for in an $(f_{7/2})^n$ coupling scheme. The 0^+ state at 3.000-MeV decay rate is fairly strong to the first excited state, indicating that it may be similar in nature to the deformed states in the Ca isotopes. Transitions involving the first 2^+ state at 0.9833 MeV also seem to show collective enhancements. The second 2^+ state at 2.420 MeV, seems to be collectively related to the 4^+ at 3.224 MeV and the (d, p) stripping data[5,20] rule out pure $(f_{7/2})^n$ wave functions[1] for this state and suggest a large $p_{3/2}$ neutron component to the wave function. Thus there does not seem to be much support for the interpretation of either of the 2^+ levels being simply $(f_{7/2})^n$ configurations, and therefore it is questionable whether the mode of decay of the 3^- state at 3.36 MeV via these two levels can furnish a good example of the proposed isospin selection rules.[23] The $E1$ rate for this state does show strong inhibition similar to the cases for Ca^{42} [11] and Ni^{58}.[25] The 4^+ states, as indicated, are collectively related to the 2^+ states, as are the 2^+ states to the 0^+ states so it is quite clear that collective admixtures are important in the low-lying levels of Ti^{48}.

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Elastic Scattering of 21-MeV Protons from Nitrogen-14, Oxygen-16, Argon-40, Nickel-58, and Tin-116

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Differential cross sections were measured for 21-MeV incident protons elastically scattered from ^{14}N, ^{16}O, ^{40}Ar, ^{58}Ni, and ^{116}Sn. An optical-model analysis was performed on these data as well as on previously measured elastic polarizations of 21-MeV incident protons scattered from the same nuclei. The optical potential used in this analysis included a real central term, a surface absorption term, and a real spin-orbit term. Good fits to both the cross sections and polarizations for all nuclei except ^{16}O were obtained by allowing the nine parameters of the optical potential to vary from nucleus to nucleus. The diffuseness parameters derived to fit the ^{14}N and ^{16}O data in general differ considerably from those derived to fit the scattering from the heavier nuclei. These results are compared with those of two other calculations in which two different sets of nonunique constant geometrical optical-model parameters were assumed and only the three potential strengths were obtained by searching. As expected, these constant-geometry calculations gave poorer agreement with the data. The disagreement with the polarizations of ^{14}N and ^{16}O was very pronounced.

INTRODUCTION

It is known[1] that the optical model can reproduce rather well the experimentally measured cross sections and polarizations, provided the parameters in the optical-model potential are allowed to vary as a function of energy and nucleus. Considerable efforts are being made to find a universal optical-model potential, or at least establish trends in the parameters of the potential with changes in incident proton energy, atomic number, and/or mass number.[2-8] In evolving such a potential, data are more useful if they include both cross sections and polarizations measured at the same incident proton energy. Consequently, in this paper elastic cross sections for 21-MeV incident protons

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were measured for five nuclei which differ considerably in mass, and for which elastic polarizations at 21 MeV were measured previously.[9] For each nucleus, an optical-model potential was determined which gave the "best fit" to both the cross section and polarization data.

EXPERIMENTAL PROCEDURE

The differential cross sections of protons elastically scattered from gas targets of ^{14}N, ^{16}O, and ^{40}Ar and foil targets of ^{58}Ni and ^{116}Sn were measured using a 21-MeV proton beam extracted from the University of Colorado four-sector, fixed-frequency alternating gradient cyclotron. Details of the experimental arrangement for obtaining these elastic angular distributions are given elsewhere.[10] Reaction-product particles were detected by lithium-drifted silicon semiconductor counters. The energy resolution of the experiment was approximately 150 keV, and the angular resolution was 1°.

The polarizations of 21-MeV protons elastically scattered from the above nuclei were measured[9] previously, using the NASA Lewis Research Center 60-in. cyclotron. The energy resolution of these polarization measurements was of the order of 1 MeV, and the angular resolution was approximately $3\frac{1}{2}°$.

EXPERIMENTAL RESULTS

Cross sections were obtained for elastic scattering ($5°<\theta<170°$) of 21-MeV incident protons from ^{14}N, ^{16}O, ^{40}Ar, ^{58}Ni, and ^{116}Sn. The experimental differential cross sections and their associated statistical counting errors are tabulated elsewhere.[10] The cross sections, plotted as the ratio to Rutherford, are pictured in Figs. 1(a)–5(a); the polarizations are pictured in Figs. 1(b)–5(b). The over-all uncertainty in the absolute cross sections was estimated to be less than 5% for all those reported in this experiment.

