Correlations and Distributions of Widths in Resonance Neutron Capture*

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Two-channel moments of amplitudes are calculated for a model in which the compound-nuclear wave function consists of the sum of two vectors each randomly oriented on the surface of its own hypersphere. The relative variances of partial radiation widths and of reduced neutron widths are calculated. Expressions are found for the correlation coefficient of reduced neutron widths and partial radiation widths, and for the correlation coefficient of partial radiation widths to pairs of bound states. It is found that the relative variance value 2 for reduced neutron widths, as well as recent experimental findings for the statistics of partial widths, can be incorporated in the present two-group model if one of the groups is composed of a vector space of a large number of dimensions.

1. INTRODUCTION

THIS paper presents an extension of previous statistical models[1–3] to describe phenomena involving resonance reduced neutron widths and partial radiation widths. Recently, it has been discovered that positive linear correlations of reduced neutron widths and partial radiation widths occur in several nuclides.[4,5] This effect has been discussed in terms of a nuclear structure picture in which the correlation measures the single-particle contributions of the resonance and bound states to the transition strength under the assumption of a Porter-Thomas distribution of widths.[5] The concurrent finding of correlations of partial radiation widths to pairs of final states[4,6] has also been considered in the same model.[5] However, strong evidence is now at hand for a narrower distribution of the partial radiation widths in several nuclides than that of the χ^2 distribution with one degree of freedom.[6,7,8] This contrasts with the now well-established consistency of reduced neutron widths with the Porter-Thomas model.

It is the purpose of the author to show how these phenomena may all be reconciled within a single statistical model. We assume that a compound-nucleus level can be expanded in terms of a complete, orthogonal set of states composed of two groups, one having a large number $(N-n\rightarrow\infty)$ and the other a smaller number (n) of members. The wave function of the smaller group is uniformly and randomly distributed on the surface of a sphere in n dimensions of radius $r\leq 1$. The larger group is randomly oriented on the surface of a sphere in $(N-n)$ space of radius $(1-r^2)^{1/2}$. We will show that the experimental findings can be explained by assuming that contributions to the reduced neutron width come exclusively from the larger group while both groups contribute to partial radiation widths. The theory does not explicitly describe the nature of the groups in nuclear structure terms but sets limitations on the number of their states. An example is a model in which resonance contributions to the reduced neutron widths stem from single-particle states while those states together with resonance two-particle–one-hole components yield partial radiation widths. The single-particle set would then come from the large-size group of states while the two-particle–one-hole states would, to some extent, come from the smaller group.

It should be noted that Rosenzweig has previously considered a theory in which more than one group of states contribute to the compound nucleus.[3] However, his results are based on the premise that only one group is associated with γ-ray emission. A positive correlation of reduced neutron widths and partial radiation widths cannot, in general, be accounted for in his theory.

In Sec. 2 we develop the mathematical formalism leading to expressions for the moments of widths which will be used in subsequent sections where predictions of our theory are considered.

2. TWO-CHANNEL MOMENTS OF AMPLITUDES AND WIDTHS

We consider the wave function of the ith resonance state X_i expanded in terms of a convenient, complete set of orthogonal basis functions $\{\psi_\alpha,\psi_\beta\}$, $1\leq\alpha\leq n$, $n+1\leq\beta\leq N$ as

$$X_i=\sum_{\alpha=1}^{n}a_{\alpha i}\psi_\alpha+\sum_{\beta=n+1}^{N}b_{\beta i}\psi_\beta, \tag{1}$$

where the $a_{\alpha i}$, $b_{\beta i}$ are the real expansion coefficients of the resonance states in terms of the basis set.

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The reduced neutron width amplitude y_{i0} is given by

$$y_{i0}=\left(\frac{\hbar^2}{2ma}\right)^{1/2}\int X_i\Phi_0^*dS, \tag{2}$$

where m is the reduced mass, a is the channel radius, and Φ_0 is the channel wave function of the compound system.

