A similar analysis for the variation of the heavy electron mass with $\mathcal{E}$ has not been attempted, since the corresponding peaks in the spectra are not resolved over a large spectral range. However, an average value of $m_{he}$ may be used for a determination of $m_l$. Considering the case of $\mathbf{H}\|[110]$ with $\mathbf{E}\perp\mathbf{H}$, we get an average value of $m_{he}=(0.352\pm0.006)m_0$ on the basis of the first 10 peaks, excluding that due to the lowest ($n=0$) exciton. Using an average value of $0.081m_0$ for $m_t$ as expected from the variation shown in Fig. 10, we find $m_l=(1.54\pm0.06)m_0$.

Table III gives a summary of the conduction-band-edge effective masses obtained from the present work along with the results of cyclotron resonance[28,34] and magnetoabsorption experiments.[6]

## ACKNOWLEDGMENTS

We wish to express our gratitude to Dr. G. Dresselhaus for many stimulating discussions, to Dr. J. Halpern and K. J. Button for useful comments, and to U. Smith for a critical reading of the manuscript. We would like to thank L. V. Sousa for technical assistance with the experiments, R. Arndt for the assembly of the digital data recording system, D. R. Nelson for the computer program for data reduction, and W. Tice for x-ray orientation of the samples.

PHYSICAL REVIEW VOLUME 180, NUMBER 3 15 APRIL 1969

# Magnetic Freeze-Out of Electrons in Extrinsic Semiconductors

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(Received 16 October 1968)

The density of states was derived and the statistics of conduction electrons were studied for the case of a strongly doped compensated semiconductor in an external magnetic field. The tail of the density of states and the spread in the energy distribution of impurity levels were investigated, and the temperature and magnetic field dependences of the concentration of electrons not localized in impurities were calculated. It is shown that, because of the tail of the density of states, this concentration approaches a finite limit when $T\to0$. The freeze-out of carriers begins when the magnetic field attains such a value that the binding energy becomes larger than the rms potential energy of an electron in the field of the impurities. For sufficiently large magnetic fields, the Fermi level will drop into the tail, although the electrons may remain degenerate. This last conclusion will also be true for uncompensated semiconductors.

## I. INTRODUCTION

IN semiconductors with shallow donor (acceptor) levels the impurity band emerges with the conduction band at comparatively low impurity concentrations. It has been pointed out previously[1–3] that the impurity band will split off in sufficiently strong magnetic fields. This is because of the disappearance of the overlap of electronic wave functions situated on neighboring impurity sites due to their constriction to a cigar-shaped region under the influence of the magnetic field. The radius of this region is of the order of the magnetic length $\lambda=(\hbar c/eH)^{1/2}$ which may become much smaller than the Bohr radius $a$. Thus, localized states with a binding energy which increases with magnetic field will appear[1,4,5] when the volume of the bound state ($\pi\lambda^2a$) becomes less than the average volume of an impurity ($N^{-1}$), i.e., $N\pi\lambda^2a<1$.

This behavior becomes evident in the freeze-out effect, i.e., the electron concentration begins to depend on the temperature and magnetic field. This effect was studied in InSb by Sladek[3] and later by Beckman *et al.*[6] and by Neuringer.[7] In earlier works,[2,3] a theory for this effect was proposed which did not take into account the shifts in the impurity levels due to the random impurity potential. This is valid only for very lightly doped semiconductors when the rms potential of impurities is small compared to the binding energy of the electrons $\mathcal{E}_b$ and the thermal energy $T$ (in units of energy).

In the present paper, we calculate the density of states of a strongly doped semiconductor in a magnetic field and construct the theory for freezing out. The tail of the density of states of electrons in a magnetic field

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was studied in the work of Tsitsishvilli.[8] However, this work ignored the bound states and assumed that the Fermi level was high in the conduction band.

