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Isobaric-Spin Impurities of ^{18}F States from ^{14}N(α, n)^{17}F(0.500)/^{14}N(α, p)^{17}O(0.871) Cross-Section Ratios*

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A pair of mirror reactions initiated with α particles was chosen to estimate the isobaric-spin impurity of the intermediate compound-nuclear states in ^{18}F. Total-cross-section ratios of the ^{14}N(α, n)^{17}F(0.500) and ^{14}N(α, p)^{17}O(0.871) mirror reactions were measured in 100-keV steps from $E_\alpha=7$ to 12 MeV by recording the isotropic deexcitation γ rays from the ^{17}F(0.500), ^{17}O(0.871), and ^{17}O(3.058) states. Considerable structure indicative of compound-nucleus formation was observed in the ratios plotted against α energy, with major maxima at $E_\alpha=8.12$, 9.5, 10.07, and 11.52 MeV. Symmetric angular distributions at three of these maxima resulted in tentative spin assignments of (2^-) at a ^{18}F excitation of 10.764$\pm$0.030 MeV, (2^-, 3^-, 3^+) at 12.234$\pm$0.030 MeV, and (2^-, 3^+) at 13.361$\pm$0.030 MeV. These states are unambiguously $T=0$, in contrast with the forbidden-reaction isobaric-spin assignments, having estimated $T=1$ impurities of 12%, 3–12%, and 2–3%, respectively. A comparison is made with earlier isobaric-spin-impurity measurements that utilized forbidden reactions, and the agreement is good after those forbidden-reaction estimates incorporating a statistical theory of the compound nucleus are reduced by a factor of $\frac{2}{3}$. This correction arises from the forbidden reactions proceeding through both impure $T=0$ and impure $T=1$ states, which are of comparable density in ^{18}F.

I. INTRODUCTION

THE strong sensitivity of total-cross-section ratios of mirror reactions to isobaric-spin admixtures in the compound nucleus serves as an excellent probe of the isobaric-spin impurities in these intermediate states. Barker and Mann[1] were first to explain the differing neutron and proton emission rates in photonuclear reactions on self-conjugate nuclei by postulating an isobaric-spin impurity in the decaying states. By their method, MacDonald[2] estimated an isobaric-spin impurity in the mirror reaction measurements of Wilkinson.[3]

The formulas of the earlier paper were generalized by Barker[4] and are presented here. Considering $T=1$ impurities in a mainly $T=0$ state of an $N=Z$ nucleus, one writes the wave function as the sum

$$\psi=\psi(0)+\sum_k \alpha_k\psi_k(1),$$

$$\alpha_k=(\psi^*(0),H_c\psi_k(1))/[E(0)-E_k(1)],$$

and the isobaric-spin impurity is defined as

$$\sum_k |\alpha_k|^2.$$

The Hamiltonian H_c represents the charge-dependent interaction causing the isobaric-spin mixing. The reduced-width amplitude for the channel labeled by M_T is

$$\theta_{M_T}\propto\theta(0)\left(\tfrac{1}{2}\,\tfrac{1}{2}\,M_T\,-M_T\,|\,00\right)+\sum_k\alpha_k\theta_k(1)\left(\tfrac{1}{2}\,\tfrac{1}{2}\,M_T\,-M_T\,|10\right)$$

$$\propto\theta(0)+(-1)^{M_T-1/2}\sum_k\alpha_k\theta_k(1).$$

The terms $\theta(0)$ and $\theta_k(1)$ are the reduced-width amplitudes for states $\psi(0)$ and $\psi_k(1)$ apart from the Clebsch-

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[1] F. C. Barker and A. K. Mann, Phil. Mag. 2, 5 (1957).
[2] W. M. MacDonald, in *Nuclear Spectroscopy, Part B*, edited by Fay Ajzenberg-Selove (Academic Press Inc., New York, 1960), p. 932.
[3] D. H. Wilkinson, Phil. Mag. 2, 83 (1957).
[4] F. C. Barker (private communication).

TABLE I. The sensitivity of total-cross-section ratios of mirror reactions to isobaric-spin impurity $\Sigma_k |\alpha_k|^2$ in the compound nucleus assuming that one intermediate state introduces the $T=1$ impurity and that $\theta_k(1)=\pm\theta(0)$. Double values of the ratio result from a sign ambiguity among terms in the ratio expression.

$\lvert\alpha_k\rvert^2$	α_k	σ_n/σ_p
0.01	$\pm$0.10	0.67, 1.49
0.02	$\pm$0.14	0.57, 1.75
0.05	$\pm$0.22	0.40, 2.50
0.10	$\pm$0.32	0.27, 3.70
0.20	$\pm$0.45	0.15, 6.85
0.50	$\pm$0.71	0.03, 34.0

Gordan coefficient. In the particular case considered by Barker and Mann, $\theta(0)=\theta_k(1)$. Thus, apart from the penetration and the phase-space factors, the total-cross-section ratios can be expressed as

$$\sigma_n/\sigma_p = |[\theta(0)-\sum_k \alpha_k\theta_k(1)]/[\theta(0)+\sum_k \alpha_k\theta_k(1)]|.$$

The sign of the "mixing amplitude" α_k depends on both the sign of the matrix element of H_c and the energy difference of the mixing states. In general, $\theta_k(1)/\theta(0)$ can also have either sign and Barker suggests that this sign is probably not correlated to α_k.

Assuming only one state introduces the $T=1$ impurity and considering the case $\theta_k(1)=\pm\theta(0)$, the ratio's sensitivity to isobaric-spin impurity is demonstrated in Table I.

If many states $\theta_k(1)$ contribute to the $T=1$ impurity with comparable values of α_k and $\theta_k(1)$, the ratio should become insensitive to the isobaric-spin impurity due to random cancellations in the sum.[4,5]

Note added in proof. Professor W. M. MacDonald has brought to the authors' attention an additional reference for the σ_n/σ_p derivation. [See A. Altman, W. M. McDonald, and J. B. Marion, Nucl. Phys. **35**, 85 (1962).] He also notes that for the case of many overlapping levels, the ratio appears to remain very sensitive to the mixture of isobaric spin states. In that case, the above expression for σ_n/σ_p does not apply because the ratio is derived for an isolated state. MacDonald notes that at high energies the R-matrix states can be strongly mixed in isobaric spin but the S matrix shows little effect of isobaric-spin mixing at its resonances if the levels which are mixed are much closer than their widths.

