Attenuation of Transverse Zero Sound in He^3 †

L. R. Corruccini,* J. S. Clarke,‡ N. D. Mermin,§ and J. W. Wilkins§
Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14850
(Received 26 December 1968)

It is pointed out that although a transverse zero sound mode very probably exists in liquid He^3, its attenuation, because of the conservation laws, will be about a hundred times stronger than longitudinal zero sound, and hence its experimental detection correspondingly more difficult.

The observation[1] of longitudinal ($m=0$) zero sound in liquid helium-3 has stimulated interest in the experimental detection of similar modes. In particular it is quite likely that transverse ($m=1$) zero sound exists in He^3, since[2] the Fermi liquid parameter F_1 is comparable to 6 at low pressure and substantially greater than 6 at high pressures. The purpose of this note is to point out that longitudinal zero sound is subject to atypically small attenuation compared to the higher modes, so that observation of transverse zero sound will be much more difficult. We find in a single relaxation-time approximation, keeping only F_0 and F_1, that the attenuation γ for the two modes is

$$P=0.28 \text{ atm:} \quad \gamma_{m=0}=\gamma_0=0.009/v_F\tau \ ,$$

$$\gamma_{m=1}=\gamma_1=0.9/v_F\tau \ .$$

$$P=27 \text{ atm:} \quad \gamma_0=0.0004/v_F\tau \ ,$$

$$\gamma_1=0.5/v_F\tau \ .$$

Here v_F is the Fermi velocity, and τ is the relaxation time. This striking disparity in attenuation is not a peculiarity of these approximations, but, as we shall show, is a consequence of energy and momentum conservation in quasiparticle collisions.

The kinetic equation describing a spin-independent oscillatory disturbance ν of the Fermi surface is[3]

$$(\eta-\cos\Theta)\nu(\Theta,\phi)-\cos\Theta\int\frac{d\Omega'}{4\pi}F(\chi)\nu(\Theta',\phi') = -I(\nu)/ikv_F \ . \quad (1)$$

Here η is ω/kv_F, the ratio of the wave velocity to the Fermi velocity, χ is the angle between (Θ,ϕ) and (Θ',ϕ'), and $I(\nu)$ is the collision integral.

In the zero sound regime ($\omega\tau\gg1$, where τ is a typical relaxation time), we may treat the right-hand side of (1) as a small perturbation on the solution ν. To derive an expression for the attenuation $\gamma=\mathrm{Im}k$ of zero sound, expand ν and η as

$$\nu=\nu_0+\nu_1+\cdots, \quad \eta=\eta_0+\eta_1+\cdots \ ,$$

where $\nu_1\ll\nu_0$, $\eta_1\ll\eta_0$. Equation (1) gives the zero- and first-order equations

$$(\eta_0-\cos\Theta)\nu_0-\cos\Theta\int\frac{d\Omega'}{4\pi}F\nu_0=0 \ , \quad (2)$$

$$\eta_1\nu_0+(\eta_0-\cos\Theta)\nu_1-\cos\Theta\int\frac{d\Omega'}{4\pi}F\nu_1 = -I(\nu_0)/ikv_F \ . \quad (3)$$

Now multiply (3) by $\nu_0/\cos\Theta$ and integrate over Ω; by virtue of (2)

$$\eta_1=-\frac{1}{ikv_F}\left\langle\frac{\nu_0 I(\nu_0)}{\cos\Theta}\right\rangle\Big/\left\langle\frac{\nu_0^2}{\cos\Theta}\right\rangle \ ,$$

where $\langle\cdots\rangle$ denotes an angular average. Since $\omega/kv_F\simeq\eta_0+\eta_1$, $\mathrm{Im}k=\gamma\simeq-(\eta_1/\eta_0)\,\mathrm{Re}k$, or

$$\gamma=-\frac{1}{v_F\eta_0}\left\langle\frac{\nu_0 I(\nu_0)}{\cos\Theta}\right\rangle\Big/\left\langle\frac{\nu_0^2}{\cos\Theta}\right\rangle \ . \quad (4)$$

Next we expand ν_0 as a series in spherical harmonics:

$$\nu_0=\sum_{l,m}a_{lm}Y_{lm}(\Theta,\phi) \ .$$

When this is substituted into (2), modes corresponding to different m are decoupled. We may thus restrict ourselves to a single mode

$$\nu_0^{(m)}=\sum_{l=m}^{\infty}a_{lm}Y_{lm}(\Theta,\phi) \ . \quad (5)$$

If we expand $F(\chi)$ as a series in Legendre polynomials,

$$F(\chi)=\sum_l F_l P_l(\cos\chi) \ ,$$

and expand $I(\nu)$ as

$$I(\nu)=-\sum\tau_l^{-1}a_{lm}Y_{lm}(\Theta,\phi) \quad (6)$$

then from (2),

$$\frac{v_0}{\cos\Theta} = \sum_{l=m}^{\infty} \frac{a_{lm}}{\eta_0}\left(1+\frac{F_l}{(2l+1)}\right) Y_{lm}(\Theta,\phi) \quad . \quad (7)$$

