Velocity-Dependent Effective N-N Potential for Shell-Model Calculations

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An effective N-N potential for shell-model calculations was previously obtained by modifying the velocity-dependent potential of Green to fit the low-lying energy levels of ^{18}O and ^{18}F. In this paper we present the low-energy spectra of ^{90}Y, ^{92}Zr, ^{210}Pb, and ^{210}Bi predicted by this potential. On the whole, agreement with the experimental spectra is good.

1. INTRODUCTION

IN this paper we describe an effective nucleon-nucleon potential for use in shell-model calculations of the energy levels of nuclei with two nucleons outside a closed shell. This effective potential gives reasonable agreement with the spectra of nuclei with mass numbers 18, 90, 92, and 210.

Our potential is derived from the velocity-dependent potential of Green[1] by varying some of the parameters, and we therefore call it the modified Green's potential V_{GM}. Green's original potential has been unsuccessfully applied by a number of authors[2–5] to shell-model calculations. This lack of success is not surprising because none of these calculations include core polarization, which Kuo and Brown[6] have shown to influence the spectra significantly. The effects of core polarization could be included empirically by finding an effective potential which reproduces the energy levels. This effective potential would be expected to vary from nucleus to nucleus and thus have no predictive value—it would simply be a way of parametrizing the experimental data.

Contrary to this expectation, we find that the effective potential we deduced previously for the mass-18 nuclei also gives reasonable fits to the energy levels of medium and heavy nuclei. Naturally slight variations of the parameters improve the fit to individual nuclei, but we emphasize that the main features of the spectra are correctly reproduced without such variations. This implies that the core polarization can be parametrized in such a way that it varies only weakly with the size of the core.

In Sec. 2 we summarize our previous results on mass-210 and mass-18 nuclei and describe the effective potential V_{GM} derived in Ref. 5 from the study of ^{18}O and ^{18}F. Section 3 contains the new results presented in this paper—the spectra of ^{90}Y, ^{92}Zr, ^{210}Pb, and ^{210}Bi calculated from V_{GM}—which lead to the conclusions reiterated in Sec. 4. The parametrization of V_{GM} is given in the Appendix.

2. SUMMARY OF PREVIOUS RESULTS

In the first paper of this series[4] we calculated the energy levels of ^{210}Po, ^{210}Pb, and ^{210}Bi using Green's potential. Agreement with experiment was poor. However, it was found that it was possible to fit the experimental levels by varying the parameters of the potential. In particular, it was necessary to strengthen the singlet even static potential, weaken the singlet even velocity-dependent potential, and strengthen and reduce the range of the static components of the triplet even potential.

Similar results were obtained in a study of ^{18}O and ^{18}F.[5] The levels calculated from Green's potential did not agree with experiment, but a potential modified in the way found necessary in Ref. 4 did give agreement. In this case we were also able to compare our matrix elements with those calculated by Kuo and Brown from the Hamada-Johnston potential. Except for the $T=1$, $J=0$ and $T=0$, $J=1$ matrix elements which are anomalously repulsive for Green's potential, there is a remarkable similarity between matrix elements of the two potentials. It is then of interest that the variation of the potential we made to obtain a fit to the energy levels has the same qualitative effect on the matrix elements as including core polarization. The even matrix elements of the effective interaction are more attractive than those of the bare potential.

The energy levels predicted by this modified Green's potential V_{GM} are compared with the experimental levels in Fig. 1.

The variation of parameters made to obtain the effective potential were qualitatively the same for $A=210$ and $A=18$. However, the potentials differ in detail and the $A=210$ potentials give rather poor fits to the levels of ^{18}O and ^{18}F.[7] We therefore restrict further discussion to the $A=18$ potential V_{GM} and in Sec. 3 compare the levels predicted by the potential for ^{90}Y, ^{92}Zr, ^{210}Pb, and ^{210}Bi with the experimental levels.

