Oscillations in the Magnetoconductivity of a Two-Dimensional Coherent Network of Coupled Orbits

J. O. STRÖM-OLSEN*†
Department of Physics, Rutgers, The State University, New Brunswick, New Jersey
(Received 28 October 1968)

Both long- and short-period oscillations have been seen in the conductivity of tin with a large magnetic field applied along the c axis. The measurements were made at 1.4°K by means of helicon propagation in fields up to 125 kG. The results agree with calculations based on a Kubo formulation due to Chambers if phase coherence is taken as complete around small orbits and partial around large orbits. The zone oscillations predicted by Pippard were not observed.

1. INTRODUCTION

THE phenomenon of magnetic breakdown (MB) has been widely studied both theoretically and experimentally, and a good review of the subject has been given by Stark and Falicov.[1] MB occurs in certain metals where the band gap is so small that a magnetic field can cause the conduction electrons to make transitions between various sheets of the Fermi surface. The transition or breakdown probability has been shown at all fields to be[2]

$$P=e^{-H_0/H}, \tag{1}$$

where $H_0 \simeq m\Delta^2/\epsilon_F he$. Here ϵ_F is the Fermi energy and Δ is the band gap.

One outcome of MB is that orbits which were previously independent are now coupled. In particular, in tin, with a magnetic field along the c axis, orbits around the center of the third and fourth zones are coupled to form the two-dimensional network shown in Fig. 1. The notation for the orbits is that used by Stark and Craven.[3] In the absence of MB, the orbits are $\epsilon_1{}^5$ and $\delta_1{}^1$, Bragg reflection occurring at points like A. When MB is complete there is just the single orbit B.

The effect of this coupling on the various transport coefficients is to introduce oscillations[4] arising from phase coherence of the electrons around their orbits. The size and type of the oscillations depend on the degree of coherence, and until recently they had been observed only from very small sheets of the Fermi surfaces. Thus for the above orientation in tin, Hays and McLean[5] observed only long-period oscillations corresponding to the orbit $\delta_1{}^1$. Young,[6] however, has now observed short-period oscillations in tin corresponding to orbits about the size of $\epsilon_1{}^5$. But this was for a one-dimensional network of orbits for which the oscillations were only pronounced at certain field values. Moreover, there were some beating effects which remained unexplained.

The present experiment shows large-amplitude, short-period oscillations clear at all fields; the theory used accounts for all their main features.

2. EXPERIMENT

The measurements were made using helicon standing waves in a square slab of pure white tin at a frequency of 2500 Hz and a temperature of 1.4°K. The slab, 10×10×1 mm, was cut perpendicular to (001) from a single crystal of residual resistance ratio 25 000. A large magnetic field was applied along the c axis (taken as the z direction). A plane electromagnetic wave polarized in the x direction was applied to both sides of the slab by one coil and the phase quadrature component of the signal along the y direction was detected by a second coil. A sufficiently accurate formula for this imaginary component of the transverse permeability is given by Chambers and Jones[7]:

$$I(\mu_t)=\frac{4u}{\pi}\sum_{n=1}^{\infty}\frac{1/n^2}{1+Q^2(\omega/\omega_n-\omega_n/\omega)^2}, \tag{2}$$

$$u=\frac{\rho_{xy}}{\rho_{xx}},\quad Q=(1+u^2)^{1/2},$$

$$\omega_n=(2n+1)^2\pi^2\rho_{xx}Q/4d^2.$$

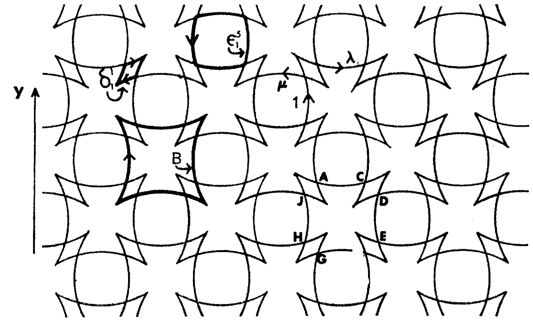

FIG. 1. Two-dimensional network in tin with the magnetic field along the c axis.

* Present address: Department of Physics, Dalhousie University, Halifax, Nova Scotia, Canada.

† Research supported by the National Science Foundation.

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[6] R. C. Young, Phil. Mag. **18**, 201 (1968).

[7] R. G. Chambers and B. K. Jones, Proc. Roy. Soc. (London) **A270**, 417 (1962).

Here d is the thickness of the slab and ω is the applied frequency. Thus resonance occurs whenever $\omega_n=\omega$.

