Phase-Shift Analysis of $^{16}O(\alpha, \alpha)^{16}O$ Scattering from 5 to 10 MeV*

JOSEPH JOHN,† J. P. ALDRIDGE, AND R. H. DAVIS
Department of Physics, Florida State University, Tallahassee, Florida 32306
(Received 18 November 1968)

Angular distributions for the elastic scattering of α particles by ^{16}O have been measured at 10-keV intervals from a bombarding energy of 5.0–12.5 MeV. Real phase shifts have been extracted by means of a partial-wave analysis of the data between 5 and 10 MeV. Twenty-seven resonances are found in the ^{20}Ne excitation energy range of 8.73–12.77 MeV. Seven of the corresponding levels in ^{20}Ne have not been previously reported. Spins and parities are assigned to 26, confirming a number of previously tentative assignments and correcting a few. Resonance widths are estimated for most of the levels.

I. INTRODUCTION

THE region of ^{20}Ne excitation energy below 15 MeV, which includes the present work, has previously been investigated by Butler,[1] Ritter,[2] Sargood and Putt,[3] Alexander *et al.*,[4] and Ferguson *et al.*[5] using the $^{19}F(d, n)^{20}Ne$ and the $^{19}F(d, n\gamma)^{20}Ne$ reactions. Of particular importance with respect to the assignment of level parameters in this region are the $^{16}O(\alpha, \gamma)^{20}Ne$ experiments of Pearson and Spear[6] and a series of experiments[7–12] performed at the Chalk River Laboratories using the $^{12}C(^{12}C, \alpha)^{20}Ne$ and $^{12}C(^{12}C, \alpha\gamma)^{20}Ne$ reactions.

The elastic scattering of α particles by ^{16}O was first studied by Ferguson and Walker[13] in 1940 using α particles from RaC′. They observed resonances at bombarding energies of 5.5 and 6.5 MeV. In 1953, Cameron[14] performed the first refined experiment in which α particles from an electrostatic accelerator were scattered from a gaseous oxygen target in the energy range 0.94–4.0 MeV. Five levels in ^{20}Ne were found in the corresponding region of excitation energy of 5.5–7.0 MeV. Also employing a gas target, McDermott *et al.*[15] extended the energy range to an excitation energy of 9.9 MeV in ^{20}Ne, and assignments were made for eight levels. A number of experiments[16–21] have been performed at energies between 15 and 65 MeV.

In the most recent work, Hunt *et al.*[22] covered the bombarding energy range 5.8–10.0 MeV, and Mehta *et al.*[23] extended the energy range to 19.0 MeV. The former measured excitation functions at ten angles and the latter at eight angles ranging from 90° to 163.8° in the c.m. system. Their measurements were made in 50-keV steps using LiOH targets which were 25–35-keV thick to 10-MeV α particles. At bombarding energies below 12.5 MeV, 29 resonances were observed. Spins and parities of most of these levels could be assigned by examining the excitation functions at angles corresponding to the zeros of Legendre polynomials for absence of structure. Several sets of states in ^{20}Ne that obeyed rotational band systematics were identified.

There were two inherent disadvantages in the experiments reported by Hunt *et al.*[22] and Mehta *et al.*[23] First, the use of thick targets and energy increments of 50 keV necessarily implies that resonances which are narrow compared to the energy resolution could be overlooked. Second, when resonances are closely spaced or overlapping, as is often true in this energy range, interference effects and structure due to resonances other than the one being considered could lead to an incorrect assignment of spins. For example, when two or more partial waves resonate at nearly the same

* Supported in part by the U.S. Air Force Office of Scientific Research, Office of Aerospace Research, U.S. Air Force, under AFOSR Grant No. AF-AFOSR-440-67; the National Science Foundation (Grant No. NSF-GP-5114).
† Present address: Gulf General Atomics, Inc., San Diego, Calif.
[1] J. W. Butler, Phys. Rev. **118**, 222 (1960).
[2] R. C. Ritter, Nucl. Phys. **80**, 377 (1966).
[3] D. G. Sargood and G. D. Putt, Australian J. Phys. **17**, 56 (1964).
[4] T. K. Alexander, J. D. Pearson, A. E. Litherland, and C. Broude, Phys. Rev. Letters **13**, 86 (1964).
[5] A. T. G. Ferguson, N. Gale, G. C. Morrison, and R. E. White, in *Proceedings of the International Conference in Direct Reactions and Reaction Mechanisms*, edited by E. Clementel and C. Villi (Gordon and Breach Science Publishers, Inc., New York, 1963), p. 510.
[6] J. D. Pearson and R. H. Spear, Nucl. Phys. **54**, 434 (1964).
[7] J. D. Pearson, E. Almqvist, and J. A. Kuehner, Can. J. Phys. **42**, 489 (1964).
[8] J. A. Kuehner and E. Almqvist, Can. J. Phys. **45**, 1605 (1967).
[9] P. J. M. Smulders, C. Broude, and A. E. Litherland, Can. J. Phys. **45**, 2133 (1967).
[10] J. S. Kuehner, in *Proceedings of the Conference on Bases for Nuclear Spin-Parity Assignment*, edited by N. B. Gove and R. L. Robinson (Academic Press Inc., New York, 1966), p. 146; C. Broude, *ibid.*, p. 162.
[11] J. A. Kuehner and R. W. Ollerhead, Phys. Letters **20**, 301 (1966).
[12] J. A. Kuehner and J. D. Pearson, Can. J. Phys. **42**, 477 (1964).
[13] A. J. Ferguson and L. R. Walker, Phys. Rev. **58**, 666 (1940).
[14] J. R. Cameron, Phys. Rev. **90**, 839 (1953).
[15] L. C. McDermott, K. W. Jones, H. Smotrich, and R. E. Benenson, Phys. Rev. **118**, 175 (1960).
[16] J. C. Corelli, E. Bleuler, and D. J. Tendem, Phys. Rev. **116**, 1184 (1959).
[17] J. C. Jodogne, P. C. Macq, and J. Steyaert, Phys. Letters **2**, 325 (1962).
[18] D. E. Blatchley and R. D. Bent, Nucl. Phys. **61**, 641 (1965).
[19] J. Aguilar, W. E. Burcham, J. Catala, J. B. A. England, J. S. C. McKee, and J. Rotblat, Proc. Roy. Soc. (London) **A254**, 395 (1960).
[20] A. I. Yavin and G. W. Farwell, Nucl. Phys. **12**, 1 (1959).
[21] B. G. Harvey, E. J. M. Rivet, A. Springer, J. R. Meriwether, W. B. Jones, J. H. Elliott, and P. Dariulat, Nucl. Phys. **52**, 465 (1964).
[22] W. E. Hunt, M. K. Mehta, and R. H. Davis, Phys. Rev. **160**, 782 (1967).
[23] M. K. Mehta, W. E. Hunt, and R. H. Davis, Phys. Rev. **160**, 791 (1967).

