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Strain Dependence of the Minority Carrier Mobility in *p*-type Germanium*

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The strain dependence of the minority carrier drift mobility in uniaxially compressed bars of *p*-type germanium has been determined at constant temperatures between 77 and 300°K. The minority carrier mobility was measured by performing a series of Haynes-Shockley drift-mobility experiments in which both the applied stress and the electron drift path were along a $<111>$ crystal direction. The observed variation in electron mobility versus strain is compared with theoretical estimates based on a strain-induced population transfer in the conduction band. This comparison yields a value for the shear deformation-potential constant $\Xi_u = 16.3 \pm 0.3$ eV. The scattering anisotropy κ_τ has also been determined from the saturated mobility values. The presence of Coulomb-type scattering, as manifested by an increase in κ_τ, is noticeable at lower temperatures. However, values for κ_τ found for minority electrons in *p*-type material are not significantly different from those obtained from saturated piezoresistance measurements using *n*-type samples having the same net impurity content.

I. INTRODUCTION

THE multivalley nature of the conduction band of germanium is firmly established.[1,2] Experiments by Smith[3] and others[4,5] have shown that an elastic strain which destroys the energy equivalence of the valleys can produce large changes in the conductivity. Herring and Vogt[6,7] have shown that two mechanisms are involved in the large elastoresistance effect: the transfer of electrons from valleys of higher energy to valleys of lower energy and a strain-induced alteration of the relaxation times for intervalley scattering. The results of Weinreich *et al.*[8] indicate that, for germanium, the latter mechanism is unimportant. Saturated piezoresistance data,[5] interpreted via the population transfer mechanism, have provided useful information with regard to scattering mechamisms in *n*-type germanium. The scattering anisotropy κ_τ determined from such experiments was found to be of the order of 1.3 or less at temperatures above 77°K. This is in agreement with values obtained from magnetoresistance measurements.[9,10] More recently, minority carrier piezodrift experiments have been carried out at room temperature using *p*-type germanium samples.[11] The scattering anisotropy determined for minority electrons was reported as 3.7 ± 0.4, indicating a marked departure from the phonon-limited value of 1.2.[7] It is to be expected[12,13] that electron-hole scattering may be important with respect to minority carrier transport, but the value of $\kappa_\tau = 3.7$ cannot be reconciled by the inclusion of carrier-carrier scattering effects alone. The present work was initiated to obtain more information on minority carrier scattering and to extend the minority carrier piezodrift measurements over a wide temperature range.

II. STRAIN DEPENDENCE OF ELECTRON MOBILITY

The theory of the population transfer effect for many-valley semiconductors was first presented by Herring.[6] A particular solution for the strain dependence of the electron mobility in germanium subjected to a uniaxial stress along a $\langle 111 \rangle$ direction has also been reported earlier.[11] The results of such a calculation can be summarized as follows. A component of the total mobility tensor may be expressed as

$$\mu_{\alpha\beta} = \sum_{i=1}^{4} \frac{n^{(i)}}{n} \mu_{\alpha\beta}{}^{(i)}, \tag{1}$$

where $\mu_{\alpha\beta}{}^{(i)}$ is a component of the single-valley mobility tensor and $n^{(i)}$ is the corresponding number density associated with that particular valley. For germanium, the mobility tensor associated with the ith $\langle 111 \rangle$ valley is diagonal when expressed in a coordinate system having one axis along that particular $\langle 111 \rangle$ direction, and has the following form[7]:

$$\boldsymbol{\mu}^{(i)} = \begin{bmatrix} e\tau_\perp/m_\perp^* & 0 & 0 \\ 0 & e\tau_\perp/m_\perp^* & 0 \\ 0 & 0 & e\tau_{\parallel}/m_{\parallel}^* \end{bmatrix}. \tag{2}$$

Substitution into Eq. (1) yields the scalar quantity

$$\mu_0 = \tfrac{1}{3} e (2\tau_\perp/m_\perp^* + \tau_{\parallel}/m_{\parallel}^*). \tag{3}$$