OPTICAL-MODEL POTENTIAL

The optical potential used for the analysis of these data can be written as

$$U(r)=V_C(r)-Vf(r,r_0,a_0)+i4Wa_i(d/dr)\times[f(r,r_i,a_i)]+(\hbar/m_\pi c)^2V_{so}(1/r)(d/dr)\times[f(r,r_{so},a_{so})]\mathbf{l}\cdot\boldsymbol{\sigma}. \quad (1)$$

$V_C(r)$ is the Coulomb potential between the incident proton and the scattering nucleus assumed to be a uniformly charged sphere of radius $1.25A^{1/3}$ F. Here $f(r,r_x,a_x)$ denotes the Woods-Saxon radial form factor and has the form

$$f(r,r_x,a_x)=\{1+\exp[(r-r_xA^{1/3})/a_x]\}^{-1}. \quad (2)$$

No volume absorption was used in this analysis, in accordance with the results of a recent analysis of 18.6-MeV proton cross-section and polarization data.[8]

OPTICAL-MODEL ANALYSES

For each of the several nuclei studied, an optical-model analysis was performed to fit simultaneously the polarization and elastic angular distributions. The calculations were performed with an optical-model computer program. The automatic search provision of this program was written by Davidon[11] and adapted for use at NASA, Lewis Research Center by Volkin and Giamati of that laboratory. It varies the nine independent parameters of the optical potential in order to minimize the quantity χ^2/N. This is defined as

$$\frac{\chi^2}{N}=\left[\sum_{i=1}^{N_\sigma}\left|\frac{\sigma_{\rm expt}(\theta_i)-\sigma_{\rm calc}(\theta_i)}{\Delta\sigma_{\rm expt}(\theta_i)}\right|^2+\sum_{i=1}^{N_P}\left|\frac{P_{\rm expt}(\theta_i)-P_{\rm calc}(\theta_i)}{\Delta P_{\rm expt}(\theta_i)}\right|^2\right](N_\sigma+N_P)^{-1}, \quad (3)$$

where $\sigma_{\rm expt}(\theta_i)$ and $P_{\rm expt}(\theta_i)$ are the experimentally measured differential cross sections and polarizations at angle θ_i, $\Delta\sigma_{\rm expt}(\theta_i)$ and $\Delta P_{\rm expt}(\theta_i)$ are the associated experimental uncertainties, $\sigma_{\rm calc}(\theta_i)$ and $P_{\rm calc}(\theta_i)$ are the calculated values, and N_σ and N_P are the number of experimental data points for the cross sections and polarizations, respectively. The experimental uncertainties of the cross sections were set equal to 10% of the measured cross sections in order to give a uniform weight to each datum point.

A series of calculations was performed for each isotope whereby the three potential strengths (V, W, and V_{so}) and the three diffusenesses (a_0, a_i, and a_{so}) were obtained by a six-parameter search. The three radii (r_0, r_i, and r_{so}), although fixed during each calculation, were varied independently in successive calculations in steps of 0.03 F within the limits

$$1.16\leq r_0\leq 1.25 \text{ F},$$
$$1.16\leq r_i\leq 1.25 \text{ F},$$
$$0.99\leq r_{so}\leq 1.14 \text{ F}.$$

The parameter values which gave the best fit to the experimental data were then used as starting values for a nine-parameter search calculation. The results of these calculations are listed in Table I and are pictured in Figs. 1–5. Quoted in Table I are the quantities χ^2/N defined by Eq. (3) and χ_σ^2/N_σ and χ_P^2/N_P