Using (5), (6), and (7), we find the attenuation (4) to be

$$\gamma = \frac{1}{\eta_0 v_F} \frac{\sum_{l=m}^{\infty} (a_{lm}{}^2/\tau_l)[1+F_l/(2l+1)]}{\sum_{l=m}^{\infty} a_{lm}{}^2[1+F_l/(2l+1)]} \quad . \quad (8)$$

Since the quasiparticle collisions conserve energy and momentum, the $l=0$ and 1 terms in the numerator of (8) are missing. We can take this into account by the convention that τ_0 and τ_1 are infinite. If we make the reasonable assumption that the τ_l for $l \geq 2$ all have the same order of magnitude, $\tau_l \simeq \tau$, $l \geq 2$ then we can conclude:

(i) For hypothetical modes with $m \geq 2$, $\gamma_m \simeq 1/\eta_0 v_F \tau$.

(ii) For the $m=0$ mode it is possible for the attenuation to be substantially less than $1/\eta_0 v_F \tau$, provided that either a_{00} or a_{10} is substantially larger than the a_{l0} for $l \geq 2$.

(iii) For the $m=1$ mode the attenuation can be substantially less than $1/\eta_0 v_F \tau$ only if a_{11} is substantially greater than the a_{l1} for $l \geq 2$.

The results for γ_0 and γ_1 quoted in the first paragraph are based on a single relaxation-time approximation ($\tau_l = \tau$, $l \geq 2$) in a model keeping only F_0 and F_1, using the experimental values[4] $F_0 = 10.77$, $F_1 = 6.25$ (0.28 atm) which imply $\eta(m=0) = 3.597$, $\eta(m=1) = 1.003$; and $F_0 = 75.63$, $F_1 = 14.35$ (27 atm) which imply $\eta(m=0) = 12.218$, $\eta(m=1) = 1.202$.[5]

Evidently the possibility (ii) is quite strikingly realized for longitudinal ($m=0$) zero sound. An examination of the solution reveals that it is the dominance of a_{10} that is responsible for the very small attenuation.

Equally clearly, possibility (iii) has *not* produced a small attenuation for the transverse mode. It is most unlikely that the large size of γ_1/γ_0 is an accident of the simple model for which we have calculated them. Even when all the F_l are kept, if we multiply (2) by $Y_{11}{}^*$ and integrate over angles we find that

$$a_{21} = [\eta_0 5^{1/2}/(1+\tfrac{1}{5}F_2)]\, a_{11} \quad . \quad (9)$$

Since $\eta_0 > 1$ is required for propagation, F_2 must be enormous to make this ratio small. Using (9) in conjunction with (8) we can set a lower bound for γ_1 in terms of the largest ($\tau_{\max}$) of the relaxation times τ_l, $l \geq 2$:

$$\gamma > \frac{1}{\eta_0 v_F \tau_{\max}}\left(1 - \frac{a_{11}{}^2(1+\frac{1}{3}F_1)}{\sum_{l=1}^{\infty} a_{l1}{}^2[1+F_l/(2l+1)]}\right)$$

$$> \frac{1}{\eta_0 v_F \tau_{\max}}\left(1 - \frac{a_{11}{}^2(1+\frac{1}{3}F_1)}{a_{11}{}^2(1+\frac{1}{3}F_1) + \frac{5\eta_0{}^2 a_{11}{}^2}{1+\frac{1}{5}F_2}}\right),$$

or

$$\gamma > \frac{1}{v_F \tau_{\max}}\left(\frac{5\eta_0}{5\eta_0{}^2 + (1+\frac{1}{3}F_1)(1+\frac{1}{5}F_2)}\right).$$

It is unlikely [6] that F_2 exceeds 5. Figure 1 indicates that η_0 will not be very much larger than unity for any plausible values of F_2. Thus under the most favorable of circumstances one might do as well as $\gamma \approx 0.3/v_F\tau$.

We conclude that since $\tau \propto 1/T^2$, one will have to go to temperatures about a factor of 10 lower than those used in observing longitudinal zero sound to achieve a comparably long attenuation length for the transverse mode.

