Particle and Photon Decay of Nuclei Following Electroexcitation

DIETER DRECHSEL*
Physics Department, The Catholic University of America, Washington, D. C. 20017
AND
H. ÜBERALL†
U. S. Naval Research Laboratory, Washington, D. C.
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Angular-correlation formulas are derived for a coincidence experiment in which an electron scattered inelastically by a nucleus is detected simultaneously with a heavy particle or photon emitted by the nucleus subsequently to its electroexcitation. The excitation of the nucleus is treated in first Born approximation; i.e., single-photon exchange between electron and nucleus is assumed. The subsequent decay depends on the properties of the emitted particle. Within the framework of the one-level resonance model (i.e., assuming that the level matrix is diagonal), the formulas derived are valid for heavy-particle as well as photon decay. Only slight modifications are necessary, however, to take into account properly the effects of coupled channels for particle decay. The present results differ from those of a previous investigation in that (a) overlapping excited levels are included, as found in most applications (we assume, however, that the level matrix is diagonal), (b) photons as well as spin-$\frac{1}{2}$ and spin-0 particles are considered as being emitted in the nuclear decay, and (c) summations over all magnetic quantum numbers were carried out. Examples considered were excitations of isolated $E1$ and $M1$ levels of a spinless nucleus, and of the overlapping $E1$ giant resonance levels of ^{12}C, evaluated by R-matrix theory.

I. INTRODUCTION

IN Barber's 1962 review article on inelastic electron scattering,[1] it was demonstrated that electron-nucleon coincidence experiments in nuclear electroexcitation (see Fig. 1) were unfeasible at the accelerators then available, the reasons being the low intensities, high backgrounds, and small duty cycles leading to an unacceptable number of accidental coincidences. In the meantime, the advent of new electron linear accelerators has brought us within close reach of feasibility of these coincidence experiments: Adaptation of the 1-GeV Stanford electron linac to superconducting operation (100% duty cycle) and construction of the 400-MeV linac at MIT are under way; construction of the 600-MeV linac at Saclay is nearly terminated; and the Los Alamos 24-MeV electron linac (a prototype for the Los Alamos meson factory[2]) is operative at a 6% duty cycle and is suitable for the coincidence experiments mentioned. With these possibilities in mind, we have derived expressions for the angular correlation between the inelastically scattered electron and a particle emitted in the subsequent nuclear decay which may be either a photon or a spin-$\frac{1}{2}$ or spin-0 nuclear particle. These expressions include the possibility of overlapping excited nuclear levels, such as are encountered in the important giant resonance region. Our results represent a generalization of the formulas for nucleon decay following electroexcitation of an isolated nuclear level as given in an earlier paper by Raphael and Überall.[3] Some errors in the transverse term given in that paper are eliminated, and the summation over magnetic quantum numbers is now carried out completely.

A generalization of the angular-correlation formula for spin-$\frac{1}{2}$ particles to overlapping levels (without explicit exhibition of the intervening resonance denominators) has been carried out by Molloy.[4,5] The special case of photon decay of electroexcited levels was considered by Hubbard and Rose[6] and by Acker and Rose,[7] although only to the ground state of the final nucleus (whereas our results are valid for any final state of the nucleus after deexcitation), but with an important extension, namely, the inclusion of terms representing the electron bremsstrahlung without nuclear excitation with which the nuclear photon decay is coherent, as well as of the corresponding interference terms.[8] Our results contain only the nuclear photon emission terms, and will thus be useful if in $(e, e'\gamma)$ coincidence experiments the photon is observed sufficiently far away from both initial and final electron directions, since, as is well known (and as demonstrated by Acker and Rose[7]), the bremsstrahlung of relativistic electrons is concentrated in a narrow cone about the electron direction (and so is the contribution of the inter-

* Supported in part by a grant of the National Science Foundation and by the Bundesministerium für Wissenschaft und Forschung. Permanent address: Institut für Theoretische Physik, Universität Frankfurt/M.

† Consultant. Permanent address: Physics Department, The Catholic University of America, Washington, D.C.

[1] W. C. Barber, Ann. Rev. Nucl. Sci. **12**, 1 (1962).

[2] Phys. Today **21**, 61 (1968).

[3] R. Raphael and H. Überall, Phys. Rev. **143**, 671 (1966).

[4] H. R. Molloy, M.S. dissertation, University of Saskatchewan, Saskatoon, Saskatchewan, 1967 (unpublished).

[5] Thanks are due to H. R. Molloy for pointing out the error in Ref. 3.

[6] D. F. Hubbard and M. E. Rose, Nucl. Phys. **84**, 337 (1966).

[7] H. L. Acker and M. E. Rose, Ann. Phys. (N.Y.) **44**, 336 (1967).

[8] In the usual noncoincidence experiments where only the outgoing electron is observed, one customarily adds to the electroexcitation terms the bremsstrahlung radiative tails in an incoherent fashion, thereby overlooking the interference between bremsstrahlung and the contribution of the (unobserved) nuclear deexcitation photons, which are important for levels below the particle threshold. It has been shown in Ref. 7 that such a procedure is adequate since the interference term becomes small when integrated over decay photon directions. This is not the case before integration, i.e., in a coincidence experiment.

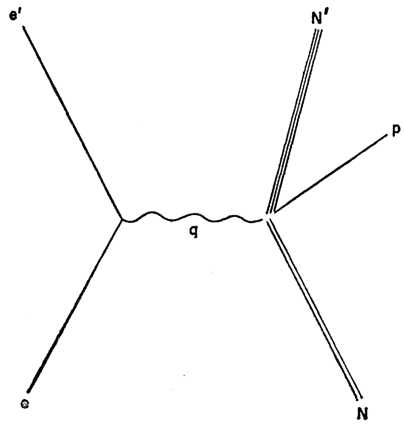


FIG. 1. Diagram for particle (or photon) decay following electroexcitation.

ference term). The electron bremsstrahlung formulas leading to an excited nuclear state are known from the work of Maximon and Isabelle.[9]

Electron-proton coincidence experiments have indeed been performed already[10] at high excitation energies and high momentum transfer. Such experiments have been described in the framework of quasi-elastic scattering, in which the passing electron interacts with an individual proton bound by the imbedding nuclear matter. This model explains very well the excitation cross section at medium energies (between 30 and 60 MeV in ^{12}C), and also at higher excitation energies if short-range correlations are taken into account.[11] It provides information on binding energies and momentum distributions of protons in nuclear shells, and of short-range correlations in particular by observing multiple coincidences[12] such as $(e, e'2p)$.

However, the model of quasi-elastic scattering does not describe the resonance structure of the electroexcitation cross section at the lower energies, in particular, in the giant resonance region which is dominated by collective states and long-range correlations. These excited states correspond to resonances of the nucleus (residual nucleus and outgoing particle) with definite multipolarity. Within each group of states with a given total spin and parity, the number of channels open for particle decay increases with excitation energy. Since these channels are coupled by the residual interaction in the nucleus, the numerical problems increase rapidly with excitation energy, thus restricting a solution of the coupled-channel problem[13,14] to low excitation energies. Further, for photoexcitation it is usually sufficient to consider electric dipole states only, also magnetic transitions and such of higher multipolarity have to be taken into account to describe electroexcitation at higher momentum transfer. Therefore, a proper description of the quasi-elastic peak in a coupled-channel formulation will pose severe numerical problems.

For low excitation energies, however, such calculations have been performed with considerable success in spite of the truncation of Hilbert space to few low-lying one-particle–one-hole states.[13,14] The photoexcitation cross section has been reproduced reasonably well, both resonance structure and background, including threshold effects.

It should be kept in mind that in R-matrix language the background is produced by contributions of faraway levels[15] and therefore will not be properly described by the one-level resonance model, i.e., if one assumes that the level matrix[15] is diagonal. Though we shall derive the formulas for electrodisintegration under this assumption, it will be shown that it is straightforward to modify the formulas to describe general R-matrix theory and the eigenchannel formulation of the coupled-channel problem as well.

In the case of coincidence experiments with α particles, the above statements apply only if one assumes that α particles are prefabricated in nuclei as clusters. A more thorough analysis would necessitate the solution of the four-particle-continuum problem first.

The electron-photon coincidence process is of second order (first electroexcitation of the nucleus to an intermediate state and subsequent γ decay; see Fig. 2). Therefore, an exact solution involves the integration over all intermediate states. Since above particle emission threshold the photon-decay width is only a small fraction of the particle-decay width, any such experiment has to be performed in the vicinity of sharp

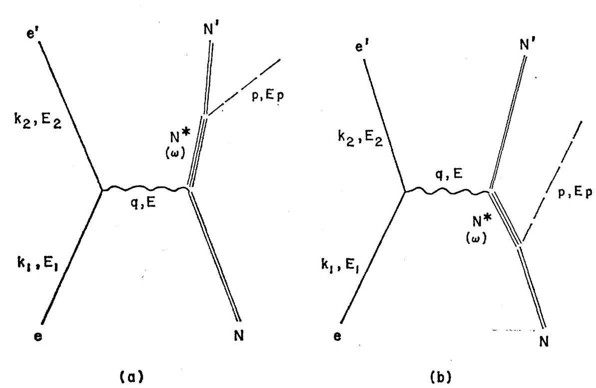


FIG. 2. Diagrams for electroexcitation of a nuclear resonance and subsequent photon decay.

[9] L. C. Maximon and D. B. Isabelle, Phys. Rev. **136**, B674 (1964).

[10] U. Amaldi, G. Campos Venuti, G. Cortellessa, C. Fronterotta, A. Reale, P. Salvadori, and P. Hillman, Phys. Rev. Letters **13**, 341 (1964).

[11] For a recent review on this process, see T. de Forest, Ann. Phys. (N.Y.) **45**, 365 (1967).

[12] Y. N. Srivastava, Phys. Rev. **135**, B612 (1964).

[13] B. Buck and A. D. Hill, Nucl. Phys. **A95**, 271 (1967).

[14] H. G. Wahsweiler, W. Greiner, and M. Danos, Phys. Rev. **170**, 893 (1968).

[15] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).

resonances to give reasonable counting rates. In such a region, however, the one-level approximation used in this paper is certainly applicable.

Finally, the advantages of coincidence experiments over an observation of the outgoing electrons alone are the following:

(a) The "radiative tail" providing a background of electrons that were degraded to a specified final electron energy by emission of a bremsstrahlung photon, rather than by excitation of the nucleus, is eliminated if an emitted heavy particle is observed in coincidence.

(b) Due to the presence of interference terms, not only between Coulomb and transverse excitations, but also between electric and magnetic transitions if there are overlapping levels, relative phases of matrix elements may be determined. Note that in the noncoincidence Born cross section, only squared matrix elements enter (even when the levels overlap), and thus phases remain undetermined.

