complex normalization factor, of the type of a undamped real sine wave, as can be seen from the bottom part of the figure. The region for $r<5$ F could thus be called the "interior region." By contrast, in this region the coupled-channel deuteron partial waves are strongly damped, which explains the difference between the values of the integrand for the two cases. This figure also illustrates that the main contribution to the C.Ch. stripping overlap integral (between 6 and 10 F) comes from distances where the nuclear deuteron or proton potentials are already quite negligible.

Figure 7 also shows that the discrepancies between the DWBA and C.Ch. values of R_J are strongly dependent on the number of nodes of the bound-state neutron wave function. The part of the overlap integrand, illustrated in the top part of the figure, which occurs for distances less than 3 F, would have opposite sign if the bound neutron p state did not have the node near 3 F. In this case the two peaked portions on either side of the node at 3 F would have nearly cancelled and the deviation of R_J from the C.Ch. value would have been

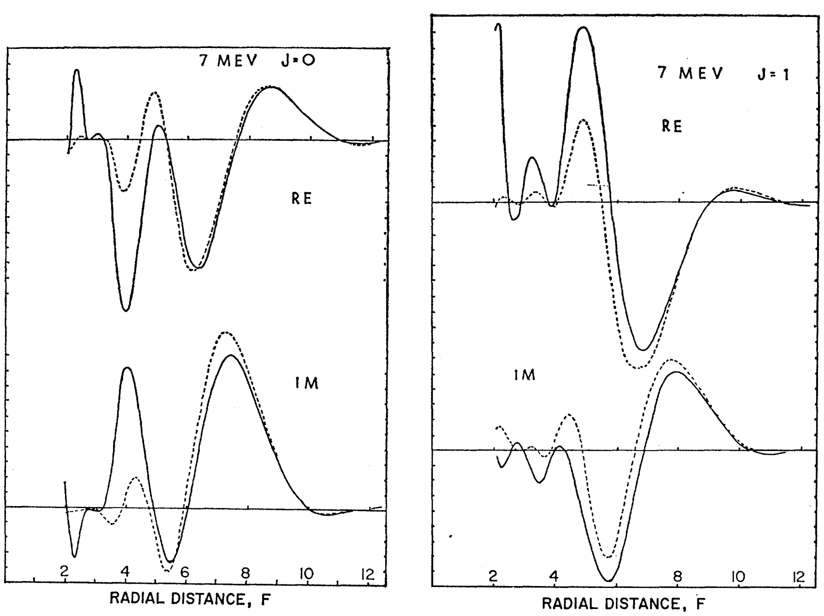

FIG. 6. Integrand $\mathcal{Y}_J(r)$ of a stripping overlap integral, defined by Eq. (12), for transitions to the $2p$ state in ^{41}Ca. The dashed and solid lines represent the C.Ch. and DWBA values, respectively. RE and IM indicate the real and imaginary parts of the integrand, respectively; J is the deuteron's angular momentum. The deuteron energy is 7 MeV. The proton's angular momentum quantum number does not appear since it has been averaged over.

much smaller. Figure 8 illustrates the integrand for the f-stripping radial overlap integrals. The degree of cancellation of the integrands of $R_{(LI)J}$ is also sensitive to the energy of the incident deuteron, the Q value of the reaction, and also to the parity of the captured neutron wave function. For odd parity, a proton angular momentum L which differs from J by a odd number enters the expression for the overlap integral. The phase difference between the proton and deuteron sinelike waves in the interior of the well is $\frac{1}{2}\pi(L-J)$ and hence the product of the two waves leads to a r-dependent structure which is quite different from the case that $L-J$ is even or odd. For the neutron captured in a $2s$ state, $L=J$ and plots of the overlap integrand similar to those in Fig. 6 do not show as pronounced an