We write Eq. (2) as

$$y_{i0}=\sum_\alpha a_{\alpha i}J_{\alpha 0}+\sum_\beta b_{\beta i}J_{\beta 0}, \tag{2'}$$

$$J_{\alpha 0}=\left(\frac{\hbar^2}{2ma}\right)^{1/2}\int\psi_\alpha\Phi_0^*dS, \qquad J_{\beta 0}=\left(\frac{\hbar^2}{2ma}\right)^{1/2}\int\psi_\beta\Phi_0^*dS. \tag{2''}$$

The partial radiation width amplitude y_{ij} of the ith resonance and jth final state is given by

$$y_{ij}=(16\pi/9)^{1/2}[k_\gamma^{3/2}/(2J+1)^{1/2}]\langle\Phi_j\,||H_{\text{mp}}||\,X_i\rangle, \tag{3}$$

where k_γ is the γ-ray wave number, J is the resonance spin, Φ_j is the final-state wave function, and H_{mp} is the multipole Hamiltonian appearing in the reduced matrix element. Equation (3) can then be written as

$$y_{ij}=\sum_\alpha a_{\alpha i}J_{\alpha j}+\sum_\beta b_{\beta i}J_{\beta j}, \tag{3'}$$

where

$$J_{\alpha j}=(16\pi/9)^{1/2}[k_\gamma^{3/2}/(2J+1)^{1/2}]\langle\Phi_j\,||H_{\text{mp}}||\,\psi_\alpha\rangle \tag{3''}$$

with a similar expression for $J_{\beta j}$.

The formal expression of the two-group hypothesis is now given. The joint distribution of the N orthogonal compound states is invariant under any orthogonal transformation in n dimensions which leaves the group of size n invariant, or in $(N-n)$ dimensions which leaves the group of size $(N-n)$ invariant. The joint distribution function is given in terms of the probability density of the components $a_{1i}\cdots b_{ni}$ of a random vector, i.e.,

$$P(a_{1i}\cdots b_{ni})\propto\int_0^1 rw(r)\delta[\sum_\alpha a_{\alpha i}^2-r^2]\times\delta[\sum_\beta b_{\beta i}^2-(1-r^2)]dr, \tag{4}$$

where we assume a superposition of spherical shells of radii r and $(1-r^2)^{1/2}$ with a weighting function $w(r)$. The physical meaning of $w(r)$ is discussed in the Appendix.

We now develop an expression for the moment $\langle y_{ij}^k y_{ij'}^m\rangle$, $j, j'=0, 1\cdots N_f$, where N_f is the number of final states considered. k and m are integers. By use of Eqs. (2′), (3′), (4), and the binomial expansion, we find

$$\langle y_{ij}^k y_{ij'}^m\rangle=\int\cdots\int rw(r)\,dr\,\delta(\sum_\alpha a_{\alpha i}^2-r^2)\delta(\sum_\beta b_{\beta i}^2-1+r^2)\prod_{\alpha,\beta}da_{\alpha i}db_{\beta i}$$
$$\times\sum_{h=0}^{k}\sum_{l=0}^{m}\frac{k!m!}{h!(k-h)!l!(m-l)!}(\sum_\alpha a_{\alpha i}J_{\alpha j})^h(\sum_\beta b_{\beta i}J_{\beta j})^{k-h}(\sum_\alpha a_{\alpha i}J_{\alpha j'})^l(\sum_\beta b_{\beta i}J_{\beta j'})^{m-l}/D, \tag{5}$$

where $D=\langle y_{ij}^0 y_{ij'}^0\rangle$. If we write

$$D\langle y_{ij}^k y_{ij'}^m\rangle=\sum_{h,l}N_{hl}k!m!/h!(k-h)!l!(m-l)!$$

and define

$$I(n,N)\equiv\int_0^1 w(r)r^n(1-r^2)^N r\,dr,$$

then by methods similar to those used by Ullah[9,10] we may evaluate N_{hl}. Replacing $a_{\alpha i}$ by $a_{\alpha i}/u$ and $b_{\beta i}$ by $b_{\beta i}/v$ and using Eq. (5) we find

$$N_{hl}u^{n+h+l-2}v^{N+k+m-n-h-l-2}=\int\cdots\int r^{-2}(1-r^2)^{-1}w(r)r\,dr\,\delta[\sum_\alpha(a_{\alpha i}^2/r^2)-u^2]\delta\{\sum_\beta[b_{\beta i}^2/(1-r^2)]-v^2\}$$
$$\times(\sum_\alpha a_{\alpha i}J_{\alpha j})^h(\sum_\beta b_{\beta i}J_{\beta j})^{k-h}(\sum_\alpha a_{\alpha i}J_{\alpha j'})^l(\sum_\beta b_{\beta i}J_{\beta j'})\prod_{\alpha,\beta}^{m-l}da_{\alpha i}db_{\beta i}. \tag{6}$$

[9] Nazakat Ullah, Nucl. Phys. **58**, 65 (1964).
[10] Nazakat Ullah, J. Math. Phys. **6**, 1102 (1965).