When stress is applied, the change in energy of the ith band-edge point, to first order in strain $\mathbf{U}$, can be expressed as

$$\delta\epsilon^{(i)} = \sum_{j,k} \Xi_{jk}{}^{(i)} U_{jk} \tag{4}$$

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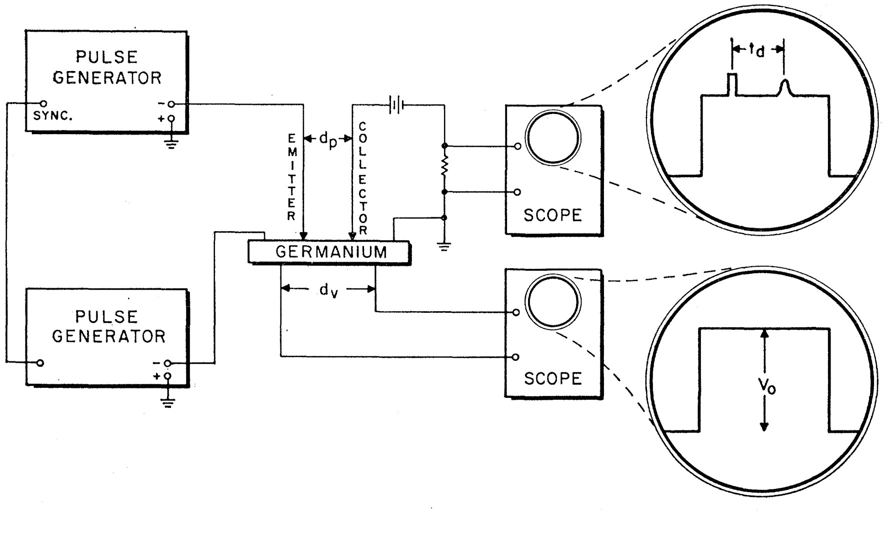

FIG. 1. Electrical circuitry used for minority carrier piezodrift experiments. The direction of applied stress is along the axis of the germanium sample.

or, equivalently, for germanium, as

$$\delta\epsilon^{(i)}=\sum_{j,k}\left(\Xi_d\delta_{jk}+k_j^{(i)}k_k^{(i)}\Xi_u\right)U_{jk}\,, \tag{5}$$

where the k_i's are normalized vectors locating the various band-edge points, and Ξ_u and $\Xi_d+\frac{1}{3}\Xi_u$ are the deformation potentials for pure shear and pure dilatation, respectively. The relative occupation numbers can then be expressed as

$$n^{(i)}=n\exp(-\delta\epsilon^{(i)}/kT)/\sum_{i=1}^{4}\exp(-\delta\epsilon^{(i)}/kT). \tag{6}$$

When the expressions given by Eqs. (5) and (6) are substituted into Eq. (1), a general expression for the strain-induced mobility variation is obtained. For a stress applied along a ⟨111⟩ direction, the normalized longitudinal component of the mobility tensor is expressed by[11]

$$\frac{\mu_L}{\mu_0}=\frac{\kappa_\tau e^{-A\delta L/L}+\frac{1}{3}(\kappa_\tau+8\kappa_m)e^{(A/3)\delta L/L}}{\frac{1}{3}(\kappa_\tau+2\kappa_m)(e^{-A\delta L/L}+3e^{(A/3)\delta L/L})}. \tag{7}$$

Here $\kappa_\tau=\tau_{\parallel}/\tau_{\perp}$ is the scattering anisotropy, $\kappa_m=m_{\parallel}^*/m_{\perp}^*$ is the effective-mass anisotropy, and

$$A=\frac{S_{44}}{S_{11}+2S_{12}+S_{44}}\frac{\Xi_u}{kT}, \tag{8}$$

where the S_{ij}'s are elastic compliance coefficients. The quantity $\delta L/L$ is the strain in the direction of applied stress. In order to compare the expression given by Eq. (7) with the experiments described herein, values of the elastic compliance coefficients as determined by McSkimin[14] were assumed, and a value of $\kappa_m=19.48$ was deduced from cyclotron resonance data.[15]

The strain dependence of the mobility may be examined for two extremes. In the low-strain limit Eq. (7) reduces to

$$\frac{\mu_L}{\mu_0}=\frac{2}{3}\frac{\kappa_m-\kappa_\tau}{2\kappa_m+\kappa_\tau}\frac{S_{44}\Xi_u}{(S_{11}+2S_{12}+S_{44})kT}\frac{\delta L}{L}+1, \tag{9}$$

which predicts a linear variation with strain and, for $\kappa_m>\kappa_\tau$, a decrease in μ_L/μ_0 for a compressive stress. For high strains, Eq. (7) reduces to

$$\mu_L/\mu_0=3\kappa_\tau/(\kappa_\tau+2\kappa_m)\,, \tag{10}$$

and indicates complete electron transfer to a single-conduction valley. Once this is achieved, the mobility ratio μ_L/μ_0 is constant.

