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$$\gamma = d(AF)/dA = F + AdF/dA,$$

where A corresponds to the surface area. For a one-component liquid $\gamma = F$. On the other hand, in the solid one has to carefully specify the conditions of measurement and usually $dF/dA \neq 0$ that that $\gamma \neq F$ (see Refs. 17 and 18). The energy required to form a surface is obviously $\int F dA$.

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Transverse Plasma-Wave Echoes

Burton D. Fried
Department of Physics, University of California, Los Angeles, California 90024,
and TRW *Systems, Redondo Beach, California*
and
Craig Olson
Department of Physics, University of California, Los Angeles, California 90024

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The concept of plasma-wave echoes, introduced by Gould, Malmberg, O'Neil, and Wharton for the case of longitudinal, electrostatic waves, is here extended to the case where one of the excitations is a transverse electromagnetic wave propagating parallel to an external magnetic field, and the echo is of the same character. (If both excitations are transverse and $k_\perp = 0$, there is no transverse echo in lowest order.) A transverse electromagnetic signal of frequency ω_1 is excited at $z = 0$, and a longitudinal electrostatic signal of frequency ω_2 is excited at $z = L$. A transverse echo with frequency $\omega_1 \pm \omega_2$ then appears at $z_0 = \omega_2 L[\omega_2 \pm (\omega_1 + \Omega_C)]^{-1}$, where Ω_C is the signed cyclotron frequency of the species involved, and ω_1 is positive for right-hand polarization of the transverse excitation, negative for left-hand polarization. The expression for the echo location is correct for both $z_0 > L$ and $z_0 < 0$. The echo in the latter case arises from particles which have $v_z < 0$, and hence see the longitudinal excitation first, followed by the transverse excitation. In contrast to the purely longitudinal case, the echo frequency may either be the sum or difference of the excitation frequencies. Echoes due to a given species and given circular polarization for the transverse excitation may have either right- or left-handed polarization, depending on the values of ω_1 and ω_2; echoes due to a given species (e.g., electrons and ions) occur at different locations and, in some cases, with different frequencies; and echoes arise for all values of ω_1 and ω_2, although the analysis is simple only if the waves associated with the excitations decay (due to Landau or cyclotron damping) in a distance less than L. A perturbation, Vlasov analysis, which takes into account all collective effects gives an expression for the echo shape in terms of a one-dimensional integral over v_z.

The plasma-wave echoes predicted theoretically by Gould *et al.*[1] and observed experimentally[2] involve only longitudinal waves (Langmuir oscillations or ion acoustic waves), but the mechanism can easily be generalized to include other waves as well. This extension appears worthwhile, not only because of the intrinsic interest of the echo phenomenon, which allows direct, quantitative measurement of mode-coupling coefficients, but because of its potential importance as a diagnostic tool for studying details of the particle distribution functions and also small-scale fluctuations.[3] We present here the results of an analysis of a class of transverse-wave echo configurations,[4] many of which should be accessible to experimental observation. As in the longitudinal wave case, we assume two localized, steady-state excitations in an infinite plasma with frequency ω_1 supplied at $z=0$ and ω_2 at $z=L$. An external magnetic field along z is imposed, and we assume plane symmetry, i.e., wave propagation only along z. Under these circumstances, simple symmetry arguments[5] show that if both signals are transverse, no transverse echo appears in lowest order (although there is a longitudinal echo[6]). If, instead, one signal is transverse and the other is longitudinal, we find a second-order transverse echo (but no longitudinal echo). We shall assume that the ω_1 signal is transverse and circularly polarized (produced, for example, by a suitable antenna or by a wave guide terminating at one end of a plasma column), and that ω_2 is a longitudinal signal (excited by a dipole grid structure or even a probe). By allowing echoes at $z_0<0$ (produced by particles having $v_z<0$) as well as at $z_0>L$ (due to particles with $v_z>0$), we include in our results the case where the transverse and longitudinal excitations are interchanged.

We find this mixed transverse-longitudinal case to be much richer in the variety of phenomena involved than the purely longitudinal case:

(1) The echo frequency may be either the sum or the difference of the excitation frequencies, depending on the specific values of ω_1 and ω_2. (In the longitudinal case, the echo occurs only at the difference frequency.)