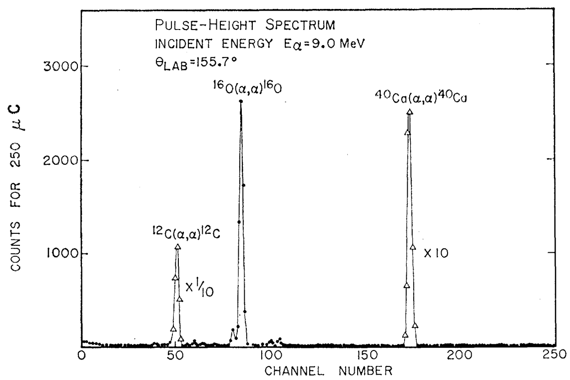

FIG. 1. A typical pulse-height distribution.

energy, an effect is seen at all angles as if there were a single resonance for the $l=0$ partial wave.

The cross sections for the elastic scattering of particles by ^{16}O have been remeasured under improved experimental conditions. Fourteen-point angular distributions were measured every 10 keV and 56-point angular distributions every 100 keV throughout the bombarding energy range 5.0–12.5 MeV. A partial-wave analysis has been performed for each angular distribution beginning at 5 MeV and the real phase shifts extracted. This analysis is truncated at 10 MeV since real phase shifts do not describe the data above that energy and the restrictions on the admissible values of the phase shifts are seriously relaxed when they are complex.

Twenty-seven resonances have been identified, and J^π assignments have been made for 26 of the corresponding levels in ^{20}Ne. Most of the spins and parities are in agreement with earlier, often tentative, assignments. Seven new states, not previously reported, have been observed. In several cases, previous assignments were found to be in error and correct parameters have been assigned.

II. EXPERIMENTAL ARRANGEMENT AND PROCEDURE

The scattering measurements were made in the 18-in.-diam stainless-steel scattering chamber described by Feldl *et al.*[24] The α-particle beam[25] from the Florida State University Tandem Van de Graaff accelerator was collimated by two 0.05×0.15 in. tantalum slits positioned at 9.75 and 27.125 in. from the target. A third slit, 0.1×0.2 in., was located 7 in. from the target to prevent the slit-scattered particles from reaching any of the detectors. During the experiment, the pressure in the scattering chamber was maintained below 1×10^{-5} Torr.

Fourteen surface-barrier detectors mounted at 10° intervals on a multiple detector ring[26] constituted the detection system. Pulses from each of these detectors were amplified using transistorized charge-sensitive preamplifiers mounted just outside the scattering chamber. The output pulses were suitably amplified and then analyzed and stored in a Technical Measurements Corp. 4096-channel analyzer. A typical pulse-height spectrum is displayed in Fig. 1.

The accuracy of the relative solid angles for the detectors was checked by comparing measured angular distribution of α particles elastically scattered by a very thin gold target with the calculated Rutherford cross sections. These measurements were made at bombarding energies below 8.0 MeV, where the scattering can be adequately accounted for by the Rutherford cross section for the full angular range.

The targets were a mixture of calcium oxide and calcium hydroxide, which served as targets for both this $^{16}O(\alpha,\alpha)^{16}O$ experiment and a $^{40}Ca(\alpha,\alpha)^{40}Ca$ experiment. Their preparation and certain other experimental details common to both experiments are described in a paper by John *et al.*[27] In all cases, the total energy loss in the target was held at less than 8 keV.

The statistical error was typically $\pm1\%$ in most of the detection systems. At angles where deep minima in the cross section were present, this error may be as high as $\pm3\%$. The systematic error of $\pm8\%$ has been assigned to the absolute cross section for oxygen.

Since the targets used are a combination of CaO and $Ca(OH)_2$, no direct measurement of the target thickness was attempted. The data were normalized to those of McDermott *et al.*[15] at 5.6, 5.7, and 5.8 MeV where the excitation function was relatively featureless. The angular distributions consisting of 56 data points at each of these energies were interpolated using a Legendre polynomial fit to obtain the values of the cross sections at c.m. angles of 90°, 125.3°, 140.8°, 149.4°, and 168.9°, where the measurements of McDermott *et al.*[15] were made. The normalized cross sections are as much as 50% larger than those obtained by Hunt *et al.*[22] and Mehta *et al.*[23] This discrepancy is probably accounted for by the fact that these authors estimated the thickness of their SiO_2 targets by weighing. The normalized cross section obtained in this experiment was compared to an absolute cross section at 8.7 MeV measured by Bisson[28] using a gas cell. The two measurements were in agreement to within 4%.

[24] E. J. Feldl, P. B. Weiss, and R. H. Davis, Nucl. Instr. Methods **28**, 309 (1964).
[25] Joseph John, C. P. Robinson, J. P. Aldridge, W. J. Wallace, K. R. Chapman, and R. H. Davis, IEEE Trans. Nucl. Sci. **NS14**, 82 (1967); Nucl. Instr. Methods **57**, 105 (1967).
[26] J. P. Aldridge, G. E. Crawford, and R. H. Davis, Phys. Rev. **167**, 1053 (1968).
[27] Joseph John, C. P. Robinson, J. P. Aldridge and R. H. Davis, Phys. Rev. **177**, 1755 (1969).
[28] A. Bisson (private communication).