There are two major advantages to using helicons. The first is that the method is probeless, so that damage to the specimen is kept to a minimum. This is especially important since, as mentioned below, phase coherence is very readily destroyed by dislocations.[8] The second is that, in contrast to direct measurement, the method may be used when the resistivity of the system is very low. The main drawback is that it is not easy to extract the various coefficients of the conductivity tensor.

Figure 2 shows the signal in arbitrary units over half a long-period oscillation under conditions for which $\omega_1 \gg \omega$. Along the abscissa are plotted both the field and the phase around $\delta_1{}^1$. The latter may be found by following up the long oscillations from low field, where they have a very characteristic nonsinusoidal shape. Two main features are evident. First is the beating effect and second is that the amplitude of the short oscillations is largest when the phase around $\delta_1{}^1$ is an odd multiple of π. Note that the fundamental and beat periods give areas which are within 2% of those found by Stark and Craven[3] for B and $\epsilon_1{}^5$ from de Haas–van Alphen measurements.

3. THEORY

There are two distinct contributions to the conductivity σ: σ_1 from the MB network and σ_2 from the rest of the Fermi surface. Since $\mathbf{H}$ lies along a fourfold axis of symmetry, σ will have $\sigma_{xx}=\sigma_{yy}$.

The theory developed here is not intended to cover all the behavior of the helicon signal, but merely to reproduce the salient features of the short-period oscillations. Since we are well beyond the fundamental resonance, this allows several approximations to be made.

The calculation of σ_1 is made with the simplifying assumption that the $\delta_1{}^1$ orbits may be neglected except

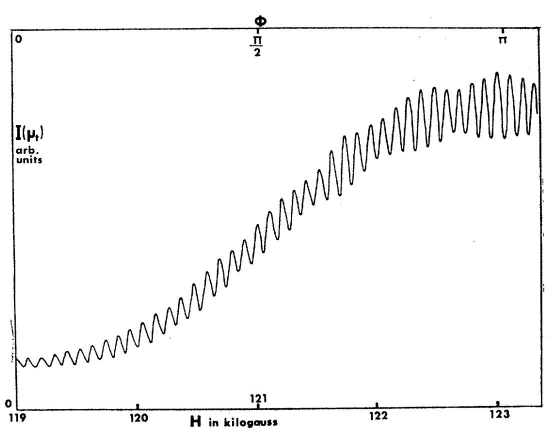

FIG. 2. Measured helicon signal.

[8] A. B. Pippard, Proc. Roy. Soc. (London) **A287**, 165 (1965).

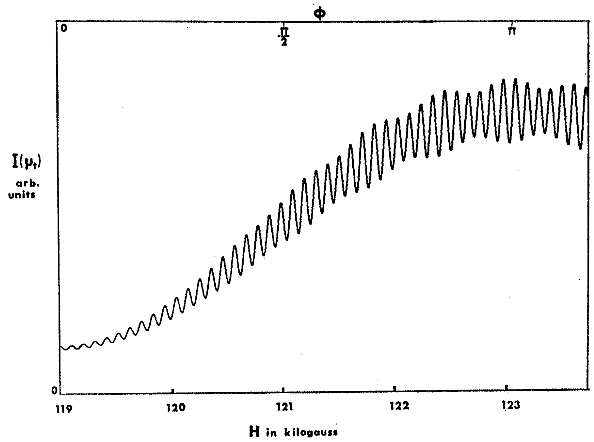

FIG. 3. Theoretical helicon signal over the same field range as Fig. 2.

insofar as they affect the switching between various branches of $\epsilon_1{}^5$ and B. The breakdown probability at points like A become λ and μ (see Fig. 1), where

$$\lambda=P^2/(1+Q^2-2Q\cos\Phi)\,, \tag{3}$$

$$\mu=2Q(1-\cos\Phi)/(1+Q^2-2Q\cos\Phi)\,, \tag{4}$$

with P given by Eq. (1) and $Q=1-P$. Φ is the phase around $\delta_1{}^1$.

The conductivity is now written in terms of an effective path for the electrons[8] which is then calculated by means of the Kubo method of Chambers.[9] We neglect impurity scattering and assume that the conductivity is limited solely by MB, a reasonable assumption in view of the size of the magnetic field and the purity of the specimen. We now apply an electric field along the x direction and consider only the electrons on AC, since those on DE and HJ are negligible and those on FG just double the contribution from AC. Suppose that the average effective path of these electrons is $\mathbf{L}=(L_x,L_y)$. Then to a sufficient degree of accuracy we may write

$$(\sigma_{xx})_1=2n'eL_x/aH\,,\quad (\sigma_{xy})_1=2n'eL_y/aH\,, \tag{5}$$

where a is the real space distance AC and n' is the effective number of carriers on that branch of the network.