Multiplying both sides by $4uv\exp(-u^2-v^2)$ and integrating with respect to u, v from 0 to ∞ yields

$$N_{hl}\Gamma[(n+h+l)/2]\Gamma[(N+k+m-n-h-l)/2]/I[n+h+l-2,\ (N+k+m-n-h-l-2)/2] \tag{7}$$

$$=\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}(\sum_\alpha a_{\alpha i}'J_{\alpha j})^h(\sum_\beta b_{\beta i}'J_{\beta j})^{k-h}(\sum_\alpha a_{\alpha i}'J_{\alpha j'})^l(\sum_\beta b_{\beta i}'J_{\beta j'})^{m-l}$$

$$\times[\exp(-\sum_\alpha a_{\alpha i}'^2-\sum_\beta b_{\beta i}'^2)]\prod_{\alpha,\beta}da_{\alpha i}'db_{\beta i}' \tag{7'}$$

$$=[\Gamma(\tfrac{1}{2})]^N\left(\frac{\partial}{\partial\Phi}\right)^h\left(\frac{\partial}{\partial\theta}\right)^{k-h}\left(\frac{\partial}{\partial\mu}\right)^l\left(\frac{\partial}{\partial\nu}\right)^{m-l}\exp\{\tfrac{1}{4}[\sum_\alpha(\Phi J_{\alpha j}+\mu J_{\alpha j'})^2+\sum_\beta(\theta J_{\beta j}+\nu J_{\beta j'})^2]\}|_{\Phi=\theta=\mu=\nu=0} \tag{7''}$$

$$=[\Gamma(\tfrac{1}{2})]^N\frac{h!l!(k-h)!(m-l)!}{2^{k+m}}$$

$$\times\sum_{p,q}\frac{2^{p+q}(\sum_\alpha J_{\alpha j}^2)^{(h-p)/2}(\sum_\alpha J_{\alpha j}J_{\alpha j'})^p(\sum_\alpha J_{\alpha j'}^2)^{(l-p)/2}(\sum_\beta J_{\beta j}^2)^{(k-h-q)/2}(\sum_\beta J_{\beta j}J_{\beta j'})^q(\sum_\beta J_{\beta j'}^2)^{(m-l-q)/2}}{[(h-p)/2]![(k-h-q)/2]![(l-p)/2]![(m-l-q)/2]!p!q!}, \tag{7'''}$$

where $a_{\alpha i}'=a_{\alpha i}/r$ and $b_{\beta i}'=b_{\beta i}/(1-r^2)^{1/2}$ in Eq. (7'). The last expression is gained by expanding { } in Eq. (7'') in a Taylor series. The integer p can have any value from 0 to the smaller of h or l provided that each of the quantities [ ] in Eq. (7'') remain integers. The value q varies to the minimum of $(k-h)$ or $(m-l)$ with a similar proviso to that for p.

From Eqs. (5) and (7) it follows that

$$\langle y_{ij}^k y_{ij'}^m\rangle=\frac{\Gamma(n/2)\Gamma[(N-n)/2]k!m!}{2^{k+m}I[n-2,\ (N-n-2)/2]}\sum_{h,l}\frac{I[n+h+l-2,\ (N+k+m-n-h-l-2)/2]}{\Gamma[(n+h+l)/2]\Gamma[(N+k+m-n-h-l)/2]}$$

$$\times\sum_{p,q}\frac{2^{p+q}(\sum_\alpha J_{\alpha j}^2)^{(h-p)/2}(\sum_\alpha J_{\alpha j}J_{\alpha j'})^p(\sum_\alpha J_{\alpha j'}^2)^{(l-p)/2}(\sum_\beta J_{\beta j}^2)^{(k-h-q)/2}(\sum_\beta J_{\beta j}J_{\beta j'})^q(\sum_\beta J_{\beta j'}^2)^{(m-l-q)/2}}{[(h-p)/2]![(k-h-q)/2]![(l-p)/2]![(m-l-q)/2]!p!q!}. \tag{8}$$