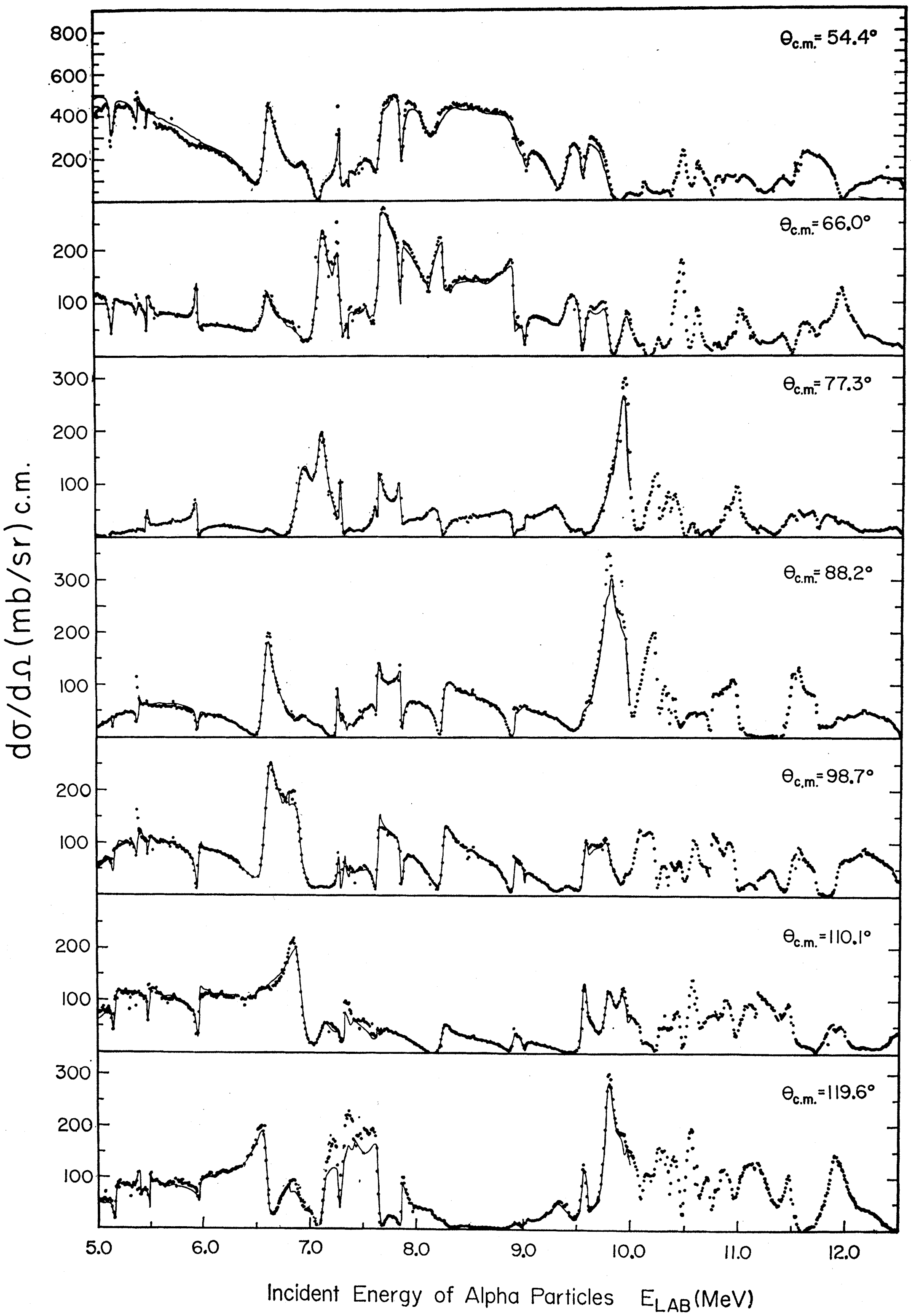

FIG. 2. Forward-angle excitation curves for $^{16}O(\alpha, \alpha)^{16}O$. The data are represented by dots. The solid curves between 5.0 and 10.0 MeV are the fits to the excitation curves obtained from the phase-shift analysis described in the text.