In earlier mirror reaction studies[5–7] of d on ^{6}Li and on ^{10}B the dominance of a stripping mechanism, due to the loosely bound character of the deuteron, masked the ratio's sensitivity to isobaric-spin impurities in the compound nucleus. By initiating the mirror reactions with α particles it was hoped the dominant reaction mechanism would be one proceeding through intermediate states. Further, by use of a thin target perhaps one or only a few compound-nuclear states of known excitation may be excited resulting in an estimate of the isobaric-spin impurity of individual levels.

The ^{14}N$(\alpha, n)^{17}$F(0.500) and ^{14}N$(\alpha, p)^{17}$O(0.871) mirror reactions were chosen to study the ratio fluctuations from $E_\alpha=7.0$ [near the (α, n_1) threshold] to $E_\alpha=12.0$ MeV. This corresponds to a range of excitation energies in ^{18}F between 9.8 and 13.7 MeV. The threshold of $T=1$ states in ^{18}F is at 1.045 MeV excitation.[8] In view of the observed levels[8] in the ^{18}O member of the $T=1$ triad, it is estimated that a large number of $T=1$ states will occupy the ^{18}F energy range of interest. The compound nucleus under study is then at a likely excitation for the simultaneous failure of both the static[9] and dynamic[10] criteria, since states of the same J^π may be close enough and live long enough to allow appreciable isobaric-spin mixing.

II. EXPERIMENTAL METHOD

The neutrons and protons emitted from the α bombardment of ^{14}N were not measured directly to obtain the ratios. Instead, the relative intensity of deexcitation γ rays from the bound mirror states, ^{17}F(0.500) and ^{17}O(0.871), and the ^{17}O(3.058) state feeding the ^{17}O(0.871) state were simultaneously measured (see Fig. 1). This method was first suggested and implemented by Wilkinson.[3] In the present experiment, all the mirror levels of interest are spin-$\frac{1}{2}$ states, therefore, the γ radiation is isotropic. A comparison of the γ-ray intensities at any angle thus yields a total-cross-section ratio of the mirror reactions. The γ rays from the feeding ^{17}F(3.10) state were not observed in our spectra. This state is unbound to proton emission by 2.5 MeV and has its threshold at $E_\alpha=10.08$ MeV.

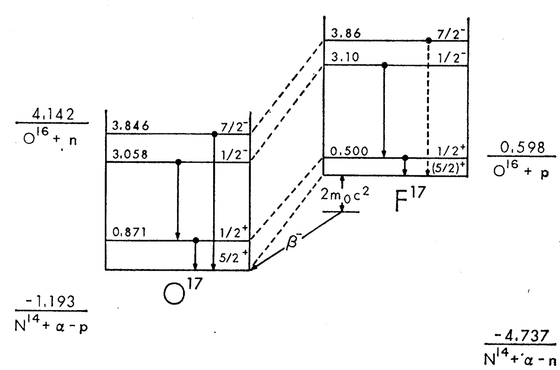

FIG. 1. Partial energy-level diagrams of ^{17}O and ^{17}F mirror nuclei with the γ-ray transitions of interest.

[5] A. K. Mann (private communication).
[6] S. M. Austin *et al.*, Bull. Am. Phys. Soc. **11**, 10 (1966).
[7] W. A. Schier *et al.*, Bull. Am. Phys. Soc. **12**, 33 (1967).

[8] T. Lauritsen and F. Ajzenberg-Selove, in *Energy Levels of Light Nuclei, Nuclear Data Sheets* (U.S. Government Printing Office, Washington, 1962), NRC 61-5, 6-261, 267.
[9] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).
[10] H. Morinaga, Phys. Rev. **97**, 444 (1955).

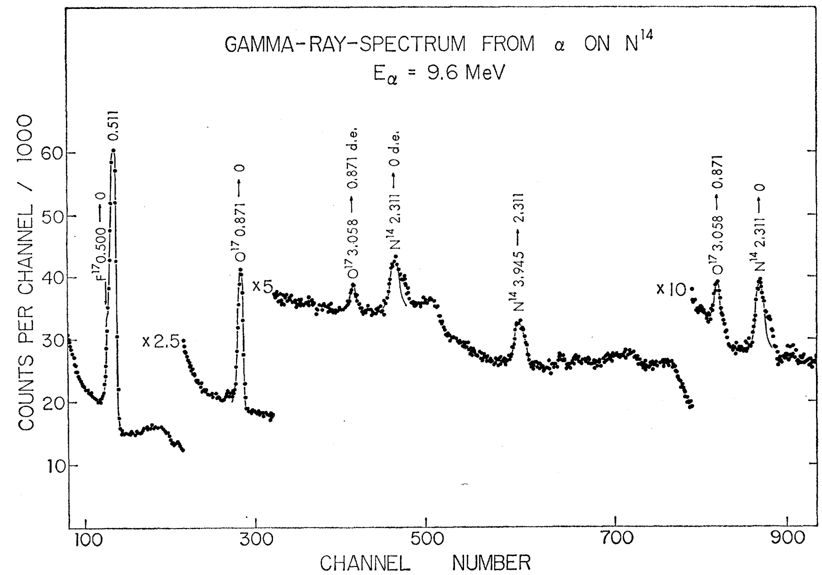

FIG. 2. γ-ray spectrum from 9.6 MeV α-particles incident on ^{14}N. The γ rays associated with the cross-section ratio measurement are the unresolved ^{17}F(0.500→0.0), the ^{17}O(0.871→0.0), and the full energy and double-escape ^{17}O(3.058→0.871) γ peaks.