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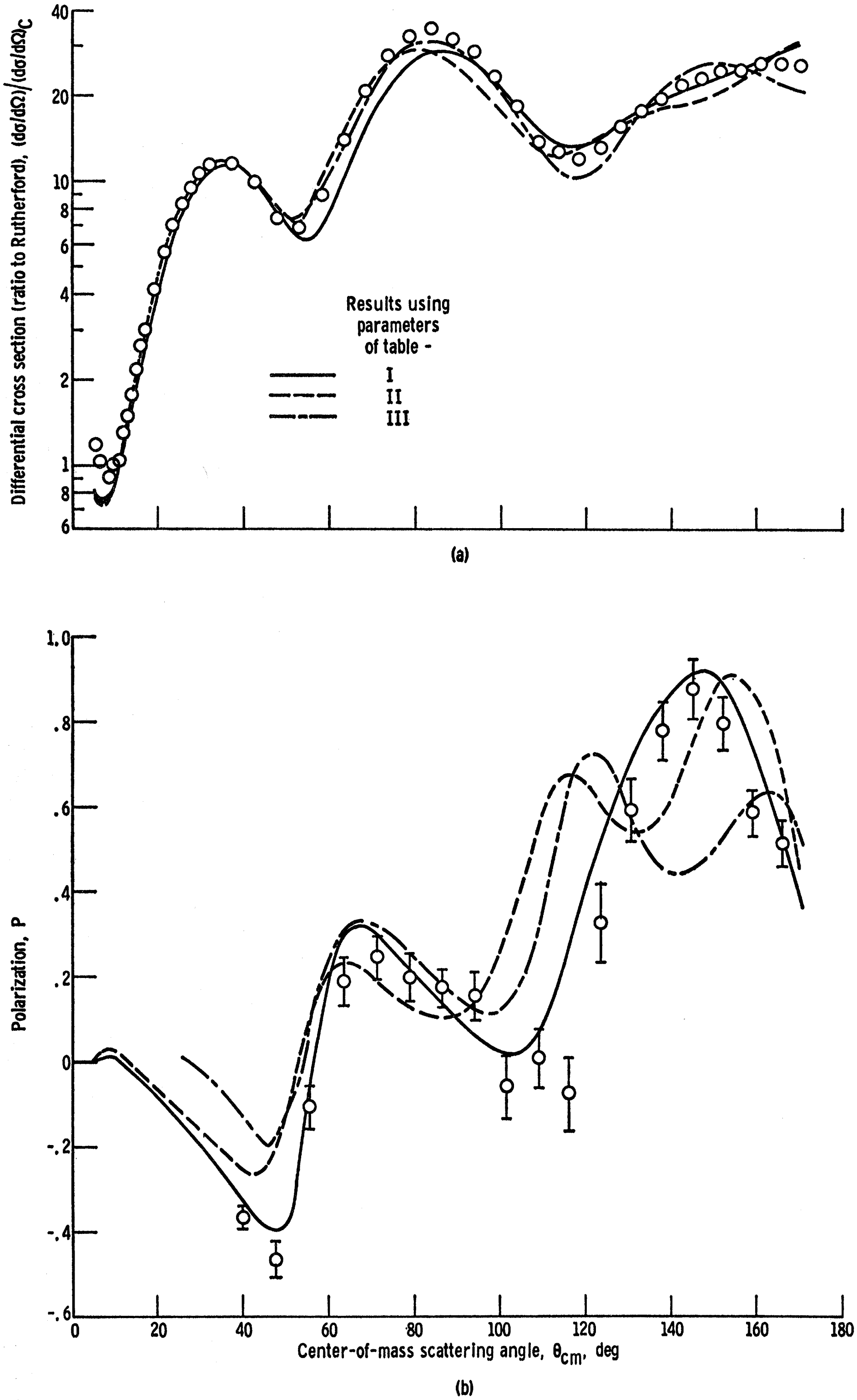

FIG. 1. Experimental and theoretical cross sections and polarizations for elastic scattering from nitrogen-14 of 21-MeV protons. (a) Elastic cross sections. Incident proton energy 21.02 MeV. (b) Elastic polarizations. Incident proton energy 20.8 MeV.