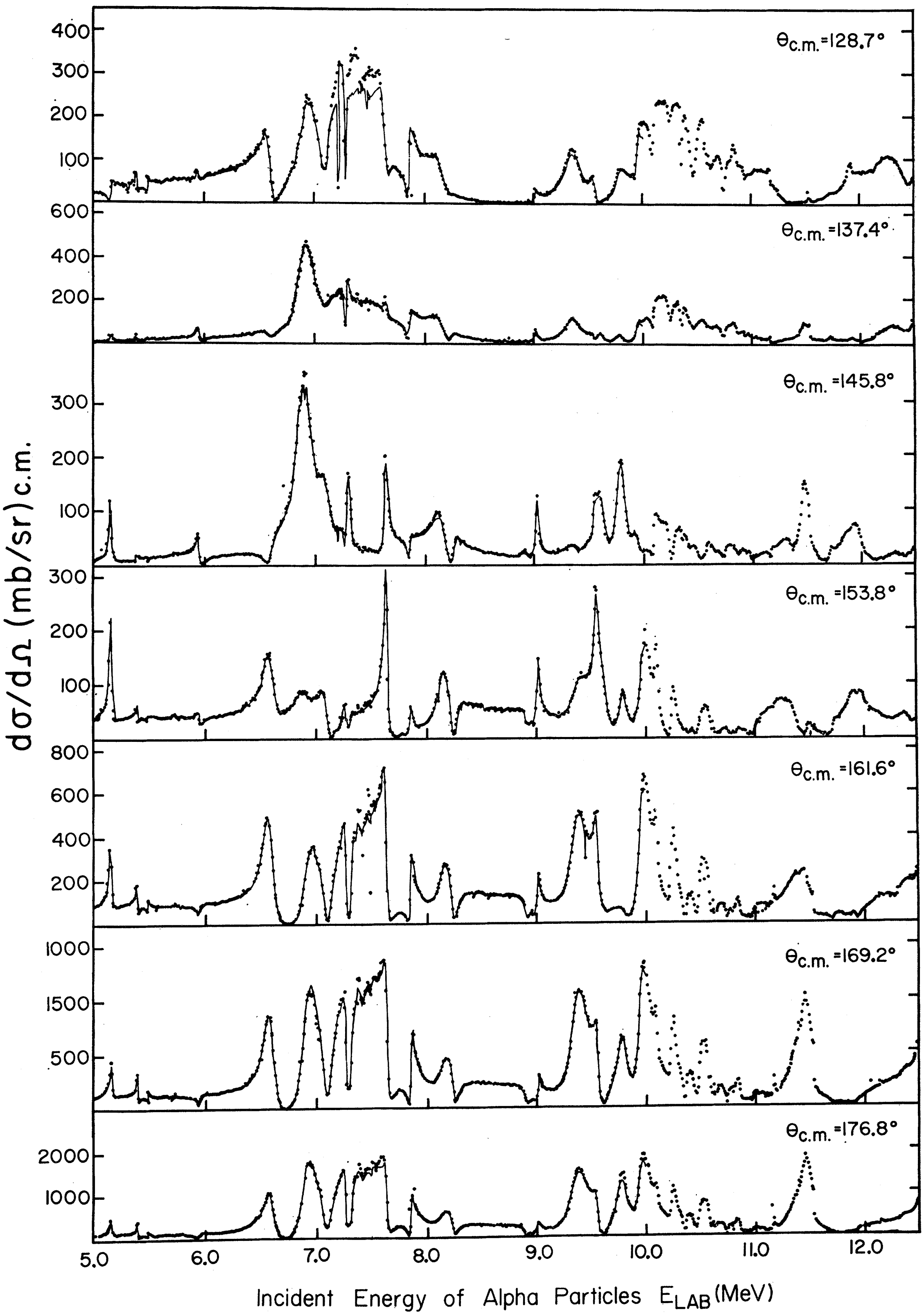

FIG. 3. Backward-angle excitation curves for $^{16}O(\alpha, \alpha)^{16}O$. The data are represented by dots. The solid curves between 5.0 and 10.0 MeV are the fits to the excitation curves obtained from the phase-shift analysis described in the text.

Excitation functions at 14 angles are displayed in Figs. 2 and 3. The solid curves between 5 and 10 MeV represent the cross sections calculated in the phase-shift analysis.

These figures display five narrow resonances below 6.0 MeV, which have been previously observed by McDermott *et al.*[15] A disagreement as much as 70 keV in the energy position of these resonances is found. For example, the level appearing at a bombarding energy of 5.96 MeV is the same 2^+ level observed by McDermott *et al.*[15] at 6.03 MeV. However, the value of 5.96 MeV found in this experiment is in good agreement with the value observed by Hunt *et al.*[22] and with the value of 5.94 MeV reported by Pearson and Spear[6] from their study of the $^{16}O(\alpha, \gamma)^{20}Ne$ reaction.

III. PHASE-SHIFT ANALYSIS

The measured angular distributions were fitted with tne usual expression for the differential cross section as a function of partial-wave phase shifts:

$$d\sigma/d\Omega = k^{-2} \left| -\tfrac{1}{2}\eta \csc^2(\tfrac{1}{2}\theta) \exp(i\eta \ln \csc^2\tfrac{1}{2}\theta) + \sum_l (2l+1) \exp[i(2\alpha_l+\delta_l)] \sin\delta_l P_l(\cos\theta) \right|^2, \quad (1)$$

where η is the Sommerfeld parameter $ZZe^2/\hbar v$, k is the wave number, and α_l is the relative Coulomb phase shift. The Florida State University CDC-6400 computer was programmed to calculate the differential cross section for a given set of real phase shifts δ_l and to measure the discrepancy between the calculated cross section and the experimental cross section by constructing the quantity χ^2 given by

$$\chi^2 = N^{-1} \sum_{i=1}^{N} \frac{[\sigma_e(\theta_i) - \sigma_c(\theta_i)]^2}{[\Delta\sigma(\theta_i)]^2}. \quad (2)$$

Here N is the number of data points, $\sigma_e(\theta_i)$ and $\sigma_c(\theta_i)$ are the measured and calculated cross sections, respectively, for the angle θ_i. The experimental error in the cross section is $\Delta\sigma(\theta_i)$.

The analysis was started at 5.0 MeV, the lowest energy at which data were obtained in this experiment. From a consideration of the hard-sphere phase shifts and the nuclear penetrabilities, it was concluded that variations in the phase shifts δ_0 up through δ_4 only would be necessary to fit the data. A computer program was used to scan the five-dimensional space of the phase shifts $\delta_0, \delta_1, \cdots, \delta_4$. The program computed χ^2 for a single set of phase shifts and then incremented the phase shifts in steps of 18°, one at a time, until the calculated cross sections for all possible combinations of the δ_l's between 0° and 180° had been compared to the data. All combinations of phase shifts that produced a χ^2 of less than 100 were used as starting values for a search to fit the data at 5.0 MeV.

The least-squares fitting program employed two search methods to locate the minimum. One method was to vary a single phase shift until a minimum value tor χ^2 was obtained. This was repeated for the other phase shifts until a minimum for variations of all phase shifts was obtained or a prescribed number of variations of all the phase shifts had been made. A second more efficient method was a modified gradient search described by Fletcher and Powell,[29] in which all phase shifts were varied simultaneously to minimize χ^2.

Most of the searches starting with the grid scan values ($\chi^2 < 100$) converged to one set of phase shifts that adequately described the data. Several combinations, however, converged to a second set of phase shifts, but the χ^2 value obtained with the latter was nearly 10 times that obtained from the first set. This indicated that the former set is probably the "correct" set of phase shifts. This was further confirmed by the fact that the values of δ_0 and δ_2 were in agreement with those deduced from the resonant phase shifts given by McDermott *et al.*[15]

The final values of δ_l obtained at 5.0 MeV were then used as starting values for a search to fit the angular distribution at 5.01 MeV. This procedure of using the final values of the phase shifts at one energy as the starting values at the next higher energy was the routine for fitting the angular distributions as functions of energy. The phase shifts $\delta_0, \delta_1, \cdots, \delta_4$ were found to vary smoothly with energy except in the neighborhood of a resonance where the phase shift corresponding to the resonant partial wave increased through nearly 180°.

Above 6.0 MeV, no combination of partial waves up through $l=4$ satisfactorily fitted the data, which indicated that δ_5 was significantly different from zero. The addition of δ_5 as a fitting parameter produced very satisfactory fits. Parametrization with phase shifts up through $l=5$ was sufficient to fit the data up to 7.0 MeV, and the inclusion of δ_6 sufficed up to 10.0 MeV.

Above 10.0 MeV, however, the fits were very poor. Variations of δ_7 and δ_8 did not improve the fits to any significant extent. Since the low-lying levels of ^{16}O can be significantly excited by inelastic α-particle scattering above 10 MeV, the scattering can no longer be accounted for by real phase shifts.

The real phase shifts determined below 10.0 MeV are the starting values being used by Meyers *et al.*[30] in a complex phase-shift analysis of the data above 10.0 MeV. Progress in this work demonstrates the difficulty with ambiguities found in a complex phase-shift analysis, where compound system states are closely spaced.

At substantially higher energies, the ambiguity

[29] R. Fletcher and M. J. D. Powell, Computer J. **6**, 163 (1963).
[30] W. J. Meyers, J. P. Aldridge, and R. H. Davis (unpublished).