The γ ray intensities were measured with a cylindrical 10-cc Ge(Li) detector placed with its axis at zero degrees and its face approximately 1.5 cm from the target. Nitrogen targets were made by evaporating adenine ($C_5H_5N_5$) onto tantalum cups. The cups were circulated at 60 rpm in a target wobbler and cooled with a stream of moist air. A cold finger extending 0.5 m along the beam axis preceded the target. It should be noted that the rotation of the target also considerably decreases the rate of hydrocarbon buildup. As a final precaution targets were replaced every two days. Target thicknesses of 50±15 and 100±30 keV were estimated from earlier experience with adenine targets.[11]

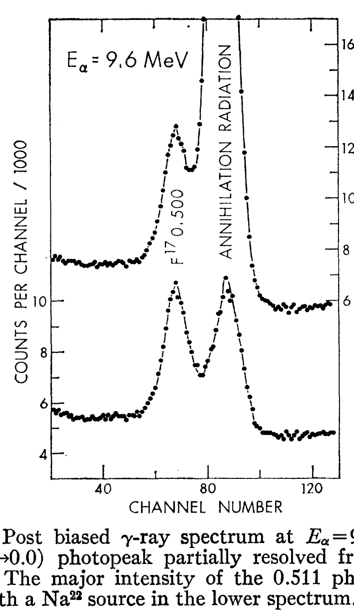

FIG. 3. Post biased γ-ray spectrum at E_α=9.6 MeV of the ^{17}F(0.500→0.0) photopeak partially resolved from annihilation radiation. The major intensity of the 0.511 photopeak is subtracted with a Na^{22} source in the lower spectrum.

[11] W. A. Schier and C. P. Browne, Phys. Rev. **138**, B857 (1965).

Beams of α^{++} were accelerated with the 5.5-MeV Van de Graaff accelerator and energy controlled on the α^+ beam. Exit slits set on the α^{++} beam, limiting the energy spread to approximately ±10 keV, removed D^+ and H_2^+ components. Beam intensities ranging from 0.02 to 0.08 μa were dictated by arbitrarily limiting the pulse-height-analyzer block time to ≤30%. The beam analyzing magnet was energy calibrated with the ^{12}C(α, n) threshold at E_α=11.346 MeV by observing the 0.511-MeV activity in a thick carbon target. An uncertainty of ±13 keV was assigned to this measurement. A 12-keV-wide resonance[12] in ^{12}C(p, p) at E_p=4.808 MeV provided a second calibration point with ±2 keV uncertainty.

A γ-ray spectrum measured at E_α=9.6 MeV is shown in Fig. 2. The γ rays of interest are the 3.058→0.871 photopeak, the double escape from this feeder γ ray, and the deexcitation γ rays from the 0.871- and the 0.500-MeV levels. To resolve the 0.500-MeV γ from annihilation radiation, this portion of the spectrum was post biased amplified and simultaneously recorded in a second pulse-height analyzer (PHA). The expanded spectrum is shown in Fig. 3. To reduce the annihilation radiation peak, after each exposure the beam was deflected, a ^{22}Na source placed near the Ge(Li) detector, and the PHA recorded in the subtract mode. The lower spectrum is the result of such a subtraction. Complete subtraction was ordinarily not made because both the width and symmetry of the residual peak afforded a check against gain shifts.

γ-ray intensities were corrected from the relative efficiency curve of Fig. 4 measured also at a source to detector distance of 1.5 cm. The efficiency points were obtained by comparing to γ-ray photopeaks in a

[12] C. W. Reich, G. C. Phillips, and J. L. Russell, Jr., Phys. Rev. **104**, 143 (1956).

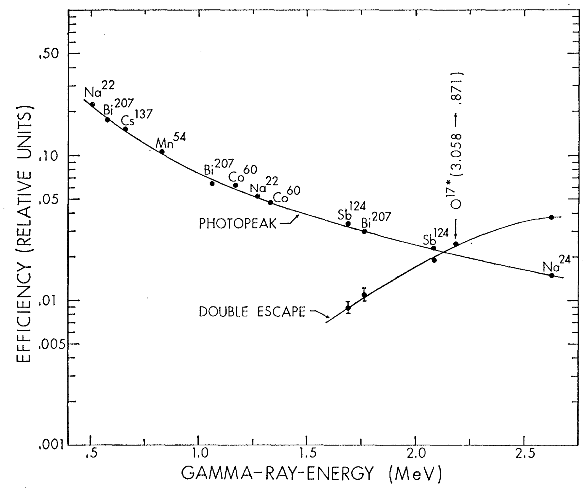

FIG. 4. Measured photo and double-escape relative efficiency curves of the 10-cc cylindrical Ge(li) detector at a source-to-detector face distance of 1.5 cm.

3×3-in. NaI detector which has a known efficiency.[13] The statistical uncertainties in these measurements are smaller than the datum symbols unless indicated. It should be noted that γ rays of interest have energies very near those from the ^{22}Na, ^{54}Mn, and ^{124}Sb sources, requiring only small extrapolations from these energies. Since the feeder γ ray introduced the major uncertainty into the ratios, both its photo- and double-escape peaks were used in the analysis to improve statistics. An efficiency point representing an average of the most accurate feeder γ-ray data normalized to the photopeak efficiency curve is also on the double-escape curve.

Angular distributions of protons from the ^{14}N(α, p)-^{17}O(0.871) reaction were measured at selected energies in an attempt to estimate penetrability effects. The protons were recorded with a four-surface-barrier detector array normalized to a monitor detector. Elastic α-particles were stopped with aluminum or molybdenum foils preceding the detectors. Targets were adenine evaporations on 0.0001-in. tantalum foils. These targets were oscillated at 180 rpm. Particulars of the scattering chamber and target oscillator have been reported elsewhere.[14] A proton spectrum measured at 9.43 MeV is shown in Fig. 5. In many runs the spectra were nearly background-free because the proton groups corresponding to ^{17}O in its ground and first excited states were the most energetic charged particles after the stopping foils. Measurements were limited at frontal angles by an intense unidentified peak. Magnetic suppression of electrons would not remove this group.