The effective path of any point on AC may be written in terms of propagators[9] which may be evaluated by a diagrammatic method. The number of diagrams depends on the path length over which the electron retains phase coherence. Coherence here is taken up to one orbit $\epsilon_1{}^5$ or B. Since we follow Chambers closely at this point, all further details of the technique may be found on the original paper. The summation of the diagrams is trivial, and leads to the result that the average effective path of the *whole* arc is

$$L_x=\tfrac{1}{2}av\frac{1-f^2}{1+f^2}\,, \tag{6}$$

[9] W. G. Chambers, Phys. Rev. **165**, 799 (1968).

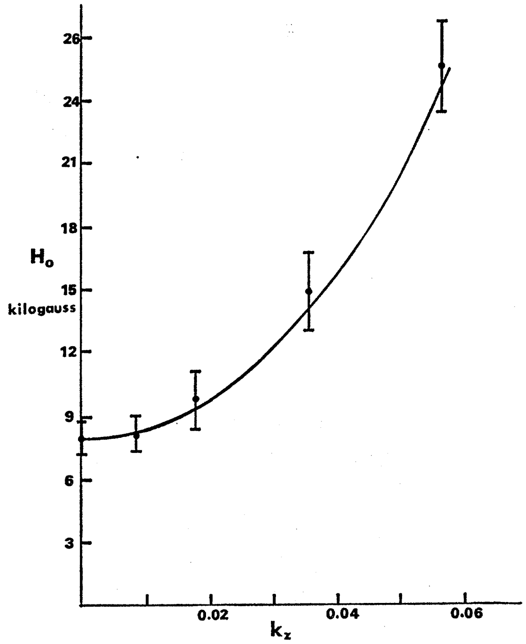

FIG. 4. Variation of the effective breakdown field in kilogauss with the height above the central plane in units of 1.08×10^8 cm^{-1}.

$$L_y = a\nu\frac{-f}{1+f^2}, \tag{7}$$

where $f=\lambda-\mu-4\lambda\mu[\lambda \cos\alpha+\mu \cos\beta]$, with α and β being the phases around B and $\epsilon_1{}^5$, respectively. ν gives the variation in density of states;

$$\nu=1+\lambda^2 \cos\alpha+\mu^2 \cos\beta. \tag{8}$$

Thus Shubnikov–de Haas oscillations are included here rather than in n'.

This formula for **L** gives an amplitude for the short-period oscillations as large as for the long-period oscillations, clearly at variance with the experimental data. This discrepancy may be removed by remembering that since dislocations are very effective in destroying coherence, there will be a fair amount of phase smearing around $\epsilon_1{}^5$ and B, even though coherence around $\delta_1{}^1$ is essentially complete. Thus the contribution from these large orbits will be reduced by some factor s, roughly the same for both, and we replace $\cos\alpha$ and $\cos\beta$ by $s \cos\alpha$ and $s \cos\beta$. Since we do not know how many dislocations are present in the specimen, s cannot be calculated but must be treated as a variable parameter. Though the absolute magnitude of the short-period oscillations depends on s, the relative magnitude for different values of Φ is insensitive to it if $s<\sim0.5$.

In principle, **L** should be expanded in $\cos\alpha$ and $\cos\beta$ to first order, since terms of higher order correspond to phase coherence around two and more orbits. However, the coefficient s renders these terms negligible.

To find σ_2, we first remember that all the orbits are closed so that at high fields the coefficients take the form

$$(\sigma_{xx})_2=A/H^2, \quad (\sigma_{xy})_2=C/H.$$

$(\sigma_{xx})_2$ falls off as H^{-2}, one order faster than $(\sigma_{xx})_1$. At these fields one will therefore expect most of the x component of current to come from $(\sigma_{xx})_1$. In fact, since we are not concerned with reproducing the absolute magnitude of the helicon signal, the only importance of $(\sigma_{xx})_2$ is that it prevents the total σ_{xx} from going to zero when $\Phi=2n\pi$. From the relative magnitude between the helicon signal at $\Phi=2n\pi$ and $(2n+1)\pi$, $(\sigma_{xx})_2$ is estimated at being no more than 20% of the maximum value of σ_{xx}.

$(\sigma_{xy})_2$, on the other hand, only falls off as H^{-1}, so that the static contribution to the Hall conductivity will always be much larger than the oscillatory contribution. Hays and McLean[5] estimated that only 2% of the total σ_{xy} was oscillatory. We therefore assume that the observed oscillations derive entirely from σ_{xx}. The coefficient C has the form $1/n''e$, where n'' is some effective number of carriers. It remains only to estimate the relative sizes of n' and n''.