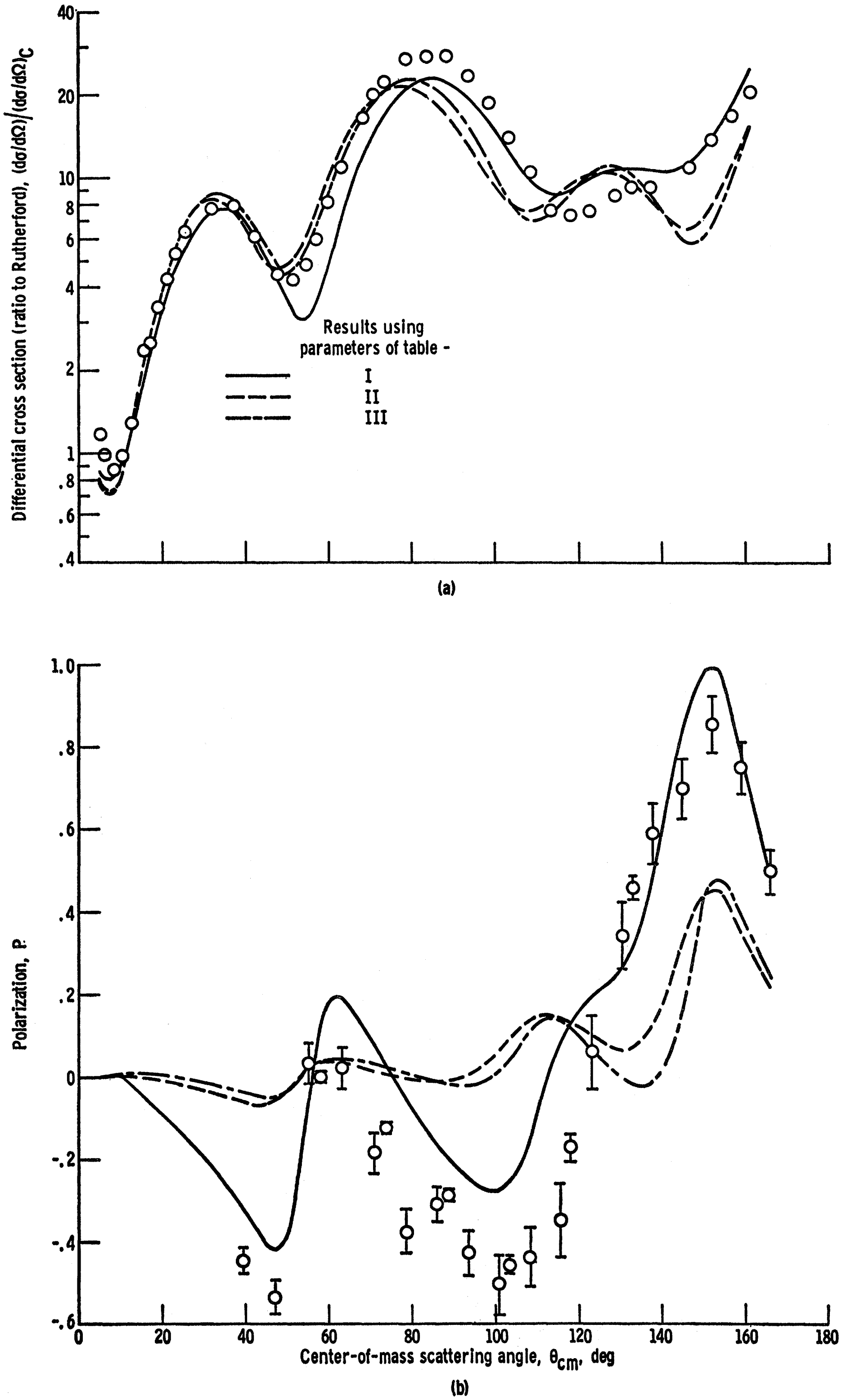

FIG. 2. Experimental and theoretical cross sections and polarizations for elastic scattering from oxygen-16 of 21-MeV protons. (a) Elastic cross sections. Incident proton energy 20.93 MeV. (b) Elastic polarizations. Incident proton energy 20.7 MeV.