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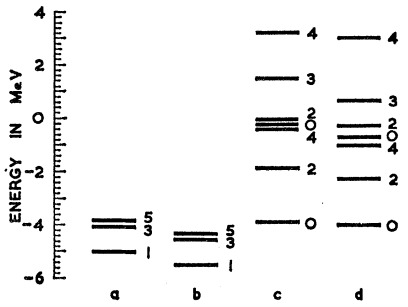

FIG. 1. Energy levels of ^{18}O, ^{18}F. (a) Experimental $T=0$ levels of ^{18}F [J. W. Olness and E. K. Warburton, Phys. Rev. **156**, 1145 (1967)]. (b) $T=0$ levels of ^{18}F calculated from V_{GM}. (c) Experimental levels of ^{18}O [P. Hawka, R. Middleton, and J. Wiza, Phys. Rev. **141**, 975 (1966)]. (d) Levels of ^{18}O calculated from V_{GM}.

3. RESULTS

A. ^{210}Pb Levels

The conventional shell-model picture of ^{210}Pb is that the doubly magic nucleus ^{208}Pb acts as an inert core whose sole role is to provide a harmonic-oscillator potential in which the two valance neutrons move.

In our calculations we include configuration mixing only for the $J=0$ state, which we find to be 80% $2g_{9/2}^2$ and 20% higher configurations ($1i_{11/2}^2$, $3d_{5/2}^2$, $4s_{1/2}^2$, $2g_{1/2}^2$, $2g_{7/2}^2$, $3d_{3/2}^2$). The other low-lying states we consider to be pure $2g_{9/2}^2$ configurations. This is supported by the calculation of Redlich,[8] which shows that these levels are more than 90% $2g_{9/2}^2$.

Our results for ^{210}Pb are shown in Fig. 2. The qualitative features are correct—the levels are correctly ordered, and the $J=0$ level is well separated from the others. Quantitatively, the fit could be improved by strengthening the singlet even static potential. As the potential V_{GM} was derived from a fit to ^{18}O this is not surprizing. *A priori* one would not have expected even the gross features of the spectrum to come out of the calculation.

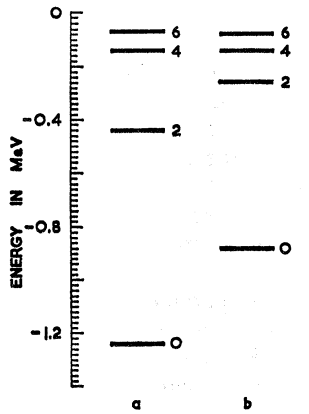

FIG. 2. Energy levels of ^{210}Pb. (a) Experimental levels of ^{210}Pb [P. Weinzierl, E. Ujlaki, G. Preinreich, and G. Eder, Phys. Rev. **134**, B257 (1964)]. (b) Levels of ^{210}Pb calculated from V_{GM}.

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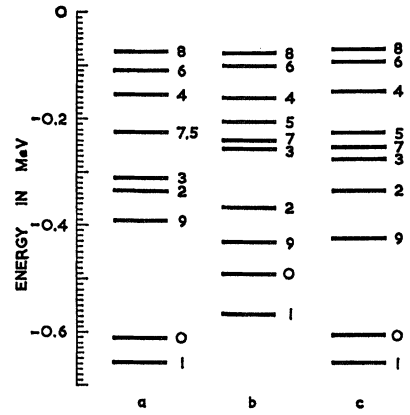

FIG. 3. Energy levels of ^{210}Bi. (a) Experimental levels of ^{210}Bi [J. R. Erskine, W. W. Buechner, and H. A. Enge, Phys. Rev. **128**, 720 (1962)]. (b) Levels of ^{210}Bi calculated from V_{GM}. (c) Levels of ^{210}Bi calculated from best-fit potential of Ref. 4.

B. ^{210}Bi Levels

The ^{210}Bi levels depend on both the $T=0$ and the $T=1$ parts of the potential, and have been discussed in some detail in Ref. 4. In Fig. 3, we compare the levels calculated from the present potential and from the "best-fit" potential of the Green type[4] with the experimental levels. Except for the $J=0$ and $J=1$ levels these two potentials give practically indistinguishable results. These two low-spin levels given by V_{GM} lie rather high, but not so high as to disturb the level ordering. The maximum discrepancy of the experimental levels and those calculated from V_{GM} is 0.12 MeV. This compares favorably with the maximum

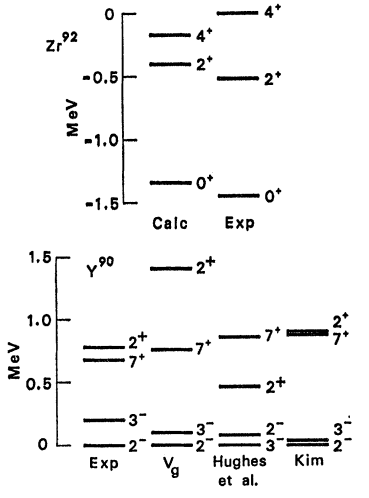

FIG. 4. Energy levels of ^{90}Y and ^{92}Zr. Note that the column labeled V_g refers to the potential V_{GM} of this paper.