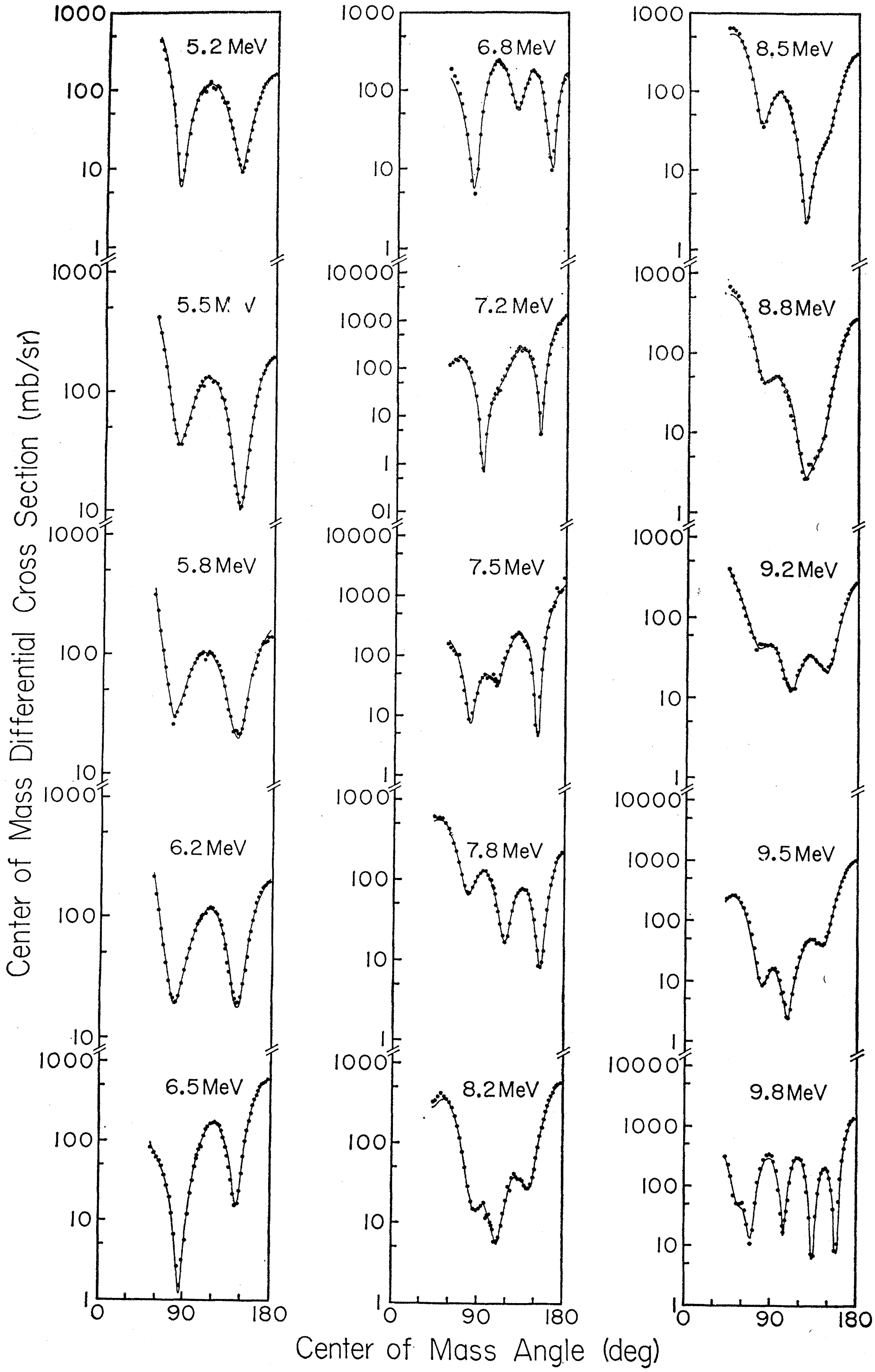

FIG. 4. Angular distributions for $^{16}O(\alpha, \alpha)^{16}O$ illustrating the quality of the phase-shift fits. The data are represented by dots and the curves are the phase-shifts fits.