## II. PHYSICAL DESCRIPTION

We will consider the case of a strongly compensated semiconductor so that the carrier concentration $n$ is much less than the charged impurity concentration $N$. (We specifically choose electrons in the conduction band but the discussion also applies to holes in the valence band.) Following the work of Keldysh and Proshko,[9] we assume some correlation between impurity positions. This correlation appears during the process of sample preparation and may be taken into account by introducing the screening radius $r_0=(\kappa T_0/4\pi Ne^2)^{1/2}$, where $\kappa$ is the dielectric constant, $e$ is the electronic charge, and $T_0$ is the temperature (in units of energy) determined by growth conditions. We assume that the electron concentration is so small that the electronic screening radius is large compared to $r_0$. Under this conditions, the impurity potential does not change during the freeze-out process.

The random impurity potential contains fluctuations of different ranges. The fluctuation in the concentration with the range $r$ ($N^{-1/3}<r<r_0$) produces a potential of the order $N(r)e/r$, where $N(r)$ is the excess number of impurities in the volume of the fluctuation, i.e., $N(r)\sim(Nr^3)^{1/2}$. The largest potentials originate in impurity fluctuations with a range of the order $r_0$. It is just these fluctuations which give the main contribution to the rms potential $(2\pi)^{1/2}(Nr_0^3)^{1/2}e/r_0$. If we study electronic states with wavelengths smaller than the range of a given fluctuation, then this fluctuation may be considered as a local variation of the bottom of the conduction band. The local downward shift of the conduction band gives rise to states in the forbidden gap in that region of the crystal. We introduce the average depth of the potential well given by

$$\Gamma=2\pi^{1/2}(e^2/\kappa r_0)(Nr_0^3)^{1/2}. \tag{1}$$

When the energy $\mathcal{E}$ does not lie too deep in the forbidden gap (a more precise statement follows later), then only such potential wells are important in the density of states whose bottoms lie below $\mathcal{E}$ by an amount of the order $\Gamma$. Thus, in the absence of a magnetic field the characteristic wavelength is $\hbar(2m\Gamma)^{-1/2}$. In order to consider the fluctuation as a local variation of the bottom of the band it is necessary for the condition

$$\hbar/(2m\Gamma)^{1/2}\ll r_0 \tag{2}$$

to be satisfied. The potential energy of electrons in the field of short-range fluctuations, for which such a description is not valid, is much less than the typical kinetic energy of an electron in a well of depth $\Gamma$, in this case, and so may be ignored. The ideas, which are the basis for the description given here, were developed by Kane,[10] Bonch-Bruevich,[11] and Keldysh and Proshko.[9]

The magnetic field begins to affect the density of states if the distance between Landau levels $\hbar\Omega$ becomes larger than the broadening of these levels due to nonhomogeneous distribution of impurities, i.e., the magnetic field is important if

$$\hbar\Omega\gtrsim\Gamma. \tag{3}$$

The condition given in Eq. (3) also means that the magnetic length which characterizes the localization of electrons in a plane perpendicular to the magnetic field is less than the wavelength along the direction of the magnetic field which is determined by the typical depth of the potential well. Thus the condition given by Eq. (2) is also sufficient to insure the smoothness of the potential variation in the magnetic-field case. Inserting Eq. (1) into Eq. (2) shows that this condition fails at high impurity concentrations. For InSb the concentration must be of the order or less than $10^{17}$ cm$^{-3}$.

Next, we discuss the electronic states localized on individual impurity centers. In the vicinity of each positively charged impurity the potential energy of electrons approaches minus infinity. However, in strongly doped semiconductors the bound states are absent if $Na^3\gg1$. It is well known[12] that bound states appear in a magnetic field and if the condition

$$N\pi\lambda^2 a<1 \tag{4}$$

is satisfied the neighboring states do not overlap. It is important to emphasize that the position of the impurity level is strongly dependent on the configuration of neighboring impurities since these states are greatly elongated along the direction of the magnetic field and they are in close proximity. If the length of the state $a$ is small compared to $r_0$ then, in the field of the most important long-range fluctuations, the impurity level will rise or fall with the motion of the bottom of the conduction band. The energy broadening of the levels over the volume of the crystal may be larger than the binding energy of the electron.