III. CROSS-SECTION RATIOS

The ratio measurements whose results are shown in Fig. 6 were made in 100-keV steps with targets ≈100 keV thick from $E_\alpha=7$ MeV (170 keV above the neutron threshold) up to $E_\alpha=12$ MeV. The energy scale is corrected for target thickness and has an estimated uncertainty of ±35 keV. The major uncertainty in the cross-section ratios arises from the uncertainty in the intensity of the feeder γ ray which above 9.5 MeV accounts for approximately half the intensity of the 0.871-MeV photopeak. Also, at these higher incident energies, Compton backgrounds from ^{14}N(α, α)^{14}N* reactions become serious so that the cross section ratios are extracted only up to $E_\alpha=10.5$ MeV. Since the 0.500/0.871 ratios of γ-ray intensities reproduce the cross-section ratio fluctuations and can be measured with considerably greater accuracy they are plotted to an α energy of 11.9 MeV. The ratios remeasured over the sharp 10.07-MeV maximum with a 50-keV target are shown in Fig. 7. Though the uncertainties in these latter ratios are greater, the maximum is reproduced both in height and width. A preliminary report of these ratios has been given in Ref. 15.

Four major maxima are observed at $E_\alpha=8.12$, 9.5, 10.07, and 11.52 MeV. These then are energies that deserve further scrutiny to estimate penetrability effects on the ratios. Angular distributions were measured at these maxima, at $E_\alpha=9.95$ MeV where the ratios pass through a minimum, and at $E_\alpha=10.32$ MeV where an isobaric-spin impure 4^+ level has been reported.[16] The energies at which angular distributions were measured are indicated in Fig. 6 by the arrows.

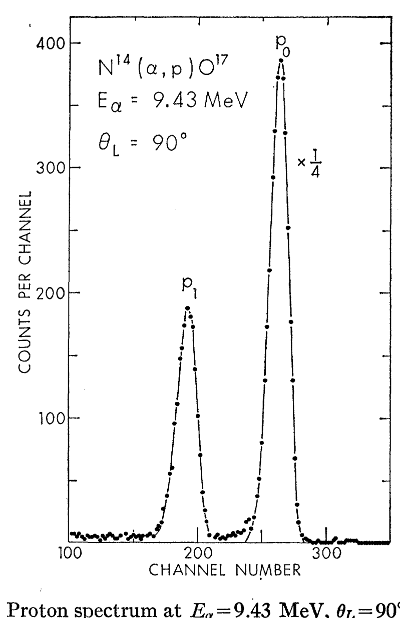

FIG. 5. Proton spectrum at $E_\alpha=9.43$ MeV, $\theta_L=90°$. Aluminum foils 35 mg/cm^2 thick preceeded the detectors to degrade the energy of the protons and remove α elastics.

[13] *Nuclear Data Tables, Part 3*, edited by J. B. Marion (U.S. Government Printing Office, Washington, D.C., 1960).
[14] R. Plattner *et al.*, Nucl. Instr. Methods **64**, 192 (1968).
[15] W. A. Schier and J. D. Reber, Bull. Am. Phys. Soc. **12**, 1198 (1967).
[16] C. M. Chesterfield and B. M. Spicer, in *Isobaric Spin in Nuclear Physics*, edited by J. D. Fox and D. Robson (Academic Press Inc., New York, 1966), p. 734.

The solid curves in Fig. 6 are the calculated[17] ratios dependent on available phase space and penetrability for s-, p-, d-, and f-wave outgoing particles assuming a nuclear radius of $1.2A^{1/3}$ fm. The ratio calculated on the square-well model has the form

$$k(n)[F_L^2(p)+G_L^2(p)]/k(p)[F_L^2(n)+G_L^2(n)],$$

where k is the wave number of the given particle and F_L and G_L are the regular and irregular Coulomb wave functions.

The square-well model is expected to overcorrect the differential cross section because the well has an unrealistically sharp cutoff. The penetrabilities[18] generated with the more realistic Woods-Saxon potential were taken to calculate the ratios shown in Fig. 8. Penetrability values were tabulated for a nuclear radius, $R=1.25A^{1/3}$ fm, and a diffuseness parameter, $a=0.65$ fm. Included in these ratios is a factor $k(n)/k(p)$ to account for differing available phase spaces. The s-, p-, d-, and f-wave square-well ratio curves are reproduced in this figure for comparison. An attractive feature of the Woods-Saxon ratios is that they are considerably less sensitive to the angular momentum of the outgoing particles at any particular energy. The energy dependence of the measured ratios, disregarding now the maxima, closely follow the trend of the Woods-Saxon ratio curves. This observation increases one confidence in the Woods-Saxon ratios, as off-maximum ratio measurements are probably due to many ^{18}F levels with the major contributors having outgoing angular momenta between 0 and 3.

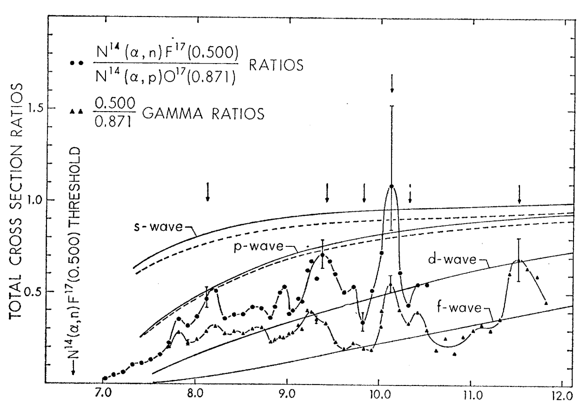

FIG. 6. Total-cross-section ratios of

$$^{14}\mathrm{N}(\alpha,n)^{17}\mathrm{F}(0.500)/^{14}\mathrm{N}(\alpha,p)^{17}\mathrm{O}(0.871)$$

mirror reactions and efficiency corrected ^{17}F(0.500)/^{17}O(0.871) γ ratios measured in 100-keV steps with 100 ± 30-keV targets. The curves are the dependence of ratios on penetrability and available phase space using a square-well potential. The solid and dashed curves were calculated for $R=1.2A^{1/3}$ and $R=1.5A^{1/3}$ fm, respectively. The arrows indicate energies at which angular distributions were measured.