(c) The coincidence cross section provides us with a means for determining the spins and parities of excited levels in a model-independent way if the measurements are carried out at fixed momentum transfer q. Note that in photonuclear angular distributions, which may similarly be used for model-independent spin and parity determinations many levels that show up in electron scattering cannot be significantly excited, whereas spin and parity assignments from noncoincident electron scattering experiments usually are model-dependent if levels overlap.

II. COINCIDENCE CROSS SECTION: GENERAL FORMALISM

Following Gourdin,[16] de Forest[11] has derived the general form of the double-coincidence electron scattering cross section in Born approximation which is valid for any type of particle emitted during the scattering process and for any mechanism of emission. The expression, which depends on four form factors, is for relativistic electrons given by

$$\frac{d^3\sigma}{d\Omega d\Omega_p dE_2}=\frac{\alpha^2}{\Delta^4}\frac{k_2}{k_1}pE_p\{V_C(\theta)W_C+V_T(\theta)W_T+V_I(\theta,\phi_p)W_I+V_S(\theta,\phi_p)W_S\}. \quad (1)$$

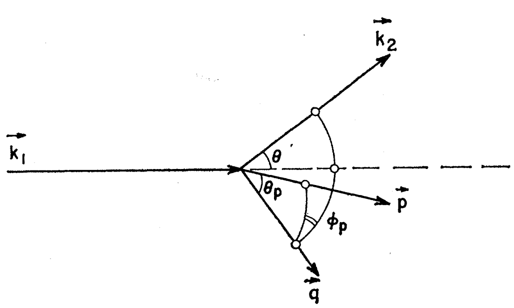


FIG. 3. Geometry of the coincidence experiment.

[16] M. Gourdin, Nuovo Cimento **21**, 1094 (1961).

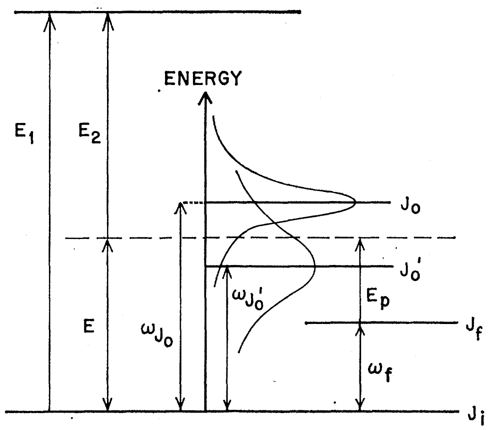


FIG. 4. Energy diagram of overlapping levels.

Here, the following designations are used: The incident and scattered electron momenta and energies are $(\mathbf{k}_1, E_1)$ and $(\mathbf{k}_2, E_2)$, respectively, whereas the momentum and total energy of the emitted particle are $(\mathbf{p}, E_p)$. The fine-structure constant is $\alpha=1/137$; the electron scattering angle is called $\theta=\measuredangle(\mathbf{k}_1, \mathbf{k}_2)$, and the corresponding element of solid angle is $d\Omega$. We introduce the momentum and energy transfer by

$$\mathbf{q}=\mathbf{k}_1-\mathbf{k}_2, \quad (2a)$$

$$E=E_1-E_2. \quad (2b)$$

The particle emission angles (θ_p, ϕ_p) are taken to refer to the direction of $\mathbf{q}$ as the polar axis (see Fig. 3), and the corresponding element of solid angle is $d\Omega_p$. Further, we have the squared four-momentum transfer $\Delta^2=q^2-E^2$. For relativistic electrons, the four kinematic functions are

$$V_C(\theta)=(\Delta^4/q^4)\beta, \quad (3a)$$

$$V_T(\theta)=(\Delta^2/2q^2)(\beta+q^2), \quad (3b)$$

$$V_I(\theta,\phi_p)=(k_1+k_2)[(\Delta^2/2q^2)V_C(\theta)]^{1/2}\cos\phi_p, \quad (3c)$$

$$V_S(\theta,\phi_p)=(\Delta^2/2q^2)(2\beta\cos^2\phi_p+q^2), \quad (3d)$$

where

$$\beta=2k_1k_2\cos^2(\tfrac{1}{2}\theta). \quad (3e)$$

The first two functions, V_C (Coulomb) and V_T (transverse), are familiar from the Born approximation formula of inelastic electron scattering.[17] Finally, the four form factors W_C, W_T, W_I, and W_S are as yet undetermined model-dependent dynamic quantities, being functions of q, E, p, and θ_p, and depending on the nature of the emitted particle, the reaction mechanism, and the nuclear model used to describe the latter. It is the aim of this paper to evaluate the functions W on the basis of the resonance model of the nuclear levels.

We characterize the initial state of the nucleus by spin, parity, and magnetic quantum number $(J_i^{\pi_i}, M_i)$.

[17] T. de Forest and J. D. Walecka, Advan. Phys. **15**, 1 (1966).

Its electroexcitation leads to the (overlapping) levels $(J_0^{\pi_0}, M_0)$ or $(J_0'^{\pi_0'}, M_0')$ of total widths $\Gamma_{J_0}{}^{\text{tot}}$ and $\Gamma_{J_0'}{}^{\text{tot}}$, respectively, centered at an excitation energy ω_{J_0} or $\omega_{J_0'}$. The decay (with emission of particle p) proceeds to the level of the final nucleus given by $(J_f^{\pi_f}, M_f)$, with energy ω_f. The energy scheme is shown in Fig. 4. In the following, the parity superscripts will be suppressed.

The transition operator for nuclear electroexcitation in Born approximation equals the electromagnetic interaction operator H, given in Eq. (25) below. Further, we introduce, following Goldfarb,[18] the "transition operator" K for the decay process. For the purpose of an angular-correlation calculation the reduced matrix elements of this operator need not be specified further and may be treated either as experimental parameters or calculated from a nuclear model.

Though we will allow for many excited levels, it is assumed in the following that the level matrix is diagonal, which gives rise to a Breit-Wigner shape of an individual level.[15] The diagrams of Fig. 2 then apply for nucleon decay of the level as well. It will be shown later that only slight modifications are necessary to generalize our results to take into account the coupled-channel effects, which may not be negligible if there are many neighboring resonances and threshold effects.[19]

Approximating the nuclear spectrum by a set of resonances with a diagonal level matrix, the cross section is given by

$$d\sigma^{J_i\to J_f}=(2\pi/v)\delta(E_1-E_2-E_p-\omega_f)$$
$$\times\frac{1}{\hat{J}_i^2}\sum_{M_iM_fm_p}\left|\sum_{J_0M_0}\frac{K_{J_fJ_0}{}^{M_fM_0}H_{J_0J_i}{}^{M_0M_i}}{E-\omega_{J_0}+\frac{1}{2}i\Gamma_{J_0}{}^{\text{tot}}}\right|^2\frac{d^3k_2d^3p}{(2\pi)^6}, \quad (4)$$

where $\hat{J}_i=(2J_i+1)^{1/2}$, v is the velocity of the incoming electrons, and m_p is the polarization index of the emitted particle. Note that the sum over J_0 also implies summation over other, unspecified, quantum numbers of the excited states.

For electron-photon coincidences Eq. (4) arises from second-order perturbation theory, the transition operator K being replaced by H, the electromagnetic interaction of the nucleons with the radiation field. We have dropped the contribution shown in Fig. 2(b), since its energy denominator shows a nonresonant behavior which will make this term negligible for practical purposes,[6,20,21] especially in the vicinity of a resonance.

In the case of heavy-particle decay, the squared matrix element of K gives the probability that the excited state (J_0, M_0) decays into the final state (J_f, M_f), again assuming that the level matrix is diagonal.

It is convenient to interpret Eq. (4) as a product of two density matrices describing the excitation and the decay parts separately. The density matrix for electroexcitation is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{M_0M_0'}^{J_i\to J_0J_0'}=(4\pi^2v)^{-1}H_{J_0J_i}{}^{M_0M_i*}H_{J_0'J_i}{}^{M_0'M_i}E_2k_2, \quad (5a)$$

so that one has a differential cross section for the electroexcitation of the individual level J_0:

$$\frac{d\sigma_{\text{tot}}{}^{J_i\to J_0}}{d\Omega}=\frac{1}{\hat{J}_i^2}\sum_{M_iM_0}\left(\frac{d\sigma}{d\Omega}\right)_{M_0M_0}^{J_i\to J_0J_0}, \quad (5b)$$

representing the total excitation probability regardless of the subsequent decay mode (i.e., all decay channels are summed over). For the decay density matrix (differential in the direction $d\Omega_p$ of the emitted particle) we have, similarly,

$$\left(\frac{dw}{d\Omega_p}\right)_{M_0M_0'}^{J_0J_0'\to J_f}=\frac{1}{4\pi^2}K_{J_fJ_0}{}^{M_fM_0*}K_{J_fJ_0'}{}^{M_fM_0'}E_pp, \quad (6a)$$

with an over-all decay probability of the individual level J_0 to the final state J_f with the emission of one particle p:

$$\Gamma_{J_0\to J_f}=\frac{1}{\hat{J}_0^2}\sum_{M_0M_fm_p}\int\left(\frac{dw}{d\Omega_p}\right)_{M_0M_0}^{J_0J_0\to J_f}d\Omega_p, \quad (6b)$$

i.e., the partial decay width into the corresponding channel. This gives for the cross section of Eq. (4)

$$\frac{d^3\sigma^{J_i\to J_f}}{d\Omega d\Omega_pdE_2}=\frac{1}{2\pi\hat{J}_i^2}\sum_{M_iM_fm_p}\sum_{J_0J_0'}\sum_{M_0M_0'}\frac{(d\sigma/d\Omega)_{M_0M_0'}{}^{J_i\to J_0J_0'}(dw/d\Omega_p)_{M_0M_0'}{}^{J_0J_0'\to J_f}}{(E-\omega_{J_0}-\frac{1}{2}i\Gamma_{J_0}{}^{\text{tot}})(E-\omega_{J_0'}+\frac{1}{2}i\Gamma_{J_0'}{}^{\text{tot}})}, \quad (7)$$

which has the form of the usual overlapping-level formula.[18] It possesses the correct normalization so that upon integration over the outgoing particle directions $d\Omega_p$ and summation over its polarizations as well as over the angular momentum projections of the final nucleus, with the help of the formula

$$\sum_{M_fm_p}\int\left(\frac{dw}{d\Omega_p}\right)_{M_0M_0'}^{J_0J_0'\to J_f}d\Omega_p=\delta_{J_0J_0'}\delta_{M_0M_0'}\Gamma_{J_0\to J_f} \quad (8)$$

[18] See, e.g., S. Devons and L. J. B. Goldfarb, in *Encyclopedia of Physics*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. XLII, p. 362; L. J. B. Goldfarb, in *Nuclear Reactions*, edited by P. M. Endt and M. Demeur (North-Holland Publishing Co., Amsterdam, 1959), Vol. 1, p. 159.