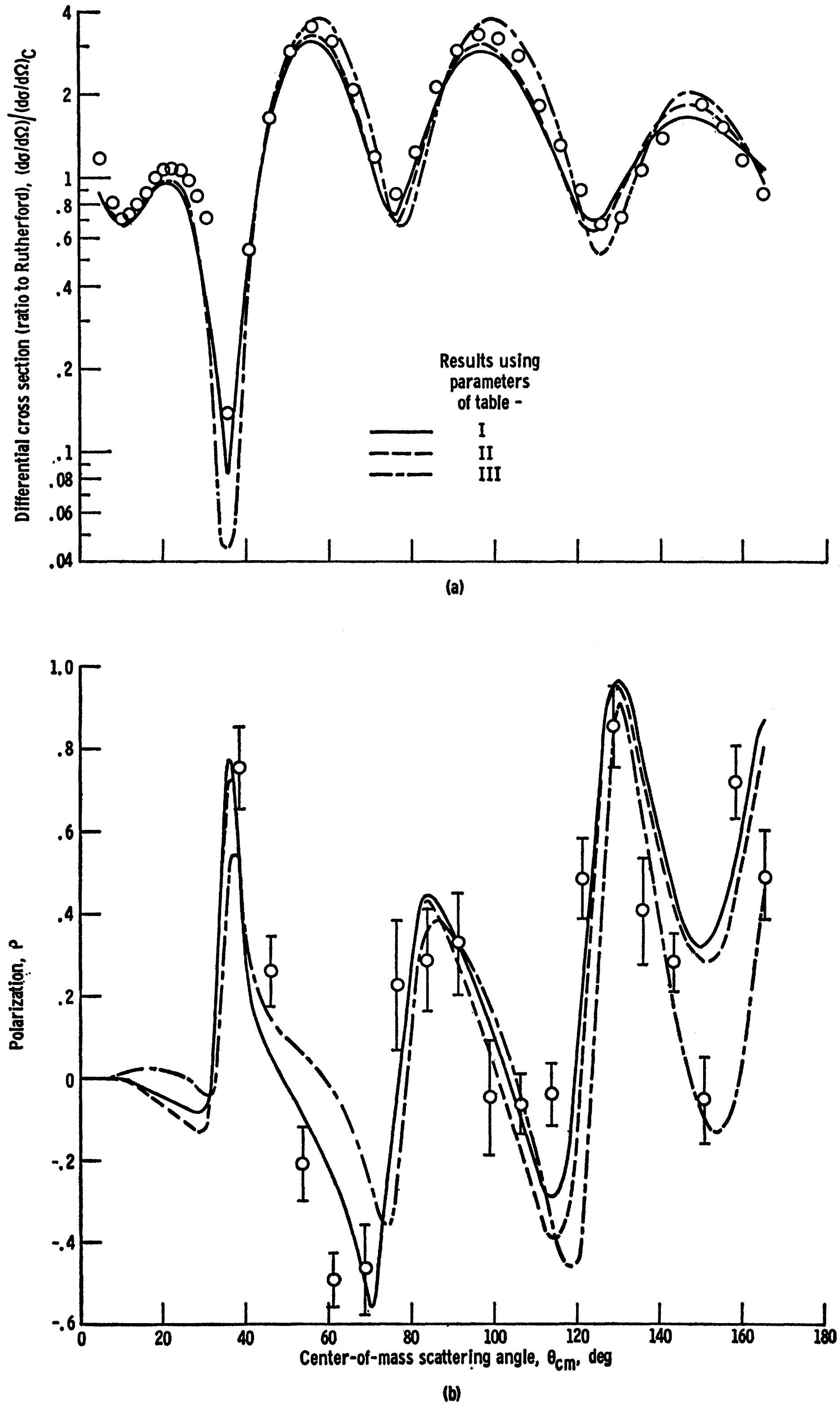

FIG. 3. Experimental and theoretical cross sections and polarizations for elastic scattering from argon-40 of 21-MeV protons. (a) Cross sections. Incident proton energy 21.13 MeV. (b) Elastic polarizations. Incident proton energy 21.0 MeV.