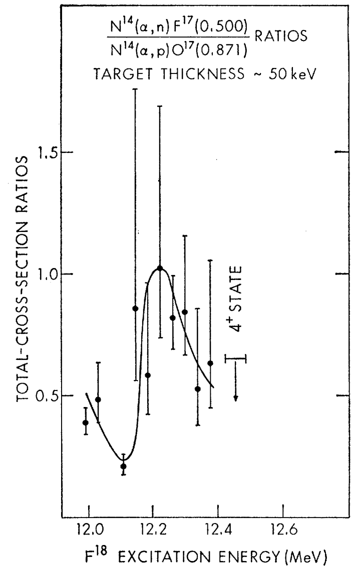

FIG. 7. The 10.07-MeV cross-section ratio maximum remeasured with a 50 ± 15 keV target in 50-keV energy increments. The position of a reported isobaric-spin impure 4^+ state is indicated.

Another attractive aspect of cross-section ratios is their relative independence of nuclear radius. Although the square-well penetrabilities are strongly dependent on the nuclear radius, (e.g., for 3.5-MeV protons, the d-wave penetrability increases by a factor of 2 and the f-wave penetrability by a factor of 4 when R is increased from $1.2A^{1/3}$ to $1.5A^{1/3}$ fm), the cross-section ratios are very insensitive to the nuclear radius (e.g., the dashed s-wave and p-wave ratio curves in Fig. 6 are calculated for $R=1.5A^{1/3}$ fm and the d- and f-wave calculations would nearly coincide with their solid curves).

IV. ANGULAR DISTRIBUTION ANALYSIS

The angular distributions from the ^{14}N(α, p)^{17}O(0.871) reaction were analyzed in the following manner. The data points in Fig. 9 were fit with Legendre polynomials by a computer program which minimized χ^2 for the curve generated with each successive order. Estimates of the largest relative angular momentum in the incident (unprimed) or outgoing (primed) channels that can be detected contributing to the angular distributions follow from the restrictions $L\leq 2l_{\max}$, $L\leq 2l'_{\max}$ on the complexity of the angular distributions. Table II lists the coefficients of the Legendre polynomials and the largest relative angular momenta detected. The asymmetric angular distributions at the $E_\alpha=9.43$-MeV ratio maximum, at the 9.95-MeV ratio minimum, and at 10.32 MeV do not lend themselves to isobaric-spin impurity estimates because the reaction proceeds through interfering levels in ^{18}F. The angular momentum composition is sufficient, though, to account for the

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[17] W. T. Sharp, H. E. Gove, and E. B. Paul, *Graphs of Coulomb Wave Functions* (Atomic Energy of Canada Limited, Chalk River, 1965).

[18] G. S. Mani, M. A. Melkanoff, and I. Iori, C.E.A. Report Nos. 2379 and 2380, Saclay, France, 1963 (unpublished).

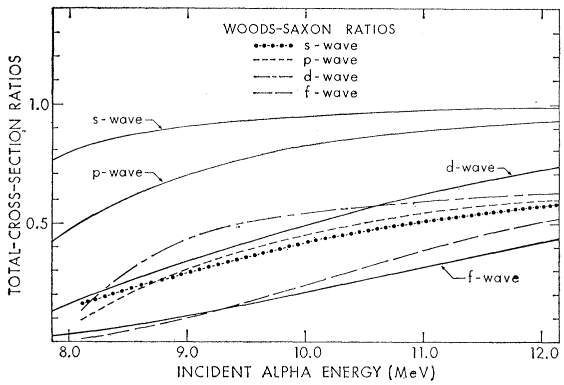

FIG. 8. Ratios calculated from penetrabilities using a Woods-Saxon potential with $R=1.25A^{1/3}$ fm and $a=0.65$ fm. A term $k(n)/k(p)$ is included in the ratios to correct for differing available phase spaces. The solid curves are s-, p-, d-, and f-wave ratios computed with a square-well potential.

cross-section ratios from penetrability considerations alone.

Since the angular distributions at 8.12, 10.07, and 11.52 MeV are quite symmetric, the single-level formula of Blatt and Biedenharn[19] was used to calculate the angular distributions assuming single levels of spin one through seven and of either parity. The normalizing constants of terms associated with a given set of $\bar{Z}$ coefficients

$$\sum_L (-1)^{s-s'}\bar{Z}(l_1J_0l_2J_0,sL)\bar{Z}(l_1'J_0l_2'J_0,s'L)P_L$$

contain the phase shifts and reduced width amplitudes in a term of the form

$$g_{\alpha sl_1}g_{\alpha sl_2}g_{\alpha's'l_1'}g_{\alpha's'l_2'}\cos(\xi_{\alpha sl_1}-\xi_{\alpha sl_2}+\xi_{\alpha's'l_1'}-\xi_{\alpha's'l_2'}).$$

Interference terms exist in this expression when nuclear states of the sequence 1^+, 2^-, 3^+, $\cdots$ are assumed, because two values of angular momenta are allowed both in the incident and the outgoing channels, resulting in a sign ambiguity in these terms. This arises from both the uncertainty in the sign of the partial-width amplitude and the unknown magnitudes of the phase shifts associated with each partial wave. In the fitting process these terms were allowed to take on either sign.

The criterion for goodness of fit was a χ^2 test defined by

$$\chi^2=N^{-1}\sum_i[(Y_i-\sigma_i)/\Delta Y_i]^2,$$

where N is the number of data, Y_i is the experimental yield, ΔY_i is the uncertainty in Y_i, and σ_i is the theoretical yield.

The theoretical curve that best fits the 8.12-MeV data shown in Fig. 10 assumes a single 2^- state in the compound nucleus. Other compound-nuclear states gave minimum χ^2 fits at least 40% larger. A tentative assignment of 2^- is given because a symmetric contribution from distant states would effect these secondary fits, although adversely in most cases.

The 10.07-MeV data in Fig. 10 can be fitted equally well with 2^-, 3^+, or 3^- level theory, as the comparable minimum χ^2's in Table III demonstrate. Other J^π assignments were rejected because their minimum χ^2's were at least 50% larger than the lowest value listed.

The 11.52-MeV data has a more pronounced deviation from symmetry. Nevertheless, the 90° peaking of the angular distribution characterizes the compound-nuclear state as a 2^- or 3^+ level. The curve shown can be generated assuming either J^π value. Since appreciable interference is present, a minimum χ^2 has little meaning for a symmetric curve and is therefore not listed in Table III.