(2) Echoes due to different species (e.g., electrons and ions) occur at different locations and, in some cases, with different frequencies. (In the longitudinal case, neither the echo location nor the echo frequency depends on the mass of particles involved.)

(3) The condition that the echo location z_0 satisfy $z_0>L$ leads to requirements of the form $\omega_2 > |\omega_1-\omega_c|$, where ω_c is a cyclotron frequency. We denote this case as tl, since the particles involved in the echo experience the transverse signal first and then the longitudinal one. If instead, $\omega_2<|\omega_1-\omega_c|$, the echo occurs at $z_0<0$, a case we denote as lt, since the echo is then due to particles (with $v_z<0$) which see the longitudinal signal first and then the transverse one. (In the longitudinal case, $z_0>L$ requires $\omega_2>\omega_1$; if $\omega_2<\omega_1$, then $z_0<0$ but this is *not* a physically different configuration if the unperturbed velocity distribution function is symmetric in v_z.)

(4) The direction of propagation of the waves comprising the echo may be towards the excitation region or away from it. (In the longitudinal case, it is always away from the excitation.)

(5) Echoes due to a given species and a given circular polarization of the transverse excitation (left handed or right handed) may have either left-handed (LH) or right-handed (RH) polarization, depending on the values of ω_1 and ω_2.[7]

For a given value of ω_1 and ω_2 and a given polarization (LH or RH) of the excitation at $z=0$, there will be two echoes in a simple plasma, one due to electrons and the other due to ions. The echo characteristics for given ω_1, ω_2 are most easily determined from Fig. 1 which indicates the classification of the echo as $tl(z_0>L)$ or $lt(z_0<0)$, its circular polarization (RH or LH), the absolute value of the echo frequency ω_3, and the direction of propagation of the echo waves (→ meaning a positive phase velocity, ← meaning a negative one). The diagram for electron echoes

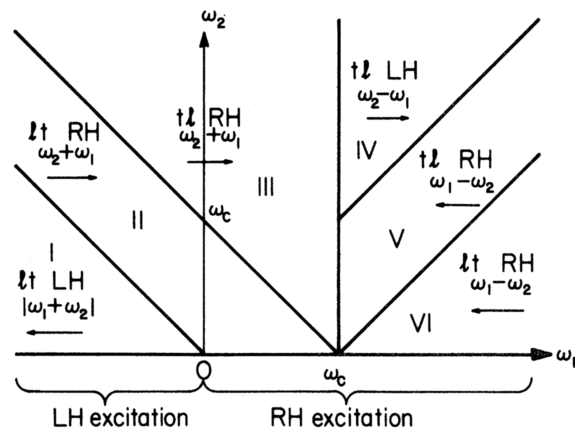

FIG. 1. Classification of electron echoes for various values of ω_1 (transverse excitation at $z=0$) and ω_2 (longitudinal excitation at $z=L$). The tl denotes an echo at $z_0>L$ (due to particles excited first by a transverse signal and then by a longitudinal one) while lt means an echo with $z_0<0$ (opposite order of excitation). The circular polarization of the echo (RH or LH) and the direction of propagation, → (along $+z$ axis) or ← (along $-z$ axis) are indicated, together with the magnitude of the echo frequency ω_3 ($|\omega_1+\omega_2|$ or $|\omega_1-\omega_2|$). (In connection with the latter, note that in the portions of regions II and III lying to the left of the ω_2 axis, ω_1 is to be considered negative so that $|\omega_3|=|\omega_2-|\omega_1||$.) If the excitation at $z=0$ has RH circular polarization, the right half of the diagram applies; for LH excitation use the left half. This same diagram can be used also for ion echoes by simply interchanging the designations RH and LH wherever they occur (including the excitation labels on the bottom of the diagram, so that the right half, $\omega_1>0$, corresponds to LH excitation); of course, ω_c must then be interpreted as ω_{ci} rather than ω_{ce}.

goes over into the one for ion echoes if the designations RH and LH are interchanged everywhere and ω_c is understood to mean ω_{ci} rather than ω_{ce}.