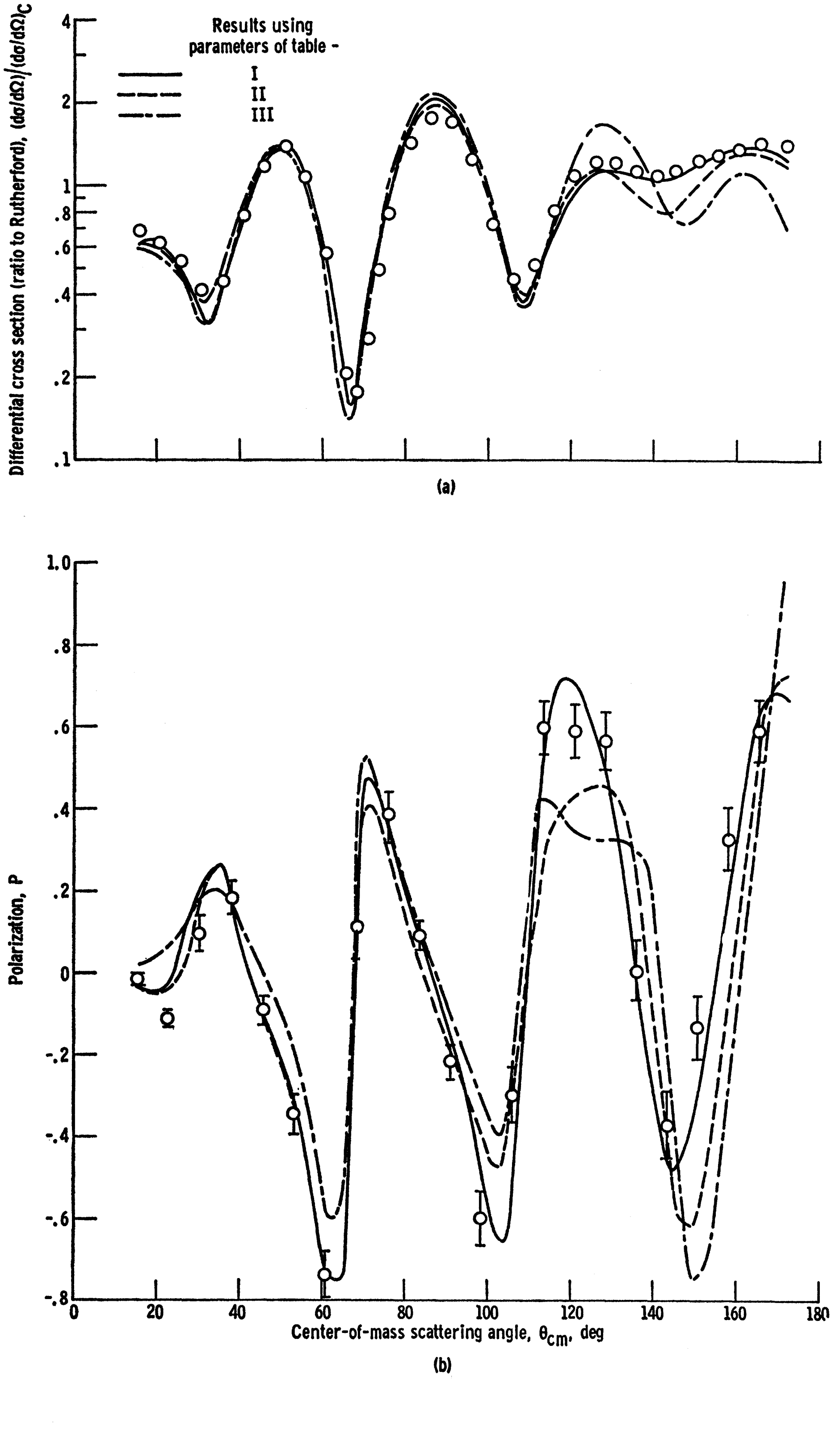

FIG. 4. Experimental and theoretical cross sections and polarizations for elastic scattering from nickel-58 of 21-MeV protons. (a) Elastic cross sections. Incident proton energy 21.3 MeV. (b) Elastic polarizations. Incident proton energy 20.9 MeV.