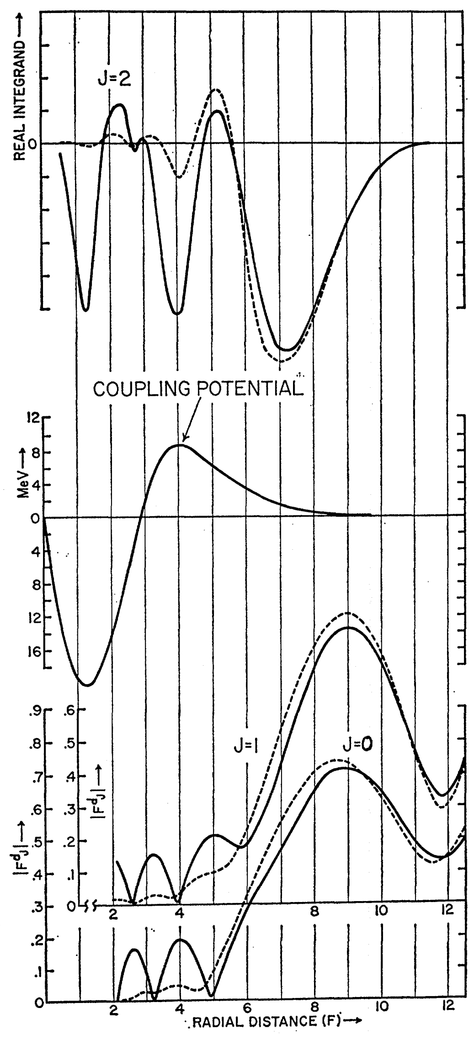

FIG. 7. The upper part of the figure represents the real part of a stripping overlap integrand $\mathcal{Y}_J$ for a $2p$ transition, the bottom part of the figure represents the absolute value of deuteron radial waves $|F_J{}^d|$ defined in Eqs. (1). The deuteron energy is 7 MeV, the solid and dashed lines represent the DWBA and the C.Ch. values, respectively. The middle part of the figure illustrates the $2p$ coupling potential $V_{l=1}$ defined in Eq. (4b).

occurrence of asymmetric peaks and valleys at small distances. This explains the inherent difference in J dependence for the R_J values corresponding to the $2s$-state stripping as compared to the $2p$ case, illustrated in the middle part of Fig. 5. The C.Ch. values for R_J are systematically larger than the corresponding DWBA results. The reason is that the contributions to the DWBA stripping overlap integral now arise mainly from the large surface peaks, since the cancellation of the contribution from the small distances is in this $(2s)$ case more complete than in the $2p$ case. The largest and most outward peak in the integrand has a different sign than its neighbor, which occurs at the smaller distance, and the magnitude of the neighboring peak is more pronounced in the DWBA case than in the C.Ch. case. Therefore a larger cancellation occurs in the DWBA integral, making its value smaller than the C.Ch. value. The difference between the C.Ch. and DWBA values of R_J for this hypothetical s-state transition is therefore connected to the difference between the corresponding deuteron radial wave functions at radial distances near the "surface" of the deuteron nuclear potential.

IV. SUMMARY AND CONCLUSIONS

In the present treatment of deuteron nucleus interaction, a stripping channel and the elastic deuteron channel are coupled to each other. As a result, some of the outgoing elastic deuteron amplitude is due to the process of pickup of a bound neutron by the outgoing proton. This emission is illustrated for the case of ^{40}Ca by the negative portions of the curves shown in Fig. 4, and is seen to occur at small distances, near 4 to 5 F. This reemission of deutrons is very likely related to the existence of a family of various deuteron optical potentials of different depths which will give rise to the same elastic scattering cross section. In the optical-model equation, outgoing deuteron waves in each partial wave are obtained by reflection from the potential gradient which occurs mainly at either the surface or at the interior of the optical potential at the turning point near the origin. The emission of deuterons, which in the C.Ch. description occurs in the vicinity of 4 F, is simulated in the optical-model description by waves of relatively large amplitude which are transmitted through the nuclear interior and are reflected near the origin. It thus appears plausible that the role of the various optical-model deuteron potentials which all give rise to the same elastic scattering cross section is to generate deuteron waves in the nuclear interior which near 4 F are in phase with the additional deuteron amplitude emitted by the mechanism of the recombination of neutrons and protons. The imaginary part of the deuteron complex potential used in the coupled equation is thus more negative at distances less than 4–5 F than the corresponding optical potential; otherwise the C.Ch. description would lead to an emission of deuterons which is too large. At distances between