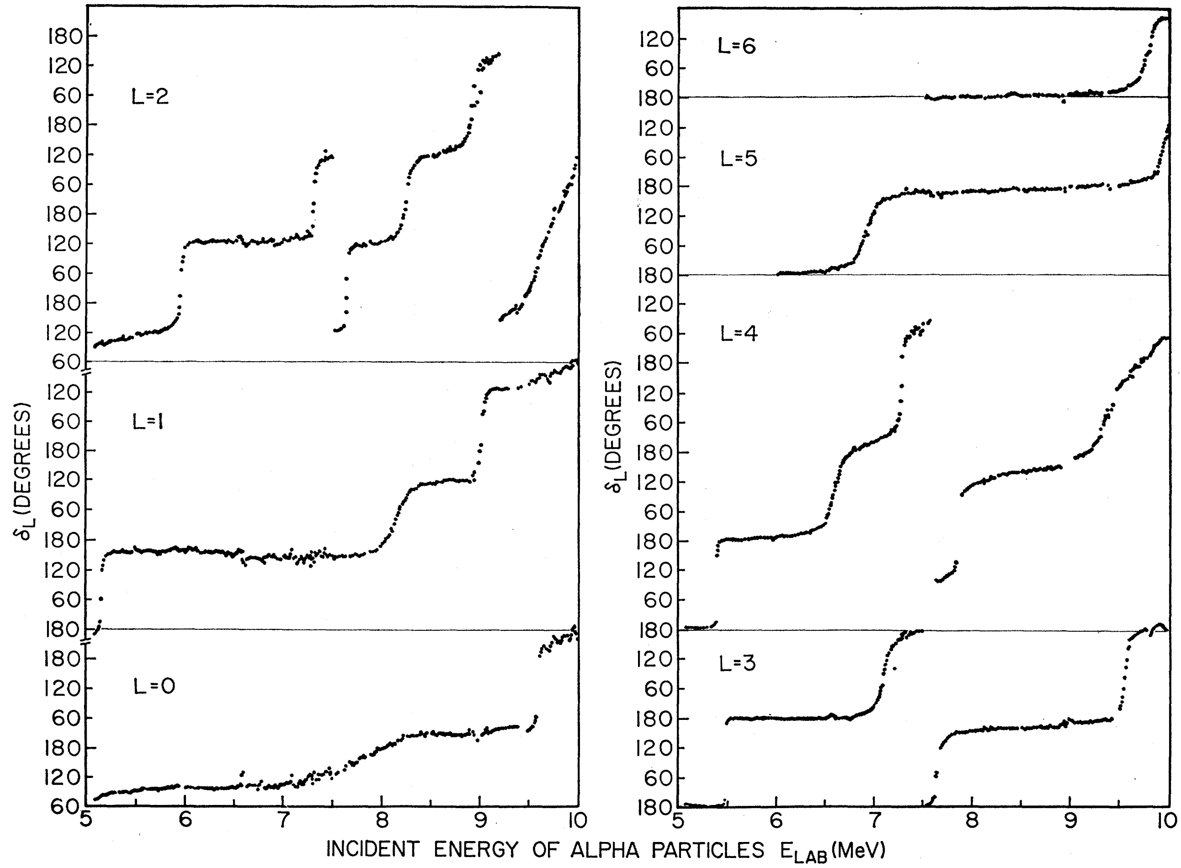

FIG. 5. Phase shifts determined for $^{16}O(\alpha, \alpha)^{16}O$ from the analysis described in the text for the $l=0$ through $l=6$ partial waves. The $l=2$ phase-shift plot is displaced by an integral number of 180° near 7.5 MeV and again at 9.0 MeV. Similar displacements of the $l=3$ and 4 phase shifts are shown near 7.5 MeV.

problem which precludes a unique phase-shift analysis can be turned to advantage in model-dependent analyses. Mehta *et al.*[23] used a smoothed, strong absorption model to compute the phase shifts except for one or more corresponding to the higher-order partial waves which were sensitive to a comparison with the data. In model-dependent analyses, the phase shifts may be computed with the assurance that the results will fall in close proximity to one or more of the ambiguous sets of complex phase shifts if the model represents the data well.

In the vicinity of narrow resonances below 10 MeV, simultaneous variation of all the phase shifts often resulted in unsatisfactory fits to the data. This caused either the quantity χ^2 to be too large to be acceptable or the phase shifts to vary irregularly with energy. The energy resolution of the experiment was approximately 8 keV, which means that the cross sections for resonances with widths less than 8 keV are averaged. Such averaged data do not correspond to single values of real phase shifts. Narrow resonances were traversed by using the first search method to vary one phase shift at a time, the resonant phase shift being varied first. Using this technique, satisfactory fits were usually obtained.

Of the 751 angular distributions that have been measured in this experiment, 500 have been analyzed in terms of the phase shifts. Fifteen of these are shown in Fig. 4, in which the continuous lines are the calculated differential cross sections. The agreement between the data and the calculated cross sections is excellent with χ^2 values typically between 1.0 and 1.5. Near a narrow resonance, however, χ^2 may be as large as 6.0.

The fits to the angular distributions displayed in Fig. 4 are quite representative of those obtained in all the 500 angular distributions which have been analyzed. The computed excitation curves are the solid lines shown in Figs. 2 and 3 between 5.0 and 10.0 MeV. The agreement is very good except perhaps at 119.6° and 128.7° between 7.0 and 7.5 MeV.

In order to make certain that another set of phase shifts which would equally well describe the observed

TABLE I. Comparison of resonant energies and J^π assignments for levels in ^{20}Ne with previously published results.

Present experiment $^{16}O(\alpha, \alpha)^{16}O$		Hunt, Mehta, Davis[a] $^{16}O(\alpha, \alpha)^{16}O$		McDermott *et al.*[b] $^{16}O(\alpha, \alpha)^{16}O$		Chalk River[c] $^{12}C(^{12}C, \alpha)^{20}Ne$ $^{12}C(^{12}C, \alpha\gamma)^{20}Ne$		Pearson and Spear[d] $^{16}O(\alpha, \gamma)^{20}Ne$	
E_x (MeV)	J^π	E_x (MeV)	J^π	E_x (MeV)	J^π	E_x (MeV)	J^π	E_x (MeV)	J^π
				~8.7	0^+				
				8.755	1^-	8.71	1^-		
				~8.8	2^+				
8.78				8.84	(5^-)	8.79	6^+		
8.85	1^-			8.905	1^-	8.87	1^-		
9.05	4^+			9.099	4^+	9.04	4^+		
9.12	3^-			9.179	3^-	9.11	3^-		
						(9.31)			
9.49	2^+	9.50	2^+	9.577	2^+	9.48	2^+	9.48	2^+
9.98	4^+	9.99	4^+					10.02	2^+
						10.24	$2^+, T=1$	10.270	$2^+, T=1$
10.26	5^-	10.30	5^-						
10.40	3^-	10.49	(3^-)						
10.55	4^+	10.55	(0^+)			10.57	4^+		
10.58	2^+								
						10.64	6^-		
10.79	4^+	~10.7	4^+						
10.837	2^+	10.83	(0^+)						
10.838	3^-								
10.97	0^+					11.0			
11.02	4^+	11.03	4^+					11.08	(4^+)
11.23	1^-							11.27	$(1^-, 2^+)$
11.33	2^+	11.29	(2^+)						
		11.6	(2^+)			11.7		11.56	(4^+)
11.87	2^+	(11.89)	$(2^+, 6^+)$						
11.96	1^-	11.99	(1^-)						
						11.99	8^+		
12.22	4^+	12.27	(4^+)			12.19	6^+	12.25	$(\leq 4^+)$
12.35	2^+								
12.37	3^-	12.39	$(0^+, 1^-, 3^-)$					12.39	
12.41	0^+								
12.56	6^+	12.58	6^+						
12.62	2^+								
12.68	5^-								
12.77[e]	4^+								

[a] Reference 22.
[b] Reference 15; see also Ref. 7.
[c] References 7–12.
[d] Reference 6.
[e] Extrapolated value. Resonance previously reported in Ref. 23.