[19] See also L. J. Crone, Ph.D. thesis, The Catholic University of America, 1968 (unpublished); L. J. Crone and C. Werntz, Nucl. Phys. (to be published).

[20] The nonresonant term also arises in nuclear photon scattering; cf. R. Silbar and H. Überall, Nucl. Phys. **A109**, 146 (1968), as an example where this term is carried through.

[21] See, e.g., E. G. Fuller and E. Hayward, in *Nuclear Reactions*, edited by P. M. Endt and M. Demeur (North-Holland Publishing Co., Amsterdam, 1959), Vol. 2, p. 113.

(to be proved later), we obtain

$$\frac{d^2\sigma^{J_i\to J_f}}{d\Omega dE_2}=\sum_{J_0}\frac{1}{2\pi}\frac{\Gamma_{J_0}{}^{\text{tot}}}{(E-\omega_{J_0})^2+\frac{1}{4}(\Gamma_{J_0}{}^{\text{tot}})^2}\frac{\Gamma_{J_0\to J_f}}{\Gamma_{J_0}{}^{\text{tot}}}\frac{d\sigma_{\text{tot}}{}^{J_i\to J_0}}{d\Omega}. \tag{9a}$$

This is a result to be expected: The interference between the levels has disappeared; their contributions must add with their individual widths. The latter are represented by the Breit-Wigner shape mentioned before, and the absolute cross section for each level J_0 (still differential in the electron direction) appears multiplied by the branching ratio $\Gamma_{J_0\to J_f}/\Gamma_{J_0}{}^{\text{tot}}$ for the decay into the single-particle emission channel to the given final nuclear state J_f. (The total width $\Gamma_{J_0}{}^{\text{tot}}$, of course, includes all types of emitted particles—even multiparticle emission—for decay of the level J_0, going to all possible final states J_f.) The energy dependence is mainly contained in the resonance denominator (the rest may be evaluated at the resonance energy $E=\omega_{J_0}$), and integration $\int dE_2$ over the width of the level J_0 then gives

$$\frac{d\sigma^{J_i\to J_f}}{d\Omega}=\sum_{J_0}\frac{\Gamma_{J_0\to J_f}}{\Gamma_{J_0}{}^{\text{tot}}}\frac{d\sigma_{\text{tot}}{}^{J_i\to J_0}}{d\Omega}, \tag{9b}$$

again the expected result.

III. DECAY DENSITY MATRIX

We shall introduce the notation

$$\rho_{J_0J_0'M_0M_0'}{}^{J_fM_f}=\sum_{m_p}K_{J_fJ_0}{}^{M_fM_0*}K_{J_fJ_0'}{}^{M_fM_0'}, \tag{10a}$$

so that the decay density matrix (6a) becomes

$$\sum_{m_p}\left(\frac{dw}{d\Omega_p}\right)_{M_0M_0'}^{J_0J_0'\to J_f}=\frac{E_p p}{4\pi^2}\rho_{J_0J_0'M_0M_0'}{}^{J_fM_f}. \tag{10b}$$

Following Goldfarb,[18] reduced decay matrix elements will be introduced according to

$$\langle J\,\|\,\Lambda\,\|\,J_0\rangle=\langle J\Lambda(J_0M_0)\mid K\mid J_0M_0\rangle, \tag{11}$$

keeping in mind the Wigner-Eckart theorem and the scalar character of K. The symbol Λ refers to the channel spin of the emitted particle; for the types of particles considered here, we have

$$\text{photon:}\quad \Lambda\equiv(\lambda,\bar{\lambda})=(l,\alpha); \tag{12a}$$

$$\text{nucleon:}\quad \Lambda\equiv(\lambda,\bar{\lambda})=(j,l); \tag{12b}$$

$$\alpha\text{ particle:}\quad \Lambda\equiv\lambda=l. \tag{12c}$$

For photons, l is the total angular momentum (or multipolarity); $\alpha=0$ for electric transitions and 1 for magnetic transitions. For nucleons, l is the orbital and j is the total angular momentum, whereas for the α particle, l is the orbital angular momentum and $\bar{\lambda}$ is absent (or zero). In Eq. (11), J and Λ are coupled to J_0 in the final state.

Since the explicit form of Eq. (10a) is

$$\rho_{J_0J_0'M_0M_0'}{}^{J_fM_f}=\sum_{m_p}\langle J_fM_f,\,\mathrm{p}m_p\mid K\mid J_0M_0\rangle^*\times\langle J_fM_f,\,\mathrm{p}m_p\mid K\mid J_0'M_0'\rangle, \tag{13}$$

the standard rotations and recoupling techniques lead to[3,18,22]

$$\sum_{M_f}\rho_{J_0J_0'M_0M_0'}{}^{J_fM_f}=\sum_{IM_I}(-1)^{-M_0'}\times(J_0M_0,J_0'-M_0'\mid IM_I)F_{J_0J_0'}{}^{J_f}(IM_I), \tag{14a}$$

where, with $\Lambda'\equiv(\lambda',\bar{\lambda}')$,

$$F_{J_0J_0'}{}^{J_f}(IM_I)=\hat{J}_0\hat{J}_0'\sum_{M_I'}\sum_{\Lambda\Lambda'}c_{IM_I'}(\Lambda\Lambda')(-1)^{I+J_f-\lambda}\times W(J_0J_0'\lambda\lambda';IJ_f)\langle J_f\,\|\,\Lambda\,\|\,J_0\rangle^*\langle J_f\,\|\,\Lambda'\,\|\,J_0'\rangle\times D_{M_IM_I'}{}^{I}(\mathcal{R}). \tag{14b}$$

Here $\mathcal{R}$ is the rotation which brings the z axis into the direction of the emitted particle. The radiation parameters $c_{IM_I'}(\Lambda\Lambda')$, in whose definition a sum over the spin states of the emitted particle is included, are given by[18]

$$\text{photon:}\quad c_{IM_I'}(l\alpha,l'\alpha')=(4\pi)^{-1}(-1)^{l'-1}\hat{l}\hat{l}'\times\tfrac{1}{2}[1+(-1)^{I+l+\alpha+l'+\alpha'}](l1,l'-1\mid I0)\delta_{M_I'0}; \tag{15a}$$

$$\text{nucleon:}\quad c_{IM_I'}(jl,j'l')=(4\pi)^{-1}(-1)^{I+j-1/2}\hat{j}\hat{j}'\times\tfrac{1}{2}[1+(-1)^{I+l+l'}](j\tfrac{1}{2},j'-\tfrac{1}{2}\mid I0)\delta_{M_I'0}; \tag{15b}$$

$$\alpha\text{ particle:}\quad c_{IM_I'}(l,l')=(4\pi)^{-1}(-1)^{l'}\times\hat{l}\hat{l}'(l0,l'0\mid I0)\delta_{M_I'0}. \tag{15c}$$

Only the case $M_I'=0$ appears, because we do not analyze the polarization of the emitted particle. Note that the projection operators $\frac{1}{2}[\cdots]$ may be written in the common form

$$\tfrac{1}{2}[1+(-1)^I\pi_0\pi_0'] \tag{15d}$$

because of parity conservation of the interaction $\pi_l=\pi_0\pi_f$ or $\pi_{l'}=\pi_0'\pi_f$, and using the fact that the parity of the transition Λ is $\pi_l\equiv(-1)^{l+\alpha}$ for photons and $(-1)^l$ for nucleons and α particles. The same factor, Eq. (15d), may be added to Eq. (15c) due to the properties of $(l0,l'0\mid I0)$. This factor [Eq. (15d)] depends only on I and on the parity of the intermediate states, and not on Λ. It shows that only even values of I appear for the cross terms from two interfering levels of the same parity, or for the square term of each one intermediate level and only odd values of I for the cross terms of two interfering levels of opposite parity.

We now may proceed to the proof of Eq. (8). Integrating Eq. (14a) over $d\Omega_p$, we obtain from the orthogonality property of

$$D_{M_I0}{}^{I}(\mathcal{R})\equiv(4\pi)^{1/2}\hat{I}^{-1}Y_{IM_I}{}^*(\hat{p}) \tag{16}$$

[22] We use here the rotation matrices $D_{mm'}{}^{I}$ and rotation R as defined in Ref. 18; these definitions also agree with those of M. E. Rose, *Elementary Theory of Angular Momentum* (Wiley-Interscience, Inc., New York, 1957).

$(\hat{p}\equiv\mathbf{p}/p)$:

$$\sum_{M_f}\int \rho_{J_0J_0'M_0M_0'}{}^{J_fM_f}d\Omega_p = 4\pi\delta_{J_0J_0'}\delta_{M_0M_0'}\sum_{\Lambda\Lambda'(\lambda=\lambda')}\hat{\lambda}^{-1}c_{00}(\Lambda\Lambda') \times\langle J_f\,||\,\Lambda\,||\,J_0\rangle^*\langle J_f\,||\,\Lambda'\,||\,J_0'\rangle. \quad (17a)$$

With

$$c_{00}(\Lambda\Lambda')\,|_{\lambda=\lambda'}=(4\pi)^{-1}\hat{\lambda}\tfrac{1}{2}[1+(-1)^{\bar{\lambda}+\bar{\lambda}'}], \quad (17b)$$

and using Eq. (17a) and the definitions Eqs. (6b) and (10b), Eq. (8) may be readily obtained. At the same time, it is seen that the quantity inside the summation sign over M_0 in Eq. (6b) is actually independent of M_0, and we also obtain the explicit expression for the decay width

$$\Gamma_{J_0\to J_f}=(2\pi)^{-2}E_p p\sum_{\Lambda\Lambda'(\lambda=\lambda')}\tfrac{1}{2}[1+(-1)^{\bar{\lambda}+\bar{\lambda}'}] \times\langle J_f\,||\,\Lambda\,||\,J_0\rangle^*\langle J_f\,||\,\Lambda'\,||\,J_0\rangle. \quad (18)$$

The particle width may also be expressed by an average of $dw/d\Omega_p$ over M_0, rather than by an integration over $d\Omega_p$ as in Eq. (6b), namely,

$$\Gamma_{J_0\to J_f}=\frac{4\pi}{\hat{J}_0^2}\sum_{M_0}\sum_{M_fm_p}\left(\frac{dw}{d\Omega_p}\right)_{M_0M_0}^{J_0J_0\to J_f}. \quad (19)$$

This means that if just one of the two possible orientation averages $\int d\Omega_p$ or $\sum_{M_0}$ is carried out, the result is already orientation-independent. This is a well-known result.

