Harmonic-Oscillator Model for Baryons

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(Received 19 December 1968)

A detailed analysis is presented of all positive-parity resonances predicted by the harmonic-oscillator model to lie in the 1500–2000-MeV mass region, particular attention being focused on their likelihood of experimental discovery. Some further comments on the model are also added.

IN our paper[1] on the harmonic-oscillator model for baryons, expressions were given for the $N\pi$ decay rates for all of the resonances occurring in the first excited odd-parity band ($n=1$), but only for some of those in the first excited even-parity band ($n=2$). The latter corresponded to P_{11}, P_{33}, F_{15}, F_{17}, and F_{37} resonances. However, other positive-parity resonances are known[2] to exist in the same energy region. We therefore present, in Table I, the $N\pi$ decay rates for all of the states in the $n=2$ band.

If it is assumed that there is only a small amount of mixing between the configurations, one can immediately deduce several likely features about the expected baryon spectrum in this region. In the first place, it is easy to see why quite a few of the predicted states may have gone without detection, should they correspond to resonances with masses less than 2 GeV, the region covered by the phase-shift analyses to date. The $N\pi$ widths of the $70,2^+$ states P_{33}, F_{35}, $P_{13}(^4D)$, $F_{15}(^4D)$ and of the $20,1^+$ states P_{11}, P_{13} would all be less than 10 MeV. These are therefore presumably broad inelastic resonances, and it is highly unlikely that they would have been detected in the usual type of analysis which is based on elastic pion-nucleon scattering. (They might be observable if their masses are indeed greater than 2 GeV.) The $70,2^+$ states P_{11}, F_{17} and the $70,0^+$ states P_{13}, P_{31} are borderline cases with $N\pi$ widths 10–20 MeV if their masses lie in the present phase-shift range (the F_{17} has possibly been seen[2] at 1983 MeV). However, we would expect all the other 11 states to be observed. It is encouraging that there are already indications of at least eight of them, namely, $P_{11}(1470)$, $P_{11}(1750)$, $P_{13}(1860)$, $P_{31}(1930)$, $P_{33}(1690)$, $F_{15}(1690)$, $F_{35}(1910)$, and $F_{37}(1950)$, leaving only three states (a P_{13}, P_{33}, and F_{15}) outstanding.[3] As regards all of the resonances, one cannot of course conclude as yet that each definitely belongs to only one configuration; it is more likely (and our explicit $N\pi$ width calculations indicate this) that they should be treated as mixtures, thus modifying the above remarks to some extent.

The mass region 1400–1500 MeV is of particular interest to many physicists at the present time. There seems to be no unanimous agreement about what is really happening here. On the one hand, the CERN phase-shift group[2] have a single P_{11} resonance in this region with a mass 1470 MeV, total width 210 MeV, and $N\pi$ width 136 MeV. On the other, experimentalists[4] frequently observe a mass enhancement in this region in πp, Kp, and pp inelastic scattering. This enhancement is found to occur, however, at positions ranging from 1385 to 1505 MeV, with total and $N\pi$ widths from 40 to 200 MeV.

In the harmonic oscillator model, we see from Table I that the state which has the most significant $N\pi$ contribution in this mass range is indeed a $P_{11}(56,0^+)$. Numerically, however, it amounts to only 37 MeV (the B factor is small since the final-state momentum k is

TABLE I. The $N\pi$ decay rates for the unmixed configurations in the first excited band ($n=2$) of positive-parity states. Here $B=(1/2835)(f_q^2/4\pi\mu^2)(k^7/\alpha^4)[\exp(-k^2/3\alpha^2)](E_N/M^*)$.

Quark configuration ($n=2$)	N^* state	$N\pi$ width
$56,0^+$	P_{11}	$437\frac{1}{2}B$
	P_{33}	$140B$
$70,0^+$	P_{11}	$140B$
	P_{13}	$17\frac{1}{2}B$
	P_{31}	$17\frac{1}{2}B$
$20,1^+$	P_{11}	0
	P_{13}	0
$56,2^+$	P_{13}	$175B$
	F_{15}	$175B$
	P_{31}	$112B$
	P_{33}	$56B$
	F_{35}	$112B$
	F_{37}	$72B$
$70,2^+$	$(^2D)P_{13}$	$56B$
	$(^2D)F_{15}$	$56B$
	P_{33}	$7B$
	F_{35}	$7B$
	P_{11}	$14B$
	$(^4D)P_{13}$	$7B$
	$(^4D)F_{15}$	$2B$
	F_{17}	$9B$

* Supported by a NATO Studentship administered by the Science Research Council, London, United Kingdom.
† Supported by the National Science Foundation under Grant No. NSF GP-9273.