The formal analysis via the second-order Vlasov equation is similar in character to that given by Gould and O'Neil,[1] so we shall only state below the results [Eqs. (2) through (10)]. However, we give first a simple picture of the physical mechanism (neglecting collective effects), adapted from the one used by Gould[8] for the longitudinal wave case. Consider electron echoes in the tl case, with RH excitation at $z=0$. In a frame rotating with the electrons at frequency ω_c, the first excitation appears to have frequency $\omega_1 - \omega_c$. For electrons with given v_z, this induces a disturbance in $\vec{v}$ with wave number $k_1 = (\omega_1 - \omega_c)/v_z$ and frequency (in the lab frame) ω_1. When the density of these electrons is modulated by the longitudinal signal of the second grid at ω_2, a current density results with frequency $(\omega_1 - \omega_c \pm \omega_2)$ in the rotating frame and hence a wave number $k_3 = (\omega_3 - \omega_c)/v_z$, where $\omega_3 = \omega_1 \pm \omega_2$ is the frequency in the lab frame. The net phase ϕ of the modulation at $z > L$ includes the part $k_1 L$ arising from $0 \le z \le L$ as well as the part $k_3(z-L)$ associated with $z > L$:

$$\phi = k_1 L + k_3(z-L) = [(\omega_1 - \omega_3)L + (\omega_3 - \omega_c)z]/v_z \ .$$

Integrating over v_z to find the total current density, we will have phase mixing except where the coefficient of $1/v_z$ vanishes, i.e., near the point

$$z_0 = (\omega_3 - \omega_1)L/(\omega_3 - \omega_c) = \omega_2 L/[\omega_2 \pm (\omega_1 - \omega_c)] \ , \tag{1}$$

which defines the echo location, the echo frequency being $\omega_3 = \omega_1 \pm \omega_2$. The width of the echo, according to this simple model, will be of the order of the wavelengths involved, i.e., $a/(\omega_3 - \omega_c)$, where a is some characteristic thermal velocity. (This is easily seen if the integration over v_z is evaluated using a saddle-point approximation.)

The inclusion of collective effects has two consequences: (1) Particles whose velocity matches the phase velocity of a plasma wave will make an enhanced contribution to the echo amplitude. This part will have a width characterized by the damping of the collective wave involved, and may result in an echo width much greater than that obtained from the single-particle model if the wave is lightly damped. (2) As in the purely longitudinal case, the theory is tractable and the results clear cut only if one can assume that the electric fields of any collective modes excited at $z=0$ are damped by collisionless mechanisms (Landau, cyclotron, etc.) in a distance of order L, and similarly, that those excited at $z=L$ are damped in a distance of order $z_0 - L$; this places restrictions on the permissible values of ω_1 and ω_2.[9] While many interesting experiments can be designed using Fig. 1 and taking into account these restrictions, one of particular interest occurs with electron echoes in region IV. Although the excitation has RH polarization and the electrons gyrate in the same sense, the echo has LH polarization[10] and hence, can resonate with ions. If $\omega_2 - \omega_1 < \omega_{ci}$, the echo will be a nearly undamped Alfven wave (or ion cyclotron wave).

To summarize the results of the Vlasov equation analysis, we assume the unperturbed velocity distribution function to be isotropic, and take the external electric field of the first excitation to be

$$E_x = E_t(z)\cos\omega_1 t, \quad E_y = E_t(z)\sin\omega_1 t \ ,$$

while that due to the second grid is $E_z = E_l(z-L) \times \cos\omega_2 t$. (With this convention, $\omega_1 > 0$ corresponds to RH polarization, $\omega_1 < 0$, to LH polarization.) With

$$j = (j_x - i j_y)/\sqrt{2} \ ,$$

we then have for the echo current in the tl case ($z > L$)

$$j(z,t) = \sum_{s=\pm 1} \int_0^\infty dv\, f(v) v^{-3} k_3 \times \exp\{i[k_3(z-z_0) - \omega_3 t]\} \times E_t(k_1,\omega_1) E_l(k_2, s\omega_2) \times A_1 A_2(v,z)/\epsilon_t(k_3,\omega_3) \ , \tag{2}$$

where z_0 is the echo location

$$z_0 = \omega_2 L[\omega_2 + s(\omega_1 + \Omega_c)]^{-1} \ ; \tag{3}$$

E_t and E_l are the self-consistent fields of the excitation signals,

$$E_t(k,\omega) = E_t(k)/\epsilon_t(k,\omega) \ , \quad E_l(k,\omega) = E_l(k)/\epsilon_l(k,\omega) \ ; \tag{4}$$