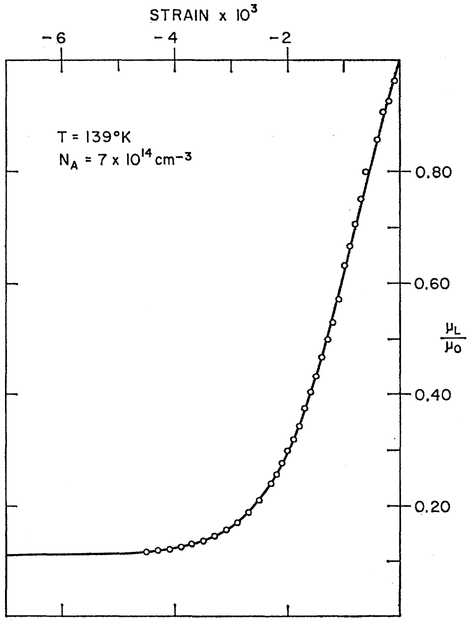

FIG. 2. Strain dependence of the normalized longitudinal mobility component μ_L/μ_0 at $T=139$°K. The solid curve is a plot of Eq. (7) with $\Xi_u=16.2$ eV and $\kappa_\tau=1.48$.

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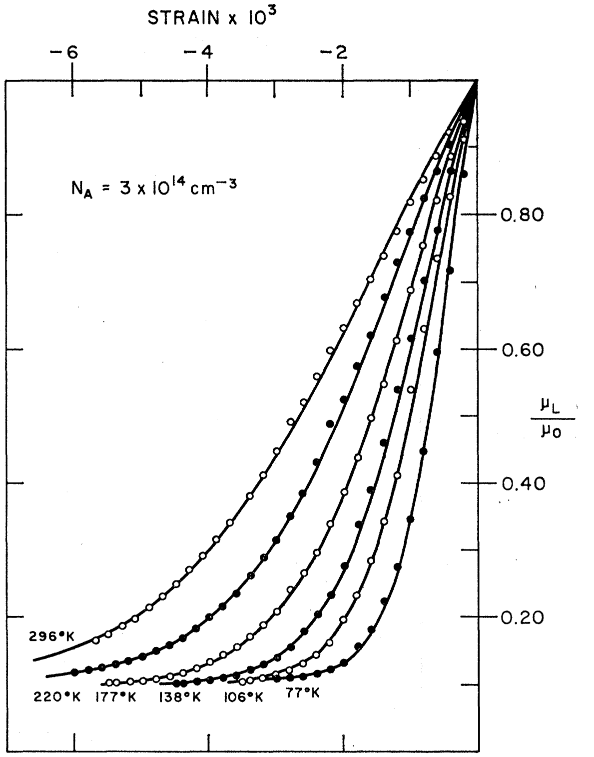

FIG. 3. Experimental measurements of μ_L/μ_0 versus strain at six temperatures for germanium samples having $N_A = 3 \times 10^{14}$ cm^{-3}. The solid curves are those obtained from Eq. (7).