By use of Eq. (8) we can write general expressions for the quantities of interest. For the correlation coefficient $C(y_{ij},\ y_{ij'})\equiv C_{jj'}$ of two amplitudes we have

$$C_{jj'}=\frac{\langle y_{ij}y_{ij'}\rangle}{(\langle y_{ij}^2\rangle\langle y_{ij'}^2\rangle)^{1/2}}$$

$$=\frac{n^{-1}\sum_\alpha J_{\alpha j}J_{\alpha j'}I[n,\ (N-n-2)/2]+(N-n)^{-1}\sum_\beta J_{\beta j}J_{\beta j'}I[n-2,\ (N-n)/2]}{\{\sum_\alpha(J_{\alpha j}^2/n)I[n,\ (N-n-2)/2]+\sum_\beta[J_{\beta j}^2/(n-2)]I[n-2,\ (N-n)/2]\}^{1/2}\times\{\sum_\alpha[(J_{\alpha j'}^2)/n]I[n,\ (N-n-2)/2]+\sum_\beta[J_{\beta j'}^2/(N-n)]I[n-2,\ (N-n)/2]\}^{1/2}}. \tag{9}$$

The widths are defined as the square of the amplitudes, i.e., $\Gamma_{ij}=y_{ij}^2$. For convenience, we write the integral $I[n+a,\ (N-n+b)/2]$ as $I_{a,b}$. The relative variance of the widths v_j is then given by

$$v_j=(\langle\Gamma_{ij}^2\rangle-\langle\Gamma_{ij}\rangle^2)/\langle\Gamma_{ij}\rangle^2$$

$$=\left\{\frac{(\sum_\beta J_{\beta j}^2)^2}{(N-n)^2}\left(3I_{-2,2}I_{-2,-2}\frac{N-n}{N-n+2}-I_{-2,0}^2\right)+\frac{2\sum_\alpha J_{\alpha j}^2\sum_\beta J_{\beta j}^2}{n(N-n)}(3I_{0,0}I_{-2,-2}-I_{-2,0}I_{0,-2})\right.$$

$$\left.+\frac{(\sum_\alpha J_{\alpha j}^2)^2}{n^2}\left(3I_{2,-2}I_{-2,-2}\frac{n}{n+2}-I_{0,-2}^2\right)\right\}\Big/\left\{\frac{(\sum_\beta J_{\beta j}^2)^2}{(N-n)^2}I_{-2,0}^2+\frac{2\sum_\alpha J_{\alpha j}^2\sum_\beta J_{\beta j}^2}{n(N-n)}I_{-2,0}I_{0,-2}+\frac{(\sum_\alpha J_{\alpha j}^2)^2}{n^2}I_{0,-2}^2\right\}. \tag{10}$$

The correlation coefficient $C(\Gamma_{ij},\ \Gamma_{ij'})$ is now given.

$$C(\Gamma_{ij},\Gamma_{ij'})=(\langle\Gamma_{ij}\Gamma_{ij'}\rangle-\langle\Gamma_{ij}\rangle\langle\Gamma_{ij'}\rangle)/(\langle\Gamma_{ij}^2\rangle-\langle\Gamma_{ij}\rangle^2)^{1/2}(\langle\Gamma_{ij'}^2\rangle-\langle\Gamma_{ij'}\rangle^2)^{1/2}$$

$$=\left\{\frac{I_{-2,2}I_{-2,-2}}{(N-n+2)(N-n)}[\sum_\beta J_{\beta j}^2\sum_\beta J_{\beta j'}^2+2(\sum_\beta J_{\beta j}J_{\beta j'})^2]-\frac{I_{-2,0}^2}{(N-n)^2}\sum_\beta J_{\beta j}^2\sum_\beta J_{\beta j'}^2\right.$$

$$+\frac{I_{0,0}I_{-2,-2}}{n(N-n)}(4\sum_\alpha J_{\alpha j}J_{\alpha j'}\sum_\beta J_{\beta j}J_{\beta j'}+\sum_\alpha J_{\alpha j}^2\sum_\beta J_{\beta j'}^2+\sum_\alpha J_{\alpha j'}^2\sum_\beta J_{\beta j}^2)$$