TABLE II. Parameters of levels in ^{20}Ne.

l	E_R (Lab keV)	E_x (MeV)	Γ_{cm} (keV)	γ^2 (keV)	$(\theta^2)^a$
0	7 800±150	10.969	576	93	0.14
0	9 605±5	12.412	≤8	≤1	$<1.5\times10^{-3}$
1	5 152±5	8 851	19	7.76	1.1×10^{-2}
1	8 132±30	11.234	172	28.5	4.2×10^{-2}
1	9 033±10	11.955	5.6	0.81	1.2×10^{-3}
2	5 955±10	9.493	24	9.6	1.4×10^{-2}
2	7 314±10	10.580	24	5.8	8.5×10^{-3}
2	7 635±5	10.837	13	2.89	4.2×10^{-3}
2	8 246±10	11.325	53	10.2	1.5×10^{-2}
2	8 930±20	11.873	46	7.83	1.1×10^{-2}
2	9 530±100[b]	12.352			
2	9 860±100[b]	12.616			
3	5 486±5	9.118	3.2	3.36	4.9×10^{-3}
3	7 092±5	10.402	81	32.4	4.8×10^{-2}
3	7 636±5	10.838	45	14.5	2.1×10^{-2}
3	9 550±10	12.368	40	7.85	1.2×10^{-2}
4	5 395±5	9.046			
4	6 569±10	9.984	97	118.1	0.17
4	7 276±5	10.550	16	12.2	1.8×10^{-2}
4	7 580±100	10.793	349	224.6	0.33
4	7 860±10	11.017	24	13.4	2.0×10^{-2}
4	9 365±20	12.221	142	44.7	6.6×10^{-2}
4	10 050±100[c]	12.768			
5	6 912±5	10.258	141	45.2	0.66
5	9 944±15	12.684	97	49.6	7.3×10^{-2}
6	9 790±10	12.560	101	155.3	0.23

[a] $\theta^2=2\gamma^2\mu R^2/3\hbar^2$. [b] $\beta_l=90°$. [c] Extrapolated value.

cross section did not exist, a grid search in 18° interval for each of the phase shifts was made at several energies. Each of the combinations of δ_l's that gave a χ^2 value of less than 100 was the starting value set for the automatic search to minimize χ^2. In most cases the search converged to the accepted set. At some energies, different minima were located but the solutions did not produce acceptable fits.

At 5.5 MeV, however, an alternate set of phase shifts was found that produced a χ^2 comparable to that obtained with the accepted set of phase shifts. This alternate set was traced as a function of energy, and varied smoothly with energy up to 5.95 MeV. At this energy the accepted set showed an $l=2$ resonance. The alternate set showed an antiresonant behavior, i.e., δ_2 dropped by nearly 40° every 10 keV. Since the phase shifts cannot decrease with energy more rapidly than the corresponding hard-sphere phases, this was not an acceptable set.

IV. RESULTS

In Fig. 5 the phase shifts obtained by fitting the data are plotted as functions of the laboratory bombarding energy. At the lowest energy δ_0 and δ_2 are significantly different from zero. This is attributed to two broad resonances, an $\iota=0$ resonance at 4.9 MeV and an $l=2$ resonance at 5.1 MeV bombarding energy observed by McDermott *et al.*[15] The phase shift corresponding to the $l=0$ partial wave remains fixed at nearly 100° up to about 7.0 MeV. It exhibits a broad resonance at a bombarding energy of about 7.8 MeV and a narrow resonance at 9.61 MeV.

The phase shift δ_1 displays three resonances at bombarding energies of 5.15, 8.13, and 9.03 MeV. Five resonances for the $l=2$ partial wave have been observed below 9.0 MeV. Above this bombarding energy there appear to be two $l=2$ resonances that are not resolved, as evidenced by the fact that δ_2 increases rapidly through 360°. A similar feature is exhibited by the $l=4$ phase shift. Resonances in the $l=3$ partial wave are found at 5.49, 7.09, 7.64, and 9.55 MeV incident energy. Phase shift δ_5 passes through two resonances and δ_6 is relatively featureless, excpet for one resonance at a bombarding energy of 9.79 MeV.

The resonance energies and J^π assignments are summarized in Table I. Included for comparison are the results of other experiments involving the α-particle bombardment of ^{16}O and the ^{12}C(^{12}C, α)^{20}Ne reaction. The levels observed in the ^{19}F(d, n)^{20}Ne and the ^{19}F(d, $n\gamma$)^{20}Ne reactions are not included in this table because of the absence of definite spin and parity assignments. The first level at 8.78 MeV was too narrow to be seen clearly. Its effects are observed at several angles at a bombarding energy of 5.06 MeV and are consistent with a high spin value (see Table I), but do not permit a determination. The next seven states listed in the table are in agreement with earlier assignments of spins and parities.

The 2^+ state in ^{20}Ne at an excitation energy of 10.58 MeV has not been reported previously. This is perhaps due to the existence of a 4^+ state at 10.55 MeV that would mask the effects of the narrow 2^+ state.

A 2^+ state is shown at 10.837 MeV and a 3^- state at 10.838 MeV excitation energy in ^{20}Ne, well within the experimental uncertainty for each energy. Because these states are less than 10 keV apart, their combined effect was tentatively interpreted as a single 0^+ state by Hunt *et al.*[22]

The broad state at 10.97 MeV excitation energy in ^{20}Ne has previously been seen in the ^{12}C(^{12}C, α)^{20}Ne experiment.[7–12] This state has now been assigned a spin and parity of 0^+. The 4^+ state at 11.02 MeV is probably the same as the 4^+ state observed by Hunt *et al.*[22] at 11.03 MeV and may be compared to the level reported by Pearson and Spear[6] at 11.08 MeV.

The five states in ^{20}Ne in the range of excitation energies 11.23–12.22 MeV have been previously reported and tentative assignments made. These have been assigned definite spins and parities.

The ^{20}Ne levels at 12.35 MeV (2^+) and 12.41 MeV (0^+) are new if the broader 3^- level at 12.37 MeV is identified with the one previously reported at 12.39 MeV. The 6^+ state at 12.56 MeV has been previously observed.

Two new states are observed at excitation energies 12.62 and 12.68 MeV. The last entry at excitation energy 12.77 MeV is a broad 4^+ level which is located by extrapolation just above the upper end of the energy range of this analysis ($E_R=10.050\pm0.1$ MeV) and has been observed by Mehta *et al.*[23]

In additions to spins, parities, and resonance energies, the phase shifts contain information about the widths of nuclear states through the energy dependence. In order to obtain this information, the resonant phase shifts β_l were obtained from the relation

$$\beta_l=\delta+\tan^{-1}(F/G_l), \qquad (3)$$

where F_l and G_l are the regular and irregular Coulomb wave functions evaluated at the interaction radius, $a=5.34$ F, following McDermott *et al.*[15]

Although a precise determination of the widths should take into account the fact that several levels of the same spin and parity are observed, an estimate of the widths is afforded by assuming that β_l is represented by

$$\beta_l=\tan^{-1}[\tfrac{1}{2}\Gamma/(E_R-E)], \qquad (4)$$

where Γ is the resonance width and E_R is the resonance energy. From Γ, the reduced width γ_λ^2 is obtained using the relation

$$\Gamma=2P_l\gamma_\lambda^2, \qquad (5)$$

where P_l is the penetrability factor. The resonance parameters extracted from the phase shifts are summarized in Table II.