IV. EXPLICIT COINCIDENCE CROSS SECTION

The Coulomb and transverse matrix elements for the excitation process in Born approximation[3,17] shall be defined here with a factor i^L included, which, due to the invariance under time reversal of the electromagnetic interaction Hamiltonian and of nuclear forces, makes them real,[23] provided we choose the nuclear wave functions so that they have the time-reversal property

$$T\,|\,J,M\rangle=(-1)^{J+M}\,|\,J,-M\rangle. \quad (20a)$$

Using the Wigner-Eckart theorem

$$\langle J_0M_0\,|\,\mathfrak{M}_{LM}\,|\,J_iM_i\rangle=\hat{J}_0^{-1}(J_iM_i,LM\,|\,J_0M_0) \times\langle J_0\,||\,\mathfrak{M}_L\,||\,J_i\rangle, \quad (20b)$$

we obtain the reduced matrix elements

$$\mathfrak{M}_L(q)\equiv\langle J_0\,||\,\mathfrak{M}_L(q)\,||\,J_i\rangle, \quad (21a)$$

$$\mathfrak{J}_L{}^{(e,m)}(q)\equiv\langle J_0\,||\,\mathfrak{J}_L{}^{(e,m)}(q)\,||\,J_i\rangle \quad (21b)$$

of the Coulomb and the transverse electric and magnetic operators

$$\mathfrak{M}_{LM}(q)=i^L\int\rho(\mathbf{r})j_L(qr)Y_{LM}(\hat{r})d^3r, \quad (22a)$$

$$\mathfrak{J}_{LM}{}^{(e)}(q)=(i^L/q)\int d^3r \times\{\mathbf{j}\cdot\boldsymbol{\nabla}\times[j_L(qr)\mathbf{X}_{LM}(\hat{r})]+q^2\boldsymbol{\mu}\cdot j_L(qr)\mathbf{X}_{LM}(\hat{r})\}, \quad (22b)$$

$$\mathfrak{J}_{LM}{}^{(m)}(q)=i^L\int d^3r\,\{\boldsymbol{\mu}\cdot\boldsymbol{\nabla}\times[j_L(qr)\mathbf{X}_{LM}(\hat{r})] +\mathbf{j}\cdot j_L(qr)\mathbf{X}_{LM}(\hat{r})\}, \quad (22c)$$

where $e\rho$, $e\mathbf{j}$, and $e\boldsymbol{\mu}$ are the nuclear charge, current, and magnetization density, and

$$\mathbf{X}_{LM}(\hat{r})=\sum_{mm'}Y_{Lm}(\hat{r})\mathbf{e}_{m'}(Lm,1m'\,|\,LM) \quad (23)$$

are vector spherical harmonics. It should be noted that the matrix elements, Eqs. (21), depend on J_i and J_0 as well as on L, although this dependence is not explicitly indicated. In the following, we shall use the notation, e.g.,

$$\mathfrak{M}_{L'}(q)\equiv\langle J_0'\,||\,\mathfrak{M}_{L'}(q)\,||\,J_i\rangle, \quad (24)$$

with J_0' understood.

The interaction of the nucleus with the electromagnetic field $(\varphi,\mathbf{A})$ of the electron is given by

$$H=e\int d^3r\,(\rho\varphi-\mathbf{j}\cdot\mathbf{A}-\boldsymbol{\mu}\cdot\boldsymbol{\nabla}\times\mathbf{A}). \quad (25)$$

For photon decay, one has $K=H$ to first order, and the decay matrix elements, Eq. (11), will likewise be given by Eq. (21). Indeed, using the well-known multipole expansion[24] of the vector potential $\mathbf{A}$ corresponding to an emitted photon, we find

$$\langle J_f\,||\,l\alpha\,||\,J_0\rangle=2\pi e\hat{J}_0^{-1}(8\pi/p)^{1/2}\mathfrak{J}_L{}^{(e,m)}(p), \quad (26)$$

p being the decay photon momentum, $\alpha=0$ (e) or 1 (m), and $\mathfrak{J}_L$ is now to be taken as

$$\mathfrak{J}_L{}^{(e,m)}(p)=\langle J_0\,||\,\mathfrak{J}_L{}^{(e,m)}(p)\,||\,J_f\rangle, \quad (27)$$

i.e., with J_f replacing J_i [Eq. (21b)], since we now deal with the decay process.

In terms of these matrix elements, we have for the excitation density matrix, as in Ref. 3,

$$\left(\frac{d\sigma}{d\Omega}\right)_{M_0M_0'}^{J_i\to J_0J_0'}=(2e^4k_2/k_1\Delta^4\hat{J}_0\hat{J}_0') \times\sum_{LML'M'}(J_iM_i,LM\,|\,J_0M_0) \times(J_iM_i,L'M'\,|\,J_0'M_0')R_{LL'}{}^{MM'}, \quad (28)$$

with $R_{LL'}{}^{MM'}$ given by Eq. (17) of Ref. 3 (in which the sign of the longitudinal-transverse interference term should be changed). If we insert Eqs. (28), (10b),

[23] R. H. Pratt, J. D. Walecka, and T. A. Griffy, Nucl. Phys. 64, 677 (1965).

[24] See, e.g., R. Raphael and H. Überall, Nucl. Phys. 85, 327 (1966).

and (14) into the cross section of Eq. (7), and introduce a "decay parameter"

$$S_{J_0J_0'}{}^{IJ_f}=\sum_{\Lambda\Lambda'}(-1)^{I+J_f-J_0-\lambda}\times c_{I0}(\Lambda\Lambda')W(J_0J_0'\lambda\lambda';IJ_f)\times\langle J_f\|\Lambda\|J_0\rangle^*\langle J_f\|\Lambda'\|J_0'\rangle,\quad(29\text{a})$$

with the properties

$$S_{J_0J_0'}{}^{IJ_f*}=(-1)^{J_0'-J_0}S_{J_0'J_0}{}^{IJ_f},\quad(29\text{b})$$

$$\Gamma_{J_0\to J_f}=(\hat{J}_0E_pp/\pi)S_{J_0J_0}{}^{0J_f};\quad(29\text{c})$$

then we obtain after some algebra the explicit form for the coincidence cross section

$$\frac{d^3\sigma}{d\Omega d\Omega_p dE_2}=\frac{\alpha^2k_2E_pp}{\pi^2k_1\Delta^4\hat{J}_i{}^2}\operatorname{Re}\sum_{J_0J_0'LL'}\frac{(-1)^L\hat{J}_0\hat{J}_0'\hat{L}\hat{L}'}{(E-\omega_{J_0}-\frac{1}{2}i\Gamma_{J_0}{}^{\text{tot}})(E-\omega_{J_0'}+\frac{1}{2}i\Gamma_{J_0'}{}^{\text{tot}})}\times\sum_I W(IJ_0'LJ_i;J_0L')\tilde{X}_{LL'}{}^{J_iI}S_{J_0J_0'}{}^{IJ_f},\quad(30\text{a})$$

with

$$\tilde{X}_{LL'}{}^{J_iI}=4\pi\sum_{M_I}\sum_{\lambda\mu\lambda'\mu'}(\hat{L}\hat{L}'/\hat{I})\hat{\lambda}'(L0,L'0\,|\,\lambda0)(\lambda\mu,\lambda'\mu'\,|\,IM_I)Y_{IM_I}{}^*(\hat{p})Y_{\lambda\mu}(\hat{q})f_{\lambda'\mu'}{}^{LL'I\lambda},\quad(30\text{b})$$

where

$$f_{\lambda'\mu'}{}^{LL'I\lambda}=\sum_{\alpha\alpha'}\begin{Bmatrix}L & L' & I\\ L & L' & \lambda\\ \alpha & \alpha' & \lambda'\end{Bmatrix}\tilde{f}_{\lambda'\mu'}{}^{LL'}(\alpha\alpha').\quad(30\text{c})$$

The sum in Eq. (30c) has contributions from the Coulomb term ($\alpha=\alpha'=0$), the interference term ($\alpha=1$, $\alpha'=0$), and the transverse term ($\alpha=\alpha'=1$):

$$\tilde{f}_{\lambda'\mu'}{}^{LL'}(00)=(\Delta^2/q^2)^2(E_1E_2+k_1k_2\cos\theta+m_e{}^2)\mathfrak{M}_L\mathfrak{M}_{L'}\delta_{\lambda'0}\delta_{\mu'0},\quad(30\text{d})$$

$$\tilde{f}_{\lambda'\mu'}{}^{LL'}(10)=-2(\Delta^2/q^2)(\tfrac{4}{3}\pi)^{1/2}\{i(\kappa/q)(E_1+E_2)Y_{\lambda'\mu'}(\hat{\kappa})\mathfrak{J}_L{}^{(e)}+[E_1k_2Y_{\lambda'\mu'}(\hat{k}_2)+E_2k_1Y_{\lambda'\mu'}(\hat{k}_1)]\mathfrak{J}_L{}^{(m)}\}\mathfrak{M}_{L'}\delta_{\lambda'1},\quad(30\text{e})$$

$$\begin{aligned}\tilde{f}_{\lambda'\mu'}{}^{LL'}(11)=&-(8\pi/3)[(\kappa^2/q^2)\mathcal{Y}_{\lambda'\mu'}(\hat{\kappa},\hat{\kappa})\mathfrak{J}_L{}^{(e)}\mathfrak{J}_{L'}{}^{(e)}+i(\kappa/q)k_2\mathcal{Y}_{\lambda'\mu'}(\hat{\kappa},\hat{k}_2)\mathfrak{J}_L{}^{(e)}\mathfrak{J}_{L'}{}^{(m)}\\&-i(\kappa/q)k_1\mathcal{Y}_{\lambda'\mu'}(\hat{k}_1,\hat{\kappa})\mathfrak{J}_L{}^{(m)}\mathfrak{J}_{L'}{}^{(e)}+k_1k_2\mathcal{Y}_{\lambda'\mu'}(\hat{k}_1,\hat{k}_2)\mathfrak{J}_L{}^{(m)}\mathfrak{J}_{L'}{}^{(m)}]+\tfrac{1}{2}\Delta^2[(\tfrac{4}{3}\pi)\mathcal{Y}_{\lambda'\mu'}(\hat{q},\hat{q})\mathfrak{J}_L{}^{(e)}\mathfrak{J}_{L'}{}^{(e)}\\&+(8\pi/3)^{1/2}(\mathfrak{J}_L{}^{(e)}\mathfrak{J}_{L'}{}^{(m)}+\mathfrak{J}_L{}^{(m)}\mathfrak{J}_{L'}{}^{(e)})Y_{\lambda'\mu'}(\hat{q})\delta_{\lambda'1}+\sqrt{3}(\mathfrak{J}_L{}^{(e)}\mathfrak{J}_{L'}{}^{(e)}+\mathfrak{J}_L{}^{(m)}\mathfrak{J}_{L'}{}^{(m)})\delta_{\lambda'0}\delta_{\mu'0}].\end{aligned}\quad(30\text{f})$$

We used the vector $\boldsymbol{\kappa}=\mathbf{k}_2\times\mathbf{k}_1$ and the abbreviation

$$\mathcal{Y}_{\lambda'\mu'}(\hat{a},\hat{b})=\sum_{\nu\nu'}(1\nu,1-\nu'\,|\,\lambda'\mu')Y_{1\nu}(\hat{a})Y_{1-\nu'}(\hat{b}).\quad(30\text{g})$$

The presence of the interference term allows relative phases to be determined, unlike in the case of electron scattering experiments without coincidences. Equations (30) generalize (and correct[5]) Eqs. (23)–(26) of Ref. 3; likewise, Eq. (18) replaces Eq. (29) of Ref. 3. It should be noted that the numerical example discussed in Sec. III of Ref. 3 remains correct.