TABLE I. Parameter values of the potential V_{GM}. Where the parameter differs from Green's original value, the original value is indicated in parentheses.

	A^C (fm^{-2})	a^C	B^C (fm^{-2})	α^C	A^P	a^P	A^T (fm^{-2})	a^T	B^T (fm^{-2})	α^T	A^{LS} (fm^{-2})	a^{LS}
SE	1.585	1.645	0.266	6	−0.74	3						
	(1.185)				(−1.14)							
TO	0		−0.089	6	0		0		−0.35	6	2.0	2.0
SO	0		−0.798	0.3	−1.3	2						
TE	2.86	1.59	0		−2.44	3.58	2.15	1.41	0		0	
	(2.6)	(2.3)			(−0.70)	(3)	(0.985)	(1.15)				

discrepancy obtained with static potentials *when the potential parameters are varied to improve the fit to the levels*. The calculation of Hughes *et al.*[9] has a maximum discrepancy of 0.15 MeV, and that of Kim and Rasmussen[10] one of 0.08 MeV.

C. ^{90}Y Levels

The lowest levels of ^{90}Y are an odd-parity doublet with spins 2 and 3. The 2^- state is the ground state. Above this doublet is an even-parity doublet, the 7^+ and 2^+ levels. The ^{88}Sr core is taken to be inert in the calculation. The dominant configuration of the 2^-, 3^- doublet is $2p_{1/2}2d_{5/2}$, while $1g_{9/2}2d_{5/2}$ dominates the even-parity doublet.

There have been many shell-model calculations of the structure of ^{90}Y. We refer particularly to those of Hughes *et al.*[9] and Kim[11] and list some of the others in Ref. 12.

It should be noted that the levels depend on both the $T=0$ and the $T=1$ parts of the potential. The results of our calculation using V_{GM} are compared with the experimental levels in Fig. 4. The results obtained by Hughes *et al.* using a static potential adjusted to fit the levels of ^{18}O and ^{18}F are also shown. V_{GM}, which is a nonstatic potential adjusted to fit the same data, gives much better results. Kim's results from a "simulated Brueckner-Gammel-Thaler potential" are shown in Fig. 4 for comparison with the present calculation.

The splitting of the 2^- and 3^- levels given by V_{GM} is too small, while that of the 2^+ and 7^+ is too large. This indicates that the central part of the triplet even force is too strong or that the tensor part is too weak. Kim's observations on the importance of the tensor force were confirmed in our calculations; without it the 3^- level is the ground-state, contradicting experiment.

D. ^{92}Zr Levels

The shell model of ^{92}Zr is that two neutrons move outside an inert ^{90}Zr core. In our calculations we have included $2d_{5/2}$, $3s_{1/2}$, $1g_{7/2}$, and $2d_{3/2}$ orbitals for the valence neutrons. Configuration mixing has been included only for the $J=0$ levels and neglected for $J>0$.

Figure 4 compares the levels with experiment. It will be seen that the agreement is good.

4. CONCLUSION

In this paper we presented the low-energy spectra of ^{90}Y, ^{92}Zr, ^{210}Pb, and ^{210}Bi calculated using a velocity-dependent effective interaction that was derived by modifying the potential of Green to fit the low-lying levels of ^{18}O and ^{18}F. Our results show that the gross features of the spectra of light, medium, and heavy nuclei can be accounted for by the same velocity-dependent effective interaction. Hughes *et al.* found that a static effective interaction fitted to the mass-18 levels did not give even the gross features of the other spectra.

This indicates that the effective interaction parametrized in a velocity-dependent form has the advantage that the parameters vary weakly with size of core and V_{GM} may be a useful first approximation in subsequent empirical studies of this dependence.