$$-\frac{I_{-2,0}I_{0,-2}}{n(N-n)}(\sum_\alpha J_{\alpha j}^2\sum_\beta J_{\beta j'}^2+\sum_\alpha J_{\alpha j'}^2\sum_\beta J_{\beta j}^2)+\frac{I_{2,-2}I_{-2,-2}}{n(n+2)}[2(\sum_\alpha J_{\alpha j}J_{\alpha j'})^2+\sum_\alpha J_{\alpha j}^2\sum_\alpha J_{\alpha j'}^2]$$

$$\left.-\frac{I_{0,-2}^2}{n^2}\sum_\alpha J_{\alpha j}^2\sum_\alpha J_{\alpha j'}^2\right\}\Big/(v_jv_{j'})^{1/2}\left\{\frac{I_{-2,0}^2}{(N-n)^2}\sum_\beta J_{\beta j}^2\sum_\beta J_{\beta j'}^2\right.$$

$$\left.+\frac{I_{-2,0}I_{0,-2}}{n(N-n)}(\sum_\alpha J_{\alpha j}^2\sum_\beta J_{\beta j'}^2+\sum_\alpha J_{\alpha j'}^2\sum_\beta J_{\beta j}^2)+\frac{I_{0,-2}^2}{n^2}\sum_\alpha J_{\alpha j}^2\sum_\alpha J_{\alpha j'}^2\right\}. \quad (11)$$

3. RELATIVE VARIANCE OF REDUCED NEUTRON WIDTHS

The formulation of Sec. 2 is quite general. Equations (9)–(11) are valid for any value of N or n. Henceforth, we assume that $N-n\to\infty$. It will be seen from further developments that this condition is necessary in order to account for the Porter-Thomas distribution of the reduced neutron widths. We proceed to the derivation of an expression for the relative variance of the reduced neutron widths.

The reduced neutron width Γ_{i0} is defined in terms of the amplitude y_{i0} as $\Gamma_{i0}\equiv y_{i0}^2$. The most general expression for y_{i0} is one which contains contributions from both groups, i.e., Eq. (2′). It is now shown that, provided n is not too large, the Porter-Thomas distribution of reduced widths is consistent only with a null contribution to the widths from the group of size n. In order to proceed we first assume the following relationship which is derived in the Appendix,

$$\int_0^1 rw(r)r^n(1-r^2)^{(N-m)/2}dr\Big/\int_0^1 rw(r)r^{n'}(1-r^2)^{(N-m')/2}=r_0^{(n-n')}(1-r_0^2)^{(m'-m)/2}. \quad (12)$$

We define a parameter R such that

$$R=\frac{r_0^2}{n}\sum_\alpha J_{\alpha0}^2\Big/\left(\frac{r_0^2}{n}\sum_\alpha J_{\alpha0}^2+\frac{1-r_0^2}{N-n}\sum_\beta J_{\beta0}^2\right). \quad (13)$$

From Eqs. (10) and (12), we find the relative variance v_0 of the reduced neutron widths to be given as

$$v_0=2[1-3R^2/(n+2)]. \quad (14)$$

It is clear that the relative variance can attain the Porter-Thomas value, 2, only for $R=0$ or for $n\to\infty$. Thus, the group of size n cannot contribute to Γ_{i0} unless $n\to\infty$. Henceforth, $R=0$ and finite n are assumed.

4. MODEL EXPRESSIONS FOR CORRELATION COEFFICIENTS AND RELATIVE VARIANCES

We now turn to partial radiation widths. Relationships will be derived for the relative variance, the correlation coefficients of partial widths and reduced widths and the correlation coefficient of partial widths to different final states.

The partial radiation width Γ_{ij} for $1\leq j\leq N_f$, where N_f is the number of final states considered, is defined in terms of the amplitude y_{ij} as $\Gamma_{ij}=y_{ij}^2$. The amplitude will generally have contributions from both groups. We will consider the amplitude contribution of the group of size $N-n$ to be proportional to the reduced neutron width amplitude y_{i0} as proposed by Lane and Lynn,[8] i.e.,

$$y_{ij}=A_jy_{i0}+\sum_\alpha a_{\alpha i}J_{\alpha j}, \quad (15)$$

where

$$y_{i0}=\sum_\beta b_{\beta i}J_{\beta 0}. \quad (16)$$

The correlation coefficient of reduced neutron width amplitudes and partial radiation width amplitudes can be found from Eq. (9) to be

$$C_{0j}=\left[A_j^2\sum_\beta J_{\beta0}^2\frac{1-r_0^2}{N-n}\Big/\left(\sum_\alpha J_{\alpha j}^2\frac{r_0^2}{n}+A_j^2\sum_\beta J_{\beta0}^2\frac{1-r_0^2}{N-n}\right)\right]^{1/2}. \quad (17)$$