V. SUM OVER MAGNETIC QUANTUM NUMBERS

In Ref. 3, the remaining summations over magnetic quantum numbers were not carried out, but this may be done as follows. Without loss of generality, we choose the z axis of our coordinate system parallel to the direction of $\mathbf{q}$ and obtain

$$Y_{\lambda\mu}(\hat{q})=\hat{\lambda}(4\pi)^{-1/2}\delta_{\mu0}.\quad(31)$$

We now pick out the λ-dependent terms in Eqs. (30a)–(30c) and define the quantity

$$F_{\lambda'\mu'}=\sum_\lambda\hat{\lambda}\hat{\lambda}'(L0,L'0\,|\,\lambda0)\times(\lambda0,\lambda'\mu'\,|\,IM_I)\begin{Bmatrix}L & L' & I\\ L & L' & \lambda\\ \alpha & \alpha' & \lambda'\end{Bmatrix}.\quad(32)$$

This sum may be evaluated by standard angular momentum algebra. Using Eqs. (6.4.3) and (6.2.8) of Edmonds,[25] we obtain

$$\begin{aligned}F_{\lambda'\mu'}=&(-1)^{\lambda'-\mu'+\alpha+\alpha'}\hat{\lambda}'\hat{\alpha}^{-2}\hat{\alpha}'^{-1}\\&\times(L'\alpha',L'0\,|\,\alpha'\alpha')\delta_{M_I\mu'}\{(L\mu'-\alpha',L'\alpha'\,|\,I\mu')\\&\times(L\alpha'-\mu',L0\,|\,\alpha\alpha'-\mu')(\alpha'\alpha',\lambda'-\mu'\,|\,\alpha\alpha'-\mu')\\&+(-1)^{\lambda'}\alpha'(L\alpha'+\mu',L'-\alpha'\,|\,I\mu')\\&\times(L\alpha'+\mu',L0\,|\,\alpha\alpha'+\mu')(\alpha'\alpha',\lambda'\mu'\,|\,\alpha\alpha'+\mu')\}.\end{aligned}\quad(33)$$

The quantity $\tilde{X}_{LL'}{}^{J_iI}$ of Eq. (30b) is given by

$$\tilde{X}_{LL'}{}^{J_iI}=\sum_{\lambda'\mu'}\sum_{\alpha\alpha'}(-1)^{\mu'}\hat{L}\hat{L}'[(I-\mu')!/(I+\mu')!]^{1/2}\times P_I{}^{\mu'}(\cos\theta_p)\exp(-i\mu'\phi_p)\tilde{f}_{\lambda'\mu'}{}^{LL'}(\alpha\alpha')F_{\lambda'\mu'},\quad(34)$$

[25] A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, N.J., 1960).

where $P_I^{\mu'}$ are the associated Legendre functions. The vector coupling coefficients appearing in Eqs. (33) limit the number of terms contributing to Eq. (34) quite substantially; namely,

Coulomb term ($\alpha=\alpha'=0$):	only $\lambda'=\mu'=0$ allowed;
interference term ($\alpha=1$, $\alpha'=0$):	only $\lambda'=1$, $\mu'=\pm1$ allowed;
transverse term ($\alpha=\alpha'=1$):	only $\lambda'=0$, 1, 2, $\mu'=0$ and $\lambda'=2$, $\mu'=\pm2$ allowed.

Expressing the functions $\mathcal{Y}_{\lambda'\mu'}$ [Eq. (30g)] explicitly by the angles of the particles, Eqs. (30e) and (30f) may be simplified for the allowed values of α, α', λ', and μ'. Since there now remain only a few terms in Eq. (34), the sum may be written explicitly.

This leads to the result

$$\begin{aligned}\tilde{X}_{LL'}^{J_iI}=&P_I(\cos\theta_p)\{(L0,L'0\,|\,I0)(\Delta^2/q^2)^2(E_1E_2+k_1k_2\cos\theta+m_e^2)\mathfrak{M}_L\mathfrak{M}_{L'}-(L1,L'-1\,|\,I0)[(\kappa^2/q^2)+\tfrac{1}{2}\Delta^2]\\&\times[\tfrac{1}{2}(1+(-1)^{I+L+L'})(\mathfrak{J}_L^{(e)}\mathfrak{J}_{L'}^{(e)}+\mathfrak{J}_L^{(m)}\mathfrak{J}_{L'}^{(m)})+\tfrac{1}{2}(1-(-1)^{I+L+L'})(\mathfrak{J}_L^{(e)}\mathfrak{J}_{L'}^{(m)}+\mathfrak{J}_L^{(m)}\mathfrak{J}_{L'}^{(e)})]\}\\&+2[I(I+1)]^{-1/2}P_I^1(\cos\theta_p)(L1,L'0\,|\,I1)(\kappa\Delta^2/q^3)(E_1+E_2)\{\cos\phi_p[\tfrac{1}{2}(1+(-1)^{I+L+L'})\mathfrak{J}_L^{(e)}\\&-\tfrac{1}{2}(1-(-1)^{I+L+L'})\mathfrak{J}_L^{(m)}]\mathfrak{M}_{L'}-i\sin\phi_p[\tfrac{1}{2}(1-(-1)^{I+L+L'})\mathfrak{J}_L^{(e)}+\tfrac{1}{2}(1+(-1)^{I+L+L'})\mathfrak{J}_L^{(m)}]\mathfrak{M}_{L'}\}\\&+[(I-1)I(I+1)(I+2)]^{-1/2}P_I^2(\cos\theta_p)(L1,L'1\,|\,I2)(\kappa^2/q^2)\{\cos2\phi_p[\tfrac{1}{2}(1+(-1)^{I+L+L'})\\&\times(\mathfrak{J}_L^{(e)}\mathfrak{J}_{L'}^{(e)}-\mathfrak{J}_L^{(m)}\mathfrak{J}_{L'}^{(m)})-\tfrac{1}{2}(1-(-1)^{I+L+L'})(\mathfrak{J}_L^{(e)}\mathfrak{J}_{L'}^{(m)}-\mathfrak{J}_L^{(m)}\mathfrak{J}_{L'}^{(e)})]\\&-i\sin2\phi_p[\tfrac{1}{2}(1-(-1)^{I+L+L'})(\mathfrak{J}_L^{(e)}\mathfrak{J}_{L'}^{(e)}-\mathfrak{J}_L^{(m)}\mathfrak{J}_{L'}^{(m)})-\tfrac{1}{2}(1+(-1)^{I+L+L'})\\&\times(\mathfrak{J}_L^{(e)}\mathfrak{J}_{L'}^{(m)}-\mathfrak{J}_L^{(m)}\mathfrak{J}_{L'}^{(e)})]\}.\qquad(35)\end{aligned}$$

For photon decay, a similar result was obtained by Hubbard and Rose.[6] These authors conjectured that the terms containing $i\sin\phi_p$ and $i\sin2\phi_p$ should be unimportant. We note that they have to vanish exactly. Indeed, since the interactions involved are parity-conserving,[26]

$$\sin\phi_p\propto[(\mathbf{q}\times\mathbf{p})\times\mathbf{q}]\cdot(\mathbf{k}_1\times\mathbf{k}_2)\qquad(36a)$$

changes sign under parity reflection, whereas

$$\cos\phi_p\propto(\mathbf{q}\times\mathbf{p})\cdot(\mathbf{k}_1\times\mathbf{k}_2)\qquad(36b)$$

does not. The detailed cancellation comes about as follows: The parity of the electroexcitation transitions is given by

$$\pi_L=(-1)^{L+\beta},\qquad(37a)$$

where $\beta=0$ for Coulomb or (e) and $\beta=1$ for (m). The factors $\frac{1}{2}[1\pm(-1)^{I+L+L'}]$ may then, because of parity conservation ($\pi_L=\pi_i\pi_0$, $\pi_{L'}=\pi_i\pi_0$), be written

$$\tfrac{1}{2}[1\pm(-1)^I\pi_0\pi_0'],\qquad(37b)$$

the lower sign appearing with the sine terms in Eq. (35) only. Not only are these projection operators independent of the multipolarity and character of the transition, but they also depend only on I and on the parity of the intermediate states just as the projection operator (15d) contained in the decay parameters S. Indeed, taken together with the latter, Eq. (37b) may be replaced by unity for the absolute and the cosine terms in Eq. (35), and reduces to zero for the sine terms, which proves their absence explicitly.