The fluctuations of the potential with a range less than $a$ cannot be considered in such a manner. However, these are not important because (a) their magnitude is less and (b) the position of the energy level is less sensitive to them. As discussed later, the impurity levels are important only in the case $r_0\gg a$.

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Thus, if Eq. (2) is satisfied, then the bottom of each Landau sub-band as well as the shallow impurity levels are shifted by an amount $V(\mathbf{r})$ equal to the potential energy of an electron at the point $\mathbf{r}$. This fact is very important for the calculation of the electron concentration in a magnetic field.

## III. DENSITY OF STATES

We begin by calculating the density of impurity states. For simplicity we assume that only the ground state located below the bottom of the lowest Landau sub-band is important. Then, according to the foregoing, if the length of this state $a$ in the direction of the magnetic field is small compared to $r_0$, the density of impurity states per unit volume at point $\mathbf{r}$ has the form

$$\rho_b(\mathcal{E},\mathbf{r})=N_D\delta(\mathcal{E}-\mathcal{E}_b-V(\mathbf{r})), \tag{5}$$

where $\mathcal{E}_b$ is the field-dependent energy of the bound level and $N_D$ is the concentration of positively charged centers.

$$V(\mathbf{r})=\sum_{j=1}^{\mathfrak{N}/2}\phi(\mathbf{r}-\mathbf{r}_j)-\sum_{j=\mathfrak{N}/2+1}^{\mathfrak{N}}\phi(\mathbf{r}-\mathbf{r}_j) \tag{6}$$

is the potential energy of an electron in the field of positively and negatively charged impurities. (We assume that the number of donors equals the number of acceptors and we ignore the electron potential.) In Eq. (6),

$$\phi(\mathbf{r})=(e^2/\kappa r)e^{-r/r_0}.$$

To obtain the full density of states, we should integrate Eq. (5) over the volume of the entire crystal. Instead we fix $\mathbf{r}$ and average Eq. (5) over all impurity configurations and thus obtain the average density of states per unit volume $\rho_b(\mathcal{E})=\langle\rho_b(\mathcal{E},\mathbf{r})\rangle$.

We introduce the probability $P(U)dU$ that the potential energy $V(\mathbf{r})$ lies in the interval from $U$ to $U+dU$.

$$P(u)=\int\frac{d^3\mathbf{r}_1}{V_0}\cdots\int\frac{d^3\mathbf{r}_{\mathfrak{N}}}{V_0}\delta(U-V(\mathbf{r})). \tag{7}$$

Then

$$\rho_b(\mathcal{E})=N_D\int_{-\infty}^{\infty}dUP(U)\delta(\mathcal{E}-\mathcal{E}_b-U). \tag{8}$$

Let us transform Eq. (7) as follows:

$$P(u)=\int\frac{d^3\mathbf{r}_1}{V_0}\cdots\int\frac{d^3\mathbf{r}_{\mathfrak{N}}}{V_0}\frac{1}{2\pi}\int_{-\infty}^{\infty}dt\,e^{i(U-V)t}$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}dt\,e^{iUt}F(t). \tag{9}$$