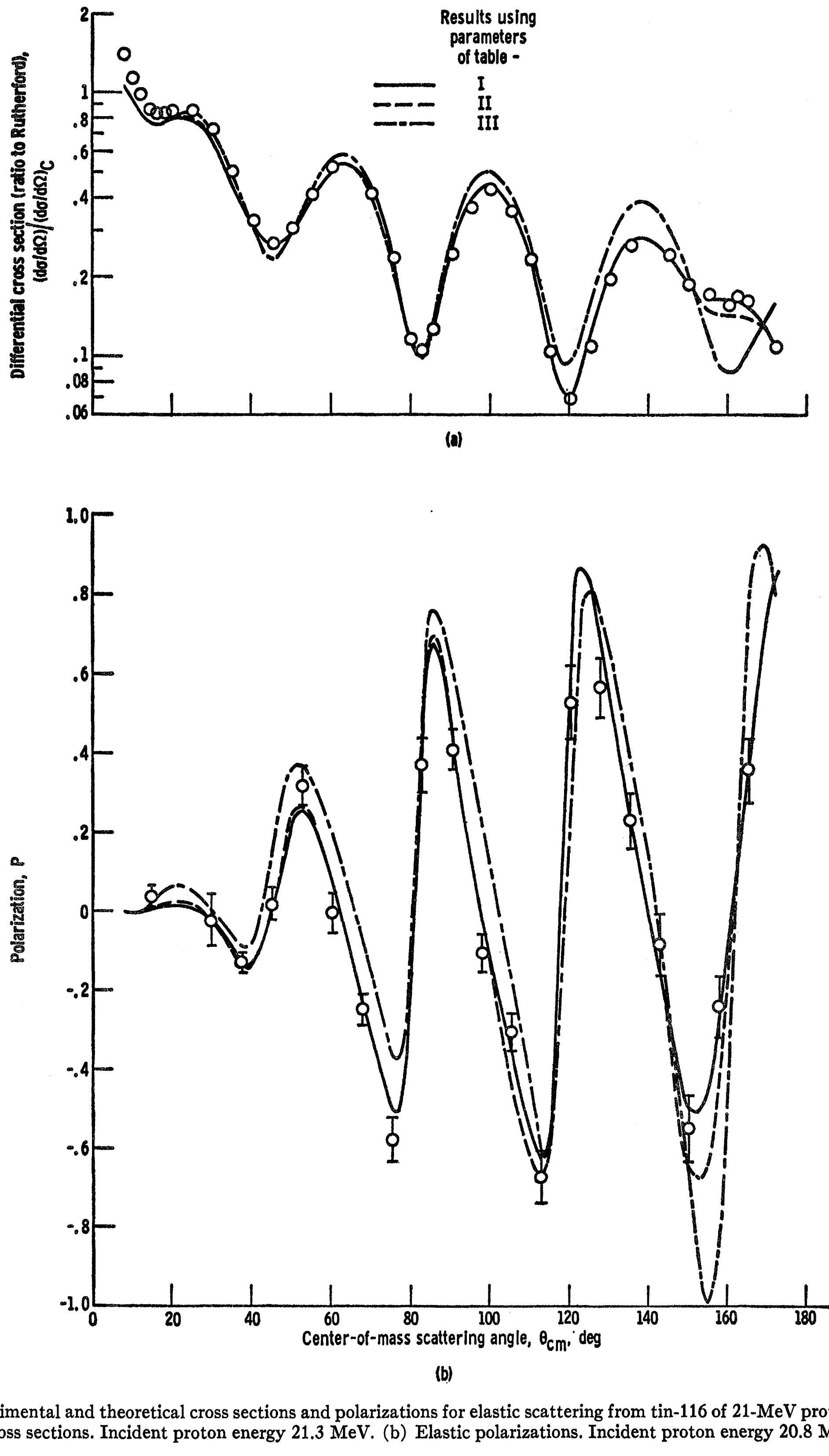

FIG. 5. Experimental and theoretical cross sections and polarizations for elastic scattering from tin-116 of 21-MeV protons. (a) Elastic cross sections. Incident proton energy 21.3 MeV. (b) Elastic polarizations. Incident proton energy 20.8 MeV.

TABLE I. Results of nine-parameter search.

	^{14}N	^{16}O	^{40}Ar	^{58}Ni	^{116}Sn
V	53.3	50.5	50.6	52.2	53.9
a_0	0.64	0.54	0.717	0.770	0.71
r_0	1.11	1.19	1.17	1.15	1.18
W	7.14	9.14	8.25	8.87	10.6
a_i	0.36	0.36	0.676	0.517	0.660
r_i	1.40	1.20	1.22	1.33	1.26
V_{so}	5.68	2.75	5.64	4.93	5.24
a_{so}	0.34	0.010	0.657	0.535	0.448
r_{so}	0.983	0.993	1.01	1.01	1.07
χ^2/N	2.08	13.95	3.88	1.93	0.705
χ_σ^2/N_σ	1.76	3.75	3.26	1.38	0.6
χ_P^2/N_P	2.83	34.4	5.26	2.81	0.905
σ (reaction)	480	490	1062	1093	1423

defined by

$$\frac{\chi_\sigma^2}{N_\sigma}=\left[\sum_{i=1}^{N_\sigma}\left|\frac{\sigma_{\text{expt}}(\theta_i)-\sigma_{\text{calc}}(\theta_i)}{\Delta\sigma_{\text{expt}}(\theta_i)}\right|^2\right]N_\sigma^{-1}, \tag{4}$$

$$\frac{\chi_P^2}{N_P}=\left[\sum_{i=1}^{N_P}\left|\frac{P_{\text{expt}}(\theta_i)-P_{\text{calc}}(\theta_i)}{\Delta P_{\text{expt}}(\theta_i)}\right|^2\right]N_P^{-1}, \tag{5}$$

respectively.

A number of three-parameter search calculations were performed to determine how well the scattering from these nuclei could be described using suitable fixed values for all the radii and diffusenesses guided by the results of the nine-parameter search. The results of these calculations are listed in Table II and pictured in Figs. 1–5 for comparison with the results of the nine-parameter search calculations. Reasonable fits were obtained for all the data except the polarizations of ^{14}N and ^{16}O. It should be noted that, since only five nuclei were studied, there is no intent here to propose that these particular values of the geometrical quantities represent a unique set of average optical-model potential parameters for 21-MeV protons.

For reference, another series of calculations was performed in which the three potential strengths were determined by searching using the fixed geometry suggested by Perey.[2] The results of these calculations are listed in Table III and pictured in Figs. 1–5.