The foregoing permits us to write for the coincidence cross section

$$\begin{aligned}\frac{d^3\sigma}{d\Omega d\Omega_p dE_2}=&\frac{\alpha^2k_2E_pp}{\pi^2k_1\Delta^4\hat{J}_i^2}\operatorname{Re}\sum_{J_0J_0'LL'}\frac{(-1)^L\hat{J}_0\hat{J}_0'\hat{L}\hat{L}'}{(E-\omega_{J_0}-\frac{1}{2}i\Gamma_{J_0}^{\text{tot}})(E-\omega_{J_0'}+\frac{1}{2}i\Gamma_{J_0'}^{\text{tot}})}\\&\times\sum_I\tfrac{1}{2}[1+(-1)^I\pi_0\pi_0']W(IJ_0'LJ_i;J_0L')X_{LL'}^{J_iI}S_{J_0J_0'}^{IJ_f},\qquad(38a)\end{aligned}$$

$$\begin{aligned}X_{LL'}^{J_iI}=&P_I(\cos\theta_p)[(L0,L'0\,|\,I0)(\Delta^2/q^2)^2(E_1E_2+k_1k_2\cos\theta+m_e^2)\mathfrak{M}_L\mathfrak{M}_{L'}\\&-(L1,L'-1\,|\,I0)((\kappa^2/q^2)+\tfrac{1}{2}\Delta^2)\sum_{\beta\beta'}\mathfrak{J}_L^{\beta}\mathfrak{J}_{L'}^{\beta'}]+2[I(I+1)]^{-1/2}P_I^1(\cos\theta_p)(L1,L'0\,|\,I1)\\&\times(\kappa\Delta^2/q^3)(E_1+E_2)\cos\phi_p\sum_\beta\mathfrak{J}_L^{\beta}\mathfrak{M}_{L'}+[(I-1)I(I+1)(I+2)]^{-1/2}P_I^2(\cos\theta_p)(L1,L'1\,|\,I2)\\&\times(\kappa^2/q^2)\cos2\phi_p\sum_{\beta\beta'}(-1)^{\beta'}\mathfrak{J}_L^{\beta}\mathfrak{J}_{L'}^{\beta'}.\qquad(38b)\end{aligned}$$

[26] The sine terms are not eliminated by the fact that in Eq. (30a) the real part of the expression has to be taken: By an exchange of dummy variables $J_0\leftrightarrow J_0'$ and $L\leftrightarrow L'$ (at the same time symmetrizing the term with P_I^1), all terms in Eq. (35) may be shown to be real. Nor are the sine terms eliminated by time-reversal-invariance arguments, which are not sufficient to render the decay parameters $S_{J_0J_0'}^{IJ_f}$ real (as shown by Watson's theorem) due to the final-state interactions between the nucleus and the emitted particle (except to first order if the latter is a photon). However, M_L and $\mathfrak{J}_L^{(l,m)}$ are real from time-reversal invariance (see Ref. 23).

Finally, the coincidence cross section may be written in the form of Eq. (1):

$$d^3\sigma/d\Omega d\Omega_p dE_2=(\alpha^2/\Delta^4)(k_2/k_1)pE_p\{V_C(\theta)W_C+V_T(\theta)W_T+V_I(\theta,\phi_p)W_I+V_S(\theta,\phi_p)W_S\}. \tag{39}$$

The kinematic functions

$$V_C=(\Delta^2/q^2)^2(E_1E_2+k_1k_2\cos\theta+m_e^2), \tag{40a}$$

$$V_T=(\kappa^2/q^2)+\tfrac{1}{2}\Delta^2, \tag{40b}$$

$$V_I=(\kappa\Delta^2/q^3)(E_1+E_2)\cos\phi_p, \tag{40c}$$

$$V_S=2(\kappa^2/q^2)\cos^2\phi_p+\tfrac{1}{2}\Delta^2, \tag{40d}$$

reduce to the expressions given by de Forest[11] in the limit of $m_e\to 0$, Eqs. (3). The generalized form factors in the partial-wave decomposition are given by

$$W_n=(\pi\hat{J}_i)^{-2}\sum_{J_0J_0'LL'}\frac{(-1)^L\hat{J}_0\hat{J}_0'\hat{L}\hat{L}'}{(E-\omega_{J_0}-\frac{1}{2}i\Gamma_{J_0}{}^{\text{tot}})(E-\omega_{J_0'}+\frac{1}{2}i\Gamma_{J_0'}{}^{\text{tot}})}$$
$$\times\sum_I \tfrac{1}{2}[1+(-1)^I\pi_0\pi_0']W(IJ_0'LJ_i;J_0L')S_{J_0J_0'}{}^{IJ_f}Z_{LL'}{}^I(n), \tag{41}$$

where

$$Z_{LL'}{}^I(C)=(L0,L'0\mid I0)\mathfrak{M}_L\mathfrak{M}_{L'}P_I(\cos\theta_p), \tag{42a}$$

$$Z_{LL'}{}^I(T)=-(L1,L'-1\mid I0)\sum_{\beta\beta'}\mathfrak{J}_L{}^\beta\mathfrak{J}_{L'}{}^{\beta'}P_I(\cos\theta_p)$$
$$-[(I-1)I(I+1)(I+2)]^{-1/2}(L1,L'1\mid I2)\sum_{\beta\beta'}(-1)^{\beta'}\mathfrak{J}_L{}^\beta\mathfrak{J}_{L'}{}^{\beta'}P_I{}^2(\cos\theta_p), \tag{42b}$$

$$Z_{LL'}{}^I(I)=[I(I+1)]^{-1/2}[(L1,L'0\mid I1)\sum_\beta\mathfrak{J}_L{}^\beta\mathfrak{M}_{L'}+(L'1,L0\mid I1)\sum_{\beta'}\mathfrak{M}_L\mathfrak{J}_{L'}{}^{\beta'}]P_I{}^1(\cos\theta_p), \tag{42c}$$

$$Z_{LL'}{}^I(S)=[(I-1)I(I+1)(I+2)]^{-1/2}(L1,L'1\mid I2)\sum_{\beta\beta'}(-1)^{\beta'}\mathfrak{J}_L{}^\beta\mathfrak{J}_{L'}{}^{\beta'}P_I{}^2(\cos\theta_p). \tag{42d}$$

The mentioned symmetrization[26] has been carried out in the interference term $Z_{LL'}{}^I$. All the quantities $Z_{LL'}{}^I(n)$ are real if the nuclear wave functions have been chosen to obey Eq. (20a).

We complete the general formalism by defining the decay parameters (29a) for the various emitted particles [the projection factor (15d) is understood here]:

photon:

$$S_{J_0J_0'}{}^{IJ_f}=(-1)^{I+J_f-J_0+1}(4\pi)^{-1}\sum_{l\alpha l'\alpha'}(-1)^{l+l'}\hat{l}\hat{l}'(l1,l'-1\mid I0)$$
$$\times W(J_0J_0'll';IJ_f)\langle J_f\|l\alpha\|J_0\rangle^*\langle J_f\|l'\alpha'\|J_0'\rangle \tag{43a}$$

[to be used in conjunction with Eq. (26)];

nucleon:

$$S_{J_0J_0'}{}^{IJ_f}=(-1)^{J_f-J_0-\frac{1}{2}}(4\pi)^{-1}\sum_{jlj'l'}\hat{j}\hat{j}'(j\tfrac{1}{2},j'-\tfrac{1}{2}\mid I0)W(J_0J_0'jj';IJ_f)\langle J_f\|jl\|J_0\rangle^*\langle J_f\|j'l'\|J_0'\rangle; \tag{43b}$$

α particle:

$$S_{J_0J_0'}{}^{IJ_f}=(-1)^{I+J_f-J_0}(4\pi)^{-1}\sum_{ll'}(-1)^{l+l'}\hat{l}\hat{l}'(l0,l'0\mid I0)W(J_0J_0'll';IJ_f)\langle J_f\|l\|J_0\rangle^*\langle J_f\|l'\|J_0'\rangle. \tag{43c}$$

In the approximation of a diagonal level matrix used in this paper, the reduced matrix elements for particle decay are given by

$$\langle J_f\|jl\|J_0\rangle=[2\pi/\surd(pE_p)]\Omega_{jl}(\Gamma_{J_0\to J_f}{}^{jl})^{1/2}, \tag{43d}$$

where Ω_{jl} is a phase factor (see Ref. 27 for definition) and $\Gamma_{J_0\to J_f}{}^{jl}$ the partial decay width of state J_0 into the state J_f of the residual nucleus and a particle with quantum numbers j and l. In a general R-matrix theory, taking coupling of the resonances into account, one has to replace in Eq. (38a) the energy denominator and the reduced matrix element (43d) by

$$(E-\omega_{J_0\alpha}-\tfrac{1}{2}i\Gamma_{J_0\alpha}{}^{\text{tot}})^{-1}\langle J_f\|jl\|J_0\alpha\rangle\to\sum_\beta A_{\beta\alpha}{}^{J_0}$$
$$\times\langle J_f\|jl\|J_0\beta\rangle. \tag{43e}$$

Here the excited states are described by (J_0,α) or (J_0,β), where α and β are additional quantum numbers to describe the intermediate states. Further, $A_{\beta\alpha}{}^{J_0}$ is the matrix element of the level matrix as defined in Ref. 15 between two states α and β (both having the

[27] H. Überall, Nuovo Cimento **41**, 25 (1966).

same spin J_0). Obviously, our old formulas are obtained if one assumes that the level matrix is diagonal:

$$A_{\beta\alpha}{}^{J_0}\longrightarrow\delta_{\alpha\beta}(E_0-\omega_{J_0\alpha}-\tfrac{1}{2}i\Gamma_{J_0\alpha}{}^{\mathrm{tot}})^{-1}.$$

In the eigenchannel theory for continuum states, no additional summation as in Eq. (43e) is necessary if we identify the excited states (J_0,α) with eigenstates of Hamiltonian and S matrix (with regard to strong interactions). One simply has to replace

$$(E-\omega_{J_0\alpha}-\tfrac{1}{2}i\Gamma_{J_0\alpha}{}^{\mathrm{tot}})^{-1}\langle J_f\,\|\,jl\,\|\,J_0\alpha\rangle \\ \rightarrow[(2\pi)^3/\sqrt{(pE_p)}]V_{J_f,jl}{}^{J_0,\alpha}\exp i\delta_{J_0}{}^{\alpha}, \quad (43\mathrm{f})$$

where $V_{J_f,jl}{}^{J_0,\alpha}$ is the amplitude of the physical channel J_f, j, l (i.e., residual nucleus in state J_f, outgoing nucleon with quantum numbers j, l) in the eigenchannel α of spin J_0, and $\delta_{J_0}{}^{\alpha}$ is the eigenphase.

The appearance of angular factors $P_I(\cos\theta_p)$, $P_I{}^1(\cos\theta_p)\cos\phi_p$, and up to $P_I{}^2(\cos\theta_p)\cos2\phi_p$ in the excitation terms goes back to the vector nature of the exchanged virtual photon (spin 1); this may be considered a generalization of the familiar Treiman-Yang test[28] to the case of exchange of a virtual particle with spin. In both the excitation and decay terms, Clebsch-Gordan coefficients appear with appropriate angular momentum projections for the spin of the particles involved; e.g., 0 for the longitudinal, ±1 for the transverse excitation terms.