ϵ_t and ϵ_l are the transverse and longitudinal dielectric functions (ratio of total $\vec{E}$ to external $\vec{E}$), e.g.,

$$\epsilon_t(k,\omega) = 1 - \omega(k^2c^2 - \omega^2)^{-1} \times ⨍ dv\,\omega_p{}^2 f(v)/(kv - \omega - \Omega_c) ; \tag{5}$$

Ω_c is the signed cyclotron frequency; $f(v)$ is the distribution function for v_z (i.e., integrated over $\vec{v}_\perp$);

$$k_1 = (\omega_1 + \Omega_c)/v_z\,, \quad k_2 = s\omega_2/v_z\,, \tag{6}$$

and $k_3 = k_1 + k_2 = (\omega_3 + \Omega_c)/v_z$

are the wave numbers of the excitations and the echo;

$$\omega_3 = \omega_1 + s\omega_2 \tag{7}$$

is the echo frequency;

$$A_1 = nq(q/m)^2/2\sqrt{2}\ , \tag{8}$$

and $A_2 = \{(d/dk)\ln[k\,e^{ik(z-L)}$

$$\times E_l(k-k_1, s\omega_2)/\epsilon_t(k,\omega_3)]\}_{k=k_3}\,. \tag{9}$$

The symbol $⨍$ denotes a sum over species, as well as integration over v; and, as indicated by the summation sign, we must also sum over the two possible values of ω_3 (although, as indicated in Fig. 1, only one gives a valid echo for any particular choice of ω_1, ω_2).

For the lt case, (9) is replaced by

$$f(v)A_2 = \frac{v\Omega_c}{\omega_1 k_3}\frac{\partial f}{\partial v} + \frac{k_1}{\omega_1}\frac{\partial}{\partial v}\langle v_\perp{}^2\rangle f\,.$$

$$\times\{(d/dk)\ln[k(k-s\omega_2/v_z)e^{ikz}$$

$$\times E_t(k-s\omega_2/v_z, \omega_1)/\epsilon_t(k,\omega_3)]\}_{k=k_3}, \tag{10}$$

the range of integration for v in (2), being now from $(-\infty)$ to 0.

The results given above are all consequences of Eqs. (2) through (10). In addition, information concerning echo shapes and amplitudes can be obtained by evaluation of (2) for any given choice of parameters. The result of this evaluation for particular cases and the extension of the analysis to anisotropic f_0 will be reported elsewhere.

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[3]T. Jensen and J. Malmberg, private communication.

[4]It should be noted that we are concerned here with *plasma-wave* echoes, which are due to particle streaming effects and do not depend in an essential way on resonance phenomena (although such effects can enhance the echo amplitude). They are to be distinguished from *cyclotron* echoes, which, like the original spin echoes, arise from interference effects among a set of highly resonant oscillators having a small spread in resonant frequencies. Particle streaming effects are generally not an essential part of the mechanism for cyclotron echoes, although under appropriate circumstances the Doppler shifts associated with particle streaming can provide the required spread in resonance frequencies, as shown by A. Y. Wong, Phys. Fluids (to be published).

[5]J. M. Cornwall, private communication.

[6]T. Kamimura and A. Hasegawa, private communication.

[7]In designating polarizations, we refer to the direction of the external magnetic field (here, the z axis) rather than the direction of the wave vector, as is customary in optics. Right-hand polarization means that the electric field rotates in the same direction as electrons gyrate in the magnetic field.

[8]R. W. Gould, private communication.

[9]However, as pointed out by R. W. Gould (private communication), there is no necessity for any of the frequencies ω_1, ω_2, ω_3 to lie in a region of propagation, since the basic mechanism, as shown above, does not depend upon collective effects.

[10]Although surprising at first sight, this is readily understood from the analogy (due to K. R. MacKenzie, private communication) of a spoke rotating at frequency ω_1 and viewed via stroboscobic light of frequency $\omega_2 \simeq \omega_1$. The apparent direction of rotation will agree with the true one if $\omega_2 < \omega_1$, but will have the opposite sense if $\omega_2 > \omega_1$.