With the aid of Eq. (6) we obtain

$$F(t)=\left[\int\frac{d^3\mathbf{r}}{V_0}e^{-i\phi(\mathbf{r})t}\right]^{\mathfrak{N}/2}\left[\int\frac{d^3\mathbf{r}}{V_0}e^{i\phi(\mathbf{r})t}\right]^{\mathfrak{N}/2}$$

$$=\left(1+\frac{f_+(t)}{V_0}\right)^{\mathfrak{N}/2}\left(1+\frac{f_-(t)}{V_0}\right)^{\mathfrak{N}/2}, \tag{10}$$

$$f_\pm(t)=\int d^3r(e^{\mp i\phi(\mathbf{r})t}-1). \tag{11}$$

In Eq. (10) take the limit $\mathfrak{N}\to\infty$ and $V_0\to\infty$ so that $\mathfrak{N}/V_0=N$, where $N$ is the concentration of charged impurities. We find that

$$F(t)=\exp[\tfrac{1}{2}N(f_++f_-)]. \tag{12}$$

If we assume that for important $\mathbf{r}$ and $t$, $\phi(\mathbf{r})t\ll1$ then we may expand the exponent in Eq. (11) to the quadratic term. Then

$$F(t)=e^{-\Gamma^2t^2/4} \tag{13}$$

and

$$\Gamma^2=2N\int d^3r\,\phi^2(\mathbf{r}). \tag{14}$$

The integral in Eq. (14) brings us to Eq. (1). Inserting Eq. (13) into Eq. (9), we have

$$P(u)=\pi^{-1/2}\Gamma^{-1}\exp(-U^2/\Gamma^2). \tag{15}$$

Then Eq. (8) takes the form

$$\rho_b(\mathcal{E})=N_D\pi^{-1/2}\Gamma^{-1}\exp^{-1}(x+x_b)^2, \tag{16}$$

where $x=(\mathcal{E}-\frac{1}{2}\hbar\Omega+\frac{1}{2}g\beta H)\Gamma^{-1}$ is the energy measured from the bottom of the lowest Landau sub-band in units of $\Gamma$, and $x_b=\mathcal{E}_b/\Gamma$, where $\mathcal{E}_b$ is the binding energy measured downwards from the bottom of the Landau sub-band ($\mathcal{E}_b>0$). Thus it happens that the impurity level is broadened by an amount on the order of $\Gamma$.

We point out that in the integral of Eq. (14), large $r$ of the order of $r_0$ are important. This is the justification of the derivation under the condition $r_0\gg a$.

It may be seen that the expansion of the exponential in Eq. (11) is justified if $|x+x_b|\ll(Nr_0^3)^{1/2}$. In the following, we assume that the condition $Nr_0^3\gg1$ is fulfilled, so that this limitation only holds in the region where the density of states is exponentially small.

Now we consider the density of states of electrons which are not localized in the vicinity of impurities. If the condition of Eq. (2) is satisfied, then

$$\rho_c(\mathcal{E},\mathbf{r})=\frac{1}{V_0}\sum_{s,p_y,p_z,M}\delta[\mathcal{E}-(M+\tfrac{1}{2})\hbar\Omega-p_z^2/2m-\tfrac{1}{2}g\beta H-V(\mathbf{r})], \tag{17}$$

where $p_y$, $p_z$, and $M$ are the quantum numbers of electrons in a magnetic field in the Landau repre-

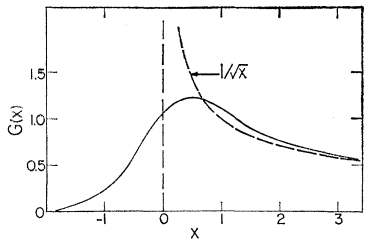


FIG. 1. Dependence of the density of states on energy (in units of $\Gamma$) near the bottom of the lowest Landau sub-band when $\hbar\Omega \gg \Gamma$ [Eq. (23)].

sentation, $g$ is the gyromagnetic ratio, and $s=\pm 1$. Performing the summation on $p_y$ and $p_z$ in Eq. (17), we obtain

$$\rho_c(\mathcal{E},\mathbf{r})=\frac{\sqrt{m}}{2\sqrt{2}\pi^2\hbar\lambda^2}\sum_{M,s}[\mathcal{E}_{Ms}-V(\mathbf{r})]^{-1/2}, \quad (18)$$

where $\mathcal{E}_{Ms}=\mathcal{E}-(M+\frac{1}{2})\hbar\Omega-\frac{1}{2}g\beta H$. In Eq. (18), each term of the sum should be considered nonzero only for such values of energy $\mathcal{E}$ for which the square root is real.