TABLE II. Results of search on V, W, V_{so} using a set of nonunique fixed geometrical parameters.

	^{14}N	^{16}O	^{40}Ar	^{58}Ni	^{116}Sn
V	48.0	47.2	49.3	49.8	53.7
a_0	0.70	0.70	0.70	0.70	0.70
r_0	1.19	1.19	1.19	1.19	1.19
W	4.37	4.94	8.37	7.08	11.1
a_i	0.64	0.64	0.64	0.64	0.64
r_i	1.25	1.25	1.25	1.25	1.25
V_{so}	6.02	1.10	4.97	5.18	5.33
a_{so}	0.55	0.55	0.55	0.55	0.55
r_{so}	1.05	1.05	1.05	1.05	1.05
χ^2/N	5.89	38.0	4.14	3.62	0.968
χ_σ^2/N_σ	1.78	5.52	3.18	1.70	0.775
χ_P^2/N_P	16.3	103	6.28	6.72	1.33
σ(reaction)	528	597	1047	1084	1392

TABLE III. Results of search on V, W, V_{so} using a set of fixed geometrical parameters suggested by Perey.

	^{14}N	^{16}O	^{40}Ar	^{58}Ni	^{116}Sn
V	45.6	44.5	45.3	47.9	49.5
a_0	0.65	0.65	0.65	0.65	0.65
r_0	1.25	1.25	1.25	1.25	1.25
W	6.09	6.66	10.7	9.51	14.7
a_i	0.47	0.47	0.47	0.47	0.47
r_i	1.25	1.25	1.25	1.25	1.25
V_{so}	7.02	0.987	3.48	4.97	5.74
a_{so}	0.65	0.65	0.65	0.65	0.65
r_{so}	1.25	1.25	1.25	1.25	1.25
χ^2/N	5.95	38.4	8.90	9.52	8.26
χ_σ^2/N_σ	1.08	5.21	6.23	5.32	6.18
χ_P^2/N_P	18.3	105	14.8	16.3	12.2
σ(reaction)	495	551	944	995	1253

DISCUSSION AND CONCLUSIONS

It is only useful to comment on calculations of the differential cross sections and polarizations, since reaction cross sections for 21-MeV protons have not been reported for these nuclei. Also, since only five nuclei were studied here, no statement can be made concerning the dependence of the optical-model potential on an isobaric-spin term.

The cross sections and polarizations for 21-MeV protons incident on ^{40}Ar, ^{58}Ni, and ^{116}Sn are well described by a nine-parameter search. The resultant radii do not differ greatly among these nuclei. Typically, the spin-orbit radius is about 16% less than the radius of the real central potential term, and is about 25% less than the radius of the absorptive-potential term. The spin-orbit diffuseness tends to decrease for heavier nuclei whereas the diffuseness of the real central potential term is relatively constant and about 0.72 F.

The constant geometrical parameters suggested by Perey[2] were determined by an optical-model analysis of 35 different proton angular distributions at five different incident proton energies ranging from 9.4 to 22.2 MeV. The general characteristics of both the polarization and cross section are reproduced for each of these three nuclei. However, the values of χ^2/N may be reduced by a factor of from 2 to 10 by using a potential derived to fit only the data presented here, as illustrated in Tables II and III.

The optical model is expected to be more successful for heavy and medium weight nuclei than for light nuclei, as they more nearly approach the limit of uniform nuclear matter with few isolated resonances to affect the interaction. For lighter nuclei, fluctuations in the parameters are expected to be larger due to nuclear structure differences and the smaller density of levels in the compound system. This problem was previously investigated by Daehnick[12] who analyzed the scattering of 13- to 19-MeV protons by ^{16}O. He observed that the interference of potential scattering with compound nuclear scattering is very strong for even 200-keV resolution. Consequently, the application of the optical model to nuclei as light as ^{14}N and ^{16}O is questionable. This is illustrated by the results of a nine-parameter search calculation on ^{14}N and ^{16}O. For these nuclei the parameters obtained from such a calculation give a very good over-all fit but in general the diffuseness parameters differ considerably from those obtained for ^{40}Ar, ^{58}Ni, and ^{116}Sn. The constant geometrical parameters listed in Tables II and III give good fits to the ^{14}N cross sections, but the polarization fits are poor. Similarly, for ^{16}O the constant geometrical parameters listed in Tables II and III result in very poor agreement with the polarization data. However, the quality of fit to this data is considerably improved by the unrestricted nine-parameter search optical-model calculation.

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