In estimating the penetrability corrections to the ratios at $E_\alpha=8.12$, 10.07, and 11.52 MeV the l' contributions to the $^{14}N(\alpha,p)^{17}O(0.871)$ total cross section are inferred from the fits to the angular distributions in Table III. Since the total cross section depends only on products of $\bar{Z}$ coefficients giving rise to P_0 terms, interference terms may be ignored and the l' composition is estimated by comparing the coefficients on the non-interference terms. Values of the outgoing angular momentum l' corresponding to each fit are listed. The ratios expected from the curves in Fig. 6 after correcting for phase space and barrier penetrability are tabulated under R_{calc}. The measured ratio is then compared with this calculated ratio in column R/R_{calc}. Again assuming

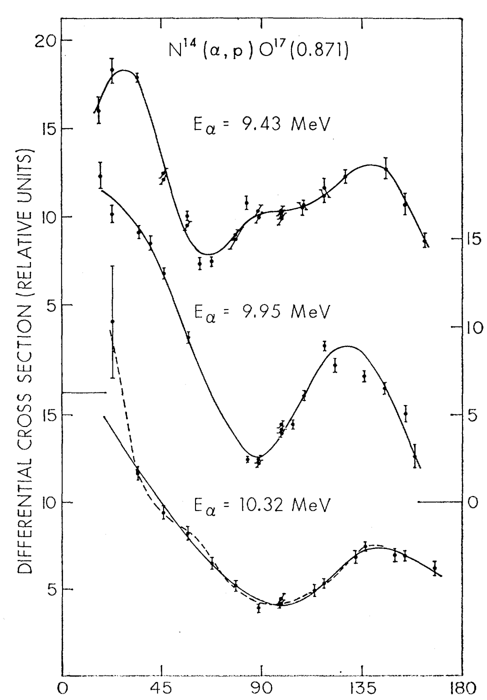

FIG. 9. Asymmetric angular distributions on $^{14}N(\alpha,p)^{17}O(0.871)$ reaction measured at $E_\alpha=9.43$, 9.95, and 10.32 MeV. The curves are the Legendre polynomial fits given in Table II.

[19] J. M. Blatt and L. C. Biedenharn, Rev. Mod. Phys. **24**, 258 (1952).

TABLE II. The Legendre polynomial fits to the asymmetric ^{14}N$(\alpha, p)^{17}$O(0.871) angular distributions. The maximum outgoing angular momentum detected follows from the relation $L \leq 2l'_{max}$.

E_α	E_x	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	$l'_{max-det}$	Plotted
9.43	11.74	1	0.11	0.32	0.38	−0.11	−0.06	−0.38	−0.16	−0.03	4	solid
9.95	12.14	1	0.59	0.76	0.72	−0.64					2	solid
10.32	12.43	1	0.41	0.71	0.21	−0.11	0.17				3	solid
		1	0.58	0.97	0.58	0.30	0.55	0.37	0.28	0.16	4	dashed

that a single state introduces the $T=1$ impurity and that $\theta_k(1) = \pm\theta(0)$, one obtains the estimate of the isobaric-spin impurity in the column headed $|\alpha|^2$(sq. well). This same procedure is followed for the Woods-Saxon ratios in Fig. 8 resulting in the isobaric-spin impurity estimates $|\alpha|^2$(Woods-Saxon).

V. COMPARISON TO FORBIDDEN-REACTION RESULTS

Several earlier experiments comparing forbidden to allowed reactions have resulted in isobaric-spin impurity estimates of ^{18}F excited states. In the ^{16}O$(d, \alpha)^{14}$N reaction studied by Browne,[20] the average yield of the forbidden group (α_1) was measured to be 7% of the average yield of the ground-state group (α_0) at $E_d=$

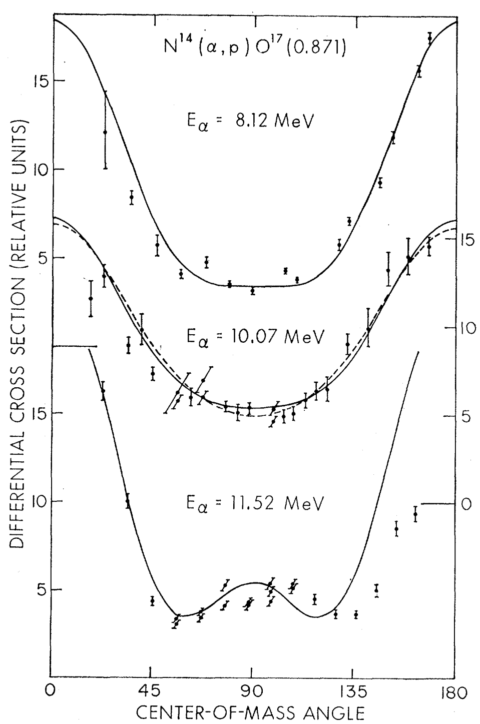

FIG. 10. Angular distributions of ^{14}N$(\alpha, p)^{17}$O(0.871) reaction which display considerable symmetry about 90°. The curves are fits in Table III generated assuming the reaction proceeds through single states in the ^{18}F compound nucleus.

[20] C. P. Browne, Phys. Rev. **104**, 1598 (1956).

7.03 MeV, corresponding to a ^{18}F excitation of 13.76 MeV. A 30° yield curve of the forbidden reaction displayed three broad resonances of comparable intensities at $E_d=6.2$, 6.6, and 7.0 MeV corresponding to ^{18}F excitations of 13.0, 12.4, and 13.8 MeV. It has been pointed out by Hashimoto and Alford[21] using the Hauser-Feshbach[22] statistical theory that because the forbidden reaction proceeds from a 0^+ state (^{16}O g.s.) to a 0^+ state (^{14}N 2.13), a statistical reduction factor of approximately 3 must be included when estimating the isobaric-spin impurity from a comparison with the allowed reaction proceeding to the ground state of ^{14}N. The impurity estimate at $E_d=7.03$ MeV would then increase to approximately 20%. The other resonances in Browne's yield curve may indicate comparable selection rule failures.