The correlation coefficient of partial radiation widths to pairs of final states j and j' is found to be

$$C_{jj'}=C_{0j}C_{0j'}+\tau_{jj'}(1-C_{0j}^2)^{1/2}(1-C_{0j'}^2)^{1/2}, \quad (18)$$

where $\tau_{jj'}$ is defined by

$$\tau_{jj'}=\sum_\alpha J_{\alpha j}J_{\alpha j'}/(\sum_\alpha J_{\alpha j}^2\sum_\alpha J_{\alpha j'}^2)^{1/2}. \quad (18')$$

We now turn to the relative variance v_j of partial widths to the jth final state. It is found from Eq. (10) to be

$$v_j=2[1-3(1-C_{0j}^2)^2/(n+2)]. \quad (19)$$

The Porter-Thomas value $v_j=2$ is approached only when $n\to\infty$ or when $C_{0j}^2\to1$.

The correlation coefficient of partial radiation widths and reduced neutron widths $C(\Gamma_{i0},\Gamma_{ij})$ is found to be

$$C(\Gamma_{i0},\Gamma_{ij})=C_{0j}^2[1-3(1-C_{0j}^2)^2/(n+2)]^{1/2}. \quad (20)$$

The theory allows only non-negative correlation coefficients to occur. We next find the correlation coefficient of partial widths of final states j, j', i.e., $C(\Gamma_{ij}, \Gamma_{ij'})$ to be given by

$$C(\Gamma_{ij}, \Gamma_{ij'}) = \frac{[C_{jj'}^2 - (2\tau_{jj'}^2+1)(1-C_{0j}^2)(1-C_{0j'}^2)/(n+2)]}{[1-3(1-C_{0j}^2)^2/(n+2)]^{1/2}[1-3(1-C_{0j'}^2)^2/(n+2)]^{1/2}}. \tag{21}$$

The coefficient may take on either positive or negative values.

For the situation in which $C_{0j}^2=0$, Eqs. (19) and (21) simply reduce to those given by Rosenzweig.[3] Indeed, one can make this statement under the more general condition that $(N-n)$ remains finite. Since $C_{0j}^2=0$ implies $\sum_\beta J_{\beta 0}^2=0$ as is noted from Eq. (17), Eqs. (10) and (11) for $j, j'\neq 0$ reduce to Rosenzweig's equations, i.e.,

$$v_j \geq 2(n-1)/(n+2) \tag{22}$$

and

$$C(\Gamma_{ij}, \Gamma_{ij'}) = 1-\tfrac{2}{3}(1-C_{jj'}^2)[(1+v_j)/v_j]. \tag{23}$$

5. ANALYSIS OF EXPERIMENTAL INFORMATION IN PRESENT MODEL

A model has now been presented that accounts for the possibility of a Porter-Thomas distribution of reduced neutron widths, narrow distributions of partial radiation widths, non-negative correlations of partial radiation widths and reduced neutron widths, and both positive and negative correlations of partial radiation widths to pairs of final states. It is not the most general model possible, but it accounts for the experimental results. One could, for example, consider a three-group case. However, extensions of our model would generally lead to the occurrence of a larger number of parameters than the n, C_{0j}^2, and $\tau_{jj'}$ that appear in (19)–(21). The extra variables could not be precisely evaluated with the experimental tools now available. In our case, on the other hand, all the parameters can, in principle, be evaluated from experimental data as is now shown. From Eqs. (19) and (20), we find an expression for C_{0j}^2 as

$$C_{0j}^2 = C(\Gamma_{i0}, \Gamma_{ij})(\tfrac{1}{2}v_j)^{1/2} \tag{24}$$

and n is given by

$$n = 2[(3/(2-v_j))(1-C(\Gamma_{i0}, \Gamma_{ij})(\tfrac{1}{2}v_j)^{1/2})^2 - 1], \tag{25}$$

while $\tau_{jj'}$ is found from (18), (21), (24), and (25).