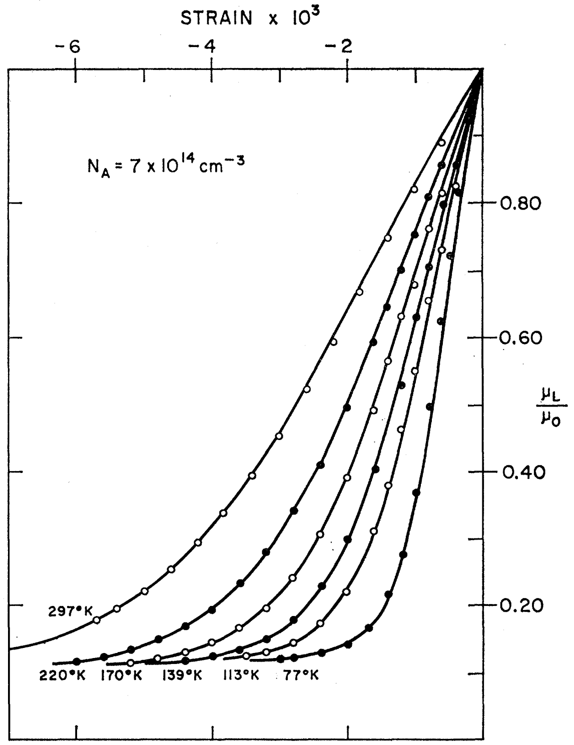

FIG. 4. Experimental values for μ_L/μ_0 versus strain for germanium samples having $N_A = 7 \times 10^{14}$ cm^{-3}. The solid curves are plots of Eq. (7).

III. EXPERIMENTAL DETAILS

The apparatus used for the application and measurement of the large uniaxial stresses required for the present experiments has been described previously.[16] Extreme care was taken in the preparation of the germanium samples for the piezodrift measurements in order to minimize local-stress centers which might give rise to premature fracture. The samples having dimensions 2×2×30 mm were cut from a suitably oriented single-crystal ingot in such a way that a ⟨111⟩ direction was along their length. The samples were lapped and polished and finally etched using the well-known CP-4A solution in order to minimize the surface recombination velocity.

The electrical circuitry used for the piezodrift experiments is shown in Fig. 1. It is similar to that used by Prince[17] in earlier drift-mobility experiments with additional precautions taken to minimize heating effects caused by the high-voltage sweep pulse. In Fig. 1, V_0 is the voltage drop associated with the sweep field, t_d is the peak-to-peak drift time, d_v is the distance between voltage probes, and d_p is the emitter-to-collector distance. A one-dimensional analysis of the diffusion and drift of a narrow pulse of excess carriers injected into the bulk of an extrinsic semiconductor has been described previously.[18] It has been shown that the measured drift mobility μ_0 can be expressed by

$$\mu_0 = (d_p d_v / V_0 t_d)\left[(1+x^2)^{1/2} - x\right], \tag{11}$$

with

$$x = (2kT/eV_0)(t_d/\tau_n + \tfrac{1}{2}), \tag{12}$$

where τ_n is the minority carrier lifetime. The quantity x is a correction term due to diffusion. Equation (12) may be used to estimate the magnitude of the drift voltage V_0 that is necessary for diffusion effects to be negligible. For the experiments described herein, this required the use of applied electric fields in excess of 10 V/cm at 300°K for the unstrained mobility measurements; fields of 30–40 V/cm were necessary at high strains. When the quantity x is negligibly small, the ratio of the longitudinal component of the mobility μ_L to the unstrained value μ_0 reduces to the simple form

$$\mu_L/\mu_0 = V_0 t_d / V_0' t_d', \tag{13}$$

where the primes indicate values obtained at a given strain. To avoid excessive specimen heating the sweep field was applied in 15-μsec pulses with a repetition rate of 60/sec; the synchronized low-level emitter pulse was less than $\frac{1}{2}$ μsec in duration. A second circuit consisting of a dc voltage source and a standard resistor in series

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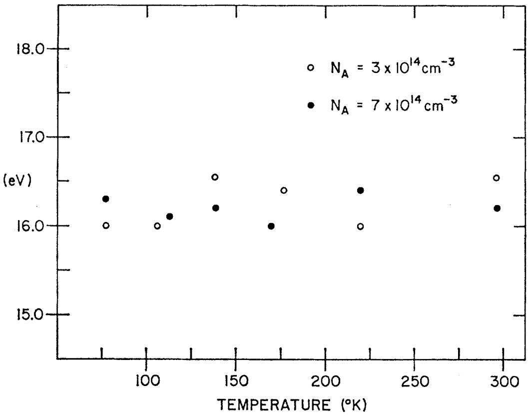

FIG. 5. Values obtained for the shear deformation potential Ξ_u between 77 and 297°K for the indicated impurity concentrations.