More recently, this comparison was made by Messelt[23] from $E_d=3$ to 5 MeV and by Jobst[24] from $E_d=5$ to 9 MeV. Messelt found the cross section of the (d, α_1) reaction to be 6–10% that of the (d, α_0) and (d, α_2) reactions on ^{16}O. Jobst estimates the isobaric-spin impurity to be 30% over the ^{18}F excitation of 11.9–15.5 MeV. His estimate is adjusted for the inhibition due to spin and parity restrictions.

Chesterfield and Spicer[16] make additional isobaric-spin impurity estimates from the ^{14}N$(\alpha, \alpha_1)^{14}$N(2.31) forbidden reaction. They have placed lower limits on the isobaric-spin impurity of 5% on the 12.47-MeV 4^+ level and 2% on the 13.32-MeV 5^- level in ^{18}F by comparing the forbidden reaction to ^{14}N$(\alpha, \alpha_0)^{14}$N yield.

Hobbie and Forbes[25] have measured the ^{12}C(^{6}Li, α_0)^{14}N, the forbidden (^{6}Li, α_1), and the (^{6}Li, α_2) cross sections at $E_{Li}=3.8$ and 4 MeV. The total cross section for the forbidden (^{6}Li, α_1) reaction is $\approx$2% that of the (^{6}Li, α_0) and $\approx$1% that of the (^{6}Li, α_2) reaction at 3.8 MeV and $\approx$3% and $\approx$4%, respectively, at 4 MeV. At 4 MeV, 20 exit channels with $T=1$ are above the Coulomb and angular momentum barrier. Comparing the 1-mb cross section to the calculated upper limit of 20 mb and adjusting for the number of available exit channels,

[21] Y. Hashimoto and W. P. Alford, Phys. Rev. **116**, 981 (1959).
[22] W. Hauser and H. Feshbach, Phys. Rev. **87**, 366 (1952).
[23] S. Messelt, in *Isobaric Spin in Nuclear Physics*, edited by J. D. Fox and D. Robson (Academic Press Inc., New York, 1966), p. 814.
[24] J. E. Jobst, Ph.D. thesis, University of Wisconsin, 1966 (unpublished).
[25] R. K. Hobbie and F. F. Forbes, Phys. Rev. **126**, 2137 (1962).

TABLE III. Isobaric-spin impurities estimated from a comparison of experimental cross-section ratios to calculated ratios corrected for barrier penetrability and available phase space. Penetrabilities using both a square-well and a Woods-Saxon potential resulted in the two columns of impurity estimates. The angular distribution fits listed were calculated from the single level formula and provided the estimate of the outgoing particles' angular momentum.

E	E_x	J	A_0	A_2	A_4	χ^2	l'	Sq well $R_{\rm calc}$	Sq well $R/R_{\rm calc}$	$\lvert\alpha\rvert^2_{\rm sq.\ well}$	$\lvert\alpha\rvert^2_{\rm Woods\text{-}Saxon}$	Plotted
8.12	10.76	2^-	1.00	1.33	0.57	28.0	1	0.53	1.0	0.0	0.12	solid
10.07	12.23	2^-	1.00	0.80	0.26	4.45	1	0.83	1.3	0.005	0.05	solid
		2^-	1.00	0.87	0.26	5.07	1, 3	0.54[a]	2.0	0.03	0.07	
		2^-	1.00	0.91	0.29	5.57	3	0.25	4.4	0.12	0.12	
		3^+	1.00	0.90	0.26	5.50	2	0.50	2.2	0.04	0.03	
		3^-	1.00	0.86	0.14	5.59	3	0.25	4.4	0.12	0.12	dashed
11.52	13.36	2^-	1.00	1.25	1.14	···	1	0.92	1.5[b]	0.01	0.03	solid
		3^+	1.00	1.25	1.14	···	2	0.68	2.1[b]	0.03	0.02	solid

[a] Average $R_{\rm calc}$. [b] Estimated ratio = twice γ ratio.

they estimate an impurity of $\approx 10\%$ at a ^{18}F excitation of 15.9 MeV.

Carlson and Heikkinen[26] extended Hobbie and Forbes measurements up to $E_{\rm Li} = 5.5$ MeV. The $(^6{\rm Li}, \alpha_1)/(^6{\rm Li}, \alpha_0)$ and $(^6{\rm Li}, \alpha_1)/(^6{\rm Li}, \alpha_2)$ ratios decrease at 4 MeV from $\approx 3\%$ and $\approx 4\%$ to $\approx 1\%$ and $\approx 0.5\%$, respectively, at $E_{\rm Li} = 5.5$ MeV (^{18}F excitation of 16.9 MeV).

Dzubay[27] estimated a $30\% \pm 15\%$ average violation from the $^{12}{\rm C}(^6{\rm Li}, \alpha_1)^{14}{\rm N}$ forbidden reaction measured at a laboratory angle of 40° over the energy interval of 3.4–6.0 MeV (^{18}F excitation 15.5–17.2 MeV). The $(^6{\rm Li}, \alpha_0)$ and $(^6{\rm Li}, \alpha_2)$ reactions were also measured at this angle and ratios of all three reactions with those calculated from the Hauser-Feshbach statistical theory were made. Since isobaric spin is not accounted for in this formula, the ratio of the forbidden to the calculated differential cross sections resulted in the impurity estimate cited above.

It is of interest to compare the present method of measuring isobaric-spin-allowed mirror reaction ratios to the above method of measuring the forbidden/allowed reaction ratios. There are two weaknesses in the mirror reaction ratio method. The angular-momentum composition must be known in order to correct for barrier penetrability effects on the ratio. It is believed that this can adequately be done in some cases (e.g., 2^- state of ^{18}F at 10.764 MeV). The second weakness lies in the relative magnitudes of the reduced width amplitudes, $\theta(0)$ and $\theta(1)$, which for the present estimates are taken to be equal in magnitude. Assuming only one state introduces the $T=1$ impurity, $|\theta(0)/\theta(1)|$ can have any value between 0 and ∞, unless there is a definite model for the states involved and this ratio can be calculated.[4] Alternatively, if it is assumed that $|\theta(0)/\theta(1)| \approx 1$, and unless the reduced width amplitudes are comparable with single-particle values, then the calculated isobaric-spin impurity may refer to only a small part of the wave function and need not be closely connected with the total isobaric-spin impurity of the level, i.e., there may be states $\psi_k(1)$ with $\theta_k(1) = 0$ but $\alpha_k \neq 0$.[1] Barker notes that a similar weakness should also exist in the forbidden/allowed reaction ratio method.