When $u\gg1$ and $\omega_1\gg\omega$, Eq. (2) may be simplified with little error to

$$I(\mu_t)=\frac{4}{\pi^2}\frac{1}{u}\left(\frac{\omega}{\omega_1}\right)^2,$$

with

$$u=\frac{\rho_{xy}}{\rho_{xx}}=\frac{\sigma_{xy}}{\sigma_{xx}}.$$

From the shape of the slow oscillations and from the shape of resonances at frequencies greater than 2500 we estimate $u\sim10$ and choose n''/n' accordingly, though it should be noted that the form of the slow oscillation is quite insensitive to u, when $u>3$. The values of σ_{xx} and σ_{xy} are now fed into (8) and the result plotted by a computer, which also scales the amplitude of the slow oscillation to match Fig. 2. Figure 3 shows the best fit over the same field range as Fig. 2, and this has $H_0=8$ kG and $s=0.08$.

It may be seen that the main features of the fast oscillations are well reproduced, namely, the variation of the amplitude with Φ and the fact that the beating is only pronounced around $\Phi=(2n+1)\pi$. This last may be seen qualitatively. The beating, which is, of course, caused by the phase difference between α and β, has an amplitude-dependent on $\lambda-\mu$, which reaches a maximum value when Φ is an odd multiple of π.

Further confirmation is provided by results for which **H** is tipped off [001] towards [100], when it is found that the beat amplitude increases. This is to be expected since H_0 increases, hence $\lambda-\mu$ increases at $\Phi=(2n+1)\pi$. Unfortunately, the appearance of open orbits makes it extremely difficult to estimate the helicon signal. However, it is possible to obtain a figure for H_0 from the beat amplitude, and a plot of this against distance from the central plane is shown in Fig. 4.

When **H** was more than 7° off [001], the helicon signal becomes damped to the point of extinction.

Finally, it should be noted that when the temperature was increased above 1.4°K, there was little change in the small oscillations up to about 1.7°K, but thereafter they were rapidly attenuated and disappeared above 2.3°K.

4. CONCLUSIONS

For the first time we have been able to observe and account for the effects of coherence in a two-dimensional network of coupled orbits. In addition, by using the oscillations resulting from this coherence, we have determined approximate values for the breakdown field away from the high-symmetry direction. The zone oscillations predicted by Pippard[10] were not observed.

ACKNOWLEDGMENT

I would like to thank Dr. W. L. McLean for many valuable discussions.

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PHYSICAL REVIEW VOLUME 180, NUMBER 3 15 APRIL 1969

Effect of Force-Constant Changes on the Coherent Neutron Scattering from Face-Centered Cubic Crystals with Point Defects*

KATJA LAKATOS† AND J. A. KRUMHANSL
Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14850

(Received 7 October 1968)

This is a study of the effects of a low concentration of random substitutional impurities on the vibrational properties of fcc monatomic harmonic crystals. Central-force-constant changes between the impurities and their nearest neighbors, as well as mass changes, are taken into account. The problem is formulated in terms of a coherent neutron-scattering experiment, although the results are useful for other experiments as well. Analytic expressions are given for the phonon self-energy function for **K** along some high-symmetry directions in the lattice, with results expressed in terms of perfect-lattice Green's functions. Numerical computations are performed on Al containing heavy impurities. The Al-impurity force constants are treated as parameters which are varied over a wide range.

I. INTRODUCTION

IN recent years much theoretical and experimental work has been done on the effects of impurities on lattice vibrations.[1–16] The theoretical work usually involves the evaluation of various lattice Green's functions or correlation functions which can be directly related to measurable quantities.

Most of the early theory was restricted to models in which only the mass of the impurities is assumed to be different from the host atoms (mass defect model).[2–4] Recently some results have been obtained on the effects of force-constant changes between the impurities and their nearest neighbors.[5–13]

In I, the motion of the impurities and their nearest neighbors as a function of frequency at low impurity concentrations in various cubic harmonic monatomic lattices were studied in detail. Mass change, as well as force-constant changes, was taken into account. The problem was formulated in terms of an incoherent neutron-scattering experiment, and the numerical results were applied to Al containing heavy impurities.

The present work is an extension of I, to give the effects of the impurities on the phonons of the lattice. The problem is formulated in terms of a coherent neu-

† Present address: Department of Physics and Astronomy, University of Rochester, Rochester, N. Y.

* Support for this work was provided by the U. S. Atomic Energy Commission through Contract No. AT(30-1)-3699, Technical Report No. NYO-3699-33, and by the Advanced Research Projects Agency through the use of the Central Facilities of the Materials Science Center, Cornell University, MSC Report No. 1039. This paper is based on a thesis submitted by K. Lakatos in partial fulfillment of the requirements for the Ph.D. degree in Physics at Cornell University.

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