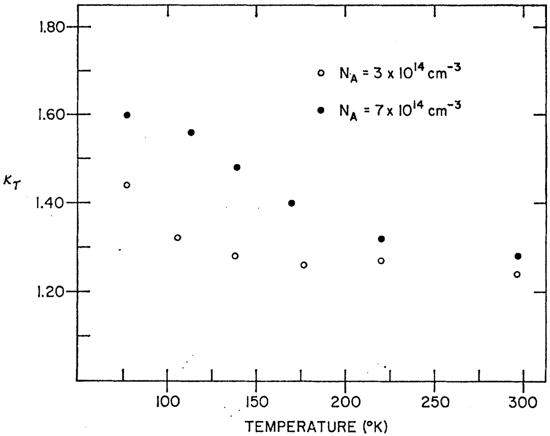

FIG. 6. Scattering anisotropy κ_τ versus temperature for minority electrons in p-type germanium with impurity concentrations as indicated.

with the germanium sample was used to monitor the specimen resistance. After the piezodrift circuitry was used to obtain an actual mobility measurement, the sample resistance was immediately determined by switching to the resistance circuit. Heating effects could then be determined from the change in sample resistance as its temperature returned to that of the ambient. In effect, the germanium itself served as a thermometer. In practice, a piezodrift measurement could be obtained in less than 1 sec. A permanent record of the drift time t_d was obtained by photographing the trace of the collector oscilloscope. The mobility ratio μ_L/μ_0 was then determined from the values of t_d and V_0 in the strained and unstrained state as indicated by Eq. (13). With the experiment performed in this manner it is estimated that an entire set of measurements, such as that shown in Fig. 2, was obtained with the sample temperature constant to within $\frac{1}{2}$°K.

IV. RESULTS OF PIEZODRIFT EXPERIMENTS

The piezodrift experiments were performed at temperatures between 77 and 297°K using p-type gallium-doped germanium samples containing 3×10^{14} and 7×10^{14} impurities per cm^3, respectively. All of the data were obtained with both the electron-drift path and the applied stress along a $\langle111\rangle$ direction. A typical set of experimental values for μ_L/μ_0 versus strain is shown in Fig. 2. A linear decrease in μ_L/μ_0, as predicted by Eq. (9), is observed at small strains. At a strain of 4×10^{-3}, the mobility ratio is constant, indicating that nearly all conduction electrons have been transferred to a single valley. The solid line shown in Fig. 2 is a plot of Eq. (7) with $\Xi_u=16.2$ eV and $\kappa_\tau=1.48$. Figures 3 and 4 display similar data acquired at six temperatures for samples having impurity concentrations as indicated. Nearly complete mobility saturation has been achieved at temperatures up to 220°K. At the higher temperatures, the fit of Eq. (7) was effected by matching the nonlinear high-strain region of the experimental data. Values for Ξ_u obtained from the mobility measurements are shown in Fig. 5. The average value of $\Xi_u=16.3\pm0.3$ eV is in agreement with other estimates[5,8,19] for the shear deformation-potential constant. The fact that the measured values of Ξ_u are quite independent of temperature, as indicated by Fig. 5, is in agreement with what is expected from theory. A plot of the scattering anisotropy versus temperature is shown in Fig. 6 for the two impurity concentrations indicated. It has been shown that a value of $\kappa_\tau=1.2$ is characteristic of pure acoustic-phonon scattering,[7] while theoretical estimates[20–22] for the scattering anisotropy associated with ionized impurity scattering give $\kappa_\tau>10$. The small values for κ_τ obtained in the present experiments indicate the predominance of acoustic-phonon scattering, although the presence of ionized impurity scattering is noticeable below 200°K, particularly for the samples having $N_A=7\times10^{14}$ cm^{-3}. The general features of the curves shown in Fig. 6 are in agreement with similar data obtained for majority electrons in n-type germanium from magnetoresistance measurements.[9,10] Values for κ_τ at 297°K, however, do not agree with the value $\kappa_\tau=3.7$ obtained by Cresswell and McKelvey from preliminary minority carrier piezodrift data[11] for samples having $N_A=3\times10^{15}$ cm^{-3}. Extrapolation of the values for κ_τ at 297°K shown in Fig. 6 to $N_A=3\times10^{15}$ cm^{-3} gives $\kappa_\tau\approx1.4$. The authors believe that this value is more nearly correct then the anomalously large value of 3.7 reported previously. Improvements in the experimental techniques[16] employed in the present experiments have resulted in an increase in accuracy of almost an order of magnitude over that reported in the preliminary experiments.