Averaging Eq. (18) according to the prescription

$$\langle(\mathcal{E}-V)^{-1/2}\rangle=\int_{-\infty}^{\mathcal{E}}dU\frac{P(u)}{(\mathcal{E}-V)^{1/2}},$$

and using Eq. (15), we find that

$$\rho_c(\mathcal{E})=A\sum_{M,s}G(x_{Ms}), \quad (19)$$

where $x_{Ms}=\mathcal{E}_{Ms}/\Gamma$, $A=m^{1/2}(2\sqrt{2}\pi^2\hbar\lambda^2\sqrt{\Gamma})^{-1}$, and

$$G(x)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{x}dy\frac{e^{-y^2}}{(x-y)^{1/2}}. \quad (20)$$

For $x<0$ the function $G(x)$ may be expressed through the MacDonald function $K_{1/4}$:

$$G(x)=\pi^{-1/2}(-\tfrac{1}{2}x)^{1/2}e^{-x^2/2}K_{1/4}(\tfrac{1}{2}x^2) \quad (x<0). \quad (21)$$

Equations (19)–(21) coincide with the results of Tsitsishvilli[8] obtained by another approach for noncompensated semiconductors.

In this case the expansion of the exponent in Eq. (11) is good for all $x>0$ if $Nr_0^3\gg 1$. For $x<0$ it is true if

$$|x|<(Nr_0^3)^{1/2}. \quad (22)$$

We begin the study of Eq. (19) for the case $\hbar\Omega\gg\Gamma$. Near the bottom of the lowest Landau sub-band, only the term with $M=0$ and $s=-1$ is important. Then

$$\rho_c(\mathcal{E})=AG(x), \quad (23)$$

where $x$ is the same as in Eq. (16). The function $G(x)$ is shown in Fig. 1. For $x\gg 1$, $G(x)\to(x)^{-1/2}$ and Eq. (23) goes into the density of states for a free electron in a magnetic field. For $x<0$ there is a tail of the density of states in the forbidden gap. The asymptotic expression has the form

$$G(x)=e^{-x^2}/\sqrt{2}|x|^{1/2} \quad (x<0,\ |x|\gg 1). \quad (24)$$

Equation (24) is valid if the condition of Eq. (22) is satisfied. There is another limitation. Values of $y$ that are close to $x$ are important in Eq. (20) for $x<0$ and $|x|\gg 1$: $|x-y|\sim|x|^{-1}$. Physically, this means that electron states with energies near the bottom of the wells are important at large negative energies. However, our description fails if the distance to the bottom of the well becomes less than $\hbar^2(mr_0^2)^{-1}$. Thus, Eqs. (21), (23), and (24) are valid for $x<0$ only if

$$|x|<\Gamma m r_0^2\hbar^{-2}. \quad (25)$$

Under the condition of Eq. (2) the right-hand side of Eq. (25) is much larger than 1. Thus, once again the limitation on Eqs. (22) and (25) apply only where the density of states is exponentially small.

In the case when $\hbar\Omega\ll\Gamma$ the magnetic field is unimportant. Changing from sum to integral in Eq. (20), we obtain

$$\rho_c(\mathcal{E})=\frac{\sqrt{2}m^{3/2}\sqrt{\Gamma}}{\pi^2\hbar^3}G_0\left(\frac{\mathcal{E}}{\Gamma}\right), \quad (26)$$

where

$$G_0(x)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{x}e^{-y^2}(x-y)^{1/2}dy. \quad (27)$$

These results are the same as those obtained previously[9,10] for strongly doped semiconductors but without magnetic field. The function $G_0(x)$ is shown in Fig. 2.