Let us apply these equations to the case of neutron capture in ^{169}Tm. The results of a correlation analysis of γ rays from eight resonances to 15 final states have been previously published.[4] The average of $C(\Gamma_{i0}, \Gamma_{ij})$ over final states, i.e., $\langle C(\Gamma_{i0}, \Gamma_{ij})\rangle_j$, was found to be 0.27. Analysis of this and other results to account for experimental error and the finite sample size indicated a "best" value with 10 and 90% confidence limits to be

$$\langle C(\Gamma_{i0}, \Gamma_{ij})\rangle_j = 0.43_{-0.23}^{+0.27}. \tag{26}$$

Although this result is based on the multivariate normal distribution of amplitudes, it was found to be relatively insensitive to the amplitude or width distributions. Analysis of the widths to obtain the χ^2 distribution with the best-fit number of degrees of freedom ν shows that[6]

$$\nu = 1.97_{-0.45}^{+0.77}. \tag{27}$$

The relationship of the relative variance to ν is given by

$$v = 2/\nu. \tag{28}$$

A value of $\nu=2$ corresponds to $v=1$. Using this and Eq. (26) in Eqs. (24) and (25), we obtain

$$\langle C_{0j}^2\rangle_j = 0.30, \qquad n = 0.97. \tag{29}$$

If we use the 10 percentile values found in Eqs. (26) and (27), we find

$$\langle C_{0j}^2\rangle_{j\,\min} = 0.16, \qquad n_{\max} = 4.3,$$

which yields an estimate of the minimum correlation coefficient and the maximum number of states in the group of smaller size. Since $n \geq 1$ and must be an integer, we have shown that

$$1 \leq n \leq 4$$

in this model with a larger probability of the smaller value being correct. Since, in our model, C_{0j}^2 measures the mean fraction of the transition strength that is due to the group of larger size, one may conclude that 16–30% of the transition strength is due to the larger size group while 70–84% of the intensity comes from a group of 1–4 states.

In the case of ^{238}U$(n,\gamma)^{239}$U, γ rays from five resonances to 18 final states have been analyzed.[11] No statistically significant correlations were found. It was shown that the best value for the number of degrees of freedom is $\nu=4$. This corresponds to a value of $\langle C_{0j}^2\rangle_j = 0$, $n=2$.

6. CONCLUSION

The two-group model of the compound nucleus we have discussed has been shown to successfully account for correlation and distribution effects of resonance neutron capture. Indeed, we have found that the parameters of the model can be quantitatively evaluated by use of presently available or obtainable experimental data.

It should be emphasized that this work has been primarily concerned with the statistical aspects of the reaction process. The underlying nuclear structure mechanism has been mentioned only briefly. The statistical and nuclear structure effects are obviously strongly interrelated. The case of caputre in ^{169}Tm can help clarify this interdependence in the present model.

We have found that approximately 16–30% of the transition intensity in Tm stems from the large size group while 70–84% comes from a group of 1–4 states. The states of the large size group are associated with the reduced neutron widths. Since the reduced widths essentially yield the single-particle strength of the compound-nucleus wave function, it is the group of

[11] D. L. Price, R. E. Chrien, O. A. Wasson, M. R. Bhat, M. Beer, M. A. Lone, and R. Graves, Nucl. Phys. **A121**, 630 (1968).

larger size which contains the single-particle component of the neutron resonances. A relatively simple interpretation (not necessarily unique) of the small sized group can also be given. We consider this set to be composed of 1–4 doorway or collective states, each of which yields the same order-of-magnitude contribution to the γ-ray transition as the single-particle component.

Interpretation of the model as applied to capture in ^{238}U can also be given. About two doorway or collective states supply the entire transition strength.

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APPENDIX: WEIGHTING FUNCTION $w(r)$

The weighting function $w(r)$ determines the overlap region of the two groups of coefficients $\{a_{\alpha i}\}$ and $\{b_{\alpha i}\}$. To clarify this, we consider two extreme situations. First suppose $w(r)=$ constant over the entire region $0\leq r\leq 1$. Equation (4) shows that $P(a_{1i}\cdots b_{ni})\propto \delta(\sum_\alpha a_{\alpha i}^2+\sum_\beta b_{\beta i}^2-1)$. Thus, this case corresponds to allowing both groups to merge into a single group the random vector of which is uniformly distributed on a unit sphere. In the second case, we consider $w(r)\propto \delta(r^2-r_0^2)$. Then

$$P(a_{ii}\cdots b_{ni})\propto\delta(\sum_\alpha a_{\alpha i}^2-r_0^2)\delta(\sum_\beta b_{\beta i}^2-(1-r_0^2)).$$

The two groups are completely independent. In general, $w(r)$ corresponds to some intermediate degree of group interdependence.