Rose has noted[6,7] that the general coincidence formulas have a certain similarity with the electron scattering cross-section expressions for aligned nuclei.[29] A coincidence experiment, therefore, is somewhat equivalent to a noncoincidence experiment with aligned nuclei (except that here it is the final nucleus which gets aligned due to the observation of the emitted particle).

Finally, it is straightforward to show that the correct noncoincidence cross section in Born approximation[17] is obtained if one integrates the coincidence cross section [Eq. (39)] over the directions of the outgoing particle $d\Omega_p$. The result is

$$\frac{d^2\sigma}{d\Omega dE_2}=\sum_{J_0}\frac{1}{2\pi}\frac{\Gamma_{J_0}{}^{\mathrm{tot}}}{(E-\omega_{J_0})^2+\frac{1}{4}(\Gamma_{J_0}{}^{\mathrm{tot}})^2} \times\frac{\Gamma_{J_0\to J_f}}{\Gamma_{J_0}{}^{\mathrm{tot}}}\frac{8\pi\alpha^2k_2}{k_1\Delta^4\hat{J}_i{}^2}\Big\{V_C(\theta)\sum_{L=0}^{\infty}|\mathfrak{M}_L|^2 +V_T(\theta)\sum_{L=1}^{\infty}[|\mathfrak{J}_L{}^{(e)}|^2+|\mathfrak{J}_L{}^{(m)}|^2]\Big\}, \quad (44)$$

which is the familiar Born-approximation scattering cross section multiplied by the branching ratio into the given decay channel, and with the contributions of the individual levels J_0, each spread out in energy according to a Breit-Wigner shape, added together incoherently. It also agrees with our general formula (9a).

This completes the general formalism. In the following, some numerical examples shall be considered.

VI. EXAMPLES

In our numerical calculations we will assume that the spin of the nucleus in the ground state is $J_i=0$, and that the excited states have spin $J_0=J_0'=1$. In that case the formulas (39)–(42) simplify considerably, and we obtain the coincidence cross section

$$\left.\frac{d^3\sigma}{d\Omega d\Omega_p dE_2}\right|_{0^+\to1^\pm}=\frac{3^{1/2}\alpha^2k_2pE_p}{\pi^2k_1\Delta^4}\sum_{ab}[(E-\omega_a-\tfrac{1}{2}i\Gamma_a{}^{\mathrm{tot}})(E-\omega_b+\tfrac{1}{2}i\Gamma_b{}^{\mathrm{tot}})]^{-1} \\ \times\{S^0[\mathfrak{M}_a\mathfrak{M}_bV_L+\mathfrak{J}_a\mathfrak{J}_bV_T]-2^{1/2}S^2[(\mathfrak{M}_a\mathfrak{M}_bV_L-\tfrac{1}{2}\mathfrak{J}_a\mathfrak{J}_bV_T)P_2(\cos\theta_p) \\ +2^{-1/2}\mathfrak{J}_a\mathfrak{M}_bV_IP_2{}^1(\cos\theta_p)\mp\tfrac{1}{4}\mathfrak{J}_a\mathfrak{J}_b(V_S-V_T)P_2{}^2(\cos\theta_p)]\} \quad (45)$$

(dropping the second superscript J_f of $S_{J_0J_0'}{}^{IJ_f}$ as well as the subscripts), where the functions V_n have been defined in Eq. (40) and the indices a and b indicate different dipole states. For magnetic dipole states ($0^+\to1^+$) the longitudinal matrix elements $\mathfrak{M}_a$ and $\mathfrak{M}_b$ vanish and, specializing to the case of an isolated magnetic dipole level, we obtain

$$\left.\frac{d^3\sigma}{d\Omega d\Omega_p dE_2}\right|_{0^+\to1^+}=\frac{\sqrt{3}\alpha^2k_2pE_p}{\pi^2k_1\Delta^4}\frac{|\mathfrak{J}_1{}^{(m)}|^2S^0}{(E-\omega)^2+\frac{1}{4}(\Gamma^{\mathrm{tot}})^2} \\ \times\left\{V_T+\frac{S^2/S^0}{2^{3/2}}V_T(3\cos^2\theta_p-1)+\frac{3S^2/S^0}{2^{3/2}}\left(\frac{\kappa}{q}\right)^2\sin^2\theta_p\cos2\phi_p\right\}. \quad (46)$$

The decay parameter S^0 is related to the partial decay width for the process under consideration [see Eq. (29c)] and is obtained from the cross section integrated over the angles of the emitted particle. The ratio S^2/S^0 may be determined by varying either the polar angle θ_p or the azimuthal angle ϕ_p, since all the other variables in Eq. (46) are well known. If the emitted particle is a photon and the nucleus is deexcited to its ground state ($J_f=0$), the decay matrix elements may be expressed by the transverse magnetic transition operators [see Eqs. (43a) and (26)], and the

[28] S. B. Treiman and C. N. Yang, Phys. Rev. Letters **8**, 140 (1962).
[29] L. J. Weigert and M. E. Rose, Nucl. Phys. **51**, 529 (1964).

ratio $S^2/S^0=2^{-1/2}$ is known. The coincidence cross section is then simply

$$d^3\sigma/d\Omega d\Omega_p dE_2\,|_{0^+\to1^+\to0^+}\propto\{V_T(1+\cos^2\theta_p)+(\kappa/q)^2\sin^2\theta_p\cos2\phi_p\},\quad(47)$$

and any deviations from such an angular distribution are due either to ordinary bremsstrahlung and higher-order processes (more photon exchanges) or to an admixture of states with other multipolarity and parity in the energy region under consideration. Figure 5 shows angular distributions expected for excitation of the 15.1-MeV magnetic dipole level in C^{12} with 200-MeV electrons. In Fig. 5(a), we have plotted the coplanar cross section (i.e., the photon is emitted in the scattering plane of the electron $\phi_p'=0$ as a function of the polar photon angle θ_p' about the beam axis (direction of $\mathbf{k}_1$), for various polar angles θ of the scattered electron. Figure 5(b) gives noncoplanar cross sections at fixed electron angle $\theta=90°$ for various aximuthal angles ϕ_p' between the direction of the emitted photon and the scattering plane of the electron. Note that the angles θ_p' and ϕ_p' are measured about the beam axis (direction of $\mathbf{k}_1$), whereas the angles θ_p and ϕ_p in the text refer to the direction of momentum transfer $\mathbf{q}$.

Similarly, we obtain for the excitation of an isolated electric dipole level

$$\frac{d^3\sigma}{d\Omega d\Omega_p dE_2}=\frac{\sqrt{3}\alpha^2k_2pE_p}{\pi^2k_1\Delta^4}\frac{S^0}{(E-\omega)^2+\frac14(\Gamma^{\rm tot})^2}\Bigg\{\mathfrak{M}_1{}^2V_L\bigg(1-\frac{S^2/S^0}{\sqrt2}(3\cos^2\theta_p-1)\bigg)-3\mathfrak{J}_1{}^{(e)}\mathfrak{M}_1(S^2/S^0)(\kappa\Delta^2/q^3)(E_1+E_2)\cos\theta_p\sin\theta_p\cos\phi_p$$
$$+(\mathfrak{J}_1{}^{(e)})^2\bigg[V_T\bigg(1+\frac{S^2/S^0}{2^{3/2}}(3\cos^2\theta_p-1)\bigg)-(3/2^{3/2})(S^2/S^0)(\kappa/q)^2\sin^2\theta_p\cos2\phi_p\bigg]\Bigg\}.\quad(48)$$

In the case of photon decay of the excited state to the ground state ($J_f=0$), the ratio $S^2/S^0=2^{-1/2}$ also for electric dipole transitions. The coincidence cross section is then

$$d^3\sigma/d\Omega d\Omega_p dE_2\,|_{0^+\to1^+\to0^+}\propto\{\mathfrak{M}_1{}^2V_L\sin^2\theta_p-\sqrt2\mathfrak{J}_1{}^{(e)}\mathfrak{M}_1(\kappa\Delta^2/q^3)(E_1+E_2)\sin\theta_p\cos\theta_p\cos\phi_p+(\mathfrak{J}_1{}^{(e)})^2\tfrac12[V_T(1+\cos^2\theta_p)-(\kappa/q)^2\sin^2\theta_p\cos2\phi_p]\}.\quad(49)$$

Thus the contribution of the longitudinal matrix element vanishes if the photon is emitted in the direction of momentum transfer ($\theta_p=0$). Therefore, with the exception of extreme forward and backward angles, the coincidence cross section will show a strong minimum for $\theta_p=0$. For particle emission the ratio S^2/S^0 has to be calculated from some nuclear model. Using R-matrix theory,[30] Raphael and Überall[3] have calculated the decay parameters for the process

$$e+{}^{12}\mathrm{C}\to e'+{}^{11}\mathrm{C}+n,\quad(50)$$

proceeding via the giant resonance of ^{12}C, and obtain the ratio

$$S_2/S_0=-0.595.\quad(51)$$

The particle-hole wave functions of Gillet and Vinh-Mau[31] were used. The coincidence cross section for that process (integrated over the resonance) has been plotted in Fig. 6(a) as a function of the particle emission angle θ_p' for various electron scattering angles θ. At forward and backward angles ($\theta=0°$ and 180°) the transverse terms dominate, giving an angular distribution symmetric about $\theta_p'=90°$, where the emission probability has a maximum (since $S^2/S^0<0$ for that process). For most other scattering angles, the longitudinal terms are much larger than the transverse ones. However, the interference term always contributes more than about 10% of the total angular distribution and thus it should be possible to determine the magnitude and relative phase of the transverse and longitudinal terms. Figure 6(b) shows the contribution of the various terms at $\theta=175°$, where Coulomb and transverse term are of the same importance.