## IV. ELECTRON CONCENTRATION

To find the concentration of electrons not localized on impurities it is necessary, as usual, to solve the equation for the chemical potential $\mu$.

$$n_c+n_b=n=N_D-N_A, \quad (28)$$

where $N_D$ and $N_A$ are the donor and acceptor concen-

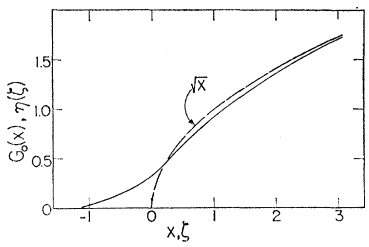


FIG. 2. Dependence of the density of states on energy (units of $\Gamma$) when $\hbar\Omega\ll\Gamma$ [Eqs. (26) and (27)]. The same plot gives the dependence of $\eta=n_c(2A\Gamma)^{-1}$ on $\zeta=\mu/\Gamma$ when the electrons are degenerate in the lowest Landau sub-band and $\hbar\Omega\gg\Gamma$.

trations, and

$$n_c=\int_{-\infty}^{\infty}\rho_c(\mathcal{E})f(\mathcal{E})d\mathcal{E},$$
$$n_b=\int_{-\infty}^{\infty}\rho_b(\mathcal{E})f(\mathcal{E})d\mathcal{E}, \tag{29}$$

where

$$f(\mathcal{E})=(e^{(\mathcal{E}-\mu)/T}+1)^{-1}.$$

The density of states $\rho_b(\mathcal{E})$ and $\rho_c(\mathcal{E})$ are given by Eqs. (16) and (19). Equation (25) cannot be solved analytically. We note some of its properties which are of the most interest from our point of view. We consider the case $\hbar\Omega\gg\Gamma$ and we suppose that all electrons are in the lowest Landau sub-band. It is seen from Eq. (19) that the density of states $\rho_c(\mathcal{E})$ is proportional to the magnetic field. The number of states in the tail ($x<0$) is of the order of $A\Gamma$, so that in sufficiently strong magnetic fields when $A\Gamma>n$, all electrons may be in the tail. On the other hand, the binding energy $\mathcal{E}_b$ increases with magnetic field.[4,5] If $\mathcal{E}_b$ becomes larger than $\Gamma$, then all electrons will go to impurity levels if the temperature is low enough. This is just the freeze-out process. The specific feature of freeze-out in strongly doped semiconductors is that the concentration $n_c$ does not tend to zero as the temperature goes to zero and if $\Gamma\gtrsim\mathcal{E}_b$, then $n_c$ is comparable with $n$.

The dependence of the concentration on magnetic field and temperature can have very different characters when the relationships among the parameters $\Gamma$, $\mathcal{E}_b$, $n$, and $N_D$ are varied. We consider certain limiting cases.

Let $\Gamma$ be larger than $\mathcal{E}_b$ (in sufficiently large magnetic fields this condition will necessarily fail). Then, if one compares Eqs. (16) and (19) and if Eq. (4) is satisfied, then $\rho_c(\mathcal{E})>\rho_b(\mathcal{E})$ and the bound states are not important. If Eq. (4) is not satisfied then the bound states are absent so that, instead of Eq. (28), one can take $n_c=n$. If $T=0$, then the chemical potential is defined by

$$n_c=A\Gamma\int_{-\infty}^{\zeta}G(x)dx=2A\Gamma G_0(\zeta), \tag{30}$$

where the function $G_0$ is defined by Eq. (27), $\zeta=\mu/T$, and the Fermi level is measured from the bottom of the lowest Landau sub-band. The plot of $\eta=n_c(2A\Gamma)^{-1}$ as a function of $\zeta$ is shown in Fig. 2. This dependence can be studied experimentally by optical experiments.[13]