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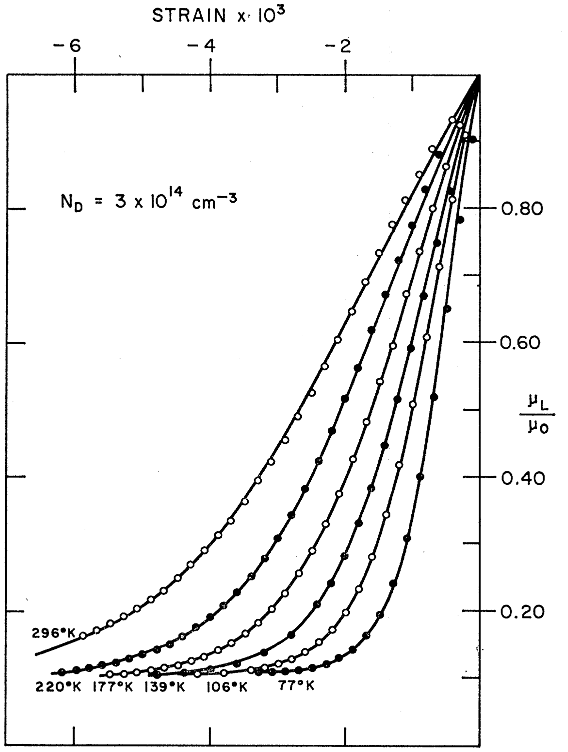

FIG. 7. Experimental values for μ_L/μ_0 versus strain for majority electrons in n-type germanium having $N_D=3\times10^{14}$ cm^{-3}. The solid curves are plots of Eq. (7).

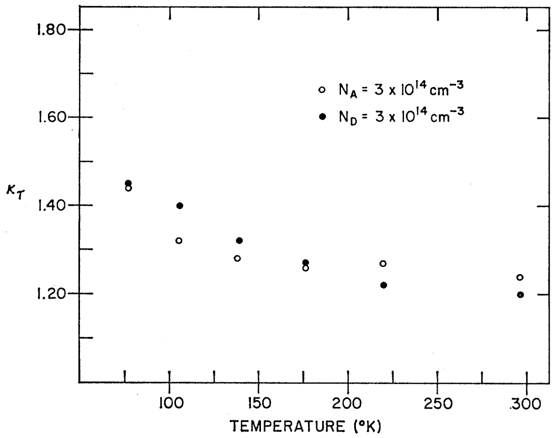

FIG. 8. Values for κ_T obtained for minority carriers in p-type germanium compared with values for κ_T for majority carriers in n-type germanium.

In order to compare minority carrier scattering versus majority carrier scattering, a series of high-stress piezoresistance experiments was performed using n-type germanium samples having $N_D=3\times10^{14}$ cm^3. The results are shown in Fig. 7. The values for Ξ_u obtained from these experiments were between 16.0 and 16.6 eV, in excellent agreement with the piezodrift measurements. The temperature variation of κ_T for majority electrons is shown in Fig. 8 along with corresponding data for minority electrons. As Fig. 8 indicates, there are no large differences in the scattering anisotropies, even though the values of κ_T for minority electrons include the effects of electron-hole scattering, which might be expected to increase the value of κ_T in the same way as Coulomb scattering from ionized impurity atoms. McLean and Paige[12,13] have given a minority carrier mobility theory which includes the effects of carrier-carrier scattering and have concluded that electron-hole scattering is significant below about 150°K. In their theory they have assumed an isotropic relaxation time. The small values of κ_T determined from the piezodrift measurements indicate that this assumption is a good one. Furthermore, the present experiments seem to indicate that for the ranges of impurity concentrations and temperature reported herein the effect of electron-hole scattering upon the relaxation time anisotropy is not significant.

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