In the examples given above it has been assumed that only one isolated resonance is being excited. This is clearly not the case in the giant resonance region where states of different internal structure and various spins and parities overlap. As a first step, we have therefore calculated coincidence cross sections as functions of the excitation energy E for the process

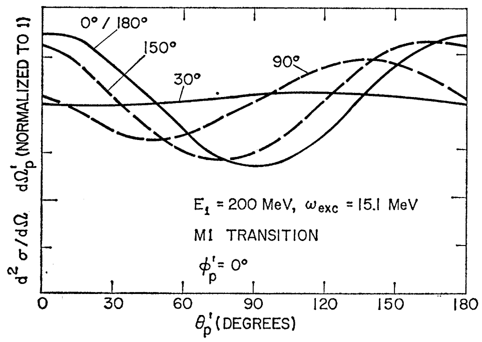
$$e+{}^{12}\mathrm{C}\to e'+{}^{12}\mathrm{C}^*\to e'+{}^{11}\mathrm{B}+p,\quad(52)$$

assuming electroexcitation of the dipole states at 17.7, 21.9, and 24.2 MeV and subsequent decay to the ground state of ^{11}B by proton emission. The longitudinal and transverse electric matrix elements as functions of momentum transfer q have been calculated[32] using Gillet's wave functions.[31] Further, we have calculated the decay parameters S^0 and S^2 along the lines of Ref. 30. (See also Ref. 27 for an outline of how to obtain the decay parameters). Besides the usual restriction of Hilbert space to few low-lying 1p-1h configurations, it is assumed in this reference that the level matrix is diagonal. Since this assumption led[30] to excellent agree-

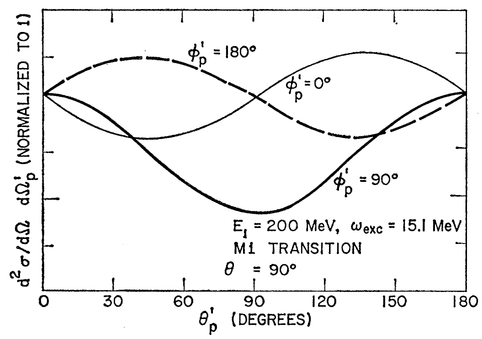
[30] See, e.g., E. Boeker, thesis, University of Amsterdam, 1963 (unpublished); a condensed version of this work appeared as E. Boeker and C. C. Jonker, Phys. Letters **6**, 80 (1963).

[31] V. Gillet and N. Vinh-Mau, Nucl. Phys. **54**, 321 (1964).

[32] Francis J. Kelly and H. Überall, Phys. Rev. **175**, 1235 (1968).



(a)



(b)

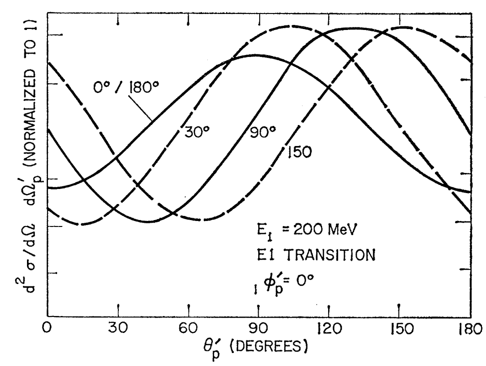
FIG. 5. Coincidence cross section $d^2\sigma/d\Omega d\Omega_p'$ as function of photon polar angle θ_p' about the beam axis for excitation of the 15.1-MeV magnetic dipole level in ^{12}C and subsequent decay by photon emission. Electron energy $E_1=200$ MeV. (a) Coplanar geometry ($\phi_p'=0$) and various electron scattering angles θ. (b) Fixed electron scattering angle $\theta=90°$ for various noncoplanarities ϕ_p'.

ment for the (γ, p) angular distributions from ^{12}C, we think that this model should also give a fair description of (e, $e'p$) coincidence experiments in the giant resonance region of ^{12}C, at least at low-to-moderate momentum transfer.

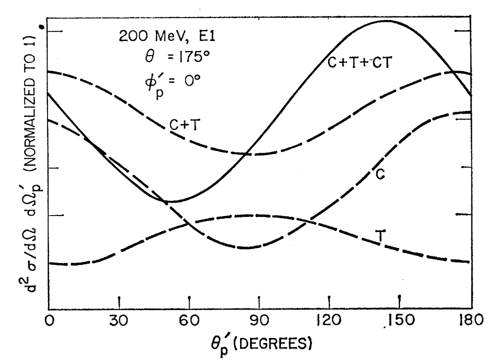
The coincidence cross section $d^3\sigma/d\Omega d\Omega_p dE$ as a function of the excitation energy E is obtained from Eq. (45) and has been plotted in Figs. 7(a)–7(c) for different geometries. Figure 7(a) shows the cross section for coplanar geometry ($\phi_p'=0$) and various polar angles θ_p' between the direction of the emitted proton and the electron beam axis. The electron energy is 200 MeV; the electron is scattered under $\theta=30°$. At this momentum transfer, the 21.9-MeV level dominates. Figure 7(b) shows that at higher momentum transfer (corresponding to $\theta=90°$) the low-lying 17.7-MeV resonance becomes more important. Finally, in Fig. 7(c), the noncoplanar cross section is given for various azimuthal angles ϕ_p'. In all cases the angular distribution of the 17.7-MeV resonance is nearly isotropic (i.e., S^2 is small at that excitation energy) since the excited particle is essentially in the $2s$ state. At higher excitation energy, however, the parameters S^0 and S^2 have comparable size, giving a strong variation of the coincidence cross section with the angles θ_p' and ϕ_p' of the emitted particle. Contributions of resonances with other spins and parities would change the pattern given in Figs. 7(a)–7(c) and described by Eq. (45). Since the form of the angular distribution is determined by spin and parity of the excited states in a model-independent way (only the magnitude of the variation with the angles θ_p' and ϕ_p' is model-dependent and determined by the decay parameters S^I), very accurate coincidence experiments will eventually enable us to project out the contributions of the various multipoles and spins according to Eqs. (39)–(42).

VII. SUMMARY

Electron scattering experiments in which an emitted particle is detected in coincidence with the scattered electron are very useful for investigating the nuclear structure. By observing a heavy particle in coincidence with the electron, one gets rid of the problems of radiative corrections above the particle emission threshold which have plagued ordinary electron scattering. This does not apply, however, if a photon is observed. In that case there occurs an interference between radiation following deexcitation of the nucleus

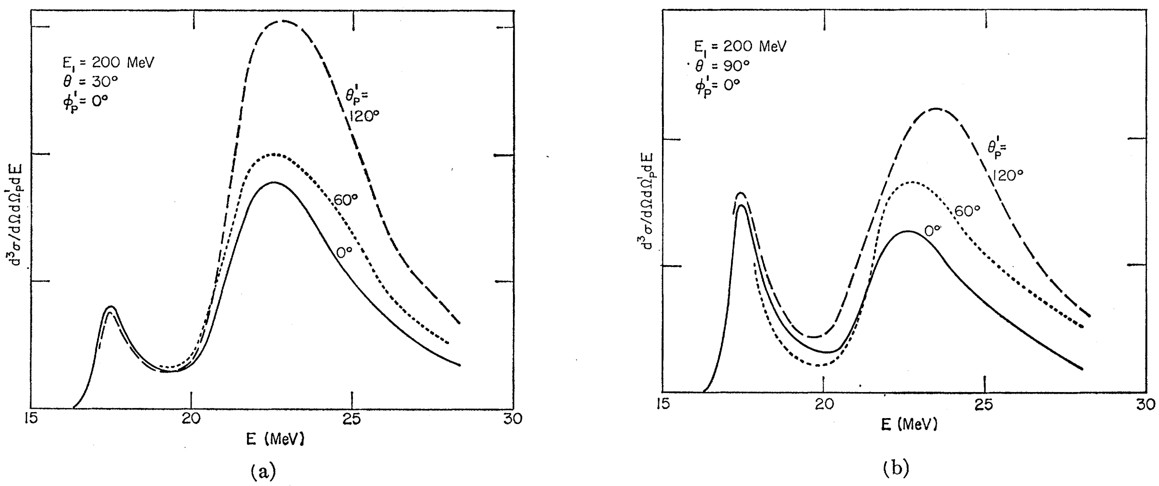


(a)



(b)

FIG. 6. Coincidence cross section $d^2\sigma/d\Omega d\Omega_p'$ as function of photon polar angle θ_p' for excitation of the giant resonance in ^{12}C and subsequent decay by neutron emission; $E_1=200$ MeV. (a) Coplanar geometry ($\phi_p'=0$) and various electron scattering angles θ. (b) Contributions of longitudinal, transverse, and interference terms at $\theta=175°$.



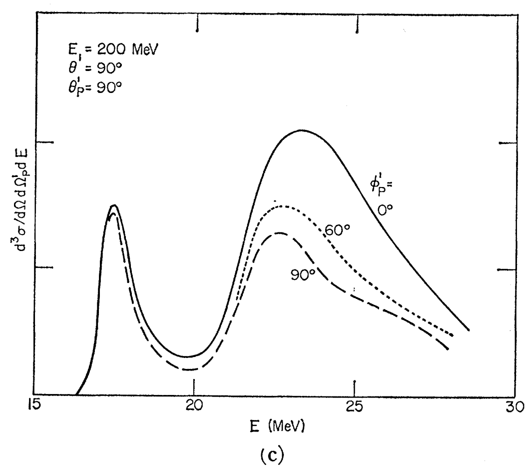


FIG. 7. Coincidence cross section $d^3\sigma/d\Omega d\Omega_p' dE$ as function of excitation energy E for excitation of the overlapping giant resonance states in ^{12}C and subsequent decay by proton emission; $E_1=200$ MeV. (a) Coplanar geometry ($\phi_p'=0$) and various polar angles θ_p'; electron scattering angle $\theta=30°$. (b) Coplanar geometry ($\phi_p'=0$) and various polar angles θ_p'; electron scattering angle $\theta=90°$. (c) Noncoplanar cross section for various azimuthal angles ϕ_p' at fixed $\theta_p'=90°$; $\theta=90°$.

and bremsstrahlung of the electron, the latter process dominating in the directions of the incident and scattered electron.[7]

While at large energy and momentum transfer electroexcitation may be described reasonably well by the model of quasi-elastic scattering,[11] i.e., neglecting the residual interaction of the ejected proton (or neutron) with the other nucleons, we consider in this paper the region of low-to-moderate energy and momentum transfer which is dominated by a resonance structure: The electron excites a (collective) nuclear motion (e.g., the giant resonance) which decays by emission of a particle. Since the nuclear resonances are classified according to their spin and parity, we have performed a partial-wave decomposition of the generalized nuclear form factors. With increasing momentum transfer, nuclear states of higher and higher multipolarity may be excited, which will eventually make the multipole expansion impracticable. Besides, at higher excitation energy the nuclear problem becomes more and more involved because of the increasing number of channels which have to be taken into account, thus limiting the calculations from low-to-moderate excitation energy.

Among our numerical examples, we have calculated the coincidence cross section for electroexcitation of the overlapping giant resonance states in C^{12} followed by proton emission. While the form of the angular distributions obtained is model-independent (i.e., depends only on spins and parities involved), the magnitude of the variations with emission angles depends strongly on the particle-hole configurations pertaining to a certain excitation energy. Eventually, coincidence experiments will make it possible to project out the contribution of resonances with a certain spin and parity and thus to compare more directly with nuclear structure calculations.

ACKNOWLEDGMENTS

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