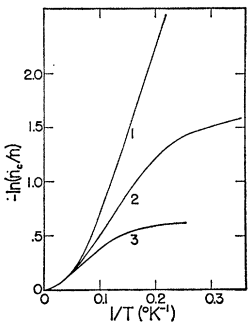


FIG. 3. Results of a numerical calculation of the dependence $n_c/n$ on temperature for $n$-InSb with concentration $n=10^{14}$ cm$^{-3}$, $N_D=5\times10^{14}$ cm$^{-3}$, and the magnetic field $H=30$ kOe. (1) Without impurity potential ($\Gamma=0$); (2) $\Gamma/\mathcal{E}_b^0=2.5$; and (3) $\Gamma/\mathcal{E}_b^0=4$, where $\mathcal{E}_b^0$ is the binding energy at $H=0$. At 30 kOe, according to Ref. 14, $\mathcal{E}_b=4.2\,\mathcal{E}_b^0$.

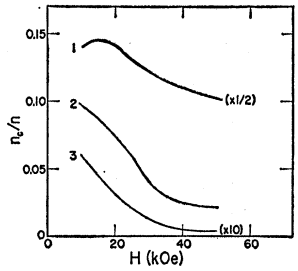


FIG. 4. Results of a numerical calculation of the dependence of $n_c/n$ on magnetic field at $T=4.1$°K. The concentractions $n_c$ and $N_D$ and the relations between $\Gamma$ and $\mathcal{E}_b^0$ are the same as in Fig. 3. The values for $\mathcal{E}_b(H)$ were taken from Ref. 14.

The condition of degeneracy in the electron tails is not the usual one. If $\Gamma\gg T$, then Eq. (30) is valid for all positive $\mu$ and for negative $\mu$ such that $|\mu|<\Gamma^2(2T)^{-1}$. If $\Gamma\ll T$, then the degeneracy condition is the usual one, $\mu\gg T$. For Boltzmann statistics the condition $-\mu-\Gamma^2(2T)^{-1}\gg T$ is necessary if $\Gamma\gg T$, and the usual condition $-\mu\gg T$ applies if $\Gamma\ll T$. In the case of Boltzmann statistics, the integral in Eq. (29) for $n_c$ can be calculated exactly and

$$n_c=n_c^{(0)}\exp(\mu/T+\Gamma^2/4T^2),$$
$$n_c^{(0)}=\frac{1}{2\sqrt{2}\pi^{3/2}}\frac{\sqrt{m}T^{1/2}}{\hbar\lambda^2}. \tag{31}$$

If $\Gamma\ll T$, then Eq. (31) gives the usual equation for electrons in a magnetic field.

Now let the magnetic field be strong enough that $\mathcal{E}_b>\Gamma$. Then at low temperatures the electrons will mainly be on the impurities. In this case one may obtain the analytical dependence of the electron concentration $n_c$ on $\mathcal{E}_b$ and $H$. The Fermi level is defined by the equation $n=n_b$ or

$$\frac{n}{N_D}=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{e^{-x^2}dx}{e(\Gamma/T)(x+\chi)+1}, \tag{32}$$

where $\chi=-\mu/\Gamma-\mathcal{E}_b/\Gamma$. Thus $\chi$ does not depend on the magnetic field. If $|x|\ll\Gamma/T$, then Eq. (32) can be transformed into

$$\Phi(-\chi)=-1+2n/N_D, \tag{33}$$

where

$$\Phi(x)=2(\pi)^{-1/2}\int_0^x\exp^{-1}(t^2)dt$$

is the probability integral. Under these conditions, $\chi$ is also independent of temperature. Under conditions where Boltzmann statistics apply, Eq. (31) is valid for $n_c$ where the chemical potential is defined by Eq. (32). Then

$$n_c=n_c^{(0)}\exp\left(-\frac{\mathcal{E}_b}{T}-\chi\frac{\Gamma}{T}+\frac{\Gamma^2}{4T^2}\right). \tag{34}$$

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The magnetic-field dependence in this case is the same as for the pure crystal but the magnitude and the temperature dependence of $n_c$ are quite different. It is worth noting that in sufficiently strong magnetic fields Eq. (34) is necessarily valid since it is only necessary to satisfy the condition $\mathcal{E}_b > \Gamma^2(2T)^{-1}$.