The mirror reaction ratio method also has several advantages. Mirror reaction ratios do not suffer the yield reduction due to isobaric-spin conservation and real spin and parity inhibitions. The sensitivity of the ratio to very small impurities can be used to establish the high isobaric-spin purity of nuclear states. It has been noted[16] that the forbidden reactions above can proceed through both $T=0(T=1)$ and $T=1(T=0)$ intermediate states, where the parenthesis denotes the admixed impurity. In cases of small isobaric-spin impurities, the mirror reaction ratio method will select out $T=0(T=1)$ intermediate states because reactions through these states will be considerably more intense than those proceeding through $T=1(T=0)$ states.

The application of Hauser-Feshbach theory to correct the forbidden cross sections for spin and parity restrictions is most valid when a large number of nuclear states are involved. Average cross sections extending over several MeV are corrected for these inhibitions to within a factor of 2.[27] It should be noted, however, that

TABLE IV. Calculated ratios of $T=1$ and $T=0$ level densities in ^{18}F at an excitation energy of 20 MeV for states of spin 0–5. The $T=1$ level densities are estimated from ^{18}O at an excitation energy of 19 MeV.

Spin	0	1	2	3	4	5
$\rho(T=1)$ / $\rho(T=0)$	0.42	0.41	0.39	0.36	0.32	0.29

[26] R. R. Carlson and D. W. Heikkinen, Phys. Letters **17**, 305 (1956).
[27] T. G. Dzubay, Phys. Rev. **158**, 977 (1967).

the forbidden reactions often proceeded through individual compound-nuclear states.[16,23,24] Were the allowed reactions likewise to proceed only through the same state (i.e., any state of the sequence 1^-, 2^+, 3^-, $\cdots$) no inhibition factor exists because only one initial and one final channel, characterized by $l=l'=J$, are open to both the forbidden and allowed reactions.[28] In this event, the better impurity estimate is simply the uncorrected forbidden/allowed ratio.

In the above cases where the statistical theory is applicable to the ^{18}F nucleus, it appears that the isobaric spin impurity is overestimated by approximately a factor of 1.5 because the forbidden reaction proceeds through both $T=0(T=1)$ and $T=1(T=0)$ states. (To convince oneself of this fact, consider a forbidden reaction and its inverse. Both reactions must proceed through the same intermediate states in accordance with the principle of detailed balance.) That there are comparable numbers of $T=0$ and $T=1$ states in the ^{18}F excitation region of interest can be concluded by noting that 17 levels are reported[8] in the 0–7.5-MeV excitation region of the ^{18}O member of the $T=1$ triad and 54 levels are reported over the 0–8.5-MeV excitation interval in ^{18}F. Again it should be noted that the threshold of $T=1$ states in ^{18}F lies at about 1 MeV. Comparable densities of $T=0$ and $T=1$ states of a particular spin are also predicted in ^{18}F from the approximate level density equation[29,30]

$$\rho(U,J)=0.0295(a^{1/2}/c^{3/2})[(2J+1)/\ U+t-\Delta)^2]$$
$$\times\exp\{-(J+\tfrac{1}{2})^2/2\sigma^2\}\exp\{2[a(U-\Delta)]^{1/2}\},$$

where $c=0.0138A^{5/3}$, $a=A/8$, $\sigma^2=ct$, $U-\Delta=at^2-t$, $\Delta_{^{18}O}\approx3$ MeV, $\Delta_{^{18}F}\approx0$, and U is the excitation energy. Table IV gives the calculated ratio of $T=1$ and $T=0$ levels in ^{18}F at an excitation of 20 MeV for levels of spin 0–5. Assuming all states have small impurity admixtures, the forbidden reaction can proceed through all the intermediate states (with real spin and parity restrictions accounted for by the Hauser-Feshbach theory), whereas the allowed reaction will proceed primarily through the $T=0(T=1)$ levels. A similar reduction in the isobaric-spin impurity estimates should be made in the other odd-odd self-conjugate nuclei because the thresholds of $T=1$ states in these nuclei are often at very low excitation energies.

[28] T. G. Dzubay (private communication).
[29] J. R. Huizenga (private communication).
[30] K. J. Le Couteur and D. W. Lang, Nucl. Phys. **13**, 32 (1959).

VI. CONCLUSION

In assigning isobaric-spin impurities in the present experiment, only those calculated from penetrabilities using the Woods-Saxon potential are used. It is stressed again that the impurity assignments are made under the assumption $\theta(1)=\pm\theta(0)$ and must be adjusted were information on these reduced width amplitudes to become available. It is concluded that the 10.674±0.030-MeV level in ^{18}F is (2^-), is $T=0$, and has an estimated $T=1$ impurity of 12%. The 12.234±0.030-MeV level in ^{18}F is given tentative J^π assignments $(2^-, 3^+, 3^-)$, is $T=0$, and has an estimated $T=1$ impurity between 3 and 12%. The 13.361±0.030-MeV level in ^{18}F is given tentative J^π assignments $(2^-, 3^+)$, is $T=0$ and has an estimated $T=1$ impurity of 2–3%.

The adjusted isobaric-spin impurity of ^{18}F states estimated from the forbidden-reactions range from the actual corss-section ratios when both forbidden and allowed reactions proceed through the same single state to $\frac{2}{3}$ the Hauser-Feshbach corrected value in the case of cross sections averaged over many nuclear levels. The forbidden-reaction measurements would then result in isobaric-spin impurity estimates in ^{18}F states $\leq20\%$ in good agreement with the 2–12% impurities in $T=0$ states estimated from mirror reaction ratios.

Eearlier isobaric-spin impurity estimates in odd-odd self-conjugate nuclei employing a statistical model should be reduced by as much as a factor of 2 because the densities of $T=0$ and $T=1$ states are comparable in these nuclei.

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