Most of the states have widths which represent only a small fraction of the single-particle limit. However, the states at 6.57, 6.91, 7.58, 7.8, and 9.79 MeV (Lab) possess widths which are a significant fraction of the Wigner limit. Although reduced widths for the 9.53 and 9.86 MeV 2^+ states are not given, it is apparent that these states possess significant widths and contribute to the 2^+ strength near 12-MeV excitation energy. The 4^+ state at 10 MeV is broad. No value for the width was extracted since the resonance energy could not be precisely located in this analysis. Parameters of the very narrow 4^+ state at 5.395 MeV are given by McDermott *et al.*[15]

V. DISCUSSION

Considerable attention has been given to an explanation of a number of the states of ^{20}Ne in terms of rotational bands.[7–12,22,23] Because of the large number of low spin ($J\leq4$) states between 11 and 12.7 MeV, the assignment of a state to rotational bands based solely on the $J(J+1)$ formula cannot be reliably made. Indeed, the abundance of these low-spin states suggests that the model in its present form is not an adequate description of them.

If the suggestion of Davis and Mayer-Böricke[31] that members of the same rotational band should have comparable reduced widths is valid, the present analysis in conjunction with the analyses of Cameron[14] and McDermott *et al.*[15] indicates that some of the band assignments given by Hunt *et al.*[22] should be revised.

[31] R. H. Davis and C. U. Mayer-Böricke, in *Proceedings of the International Conference on Nuclear Physics*, edited by P. Gugenberger (Centre National De La Recherche Scientifique, Paris, 1964), Vol. II.

In particular, the band built on the 6.722-MeV 0^+ state should be $6.722(0^+)$, $7.434(2^+)$, $9.984(4^+)$, $12.560(6^+)$, with reduced widths of 130, 28, 118, and 155 keV, respectively, and the band built on the 7.202-MeV 0^+ state should be $7.202(0^+)$, $7.838(2^+)$, $10.550(4^+)$, $13.936(6^+)$, with reduced widths of 7, 4, and 12 keV for the first three states. The width for the 6^+ member of the band is not known accurately. Even with the assumption concerning the reduced widths, these assignments are not unique. There are states with reduced widths comparable to those assigned to the bands which are not assigned to any band.

ACKNOWLEDGMENTS

The authors thank H. Kaufmann for pointing out the work of Fletcher and Powell and for valuable discussions regarding the search program. The experiment was performed with the aid of Dr. P. Robinson, A. Bisson, J. Frickey, K. Knuth, J. Wallace, and B. Watson.

PHYSICAL REVIEW VOLUME 181, NUMBER 4 20 MAY 1969

Recoil Broadening of Secondary Transitions in Neutron-Capture Gamma-Ray Cascades

K. J. WETZEL*

Argonne National Laboratory, Argonne, Illinois 60439

(Received 12 December 1968; revised manuscript received 17 March 1969)

Recoil Doppler broadening has been observed in secondary γ-ray transitions (4–9 MeV) in cascades following thermal-neutron capture in ^{10}B and ^{14}N. An increase in linewidth (typically 0.5–1.5 keV) is measured by use of a Ge(Li) pair spectrometer. An ambiguity in the γ_1-γ_2 angular correlation $W(\theta)$ exists in most cases and introduces an uncertainty in the calculated linewidths, but measured widths generally fall within the range of the calculations. A comparison of results in ^{15}N from a gas sample and from a Melamine sample shows an attenuation of the effect for the 5271- and 5299-keV levels, and estimates of their lifetimes are consistent with known values. For ^{15}N levels showing no attenuation, the estimated lower limits on the lifetimes are also consistent with previous measurements; the lifetime for the 8313-keV level is an improvement over existing information. A more accurate value is also deduced for the lifetime of the 6741-keV level in ^{11}B from a comparison of the calculated and the observed broadening. It is also shown that in certain cases the Doppler broadening could provide information about the spin of the capturing state through the dependence of $W(\theta)$ on J_c.

1. INTRODUCTION

In most measurements of γ-ray spectra, a knowledge of the experimental line shape is necessary in order to obtain maximum information from measured data. This is particularly true for Ge(Li) spectrometry in which the inherent resolution has allowed one to obtain increasingly detailed and accurate information in many fields of nuclear physics—among them, decay schemes of radionuclides, lifetime measurements, and nuclear reactions. In the latter category, neutron-capture γ-ray studies have profited greatly from technological developments in Ge(Li) detectors and low-noise electronics. A large number of thermal-neutron-capture measurements with Ge(Li) spectrometers, both high- and low-energy spectra, as well as data from capture in and between individual resonances, have been reported in the literature.[1]

Bollinger and Thomas[2] have recently reported a new technique for simultaneously observing capture in a band of epithermal neutron energies, often containing a large number of resonances. In the high-energy ($E_\gamma \approx 5$–10 MeV) γ-ray spectrum of primary transitions, the effect of the kinetic energies (in the keV region) of neutrons captured under such conditions is observed[3] as a broadening of the peak in the Ge(Li)

* Work performed under the auspices of the U.S. Atomic Energy Commission.

[1] G. A. Bartholomew, A. Doveika, K. M. Eastwood, S. Monaro, L. V. Groshev, A. M. Demidov, V. I. Pelekhov, and L. S. Sokolovskii, Nucl. Data **A3**, 367 (1967); L. V. Groshev, A. M. Demidov, V. I. Pelekhov, L. L. Sokolovskii, G. A. Bartholomew, A. Doveika, K. M. Eastwood, and S. Monaro, *ibid.* (to be published); L. M. Bollinger, in *Slow Neutron Physics*, edited by J. A. Harvey (Academic Press Inc., New York, to be published).

[2] L. M. Bollinger and G. E. Thomas, Phys. Rev. Letters **18**, 1143 (1967).

[3] L. M. Bollinger and G. E. Thomas, Phys. Rev. Letters **21**, 233 (1968).