Let us now consider the conditions for the validity of Eq. (12). For this purpose, we examine the integral

$$I[n,\,(N-m)/2]=\int_0^1 w(r)r^n(1-r^2)^{(N-m)/2}rdr. \quad (\text{A1})$$

In the case $w(r)=\delta(r^2-r_0^2)$, the integral is proportional to $r_0^n(1-r_0^2)^{(N-m)/2}$ and Eq. (12) immediately results. We will now show that Eq. (12) is correct for any weighting function $w(r)$ satisfying the following conditions: (a) $m,\ n\ll(N-m)/2$; $m',\ n'\ll(N-m')/2$; (b) $w(r)$ is not proportional to $(1-r^2)^{-N/2}$; (c) $w(r<a)=dw(r<a)/dr=\cdots=d^lw(r<a)/dr^l=\cdots=0$.

Integrating Eq. (A1) by parts yields

$$I[n,\,(N-m)/2]=\frac{w(a)a^n(1-a^2)^{(N-m+2)/2}}{N-m+2}+\frac{1}{N-m+2}\times\left(\int_a^1\frac{dw}{dr}r^{(n-1)}(1-r^2)^{(N-m+2)/2}d(r^2)+n\int_a^1 wr^{(n-2)}(1-r^2)^{(N-m+2)/2}d(r^2)\right). \quad (\text{A2})$$

Integration by parts a second time gives us

$$I[n,\,(N-m)/2]=\frac{w(a)a^n(1-a^2)^{(N-m+2)/2}}{N-m+2}+\frac{nw(a)a^{(n-2)}(1-a^2)^{(N-m+4)/2}}{(N-m+2)(N-m+4)}+\left.\frac{dw}{dr}\right|_{r=a}\times\frac{a^{n-1}(1-a^2)^{(N-m+4)/2}}{(N-m+2)(N-m+4)}+\frac{1}{(N-m+2)(N-m+4)}\int\cdots. \quad (\text{A3})$$

For the case in which $w(a)\neq0$ we note that the ratio of the second term to the first term is

$$n(1-a^2)/(N-m+4)a^2, \quad (\text{A4})$$

which is small under the stated conditions. The third term, too, is small relative to the first term. If, on the other hand, $w(a)=0$, $dw/dr|_{r=a}\neq0$ then the main contribution to the integral comes from the third term. In general, if all derivatives of order $<l$ are zero-valued at $r=a$ and $d^lw/dr^l|_{r=a}\neq0$, then

$$I[n,\,(N-m)/2]=[d^lw(a)/dr^l]\times\{a^{n-l}(1-a^2)^{[N-m+2(1+l)]/2}/\prod_{j=1}^{l}[N-m+2(j+1)]\}. \quad (\text{A5})$$

If it is assumed that $l\ll(N-m)/2$, then setting $r_0=a$, Eq. (12) immediately follows from Eq. (A5).

There is another weighting function type of interest, i.e., where $w(r)$ fulfills conditions (a) and (b) but either $w(0)$ or one of its derivatives at $r=0$ has a nonzero value. We will show that under these conditions the results are indistinguishable from those of the Krieger-Porter model.[2,5] Let us consider the situation in which $w(0)\neq0$. Then, by partial integration, we find that

$$I[n,\,(N-m)/2]=(n!!/\prod_{j=1}^{n+1}[N-m+2j])\times\int_0^1 w(r)r^\alpha(1-r^2)^{[N-m+2(n+1)]/2}d(r^2), \quad (\text{A6})$$

where $\alpha=0$ for n an odd, and $\alpha=1$ for n an even integer. It is not difficult to show that under the stated conditions the integral in Eq. (A6) has the same order of magnitude value for $n\rightarrow n+2$ as for n. Therefore, we are justified in stating that

$$I[n+2,\,(N-m)/2]/I[n,\,(N-m')/2]\sim1/N, \quad (\text{A7})$$

a value much smaller than unity. Under these circumstances, Eqs. (19) and (20) reduce to

$$v_j=2 \quad (\text{A8})$$

and

$$C(\Gamma_{ij},\,\Gamma_{i0})=C_{J0}^2, \quad (\text{A9})$$

as found in the case with only one group with a very large number of members.