If the Fermi level given by Eq. (32) is such that the nonlocalized electrons are degenerate, then Eq. (30) or the plot in Fig. 2 may be used to find the concentration $n_c$. These values for $n_c$ are the limiting values as $T \to 0$ and they are strongly dependent on the magnetic field.

The most interesting region is when $\mathcal{E}_b \sim \Gamma$. Unfortunately, in this region it is necessary to obtain $n_c(H,T)$ by numerical calculation. This calculation was performed for $n$-InSb for two values of $\Gamma/\mathcal{E}_b^0$, where $\mathcal{E}_b^0$ is the binding energy in zero magnetic field. The results are shown in Figs. 3 and 4. For comparison the curve without impurity potential ($\Gamma = 0$) is also shown.

The magnetic-field dependence of $n_c$ arises from two competing processes: the linear increase in the density of states $\rho_c(\mathcal{E})$ with magnetic field and the increase in binding energy $\mathcal{E}_b$ with magnetic field. The maximum in curve 3 of Fig. 4 reflects this competition. The calculated values of $n_c$ are very sensitive to the magnetic-field dependence of $\mathcal{E}_b$. In these calculations we used the experimental values determined by Kaplan.[13]

## V. CONCLUSION

There are several possible ways to compare this theory with experiment. The most direct method for the determination of $\Gamma$ is by optical experiments. This question is discussed in an accompanying paper.[14] Here we discuss the magnetic-field and temperature dependences of the Hall constant. There are two characteristic regions in the temperature dependence of the concentration $n_c$: the high-temperature region $T \gg \Gamma$ where $n_c$ does not depend on $\Gamma$ and the saturation region where $n_c$ depends strongly on $\Gamma$. Unfortunately, the freeze-out in magnetic field has not been extensively studied experimentally. The temperature dependence of the Hall constant in $n$-InSb was studied by Sladek[3] in fields to 28 kOe. It is difficult to compare our theory with his experiments because of the conditions of the experiment. In his experiment the electron mobility in the impurity band was important and the compensation of his samples was small. Under these conditions $\Gamma$ will depend on temperature and magnetic field. This was not taken into account in our theory. The experiments of Beckman *et al.*[6] and Neuringer[7] were done in magnetic fields to 200 kOe. The observed magnetic field dependence was $\exp^{-1}(\mathcal{E}_b/T)$ and under their conditions this term should be vanishingly small since the ratio $\mathcal{E}_b/T$ is of the order of 20 at 100 kOe. However, the Hall constant was only changed by two or three orders of magnitude at this field. This difference is explained by Eq. (34), which contains the large term $\exp(\Gamma^2/4T^2)$ as well as the factor $\exp^{-1}(\mathcal{E}_b/T)$. This first term does not depend on magnetic field. Experimental measurements of the temperature dependence of the Hall constant are necessary to verify this conclusion.

Qualitatively our results are valid for noncompensated semiconductors. In this case the Fermi level will also fall into the tail with increasing magnetic field. Quantitative considerations are complicated by the fact that the electron concentration becomes inhomogeneous and the electronic potential must be taken into account.

It is also necessary to mention that if the conduction electrons are in the far tail they are indeed localized in regions of the order of $r_0$. In this situation the crystal is very inhomogeneous and questions about kinetic effects, in particular, the question of the Hall constant, must be studied separately.[15] An experimental method free from these objections would be the measurement of the Faraday effect under the condition $4\pi\sigma_{xy}/\kappa \ll \omega \ll \mathcal{E}_b/\hbar$.

## ACKNOWLEDGMENTS

We thank V. I. Perel for helpful discussions of several topics included in this work and N. B. Brovtzina for assistance with the numerical calculations.

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