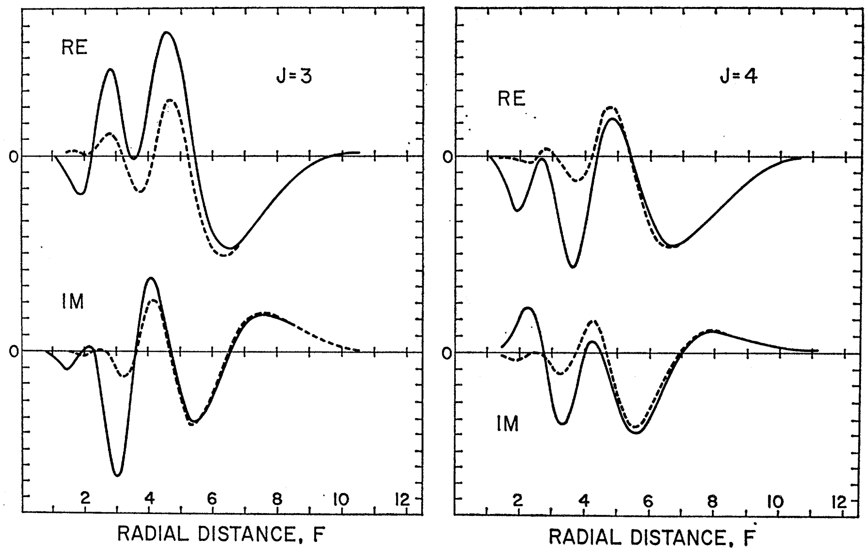

FIG. 8. Same as Fig. 6 for stripping to the $1f$ state in ^{41}Ca. The incident deuteron energy is 11 MeV.

5 and 9 F the incident deuterons are absorbed and give rise to outgoing protons. In this region the imaginary part of the deuteron potential used in the coupled equations is smaller than the imaginary part of the optical potential.

The effect which the stripping channel has upon the deuteron channel can also be illustrated in terms of the "trivially equivalent local potential" $V_{\mathrm{TEQ},J}{}^{d}$ illustrated in Fig. 3 for values of the deuteron angular momentum J of 0, 1, 2, and ∞. The deviations of the curves from that marked $J=\infty$ describe the effect of the coupling to the stripping channels. Figure 3 shows that the effect at large distances is mainly that of an attractive real potential together with an attractive negative surface imaginary part. The systematic deviation of the curves in Fig. 3 from those marked $J=\infty$ is due to the existence of internal degrees of freedom of the deuteron. It is therefore concluded that the method of obtaining the optical dueteron potential from the neutron-nucleus and proton-nucleus optical potentials averaged over the static internal deuteron wave function will not lead to an accurate result, as is borne out by numerical calculation.[11] Differences in the calculation of stripping cross sections arise from the differences in the C.Ch. and optical-model deuteron wave function in the nuclear interior. This is illustrated in Figs. 6–8. It is seen from these figures that the contribution from the interior depends on the energy of the incident deuteron and the Q value of the reaction as well as on the number of nodes of the bound neutron wave function. A quantity which shows at a glance the differences between the two methods of calculation are the absolute value squared of the overlap integrals averaged over the various proton oribtal angular momenta which are coupled to the same total angular momentum value J. These quantities are illustrated in Fig. 5.

The maximum of the stripping cross section calculated by the C.Ch. and DWBA methods are compared in Fig. 2. The differences are particularly pronounced for the f-stripping case. For this case the DWBA result is larger than the C.Ch. result because of contributions to the stripping overlap integrals from the nuclear interior which are present in the DWBA calculation and absent in the C.Ch. treatment. This type of behavior is consistent with the results of Lee *et al.*,[4] who find that a cutoff has a much larger effect on the f-stripping than on the p-stripping cross section.

Comparison of the present results with experiment for the ^{40}Ca$(d,p)^{41}$Ca reaction is still premature because stripping to the f states has not yet been explicitly incorporated into the coupled equations. The general conclusion from the present study is that the coupling of stripping channels to the elastic deuteron channel could introduce modifications in the value of the spectroscopic factor by about 30% or more compared to the DWBA results. These corrections appear to decrease with increasing incident deuteron energy.

ACKNOWLEDGMENTS

The authors are grateful to the group at Edmonton, quoted in Ref. 5, for helpful correspondence concerning their experimental results, prior to publication. The calculations were carried out mostly at the Computer Center of the University of Connecticut, which is supported in part by Grant No. GP-1819 of the National Science Foundation.

[11] F. G. Perey and G. R. Satchler, Nucl. Phys. **A97**, 515 (1967).