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APPENDIX

The velocity-dependent potential of Green is defined as[1]

$$V = V_{SE}P_{SE} + V_{SO}P_{SO} + V_{TE}P_{TE} + V_{TO}P_{TO},$$

where P_{SE}, P_{SO}, P_{TE}, and P_{TO} are projection operators

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onto the singlet even, singlet odd, triplet even, and triplet odd states, respectively, and

$$V_{SE}=V_{SE}^{C}(r)+U_{SE}(r,p^2),$$
$$V_{SO}=V_{SO}^{C}(r)+U_{SO}(r,p^2),$$
$$V_{TE}=V_{TE}^{C}(r)+U_{TE}(r,p^2)+V_{TE}^{T}S_{12},$$
$$V_{TO}=V_{TO}^{C}(r)+V_{TO}^{LS}\mathbf{l}\cdot\mathbf{S}+V_{TO}^{T}S_{12}.$$

The superscripts C, LS, and T designate the static central, spin-orbit, and tensor parts of the potential. The operators $\mathbf{l}\cdot\mathbf{S}$ and S_{12} are the usual spin-orbit and tensor operators. We have written

$$U_{SE}(r,p^2)=m^{-1}[p^2V_{SE}^{P}(r)+V_{SE}^{P}(r)p^2],$$

U_{SO} and U_{TE} being similarly defined. The symbol m denotes the nucleon mass, r is the relative coordinate $|\mathbf{r}_1-\mathbf{r}_2|$, and p is the relative momentum $\frac{1}{2}|\mathbf{p}_1-\mathbf{p}_2|$. The superscript P indicates that the radial form factor is part of the momentum or velocity-dependent potential.

The radial functions $V^X(r)$ are given by

(i) for $X\neq$P,

$$V^X(r)=-A^X\exp[-(0.6772a^X\mu r)^2]$$
$$-B^X(e^{-\mu r}/\mu r)[1-\exp(-\alpha^X\mu r)],$$

(ii) for $X=$P,

$$V^P(r)=-A^P\exp[-(0.6772a^P\mu r)^2].$$

Here μ is the inverse pion Compton wavelength, $\mu=0.7082$ fm^{-1}.

The parameters of V_{GM} are listed in Table I, and where the parameters differ from those of Green's potential, $V_{SE}(a)+V_{SO}(a)+V_{TE}(a)+V_{TO}(a)$, the parameters of the latter potential are quoted in parentheses. The parameters A^X and B^X for $X\neq$P have the dimensions of energy. The quoted values are in units of fm^{-2}, i.e., they are to be multiplied by $\hbar^2/m=41.469$ MeV fm^2 to convert to MeV. All other parameters are dimensionless.

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Effect of Woods-Saxon Wave Functions on the Calculation of $A=18$, 206, 210 Spectra with a Realistic Interaction*

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The spectra of the nuclei O^{18}, F^{18}, Pb^{206}, Pb^{210} are calculated using realistic forces and a Woods-Saxon form for the shell-model average field. The substitution of Woods-Saxon for harmonic-oscillator single-particle wave functions leads to appreciable upward shifts in the calculated positions for many low-lying levels in the $A=18$ nuclei. In particular the $T=0$, $J=1^+$ and $T=1$, $J=0^+$ binding energies are decreased by 0.6–1.7 MeV. In the heavier nuclei, one finds significant changes in observed energy levels perhaps only for the ground states. Nevertheless, the introduction of the more realistic single-particle average field in the Pb isotopes and in neighboring nuclei permits one to improve the conceptual basis upon which this field is erected.

I. INTRODUCTION

RECENTLY, many authors[1–5] have attempted to derive the residual interaction of the shell model directly from the free nucleon-nucleon force. The spectra obtained with such interactions are invariably calculated using harmonic-oscillator single-particle wave functions. Several authors[6–9] have suggested the changes to be expected if one employs instead the more realistic single-particle Woods-Saxon (WS) wave functions.[10] The present authors deem it necessary to do a somewhat more complete study of the Woods-Saxon shell model. This is especially true in light of our results which indicate that for mass-18 nuclei, the dislocations in level positions resulting from the use of

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