[1] D. Faiman and A. W. Hendry, Phys. Rev. **173**, 1720 (1968).

[2] C. Lovelace, in *Proceedings of the Heidelberg International Conference on Elementary Particles*, edited by H. Filthuth (Wiley-Interscience, Inc., New York, 1968), p. 79; A. Donnachie, R. Kirsopp, and C. Lovelace, Phys. Letters **26B**, 161 (1968).

[3] Although one should be wary of interpreting the detail of the phase-shift Argand diagrams, we note that both the Saclay analysis (Ref. 5) and the Berkeley analysis (Ref. 6) display additional structure (not yet interpretable as a resonance) in the P_{13} wave at 1400–1500 MeV, in the F_{15} wave at 1800–1900 MeV, and in the P_{33} wave at 1900–2000 MeV.

[4] For a summary of the experimental situation, see J. G. Rushbrooke, in *Proceedings of Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, Switzerland, 1968), p. 158.

small) for a mass of 1470 MeV, which is much less than the CERN estimate but not too far from some of the experimental measurements. Before a definite comparison can be made, the discrepancies between the phase shifts and direct experimental observation will have to be cleared up. Certainly, none of the present estimates should be regarded as final. In particular, it is interesting to note that in both the Saclay[5] and Berkeley[6] phase shifts which are obtained by an energy-independent method, the P_{13} wave shows a marked unstable behavior (not yet interpretable as a resonance) in this region. The possibility of there being more than one resonance in the 1400–1500-MeV region cannot therefore be excluded at present, and may in fact offer a resolution of some of the experimental observations. A detailed analysis on the experimental final-state distributions, which at the moment are only "consistent with" P_{11}, would obviously be very helpful in determining the content of the enhancements.

Some further comments on the harmonic-oscillator model itself also seem to be in order at this point in view of various remarks in the literature[7] concerning the importance of recoil. It is important to remember what the basic tools of the model are, namely, an orthonormal set of wave functions which seem to be in one-to-one correspondence with the observed baryon resonances, and a phenomenological decay Hamiltonian. The interpretation[8] of the wave functions is of special interest. They are, of course, not necessarily eigenfunctions of only the Schrödinger equation. As an illustration of this point, consider a one-oscillator system. The appropriate eigenvalue equation is

$$(-\nabla^2+M^2\omega^2r^2)\Psi=\lambda\Psi\,.$$

The eigenfunctions are precisely of the kind we have been using, with eigenvalues $\lambda_n=2M\omega(n+\frac{3}{2})$. If λ is interpreted as the quantity $2ME$, the above equation is just the Schrödinger equation. The experimental spacing of the energy levels then determines ω, which, when taken with the value of $\alpha^2=M\omega$ as found by comparing the computed and experimental decay rates, gives the quark mass M. As shown in Ref. 1, such an argument leads to $M\sim400$ MeV. If the quark were really so light, the model as such becomes highly suspect. It would then be very puzzling indeed why the observed baryon spectrum resembles so closely the energy level spectrum of the nonrelativistic shell model, and why our computed decay rates yield numbers of reasonable magnitude over such a wide energy range.

However, one can interpret λ in many other ways. For example, taking $\lambda=E^2-M^2$ leads to the equation

$$(E^2-p^2-M^2)\Psi=M^2\omega^2r^2\Psi\,,$$

which is a Klein-Gordon type of equation with a potential term. The wave functions are just the same as in the Schrödinger case, but now the energy spectrum determines not ω but $M\omega$. Thus M is undetermined in such a theory.

Of course, we do not know what the correct equation is to describe the motion of quarks, but if the nonrelativistic type of calculation in Ref. 1 is to have any validity, it seems that the quarks should be heavy and therefore that recoil effects would be of secondary importance.

The authors wish to acknowledge useful discussions with many of our colleagues at the University of Illinois and with Professor R. H. Dalitz.

[5] P. Bareyre, C. Bricman, and G. Villet, Phys. Rev. **165**, 1731 (1968).

[6] C. H. Johnson, P. D. Grannis, M. J. Hansroul, O. Chamberlain, G. Shapiro, and H. M. Steiner, as reported by C. Lovelace (Ref. 2); C. H. Johnson, University of California Radiation Laboratory Report No. UCRL-17683, 1967 (unpublished).

[7] G. Morpurgo, *Proceedings of Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, Switzerland, 1968), p. 225.

[8] The authors wish to thank Professor R. H. Dalitz for discussions on this matter.