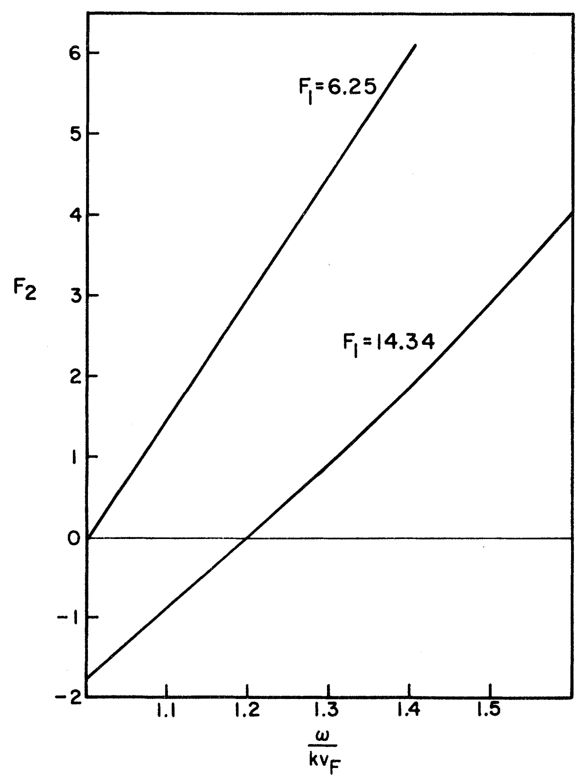

FIG. 1. The dependence of $\eta_0 = \omega/kv_F$ on F_2 is shown for the transverse ($m=1$) mode in a model that keeps only F_1 and F_2 for the experimental values of F_1 at 0.28 and 27 atm.

†Work supported in part by the National Science Foundation through contracts GP-8148 and GP-6594, and by the Office of Naval Research through contract NONR-401(38), Technical Report No. 30.

*NSF Predoctoral Fellow.

‡New York State Regents Predoctoral Fellow.

§Alfred P. Sloan Foundation Fellow.

[1]B. E. Keen, P. W. Matthews, and J. Wilks, Proc. Roy. Soc. (London) A284, 125 (1965); W. R. Abel, A. C. Anderson, and J. C. Wheatley, Phys. Rev. Letters 17, 74 (1966).

[2]L. D. Landau, Zh. Eksperim. i Teor. Fiz. 32, 59 (1957) [English transl.: Soviet Phys. – JETP 5, 101 (1957)].

[3]A. A. Abrikosov and I. M. Khalatnikov, Zh. Eksperim. i Teor. Fiz. 33, 110 (1957) [English transl.: Soviet Phys. – JETP 6, 84 (1958)]. This equation assumes ν depends only on (Θ, ϕ) at the Fermi surface; it is most unlikely that the neglected dependence of ν on $(\epsilon - \mu)/kT$ through $I(\nu)$ could alter our result $\gamma_1/\gamma_0 \simeq 10^2$ by even a single order of magnitude.

[4]J. C. Wheatley, in Quantum Fluids, edited by D. F. Brewer (North Holland Publishing Co., Amsterdam, 1966), p. 206.

[5]It is essential, in doing this calculation, to use a numerical value for η which preserves its precise analytical dependence on F_0 and F_1 to a high degree of accuracy, since a delicate cancellation occurs in yielding a small value of γ_0. Alternatively, if one wishes to demonstrate that ν_0 is dominated by a_{10} one must again use an η that contains the precise dependence on the F_l's to a very high order. If this is not done a spurious divergence in the a_l will be found for higher l.

[6]See, for example, G. A. Brooker, Proc. Phys. Soc. (London) 90, 411 (1967).

PHYSICAL REVIEW VOLUME 180, NUMBER 1 5 APRIL 1969

Stimulated Thermal Scattering in the Second-Sound Regime

R. H. Enns and I. P. Batra
Department of Physics, Simon Fraser University, Burnaby, Canada
(Received 28 October 1968)

The theory of stimulated thermal scattering in liquids has been extended to the second-sound problem in crystals. In contrast to the diffusion limit, in the second-sound case the stimulated thermal Brillouin scattering has a much larger gain per unit length than the stimulated thermal Rayleigh peak. Further, we also obtain asymmetry in the gain factors, which actually exists in the diffusion limit as well but has not been reported.

I. INTRODUCTION

Although the detection of second sound in crystals by light scattering has been much discussed in the recent literature,[1,2,3] second sound has not been observed by this method. One of the fundamental reasons is that the ratio of the intensity of the second-sound doublet to that of the Brillouin doublet is equal to $\gamma - 1 \approx 10^{-3}$ for crystals at the very low temperature needed for second-sound propagation. Here γ is the usual ratio of specific heats. It has been suggested by Griffin[3] that one might extend the theory of Herman and Gray[4] on stimulated thermal scattering to the second-sound problem and thus arrive at a situation which is experimentally more favorable. In this paper we present such an analysis. The procedure adopted here resembles closely that of Herman and Gray.[4]

II. CALCULATION

In stimulated scattering, we are interested in calculating the power gain per unit length, G, for the various physical processes that may take place. If we adopt the convention that a positive frequency has a time dependence $e^{+i\omega_s t}$, then following Bloembergen,[5] it can be shown that G is related to the imaginary part of the nonlinear susceptibility $\chi_{NL}(\omega_s)$ through the relation $G = 4\pi k_s \mathrm{Im}\chi_{NL}(\omega_s)$, where the subscript s refers to the scattered wave and k is the wave number. Then $G > 0$ corresponds to amplification of the scattered wave, $G < 0$ to attenuation. To calculate the nonlinear susceptibility χ_{NL}, we start with a set of